

# Precision predictions for LC top quark physics



Jürgen R. Reuter, DESY

**HELMHOLTZ**  
RESEARCH FOR GRAND CHALLENGES



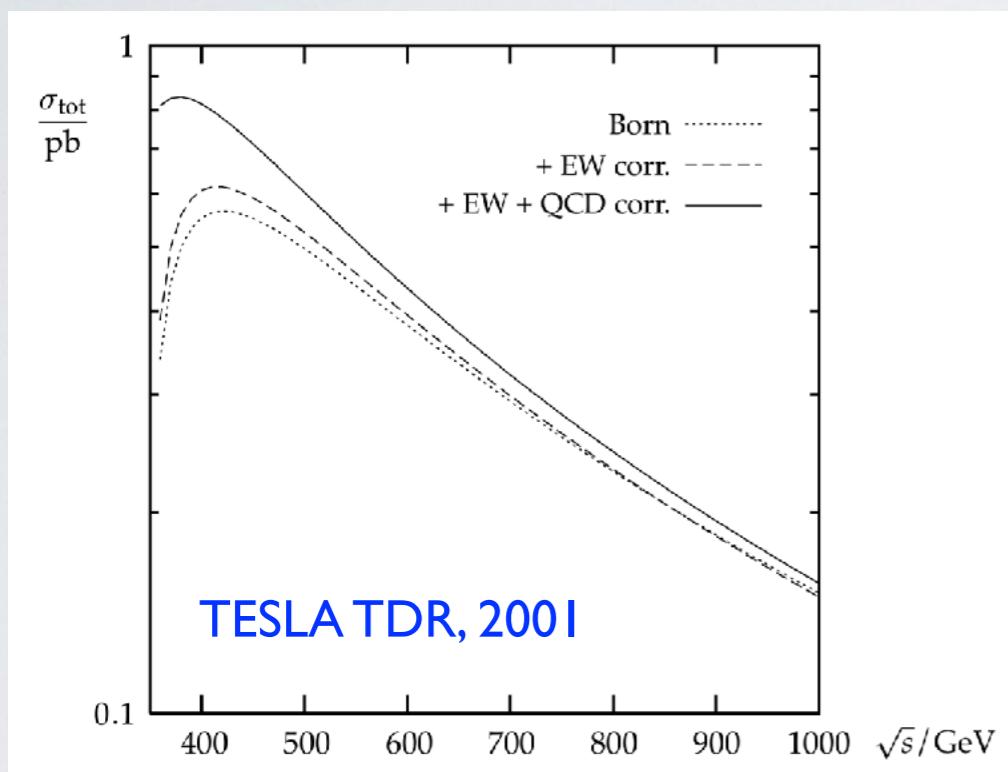
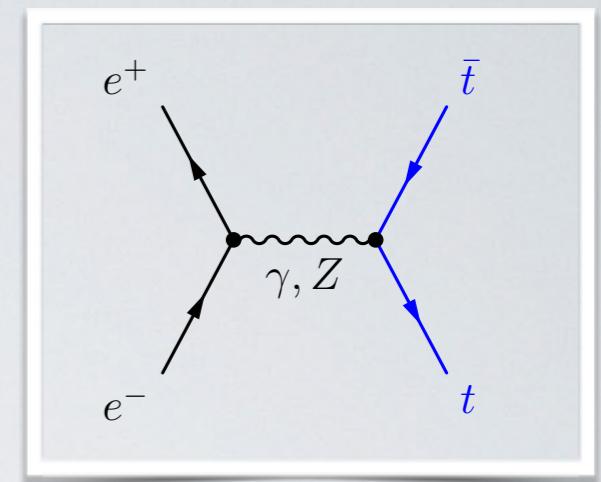
J.R.Reuter

Precision prediction for top physics

LCWS 2019, Sendai, 31.10.10.19

# Top Quark Production

- s-channel production of top-quark pairs
- Top decay happens before hadronization
- Electroweak couplings / polarization studies possible
- Access to vector- and axial-vector  $t\bar{t}Z$  couplings



- Production cross section: 0.1 - 1 pb (sub-TeV)
- 100 fb<sup>-1</sup>: 100,000 top pairs @ threshold
- 2 ab<sup>-1</sup>: 1,400,000 top pairs @ 500 GeV
- Top helicities available from lepton / jet distributions

- ⌚ Paradigm processes : precision determination of top properties
- ⌚ Major bkgd for EW measurements (VVV and VBS); any [most] BSM searches

# Top Production & Decay: Precision

On-Shell process:  $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993; Akhundov/Bardin/Leike, 1991]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]

# Top Production & Decay: Precision

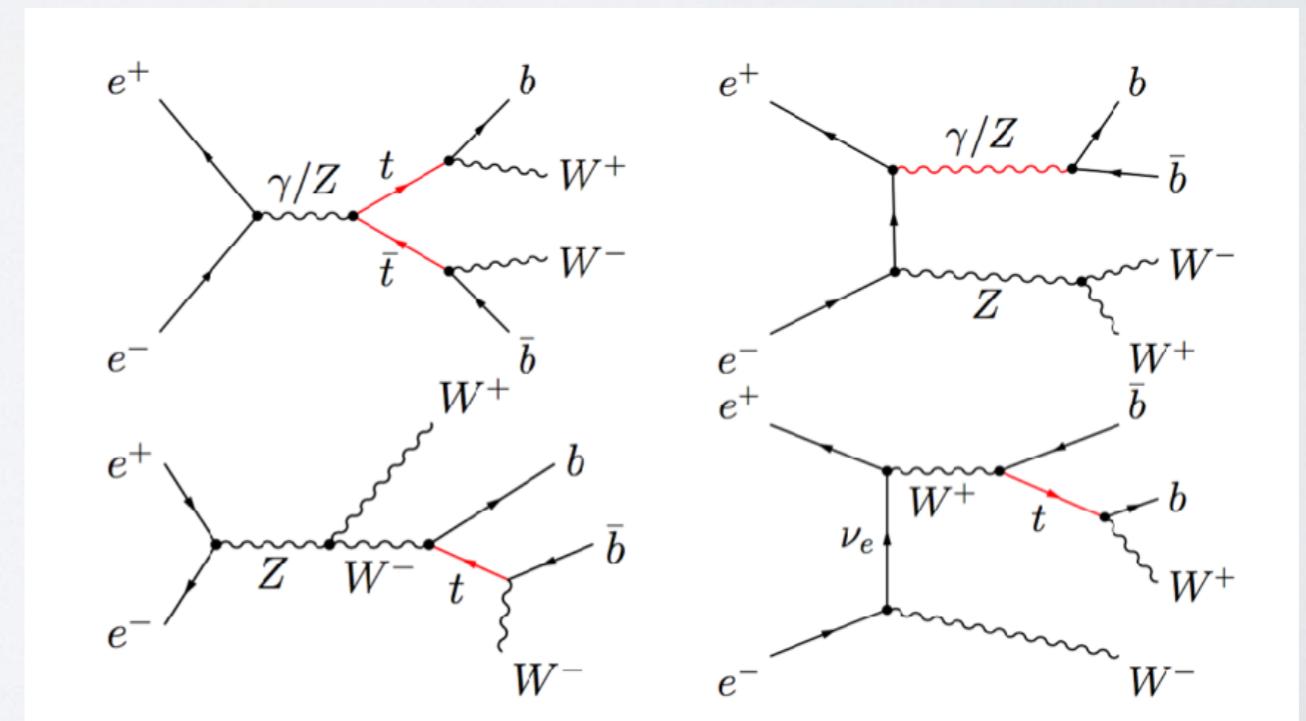
On-Shell process:  $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993; Akhundov/Bardin/Leike, 1991]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]

Off-Shell process:  $e^+e^- \rightarrow W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] X
- NLO QCD diff. [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanastasiou, 2015;  
Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, 2016]

- Properly model the threshold
- Assess selection efficiencies
- Assess systematic uncertainties



# Top Production & Decay: Precision

On-Shell process:  $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993; Akhundov/Bardin/Leike, 1991]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]

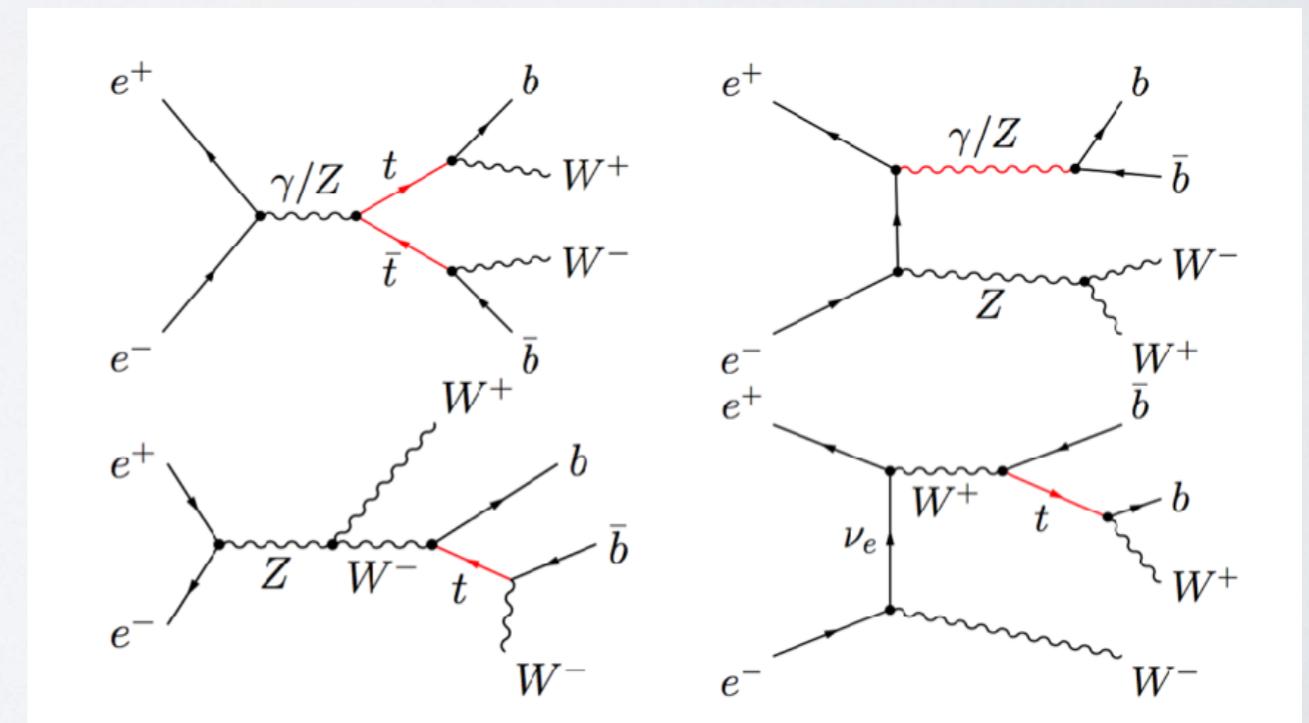
Off-Shell process:  $e^+e^- \rightarrow W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] X
- NLO QCD diff. [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanastasiou, 2015;  
Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, 2016]

- Properly model the threshold
- Assess selection efficiencies
- Assess systematic uncertainties

Top width:  $t \rightarrow W^+b$

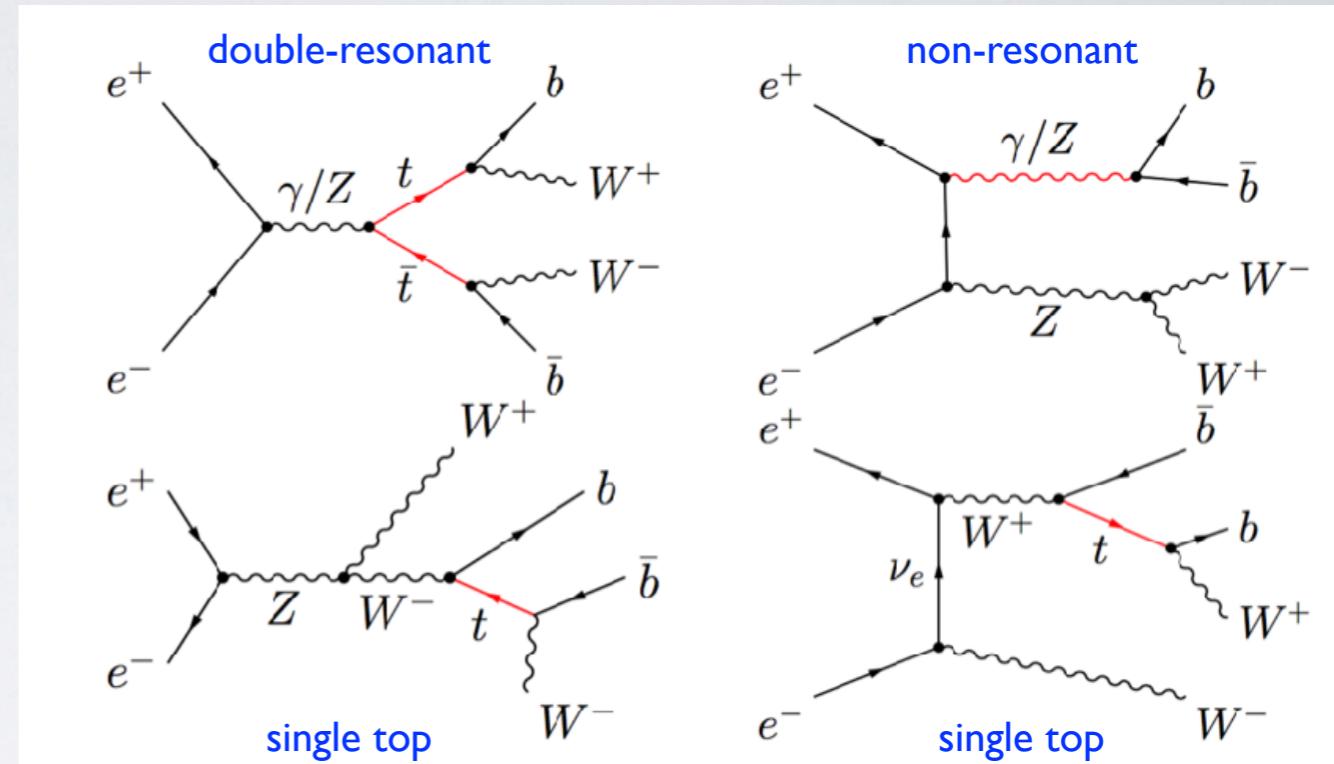
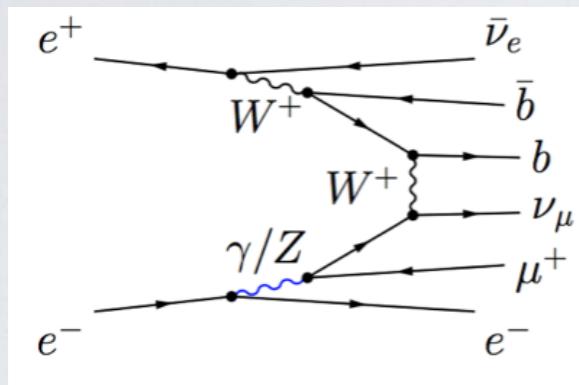
- NLO QCD [Jezabek/Kühn, 1989]
- NNLO QCD [Guo/Li/Zhu, 2012]



# $e^+e^- \rightarrow tt$ off-shell in the continuum

4 / 29

- NLO QCD  $2 \rightarrow 2$  and  $2 \rightarrow 4$  calculated with WHIZARD, Sherpa & Munich
- Using massive  $b$  quarks:  
no cuts necessary for  $e^+e^- \rightarrow W^+W^-bb$
- Full process  $e^+e^- \rightarrow \mu^+\nu_\mu e^-\nu_e bb$  exhibits Coulomb singularity:

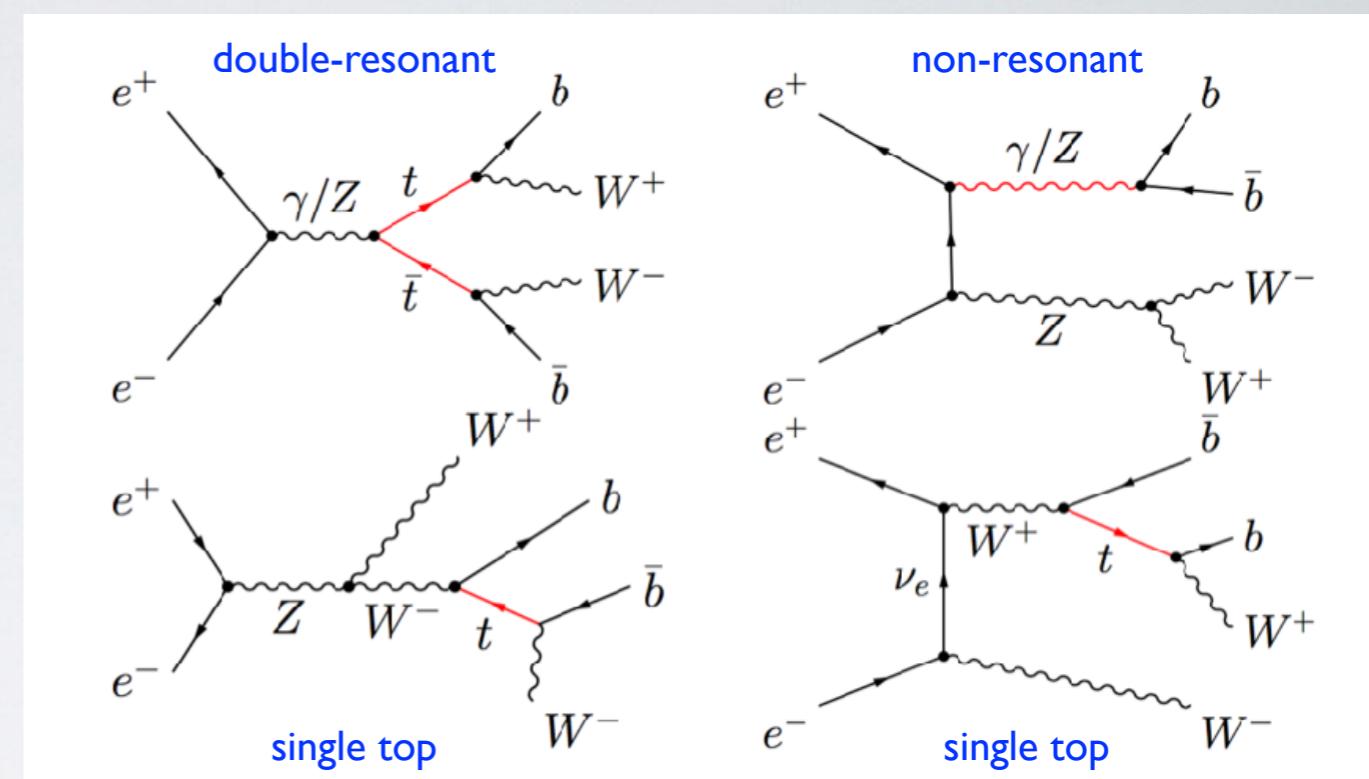
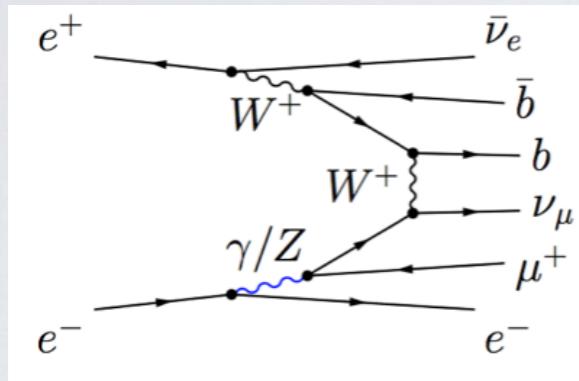


# $e^+e^- \rightarrow tt$ off-shell in the continuum

- NLO QCD  $2 \rightarrow 2$  and  $2 \rightarrow 4$  calculated with WHIZARD, Sherpa & Munich

- Using massive  $b$  quarks:  
no cuts necessary for  $e^+e^- \rightarrow W^+W^-bb$

- Full process  $e^+e^- \rightarrow \mu^+\nu_\mu e^-\nu_e bb$  exhibits Coulomb singularity:



## INPUT PARAMETERS:

$$m_Z = 91.1876 \text{ GeV}, \\ m_b = 4.2 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV} \\ m_t = 173.2 \text{ GeV.}$$

$$\Gamma_{t \rightarrow Wb}^{\text{LO}} = 1.4986 \text{ GeV}, \\ \Gamma_{t \rightarrow f\bar{f}b}^{\text{LO}} = 1.4757 \text{ GeV},$$

$$m_H = 125 \text{ GeV}$$

$$\Gamma_H = 0.00431 \text{ GeV}$$

$$\Gamma_Z^{\text{LO}} = 2.4409 \text{ GeV}, \\ \Gamma_W^{\text{LO}} = 2.0454 \text{ GeV},$$

$$\Gamma_Z^{\text{NLO}} = 2.5060 \text{ GeV}, \\ \Gamma_W^{\text{NLO}} = 2.0978 \text{ GeV.}$$

## Complex Mass Scheme (CMS):

$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

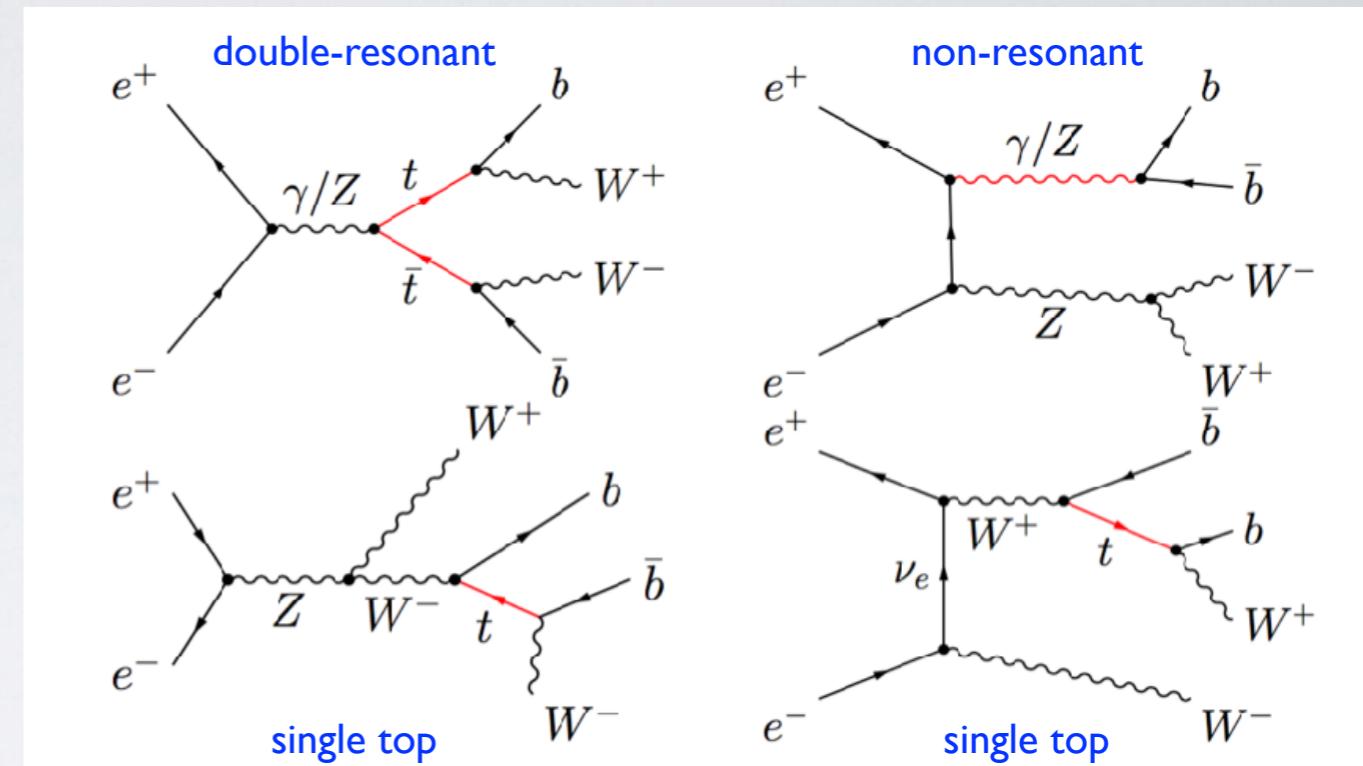
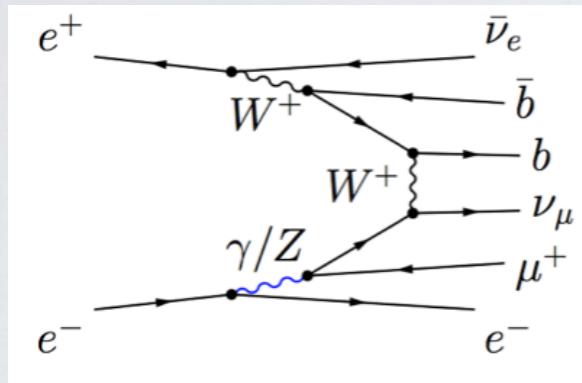
$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_w^2}{\mu_Z^2}$$

# $e^+e^- \rightarrow tt$ off-shell in the continuum

- NLO QCD  $2 \rightarrow 2$  and  $2 \rightarrow 4$  calculated with WHIZARD, Sherpa & Munich

- Using massive  $b$  quarks:  
no cuts necessary for  $e^+e^- \rightarrow W^+W^-bb$

- Full process  $e^+e^- \rightarrow \mu^+\nu_\mu e^-\nu_e bb$  exhibits Coulomb singularity:



## INPUT PARAMETERS:

$$m_Z = 91.1876 \text{ GeV}, \\ m_b = 4.2 \text{ GeV},$$

$$\Gamma_{t \rightarrow Wb}^{\text{LO}} = 1.4986 \text{ GeV}, \\ \Gamma_{t \rightarrow f\bar{f}b}^{\text{LO}} = 1.4757 \text{ GeV},$$

$$m_H = 125 \text{ GeV}$$

$$\Gamma_Z^{\text{LO}} = 2.4409 \text{ GeV}, \\ \Gamma_W^{\text{LO}} = 2.0454 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV} \\ m_t = 173.2 \text{ GeV.}$$

$$\Gamma_{t \rightarrow Wb}^{\text{NLO}} = 1.3681 \text{ GeV}, \\ \Gamma_{t \rightarrow f\bar{f}b}^{\text{NLO}} = 1.3475 \text{ GeV.}$$

$$\Gamma_H = 0.00431 \text{ GeV}$$

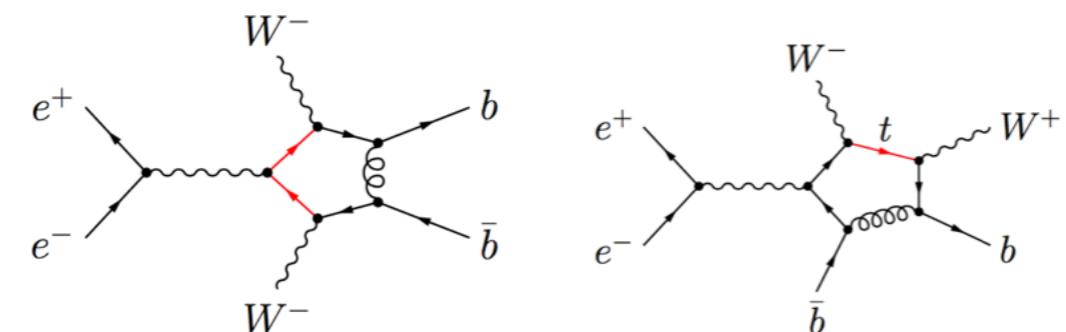
$$\Gamma_Z^{\text{NLO}} = 2.5060 \text{ GeV}, \\ \Gamma_W^{\text{NLO}} = 2.0978 \text{ GeV.}$$

## Complex Mass Scheme (CMS):

$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

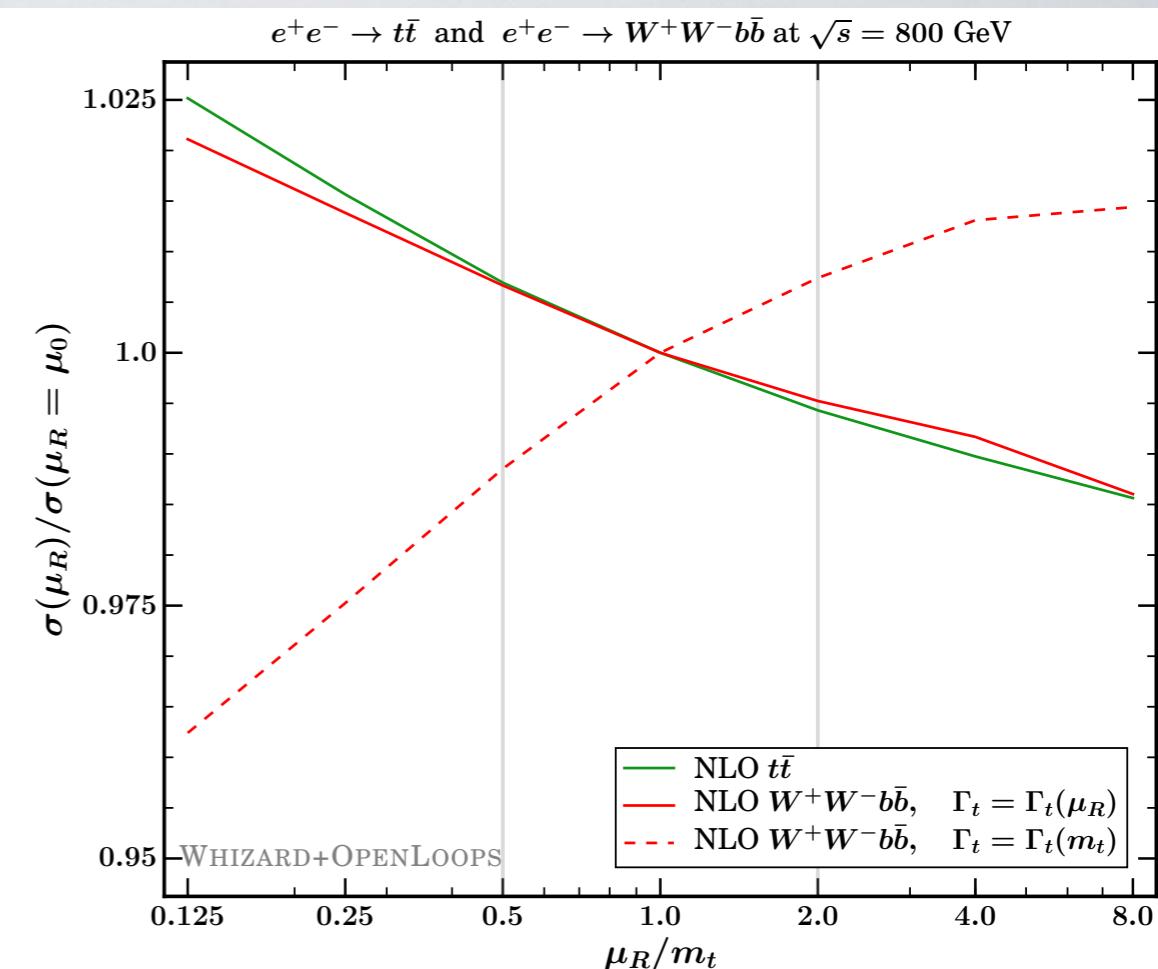
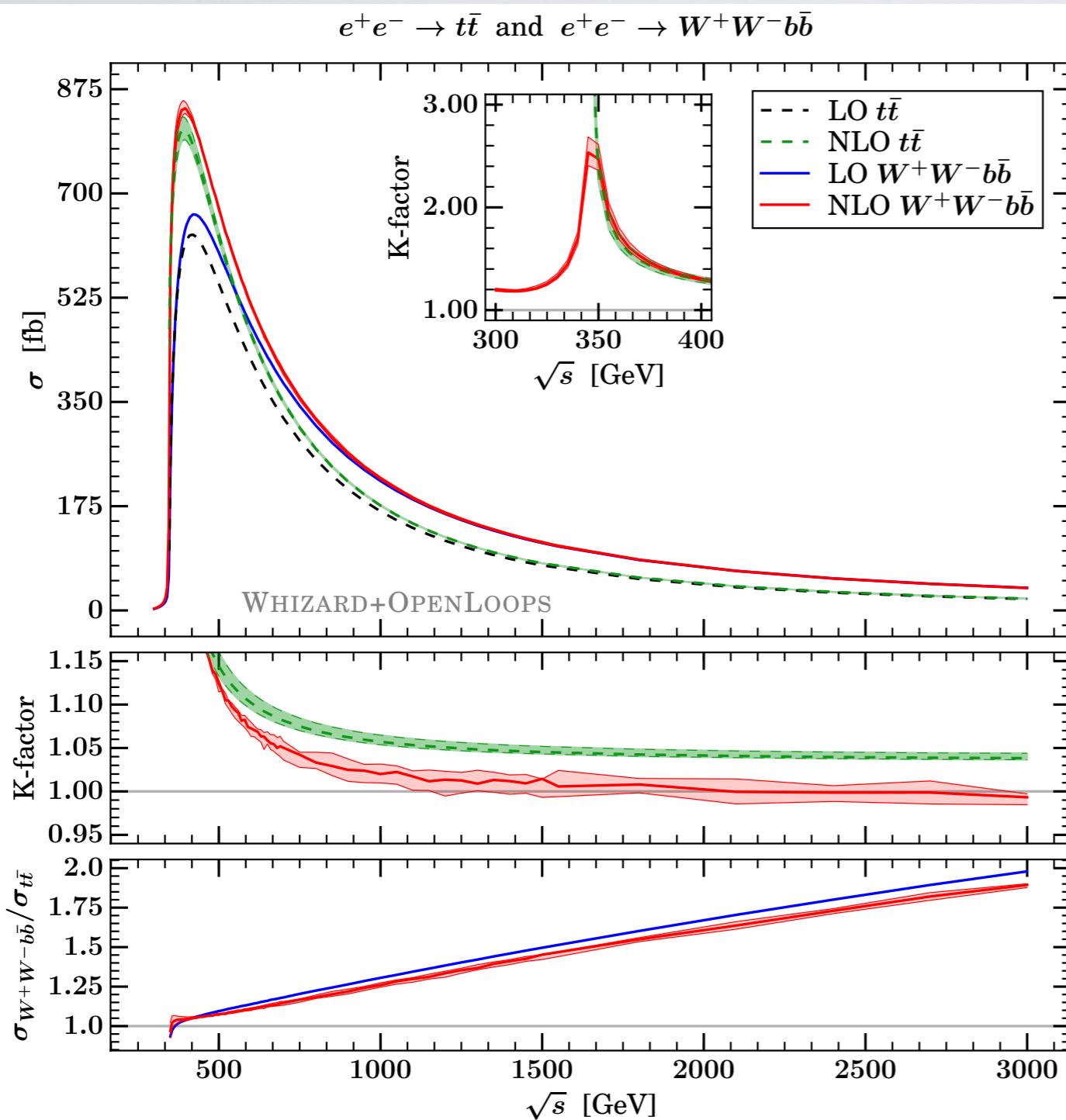
$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_w^2}{\mu_Z^2}$$

## Typical pentagons:



# NLO QCD Results for off-shell $e^+e^- \rightarrow tt$

5 / 29



Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390



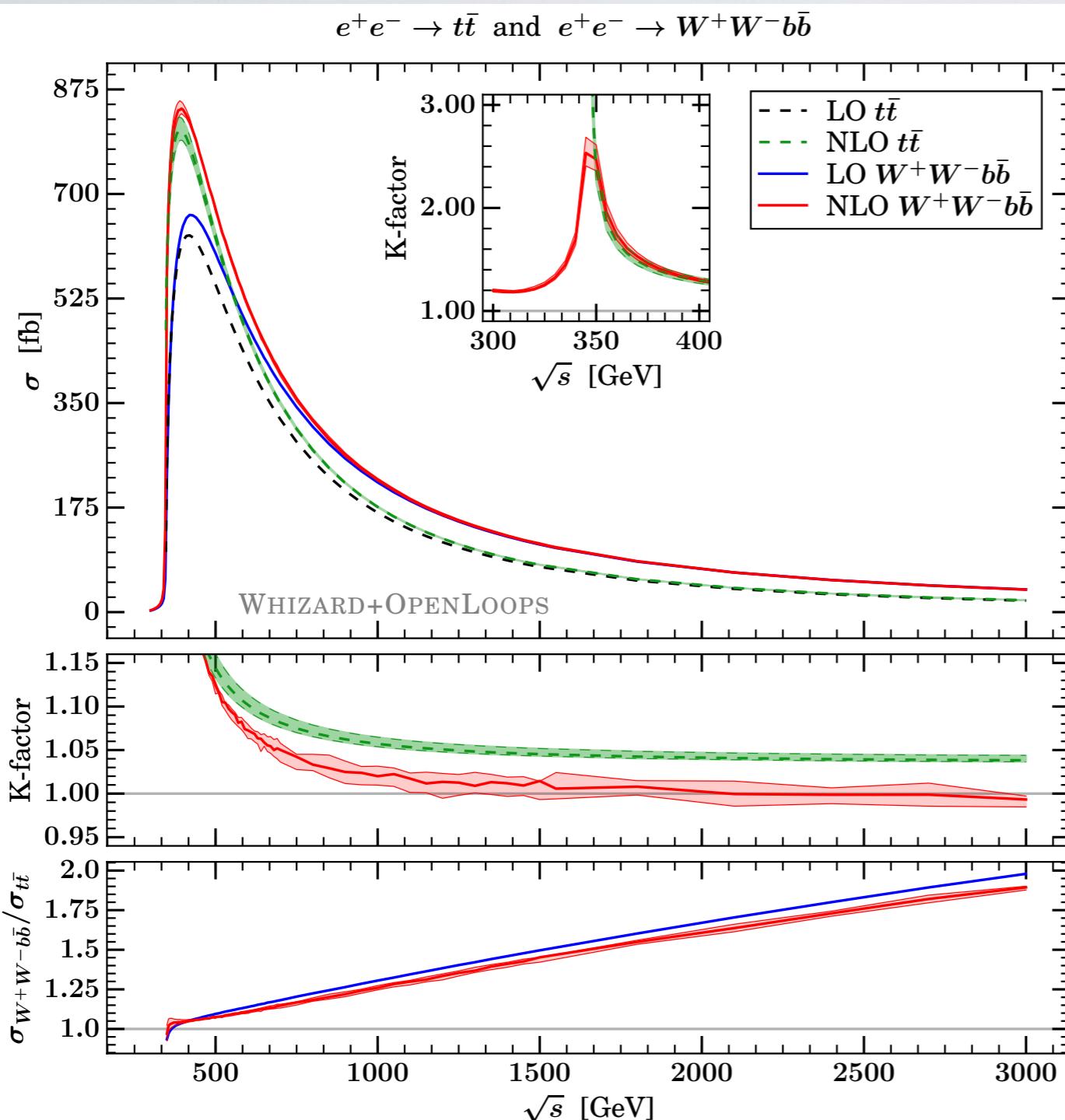
J.R.Reuter

Precision prediction for top physics

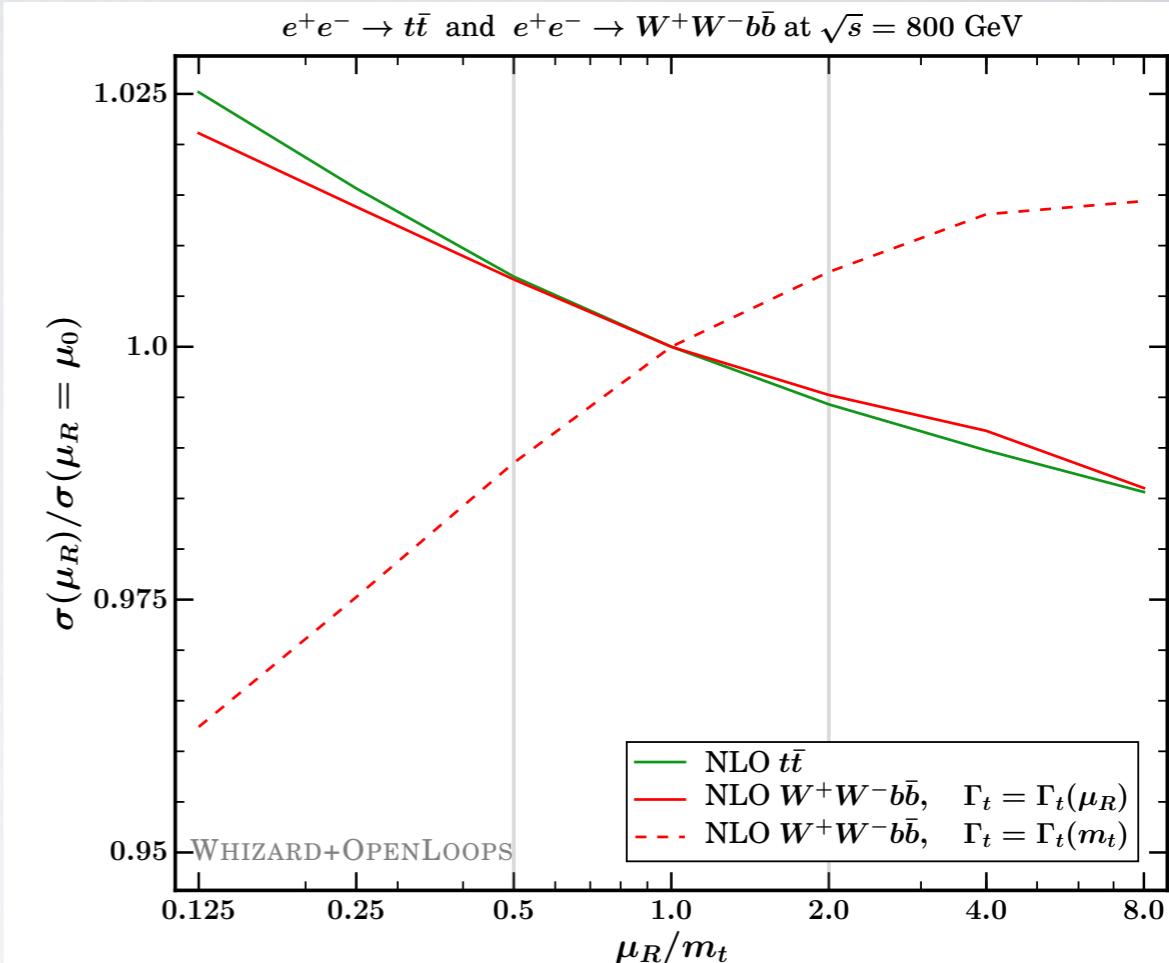
LCWS 2019, Sendai, 31.10.10.19

# NLO QCD Results for off-shell $e^+e^- \rightarrow tt$

5 / 29



Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

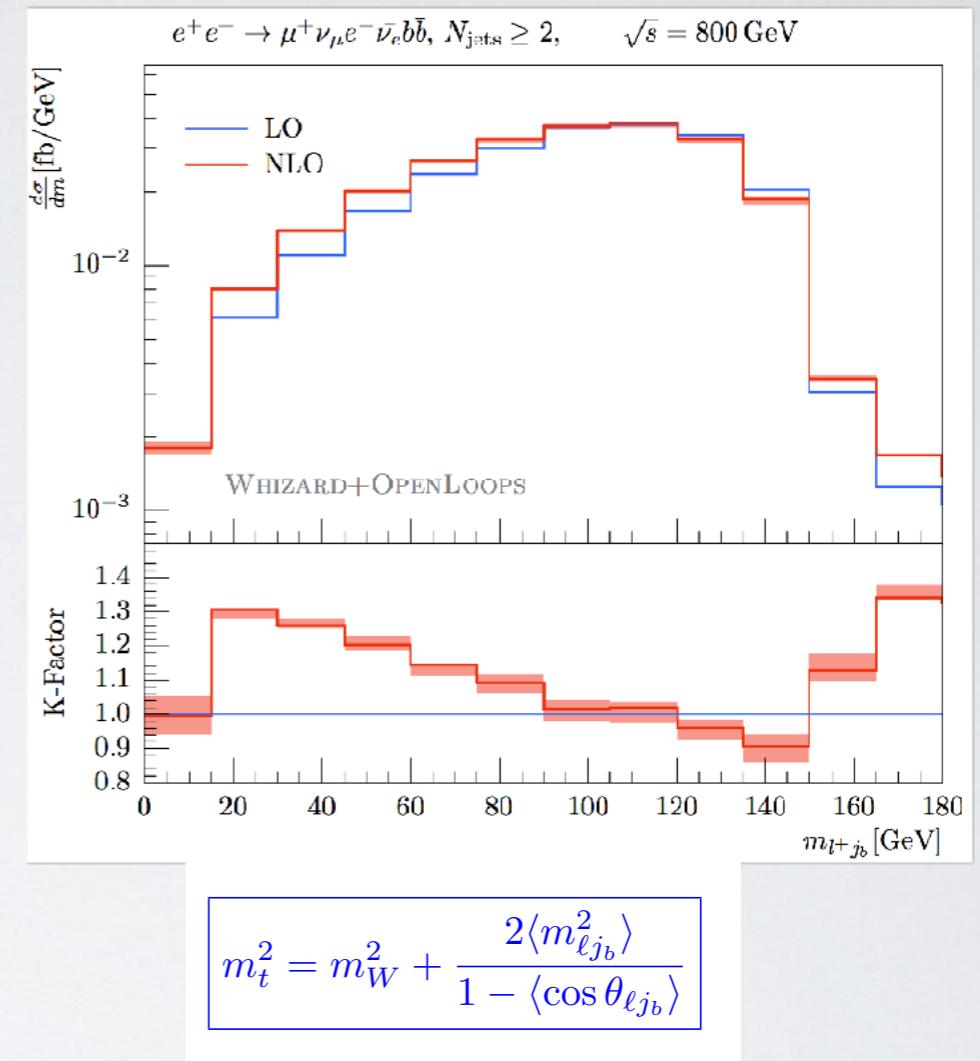
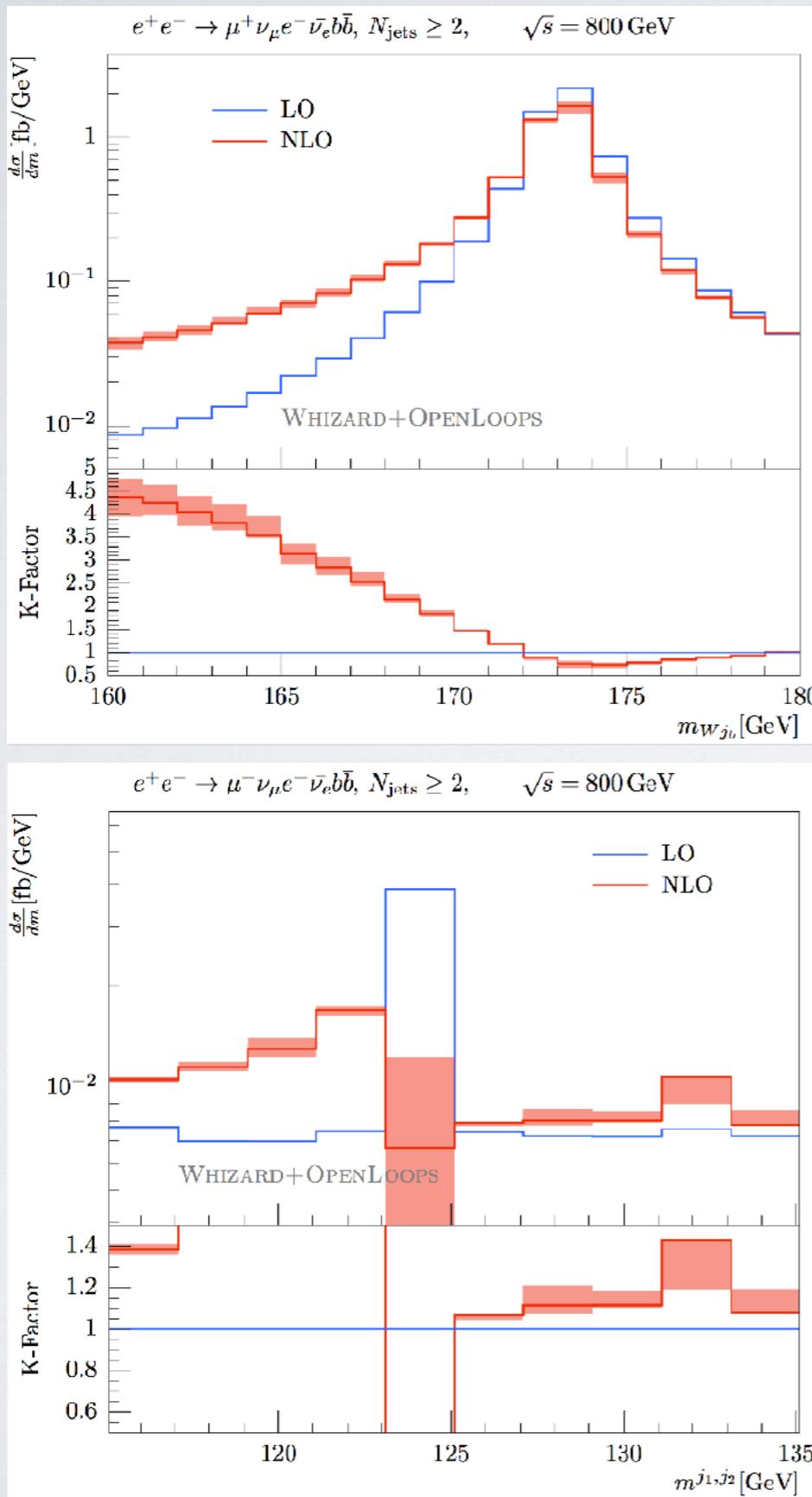


$\sqrt{s}$ [GeV]	$e^+e^- \rightarrow t\bar{t}$			$e^+e^- \rightarrow W^+W^-b\bar{b}$		
	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993



# Differential Results for off-shell $e^+e^- \rightarrow tt$

6 / 29

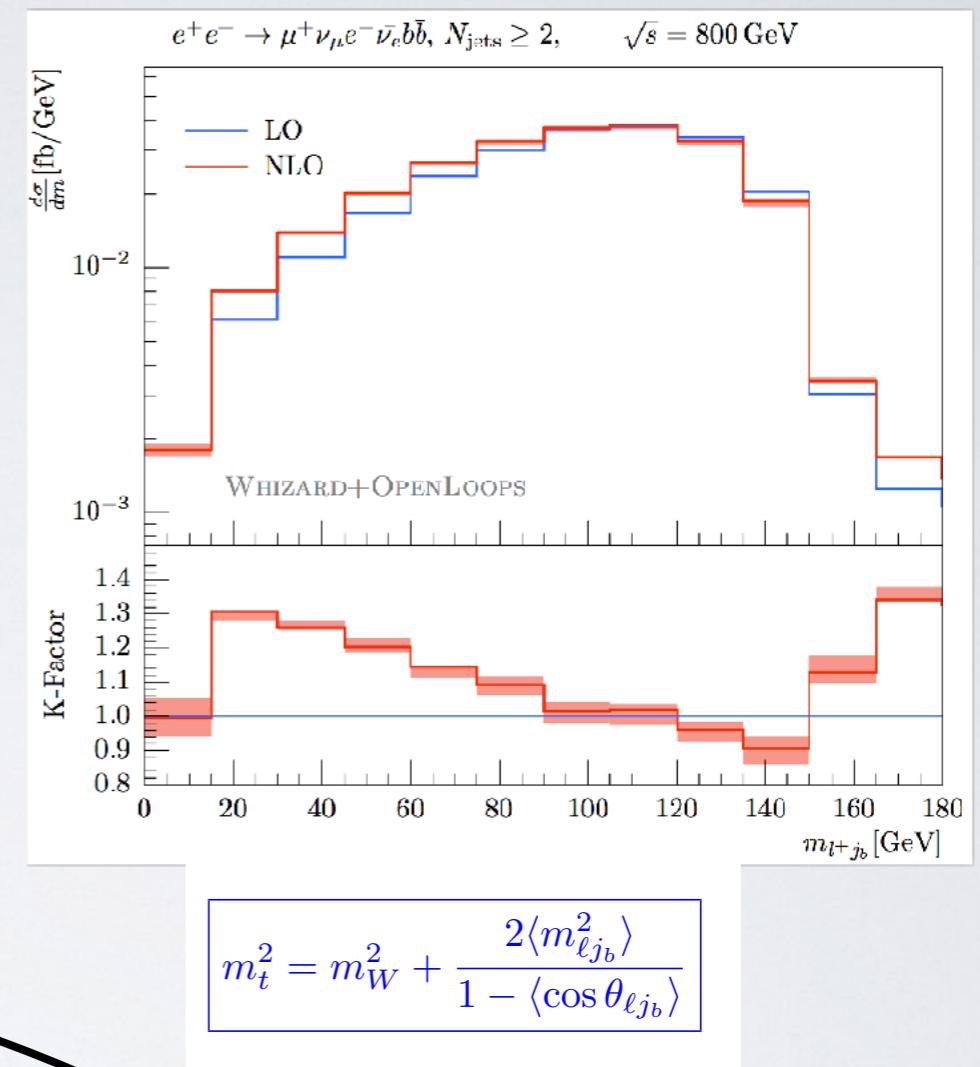
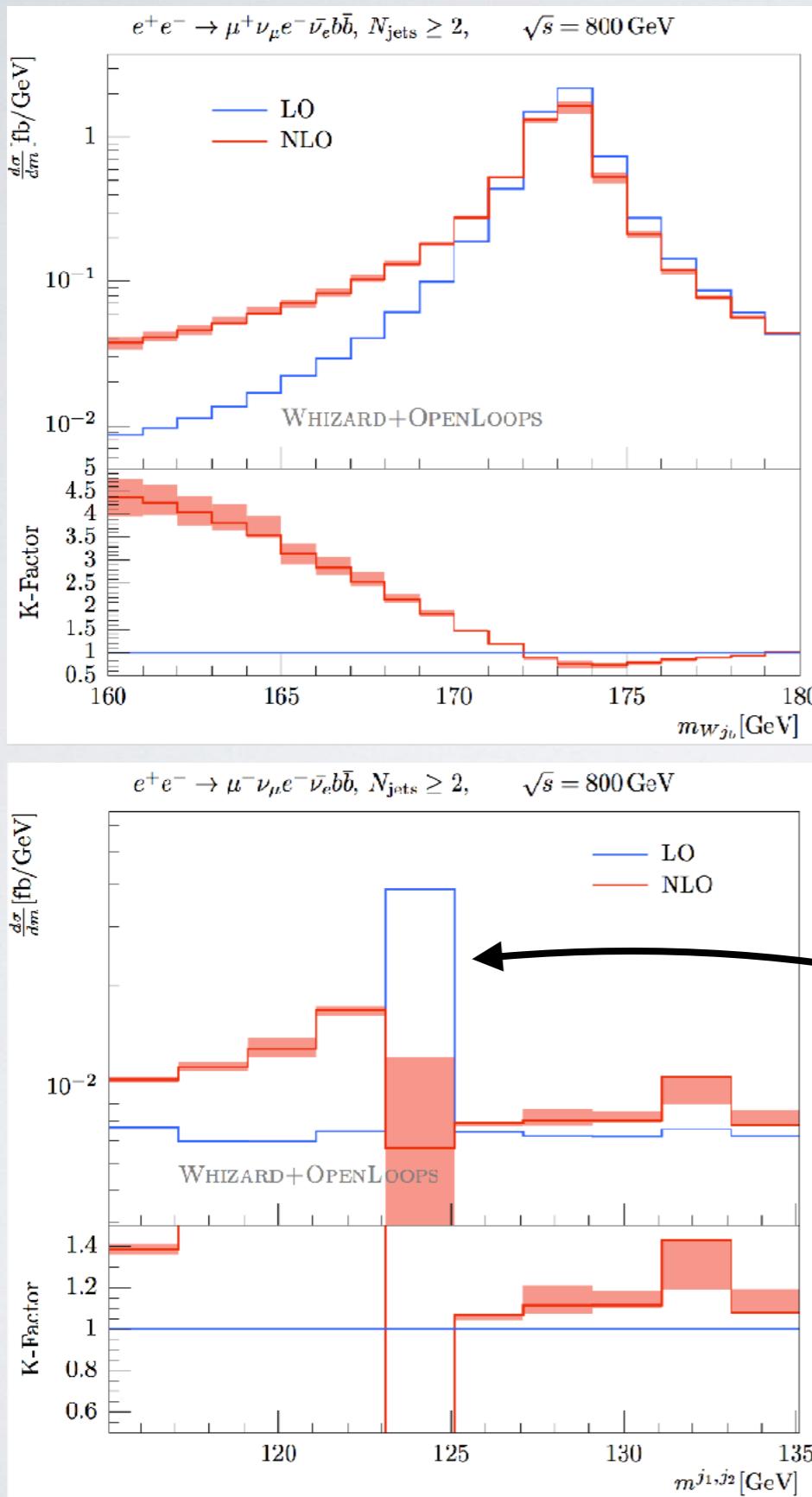


Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390



# Differential Results for off-shell $e^+e^- \rightarrow tt$

6 / 29



Full process  $e^+e^- \rightarrow \mu^+\nu_\mu e^-\bar{\nu}_e b\bar{b}$  contains also  $e^+e^- \rightarrow W^+W^- H$  (!)

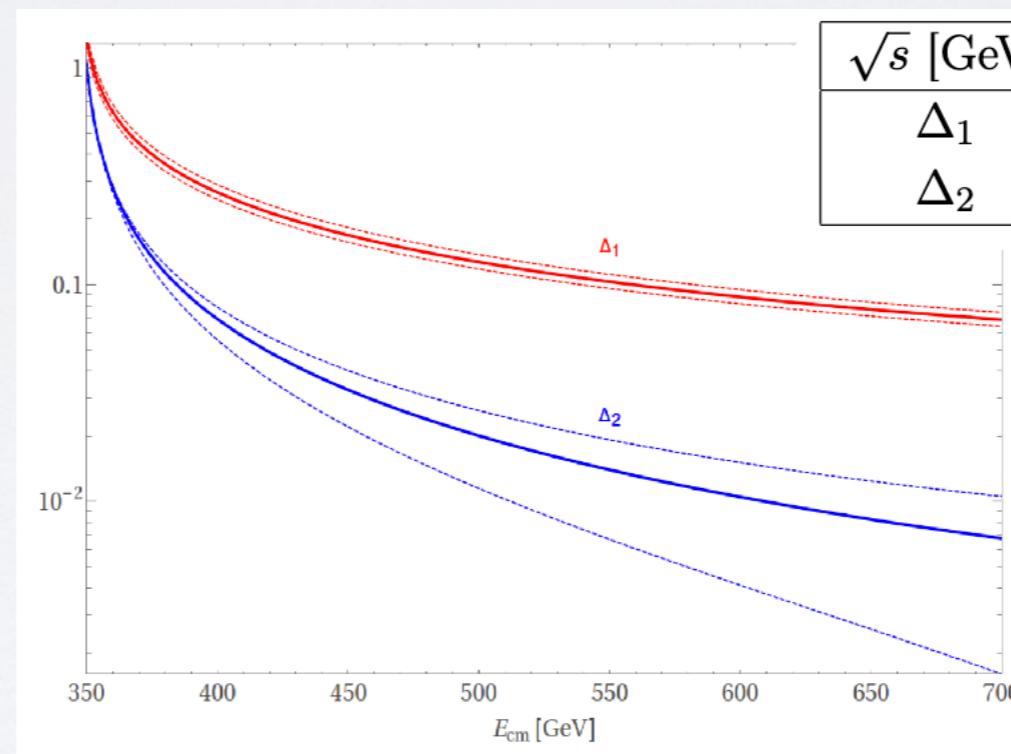
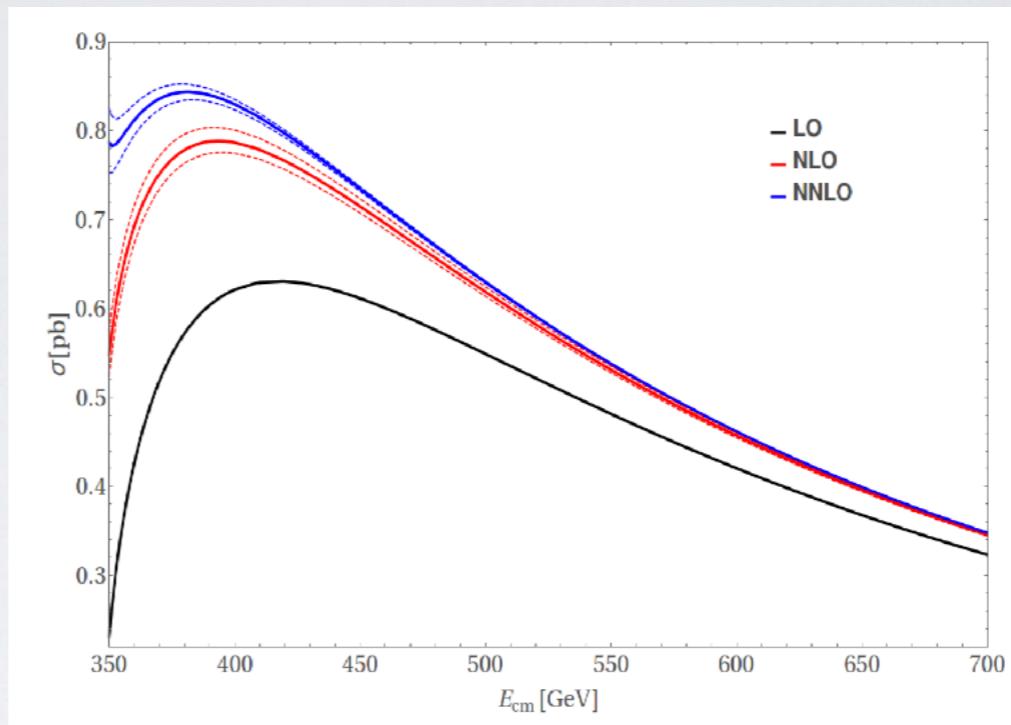
Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

- NNLO QCD on-shell for all infrared-safe observables
- Use of antenna subtraction

Chen/Dekkers/Heisler/  
Bernreuther/Si, 1610.07897

- NNLO QCD on-shell for all infrared-safe observables
- Use of antenna subtraction

Chen/Dekkers/Heisler/  
Bernreuther/Si, 1610.07897



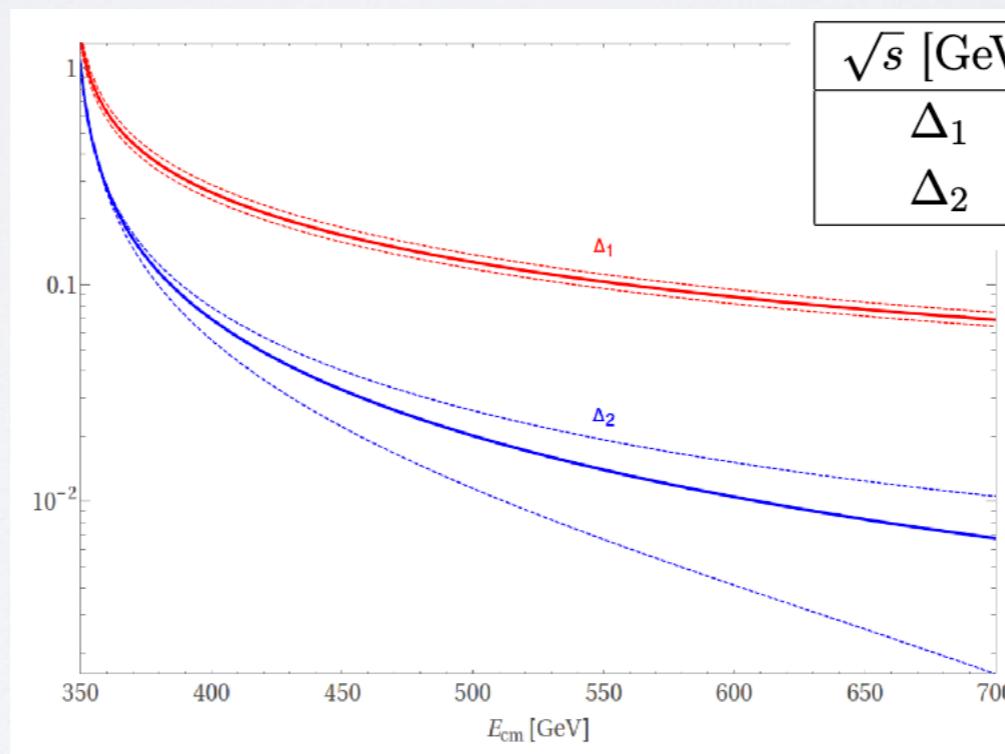
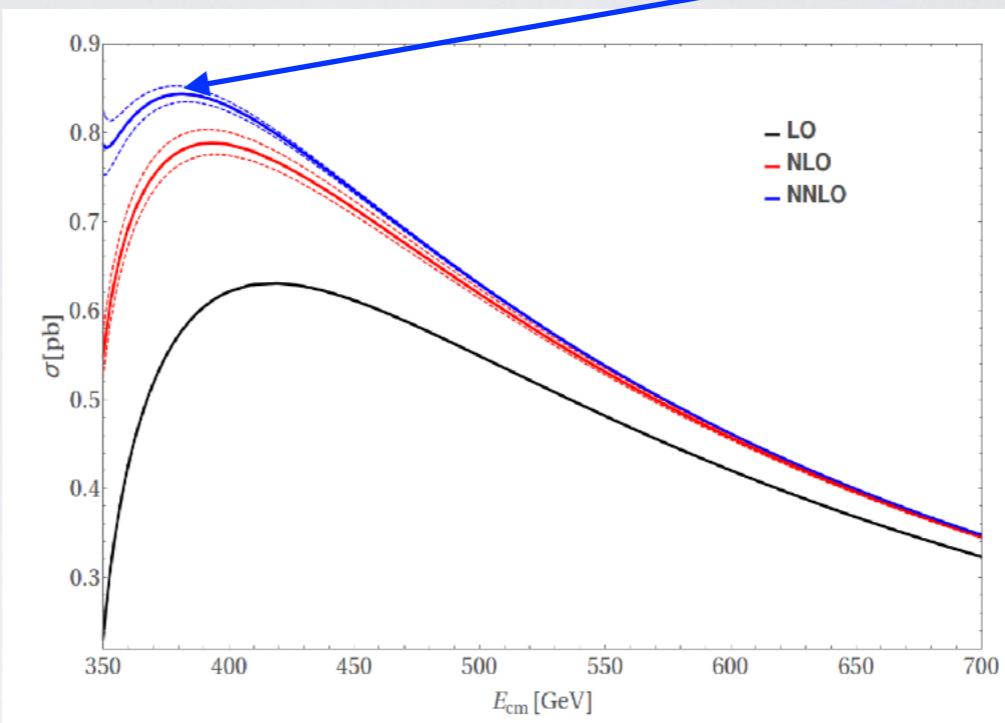
$$\sigma_{\text{NNLO}} = \sigma_{\text{LO}} (1 + \Delta_1 + \Delta_2)$$

- NNLO QCD on-shell for all infrared-safe observables

Maximal NNLO xsec. @ 381.3 GeV

- Use of antenna subtraction

Chen/Dekkers/Heisler/  
Bernreuther/Si, 1610.07897



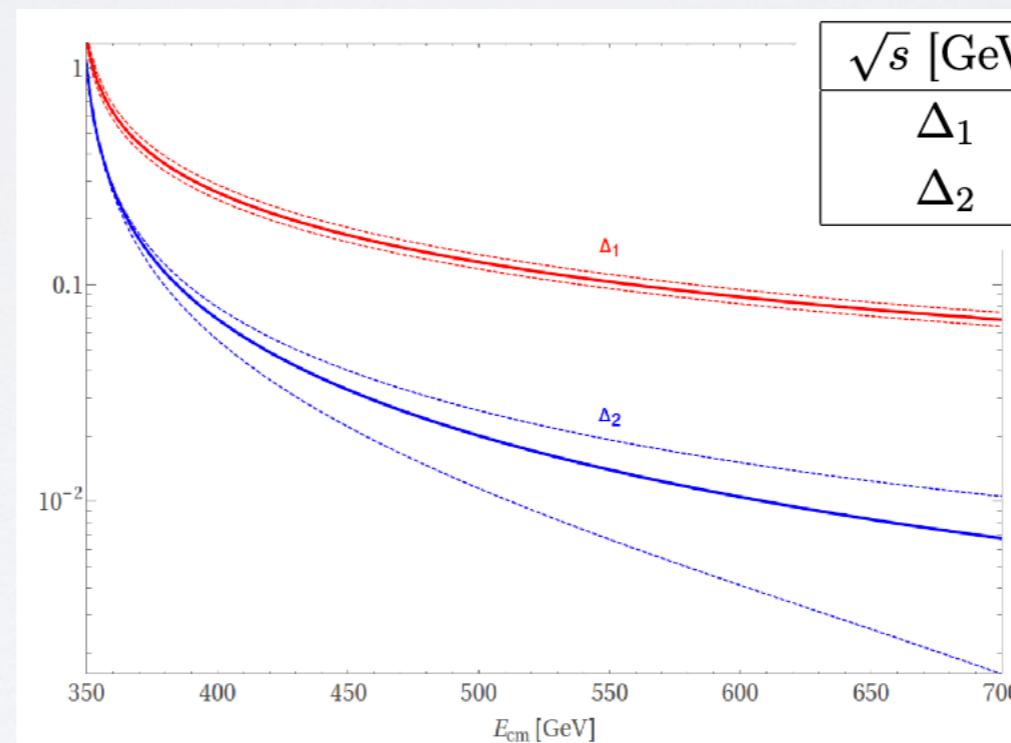
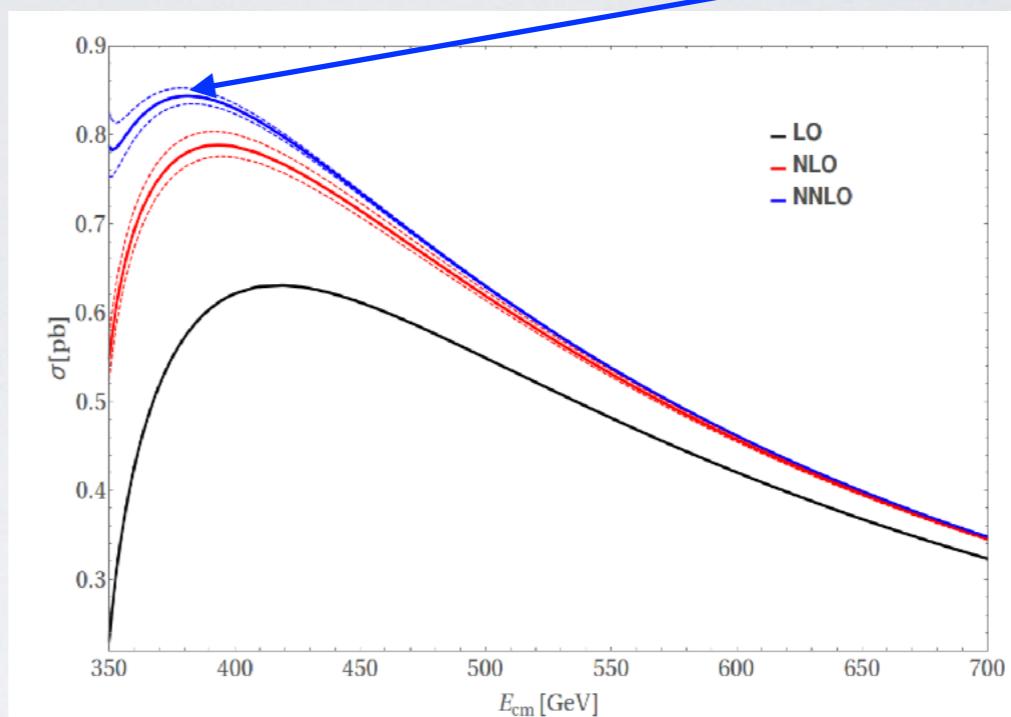
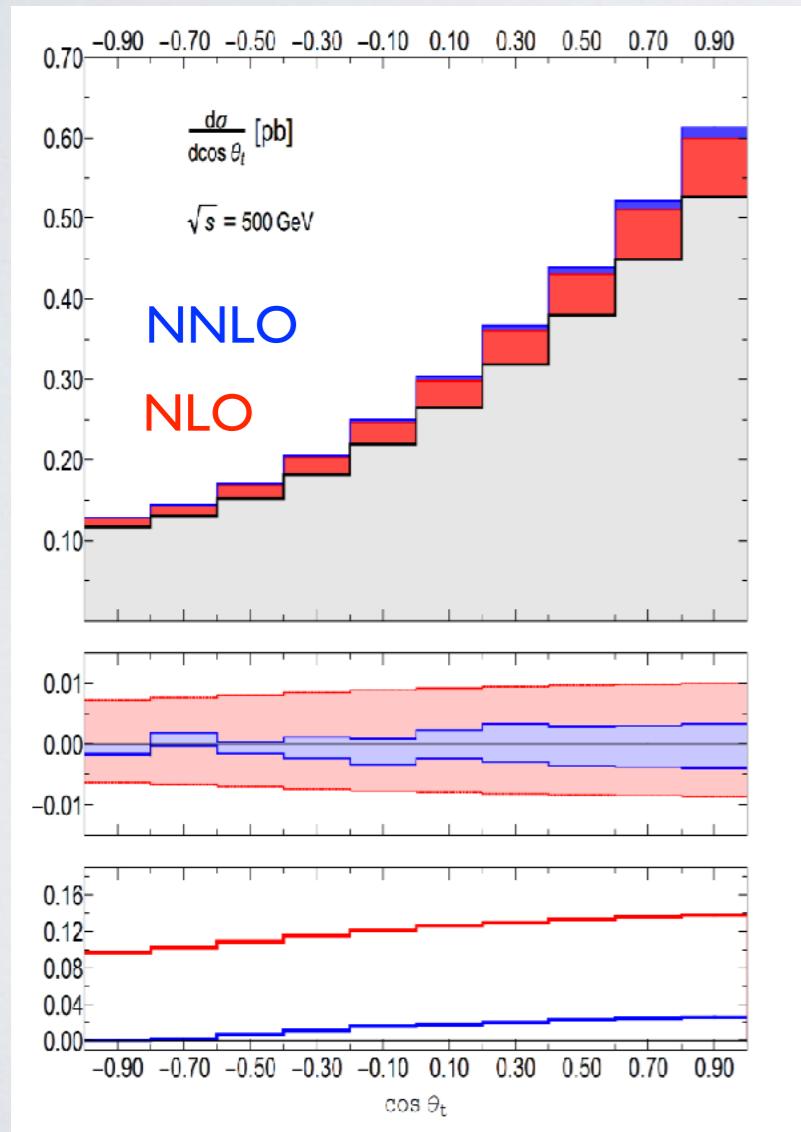
$$\sigma_{NNLO} = \sigma_{LO} (1 + \Delta_1 + \Delta_2)$$

- NNLO QCD on-shell for all infrared-safe observables

Maximal NNLO xsec. @ 381.3 GeV

- Use of antenna subtraction

Chen/Dekkers/Heisler/  
Bernreuther/Si, 1610.07897



$$\sigma_{NNLO} = \sigma_{LO} (1 + \Delta_1 + \Delta_2)$$

# THE BEST OF BOTH SIDES OF THE DETECTOR: THE FORWARD- BACKWARD ASYMMETRY

# Top-Forward Backward Asymmetry

$$\frac{d\sigma(e^+e^- \rightarrow t\bar{t})}{d\Omega_{CM}} = \frac{\alpha^2}{4s} \sqrt{1 - \frac{4M_t^2}{s}} \left\{ \left( 1 + \cos\theta^2 + \frac{4M_t^2}{s} \sin^2\theta \right) G_1(s) - \frac{8M_t^2}{s} G_2(s) + \sqrt{1 - \frac{4M_t^2}{s}} 2\cos\theta G_3(s) \right\}$$

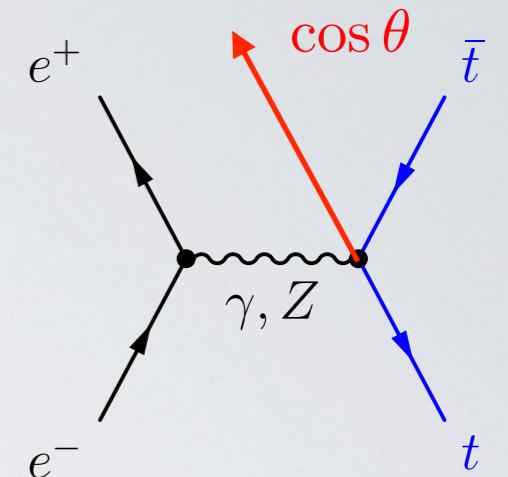
$$G_1(s) = Q_e^2 Q_t^2 + 2Q_e Q_t V_e V_t \operatorname{Re}[X_Z(s)] + (V_e^2 + A_e^2)(V_t^2 + A_t^2) |X_Z(s)|^2$$

$$G_2(s) = (V_e^2 + A_e^2) A_t^2 |X_Z(s)|^2$$

$$G_3(s) = 2Q_e Q_t A_e A_t \operatorname{Re}[X_Z(s)] + 4V_e V_t A_e A_t |X_Z(s)|^2$$

$$X_Z(s) = \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}$$

$$-\frac{g}{2c_W} \bar{f} [V_f \gamma^\mu - A_f \gamma^\mu \gamma^5] f Z_\mu$$



- ▶ Axial vector photon-Z and vector—axial-vector interference
- ▶ Linearly dependent term generates Forward-Backward Asymmetry

# Top-Forward Backward Asymmetry

$$\frac{d\sigma(e^+e^- \rightarrow t\bar{t})}{d\Omega_{CM}} = \frac{\alpha^2}{4s} \sqrt{1 - \frac{4M_t^2}{s}} \left\{ \left( 1 + \cos\theta^2 + \frac{4M_t^2}{s} \sin^2\theta \right) G_1(s) - \frac{8M_t^2}{s} G_2(s) + \sqrt{1 - \frac{4M_t^2}{s}} 2\cos\theta G_3(s) \right\}$$

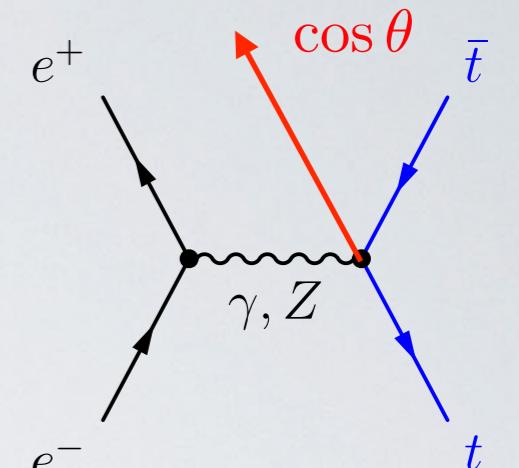
$$G_1(s) = Q_e^2 Q_t^2 + 2Q_e Q_t V_e V_t \operatorname{Re}[X_Z(s)] + (V_e^2 + A_e^2)(V_t^2 + A_t^2) |X_Z(s)|^2$$

$$G_2(s) = (V_e^2 + A_e^2) A_t^2 |X_Z(s)|^2$$

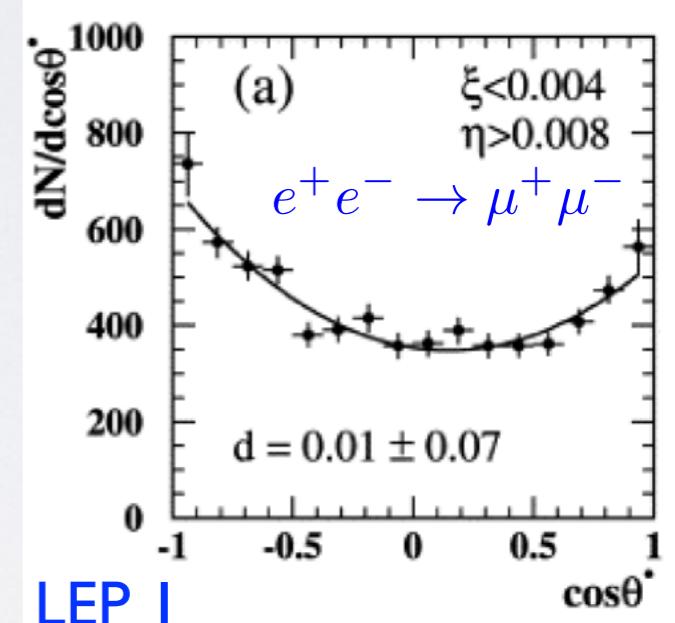
$$G_3(s) = 2Q_e Q_t A_e A_t \operatorname{Re}[X_Z(s)] + 4V_e V_t A_e A_t |X_Z(s)|^2$$

$$X_Z(s) = \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}$$

$$-\frac{g}{2c_W} \bar{f} [V_f \gamma^\mu - A_f \gamma^\mu \gamma^5] f Z_\mu$$



- ▶ Axial vector photon-Z and vector—axial-vector interference
- ▶ Linearly dependent term generates Forward-Backward Asymmetry



$$A_{FB} = \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)}.$$

Asymmetry is function of collider energy

# Top-Forward Backward Asymmetry

$e^+e^- \rightarrow$	$A_{FB}^{\text{LO}}$	$A_{FB}^{\text{NLO}}$	$A_{FB}^{\text{NLO}}/A_{FB}^{\text{LO}}$	
$A_{FB}$	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	-0.402	-0.387	0.964
$\bar{A}_{FB}$	$t\bar{t}$	0.535	0.539	1.013
	$W^+W^-b\bar{b}$	0.428	0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.415	0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	0.377	0.350	0.928

## NLO QCD Corrections

Gluon emission symmetric in  $\theta \Rightarrow$

NLO QCD corrections small

[Djouadi/Lampe/Zerwas, hep-ph/9411386](#)

[Bardin/Christova/Jack/Kalinovskaya/Olchevski/S. Riemann/T. Riemann, hep-ph/9908433](#)

[Altarelli/Lampe, NPB391 \(1993\) 3](#)

[Ravindran/van Neerven, hep-ph/9809411](#)

[Catani/Seymour, hep-ph/9905424](#)

$A_{FB}$  of the top quark

Forward-backward asymmetry

$$A_{fb} := \frac{\sigma(p_z^t > 0) - \sigma(p_z^t < 0)}{\sigma(p_z^t > 0) + \sigma(p_z^t < 0)}$$

# Top-Forward Backward Asymmetry

$e^+e^- \rightarrow$	$A_{FB}^{\text{LO}}$	$A_{FB}^{\text{NLO}}$	$A_{FB}^{\text{NLO}}/A_{FB}^{\text{LO}}$	
$A_{FB}$	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	-0.402	-0.387	0.964
$\bar{A}_{FB}$	$t\bar{t}$	0.535	0.539	1.013
	$W^+W^-b\bar{b}$	0.428	0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.415	0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	0.377	0.350	0.928

## NLO QCD Corrections

Gluon emission symmetric in  $\theta \Rightarrow$

NLO QCD corrections small

[Djouadi/Lampe/Zerwas, hep-ph/9411386](#)

[Bardin/Christova/Jack/Kalinovskaya/Olchevski/S. Riemann/T. Riemann, hep-ph/9908433](#)

[Altarelli/Lampe, NPB391 \(1993\) 3](#)

[Ravindran/van Neerven, hep-ph/9809411](#)

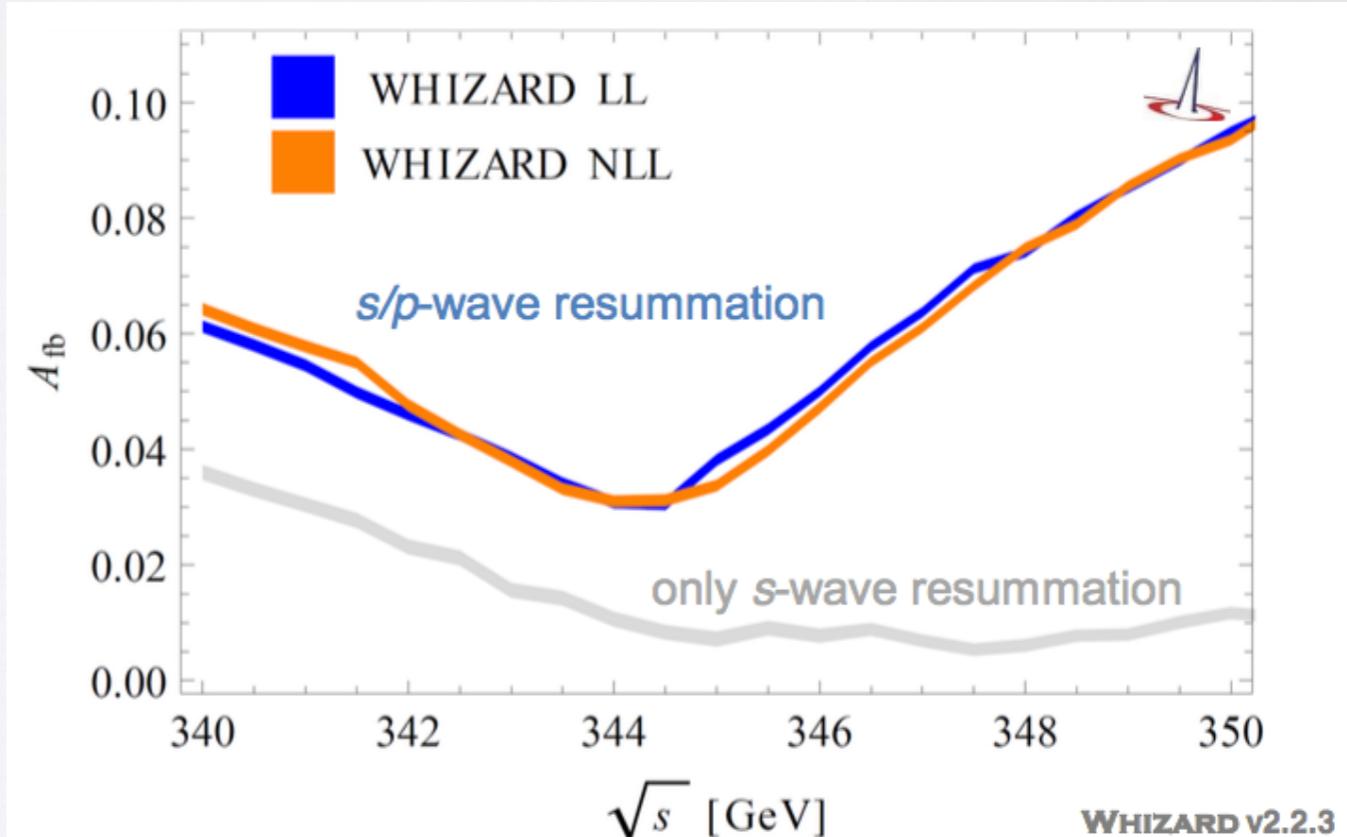
[Catani/Seymour, hep-ph/9905424](#)

$A_{FB}$  of the top quark

Forward-backward asymmetry

$$A_{fb} := \frac{\sigma(p_z^t > 0) - \sigma(p_z^t < 0)}{\sigma(p_z^t > 0) + \sigma(p_z^t < 0)}$$

Threshold region:  $P$ -wave (axial vector) resummation important



WHIZARD v2.2.3



# Top-Forward Backward Asymmetry

$e^+e^- \rightarrow$	$A_{FB}^{\text{LO}}$	$A_{FB}^{\text{NLO}}$	$A_{FB}^{\text{NLO}}/A_{FB}^{\text{LO}}$	
$A_{FB}$	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	-0.402	-0.387	0.964
$\bar{A}_{FB}$	$t\bar{t}$	0.535	0.539	1.013
	$W^+W^-b\bar{b}$	0.428	0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.415	0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	0.377	0.350	0.928

## NLO QCD Corrections

Gluon emission symmetric in  $\theta \Rightarrow$

NLO QCD corrections small

[Djouadi/Lampe/Zerwas, hep-ph/9411386](#)

[Bardin/Christova/Jack/Kalinovskaya/Olchevski/S. Riemann/T. Riemann, hep-ph/9908433](#)

[Altarelli/Lampe, NPB391 \(1993\) 3](#)

[Ravindran/van Neerven, hep-ph/9809411](#)

[Catani/Seymour, hep-ph/9905424](#)

[Chen/Dekkers/Heisler/Bernreuther/Si, 1610.07897](#)

**$A_{FB}$  of the top quark**

$$A_{FB}^{\text{NNLO}} = A_{FB}^{\text{LO}}(1 + A_1 + A_2)$$

$\sqrt{s}$ [GeV]	$A_{FB}^{\text{LO}} [\%]$	$A_{FB}^{\text{NLO}} [\%]$	$A_{FB}^{\text{NNLO}} [\%]$	$A_1 [\%]$	$A_2 [\%]$	$\delta A_{FB}^{\text{NNLO}} [\%]$
360	14.94	$15.54^{+0.05}_{-0.04}$	$16.23^{+0.12}_{-0.10}$	$4.01^{+0.35}_{-0.29}$	$4.58^{+0.46}_{-0.38}$	$\pm 0.59$
400	28.02	$28.97^{+0.08}_{-0.07}$	$29.63^{+0.11}_{-0.10}$	$3.41^{+0.29}_{-0.25}$	$2.36^{+0.11}_{-0.11}$	$\pm 0.27$
500	41.48	$42.42^{+0.08}_{-0.07}$	$42.91^{+0.08}_{-0.07}$	$2.28^{+0.19}_{-0.16}$	$1.18^{+0.01}_{-0.01}$	$\pm 0.13$
700	51.34	$51.81^{+0.04}_{-0.03}$	$52.05^{+0.04}_{-0.04}$	$0.91^{+0.07}_{-0.06}$	$0.47^{+0.01}_{-0.01}$	$\pm 0.06$

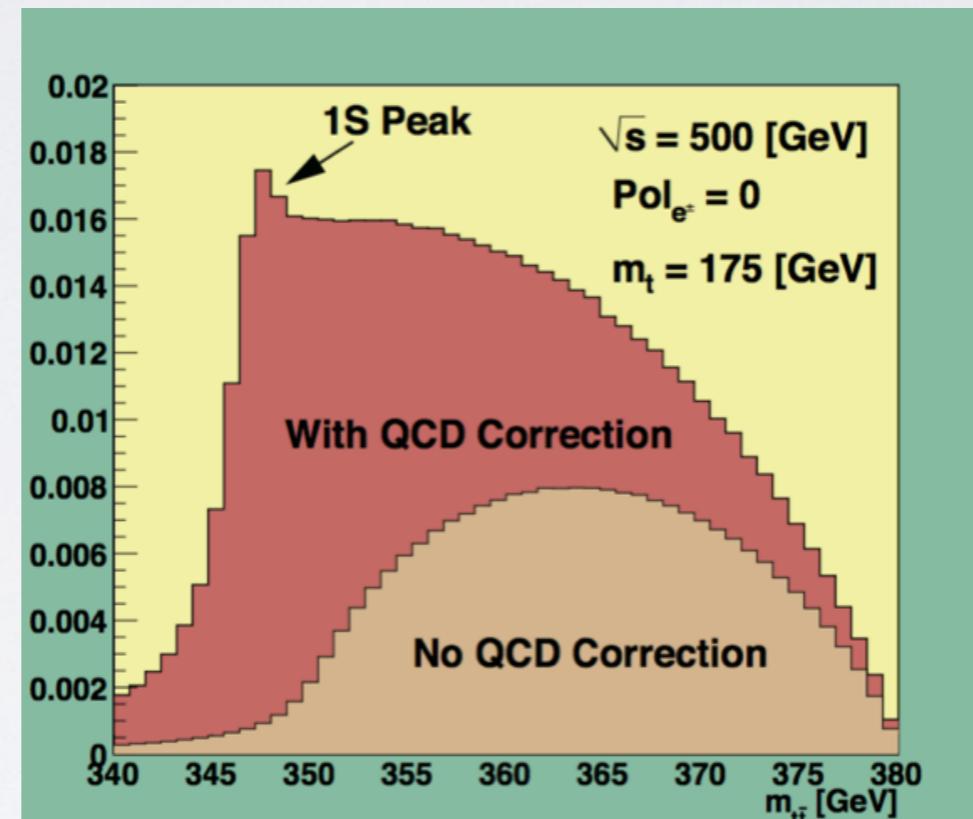
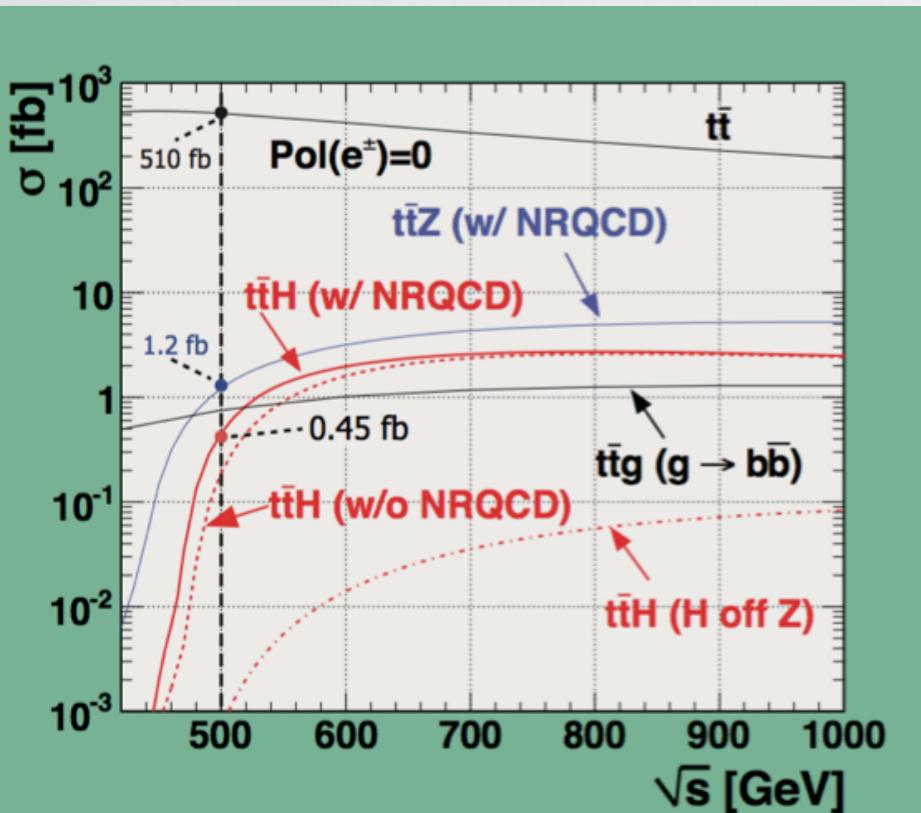
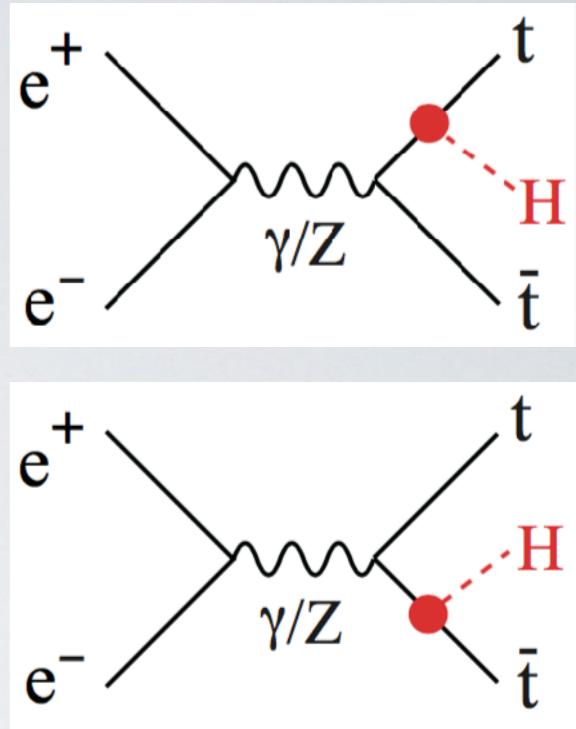


# OFF THE TOP OF MY HIGGS: TOP YUKAWA COUPLING



# $e^+e^- \rightarrow ttH$ : the Top Yukawa coupling

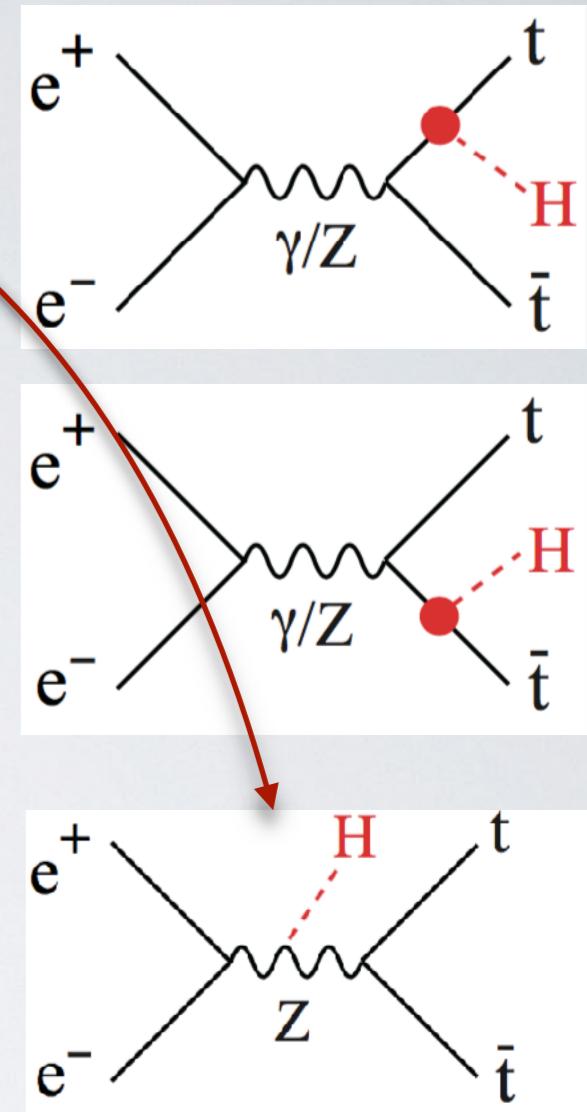
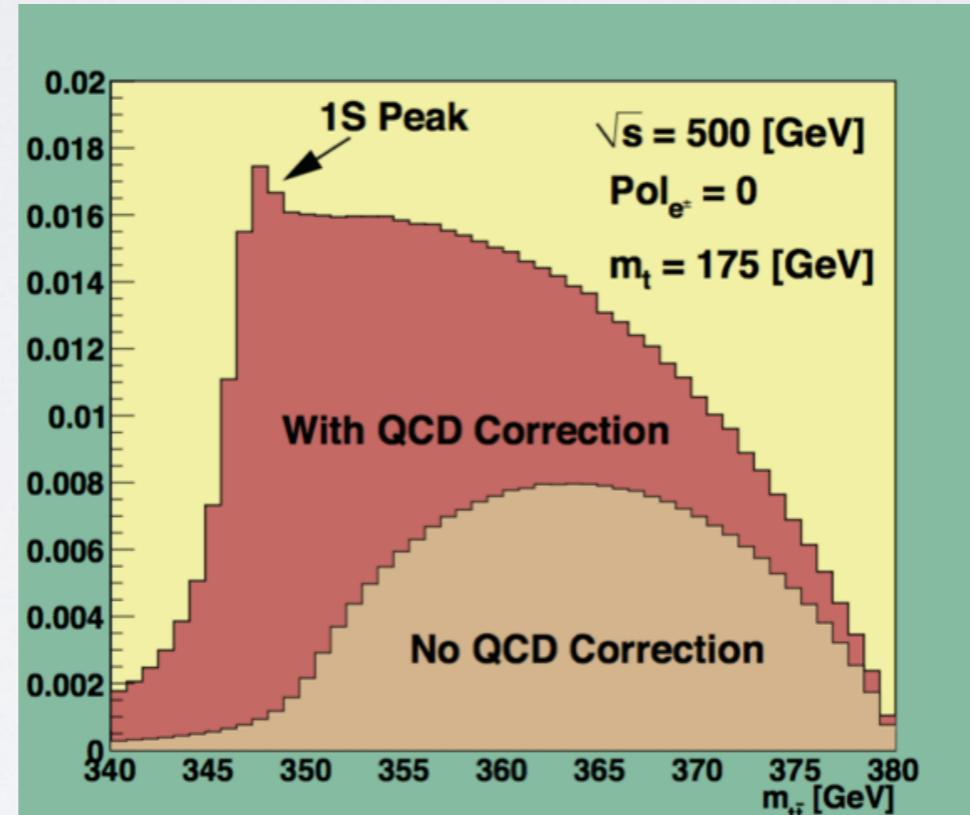
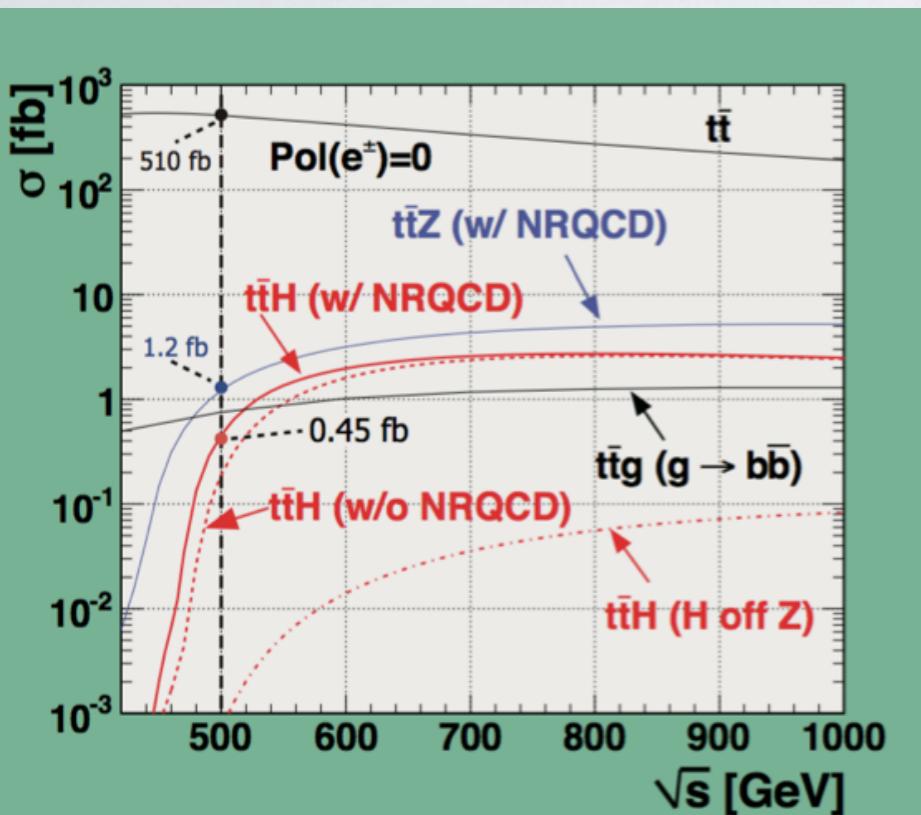
- Measurement possible for  $\sqrt{s} \gtrsim 500$  GeV [ttH associated production]
- Contamination from Higgsstrahlung,  $Z^* \rightarrow tt$
- Cross section rises by factor of 4 between 500 and 550 GeV (!)
- Slight enhancement at threshold due to QCD Coulomb gluons
- Minor sensitivity already at threshold ( $\rightarrow$  later)



Yonamine/Ikematsu/Tanabe/Fujii/Kiyo/Sumino/Yokoya, 2011

# $e^+e^- \rightarrow ttH$ : the Top Yukawa coupling

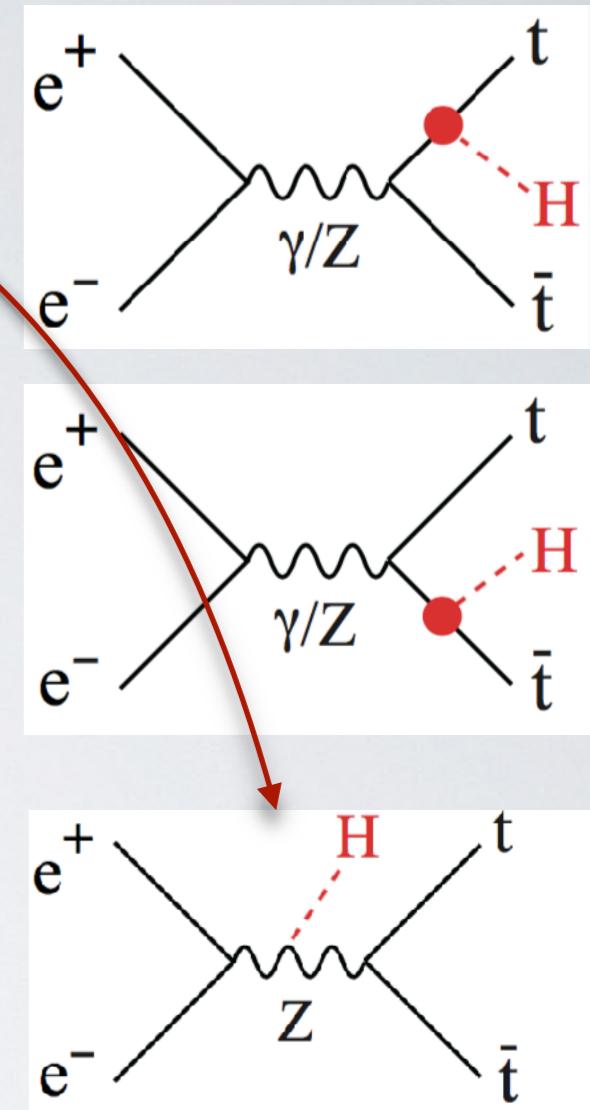
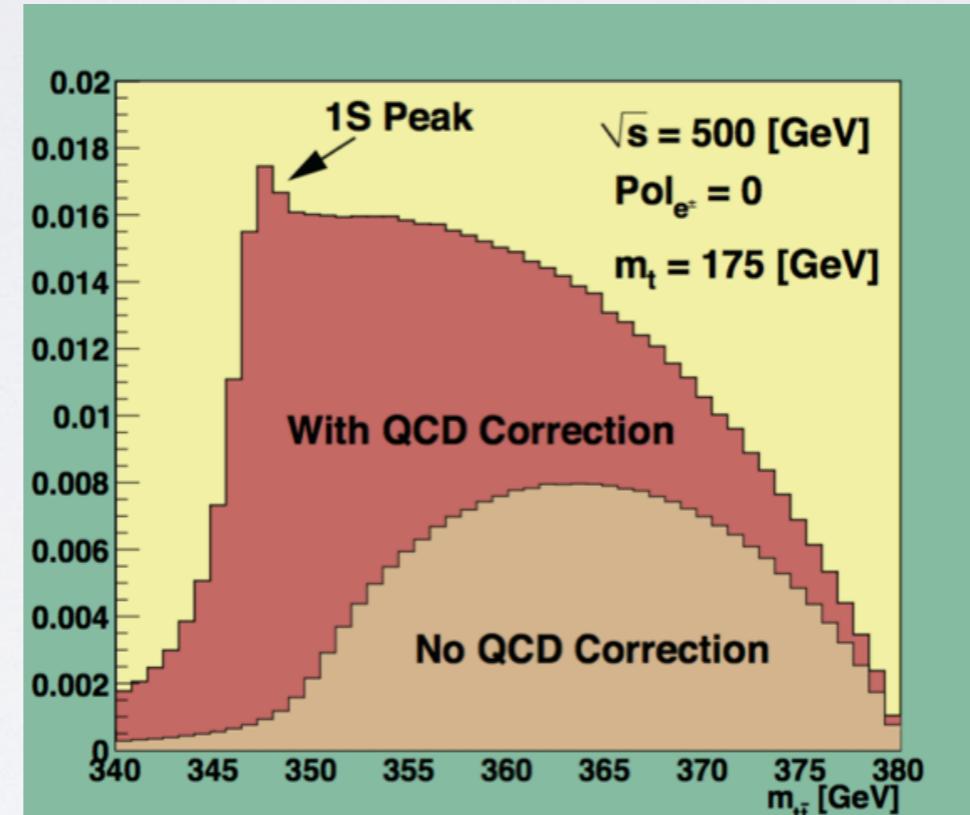
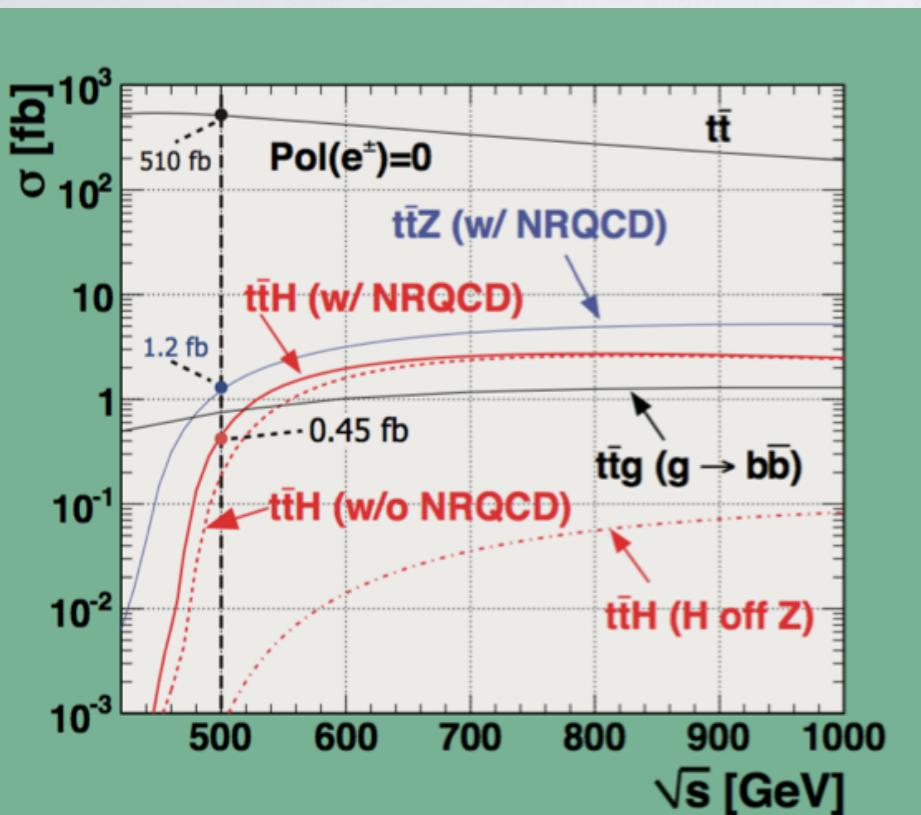
- Measurement possible for  $\sqrt{s} \gtrsim 500$  GeV [ttH associated production]
- Contamination from Higgsstrahlung,  $Z^* \rightarrow tt$
- Cross section rises by factor of 4 between 500 and 550 GeV (!)
- Slight enhancement at threshold due to QCD Coulomb gluons
- Minor sensitivity already at threshold ( $\rightarrow$  later)



Yonamine/Ikematsu/Tanabe/Fujii/Kiyo/Sumino/Yokoya, 2011

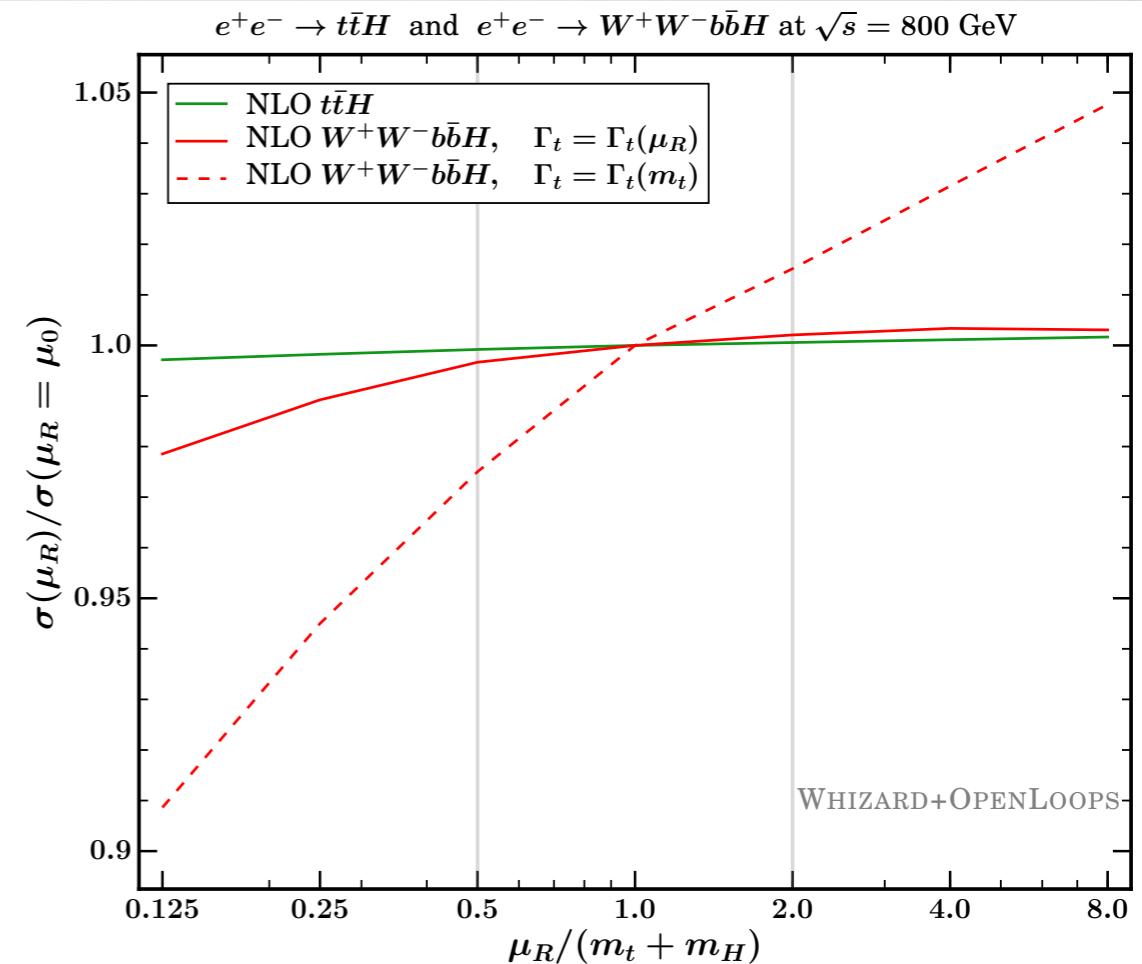
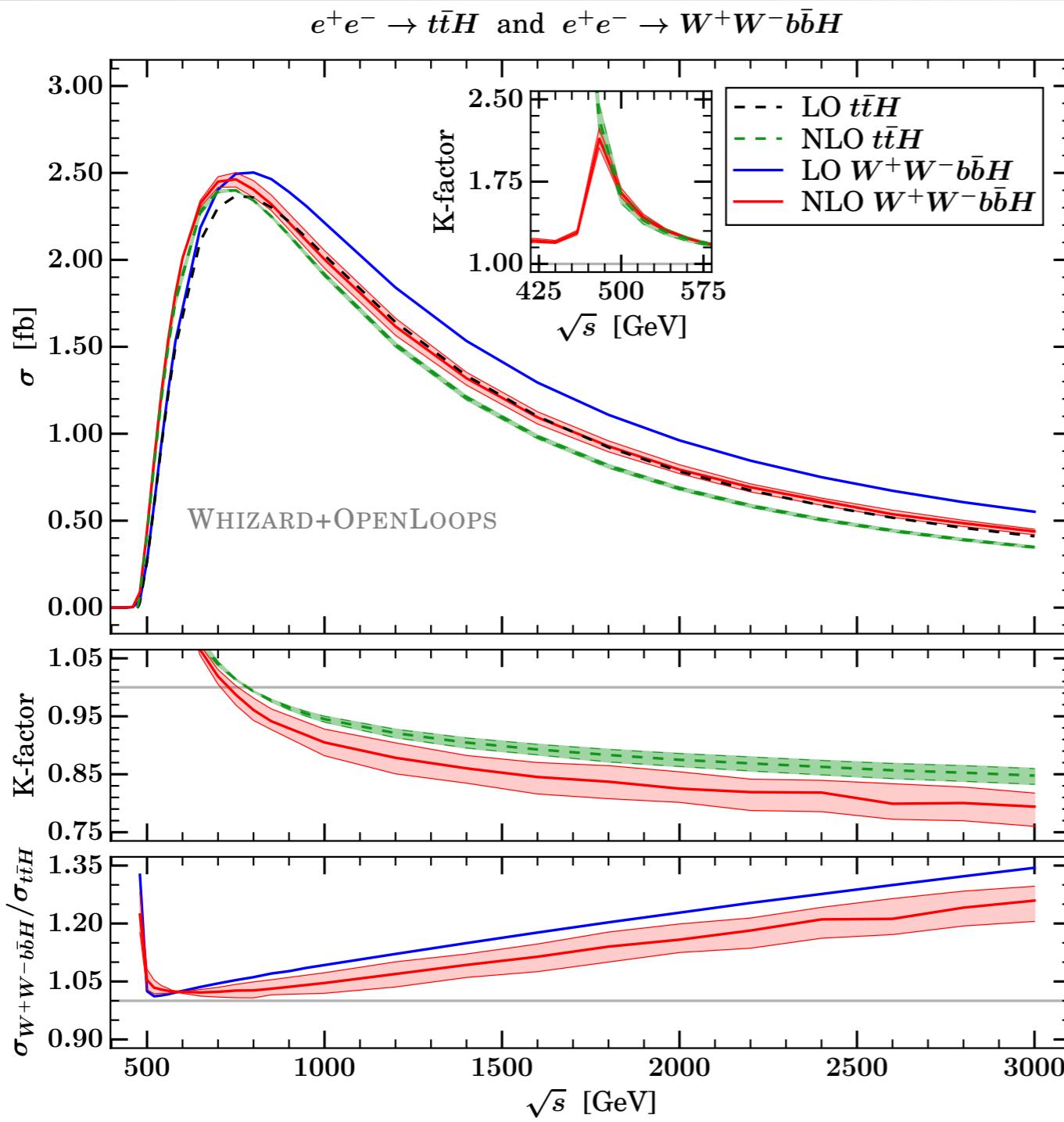
# $e^+e^- \rightarrow ttH$ : the Top Yukawa coupling

- Measurement possible for  $\sqrt{s} \gtrsim 500$  GeV [ttH associated production]
- Contamination from Higgsstrahlung,  $Z^* \rightarrow tt$
- Cross section rises by factor of 4 between 500 and 550 GeV (!)
- Slight enhancement at threshold due to QCD Coulomb gluons
- Minor sensitivity already at threshold ( $\rightarrow$  later)

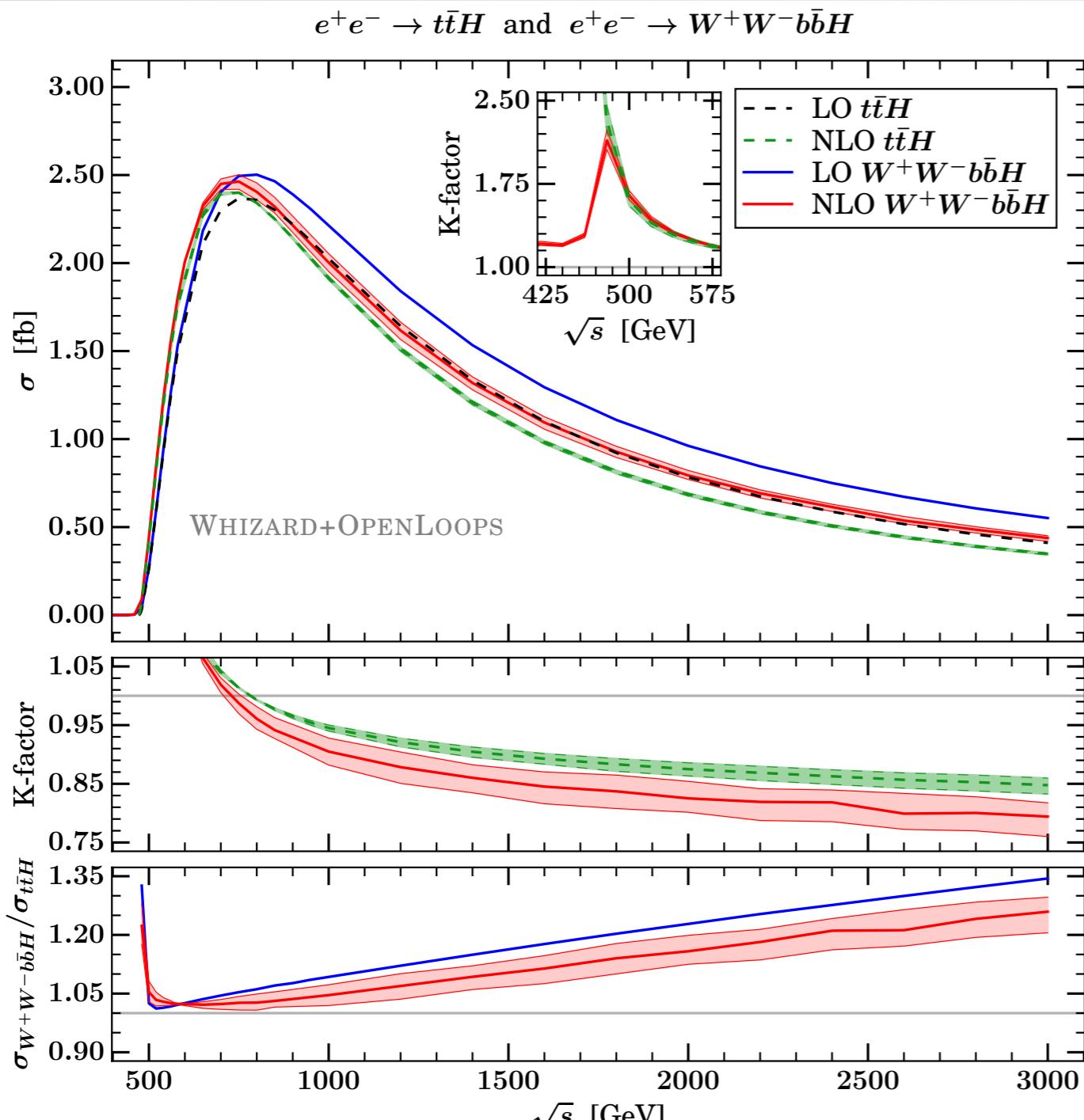


Yonamine/Ikematsu/Tanabe/Fujii/Kiyo/Sumino/Yokoya, 2011

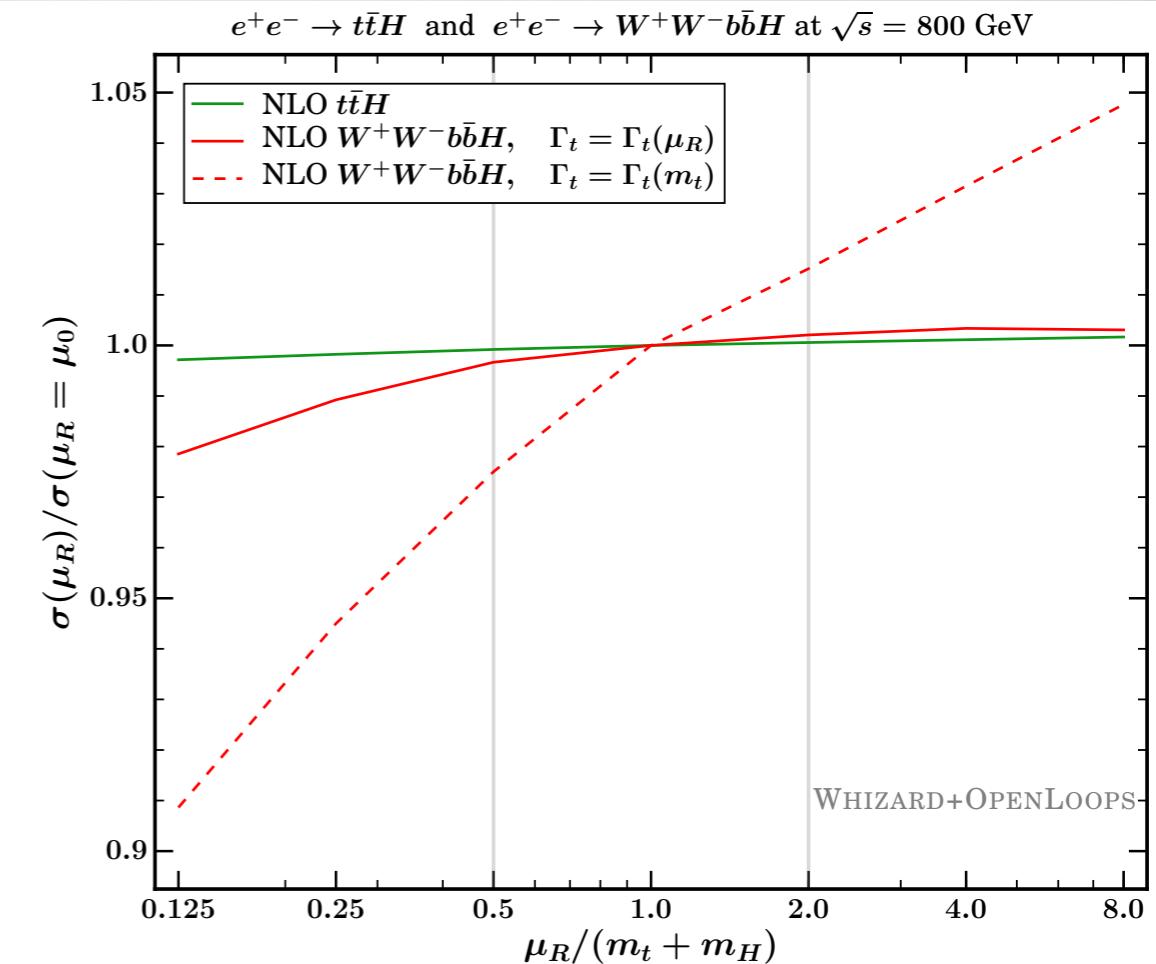
- Much gain from higher center-of-mass energies like 0.8-1.0 TeV [CLIC: 1.5 TeV !]
- Larger cross section, (much) less  $t\bar{t}$  background



Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390



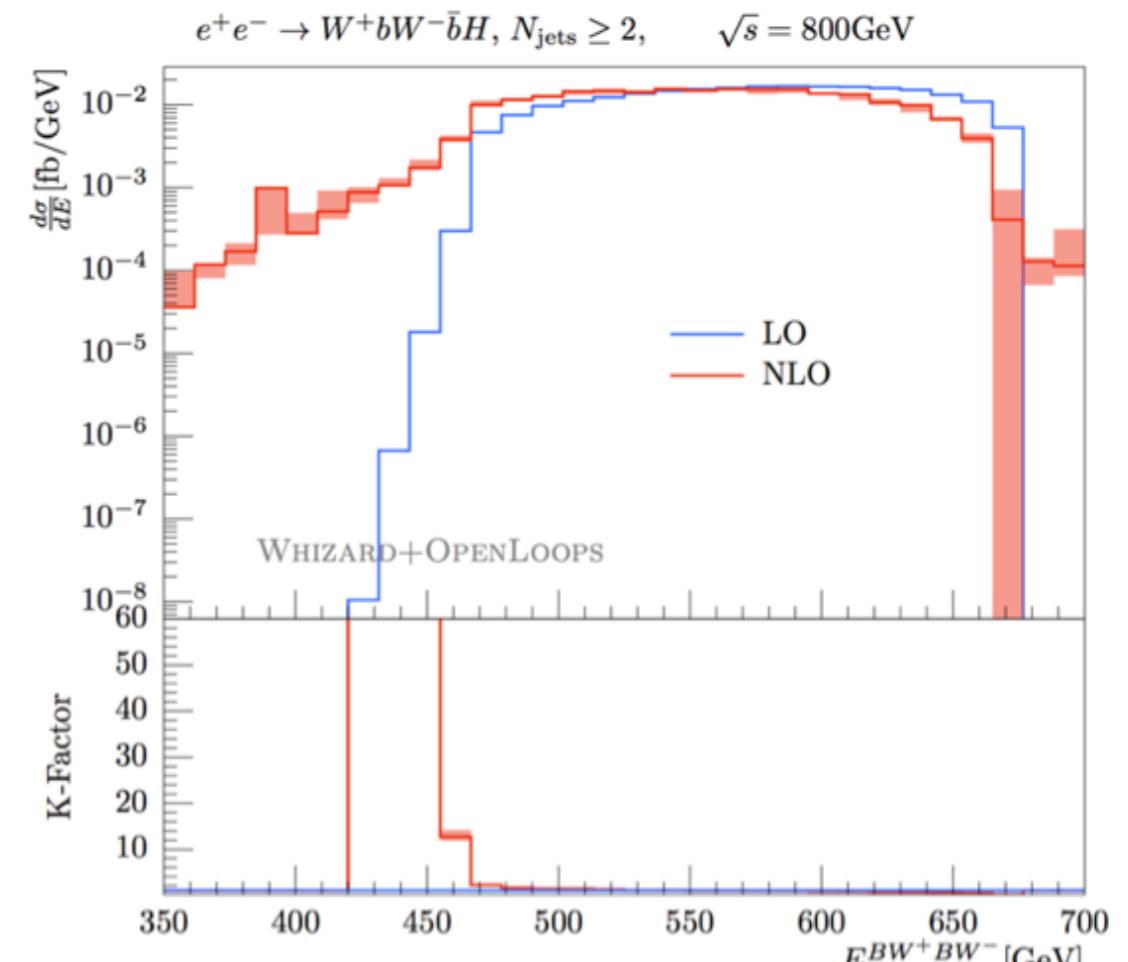
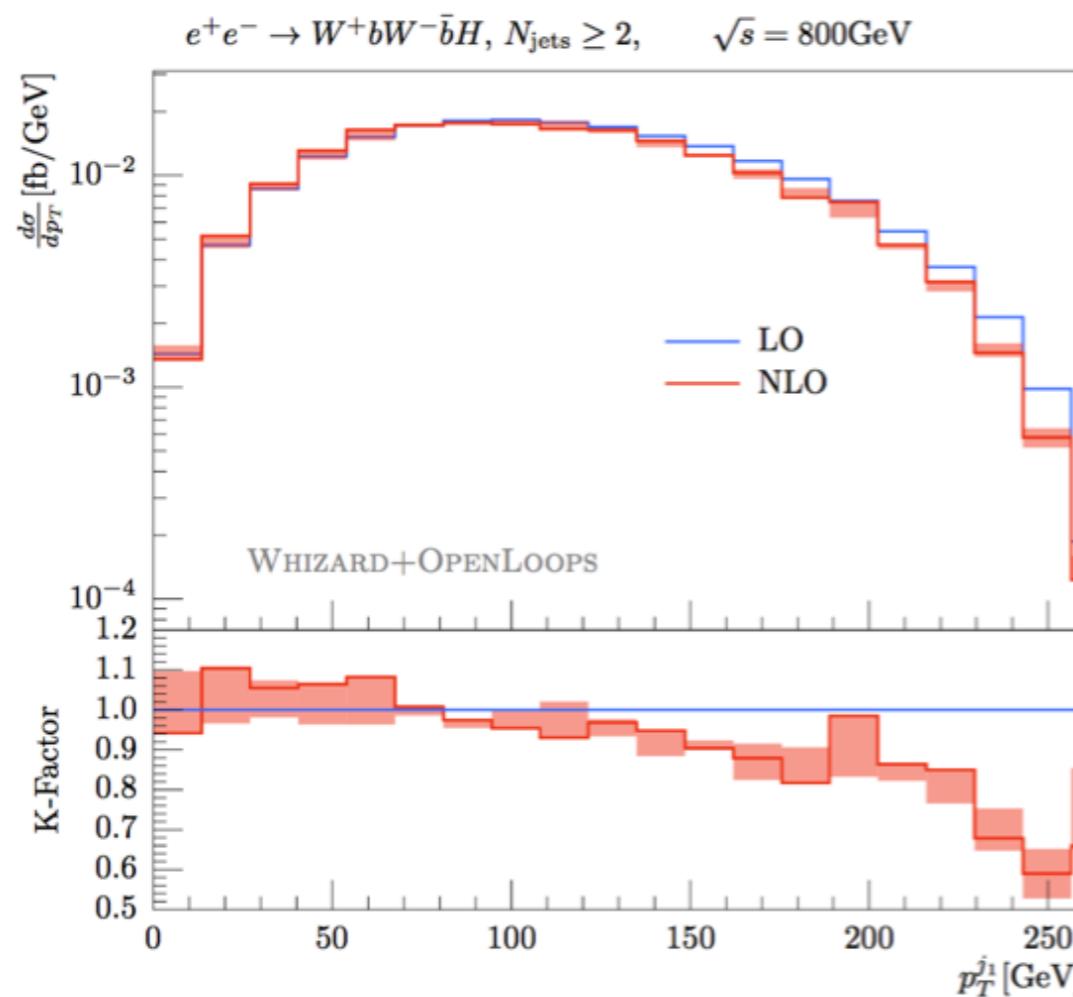
Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390



$\sqrt{s}$ [GeV]	$e^+e^- \rightarrow t\bar{t}H$			$e^+e^- \rightarrow W^+W^-b\bar{b}H$		
	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor
500	0.26	$0.42^{+3.6\%}_{-3.1\%}$	1.60	0.27	$0.44^{+2.6\%}_{-2.4\%}$	1.63
800	2.36	$2.34^{+0.1\%}_{-0.1\%}$	0.99	2.50	$2.40^{+2.1\%}_{-1.9\%}$	0.96
1000	2.02	$1.91^{+0.5\%}_{-0.5\%}$	0.95	2.21	$2.00^{+2.5\%}_{-2.5\%}$	0.90
1400	1.33	$1.21^{+0.9\%}_{-1.0\%}$	0.90	1.53	$1.32^{+2.6\%}_{-3.0\%}$	0.86
3000	0.41	$0.35^{+1.4\%}_{-1.8\%}$	0.84	0.55	$0.44^{+2.9\%}_{-4.3\%}$	0.79

# Differential Results for off-shell $t\bar{t}H$

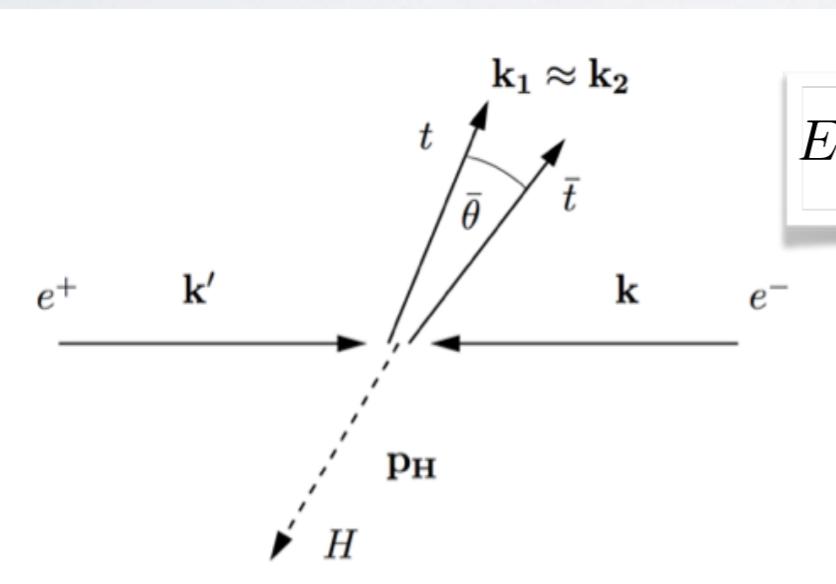
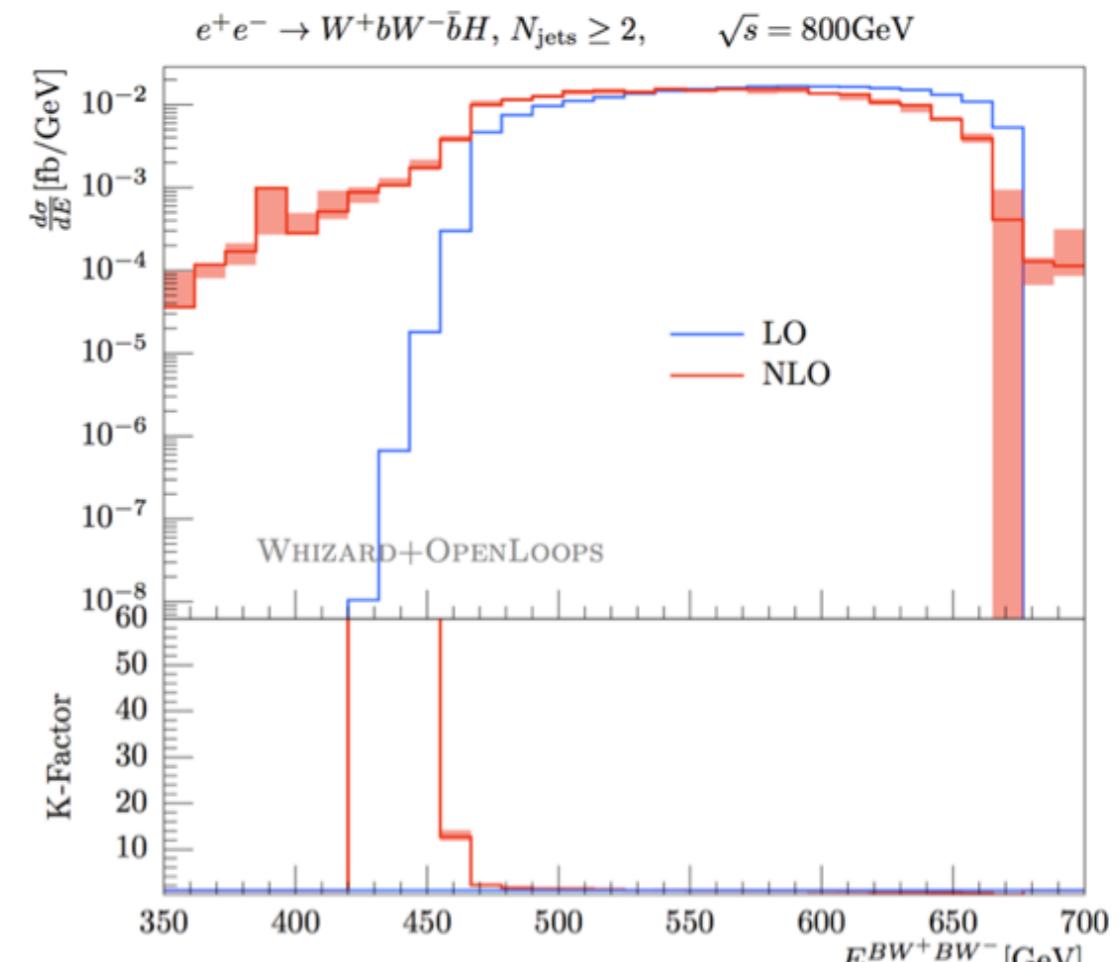
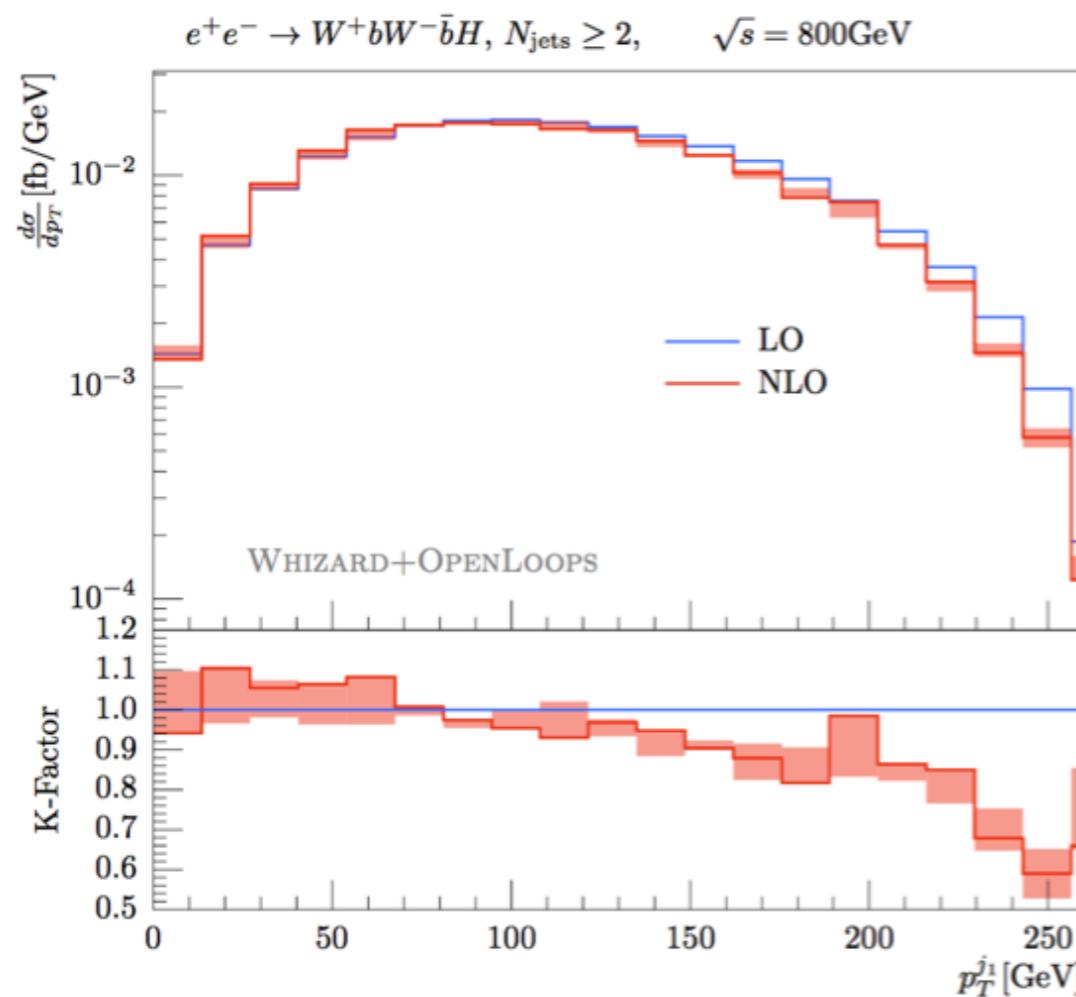
14 / 29



Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

# Differential Results for off-shell $t\bar{t}H$

14 / 29



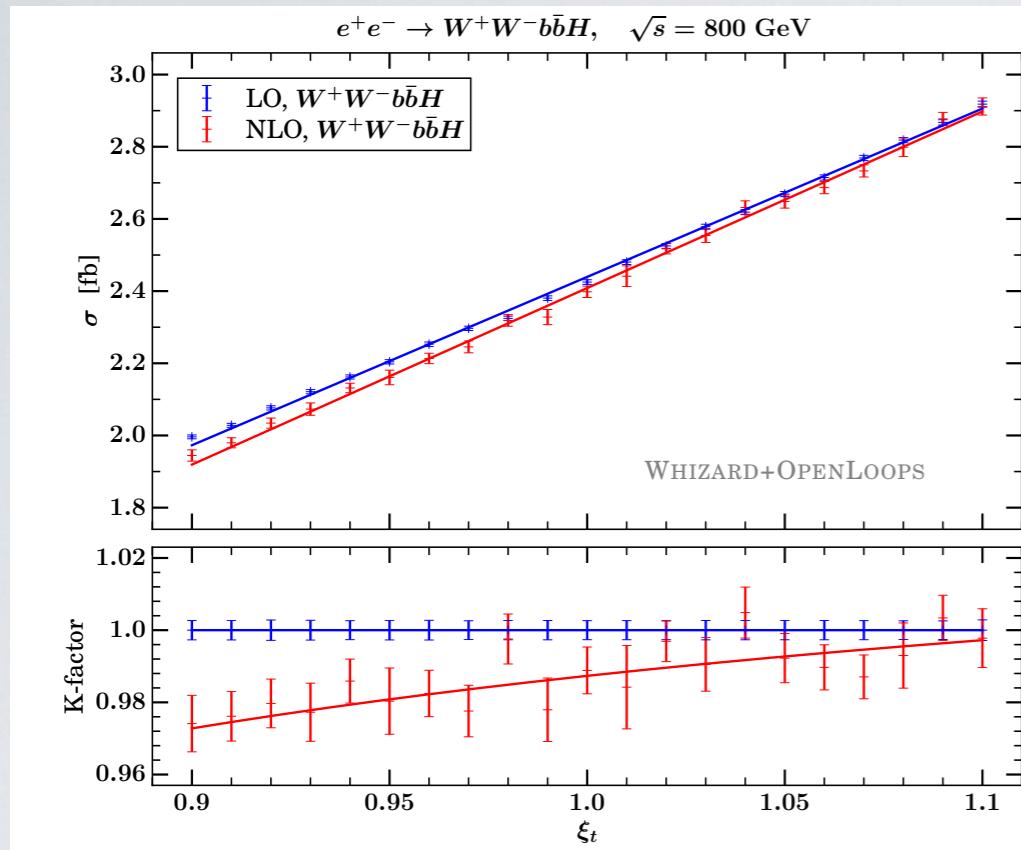
$$E_h = \frac{1}{2\sqrt{s}} [s + M_h^2 - (k_1 + k_2)^2] \xrightarrow{E_h \text{ large}} \frac{1}{2\sqrt{s}} [s + M_h^2 - 4m_t^2]$$

Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

# Determination of Top-Yukawa coupling

15 / 29

Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

SM signal-strength /  
coupling modifier:

$$\lim_{\xi_t \rightarrow 1} \sigma(\xi_t) \left[ \frac{d\sigma(\xi_t)}{d\xi_t} \right]^{-1} = \frac{S + I + B}{2S + I} = \frac{1}{2} + \frac{I/2 + B}{2S + I}.$$

	$ttH$	$W^+W^-b\bar{b}H$
LO	$0.514 \pm 0.0002$	$0.520 \pm 0.001$
NLO	$0.485 \pm 0.0002$	$0.497 \pm 0.002$

cf. also CLIC Top Report, 1807.02441

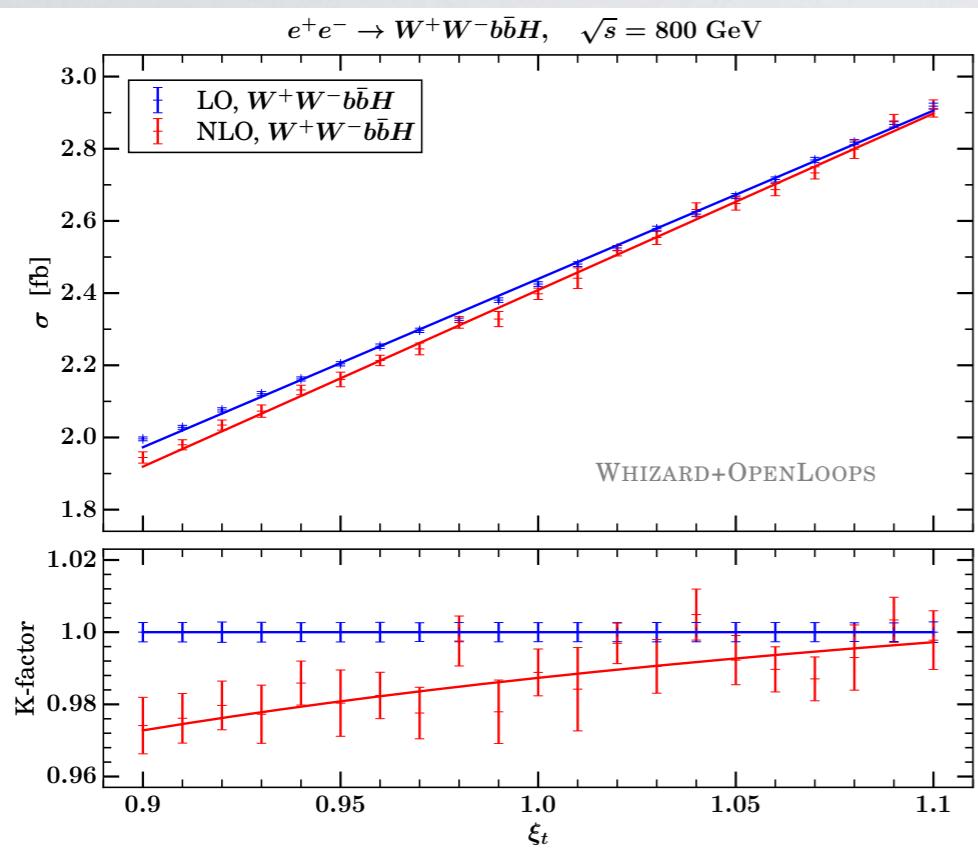
# Determination of Top-Yukawa coupling

15 / 29

Chokouf  /Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

SM signal-strength /  
coupling modifier:

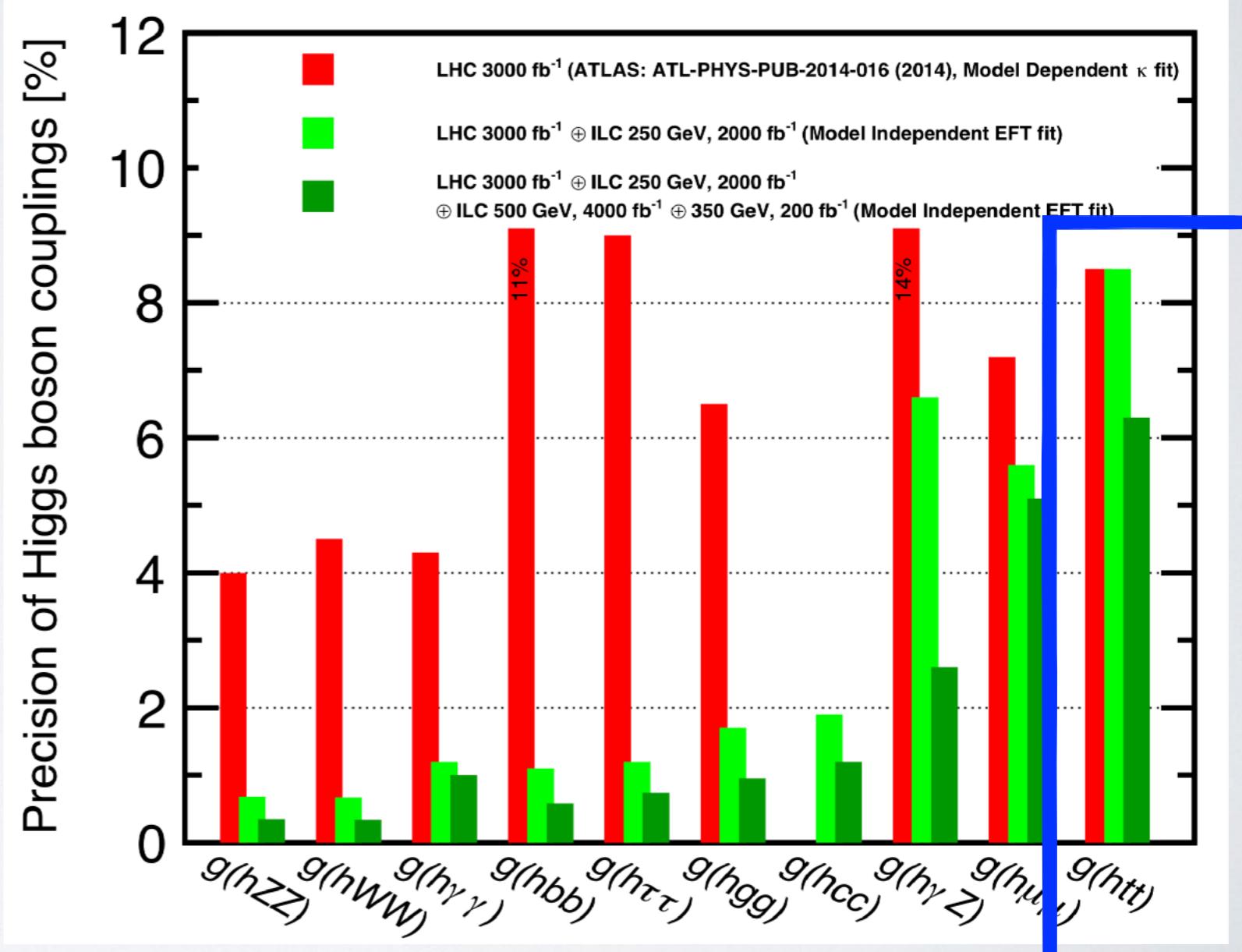
$$\lim_{\xi_t \rightarrow 1} \sigma(\xi_t) \left[ \frac{d\sigma(\xi_t)}{d\xi_t} \right]^{-1} = \frac{S + I + B}{2S + I} = \frac{1}{2} + \frac{I/2 + B}{2S + I}.$$



	$t\bar{t}H$	$W^+W^-b\bar{b}H$
LO	$0.514 \pm 0.0002$	$0.520 \pm 0.001$
NLO	$0.485 \pm 0.0002$	$0.497 \pm 0.002$

cf. also CLIC Top Report, 1807.02441

from 1710.07621

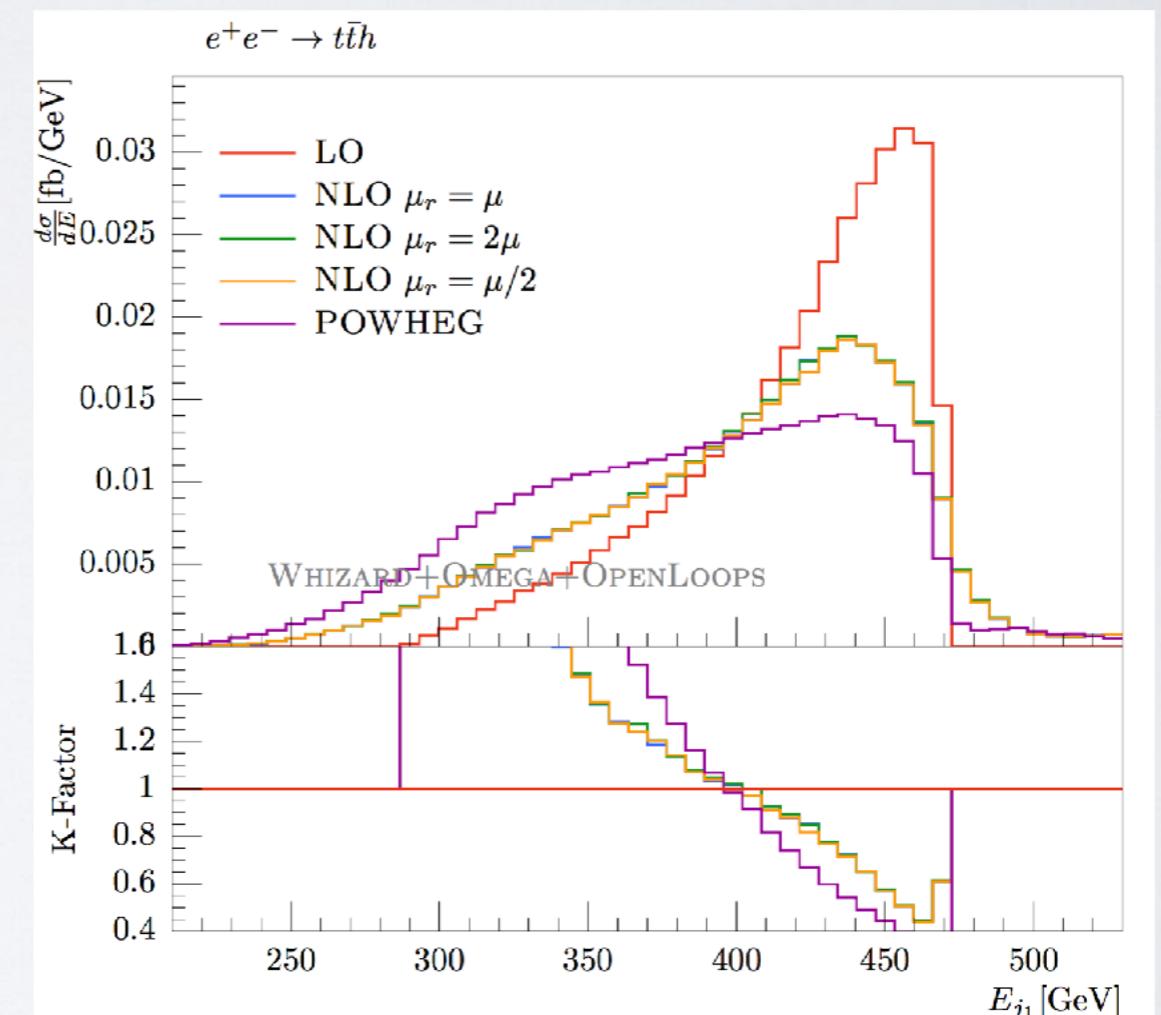
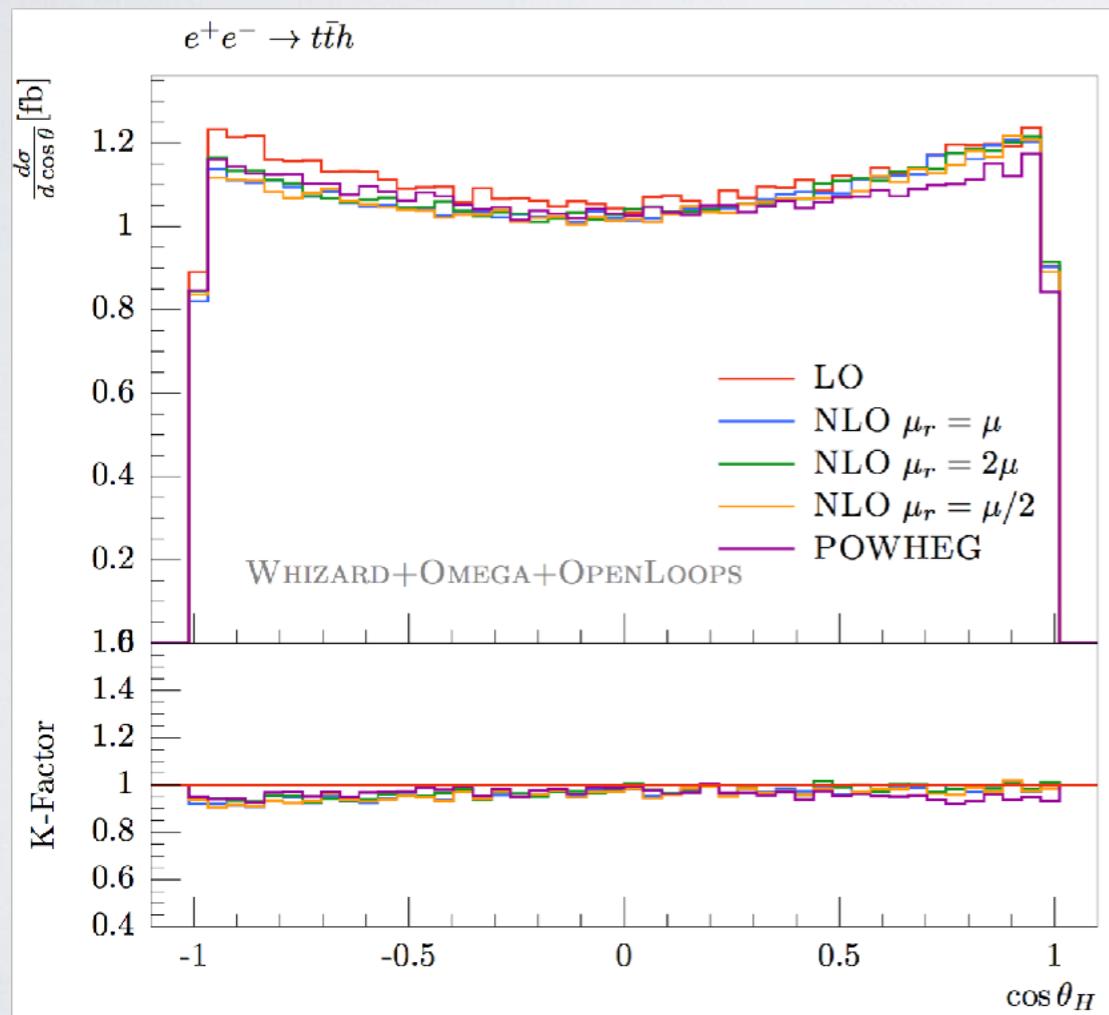


- Precise predictions of multi-parton final states require properly matched samples
- NLO QCD including POWHEG matching already available [WHIZARD+OpenLoops]
- All descriptions at NLO at the moment for the on-shell process
- Even LO simulations are demanding, e.g.:  $e^+e^- \rightarrow b\bar{b}b\bar{b}jj\ell\nu_\ell$ ,  $b\bar{b}jjjjjj\ell\nu_\ell$

# Matched NLO QCD results

16 / 29

- Precise predictions of multi-parton final states require properly matched samples
- NLO QCD including POWHEG matching already available [WHIZARD+OpenLoops]
- All descriptions at NLO at the moment for the on-shell process
- Even LO simulations are demanding, e.g.:  $e^+e^- \rightarrow b\bar{b}b\bar{b}jj\ell\nu_\ell$ ,  $b\bar{b}jjjjjj\ell\nu_\ell$



[Chokoufe/JRR/Weiss]



Process	$\sigma^{\text{LO}}[\text{fb}]$	MG5_AMC $\sigma^{\text{NLO}}[\text{fb}]$	$K$	$\sigma^{\text{LO}}[\text{fb}]$	WHIZARD $\sigma^{\text{NLO}}[\text{fb}]$	$K$
$e^+e^- \rightarrow t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \rightarrow t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \rightarrow t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \rightarrow t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \rightarrow t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \rightarrow t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \rightarrow t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \rightarrow t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \rightarrow t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \rightarrow t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \rightarrow t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \rightarrow t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \rightarrow t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \rightarrow t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \rightarrow t\bar{t}W^\pm jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \rightarrow t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \rightarrow t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \rightarrow t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \rightarrow t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \rightarrow t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \rightarrow t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \rightarrow t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

Process	$\sigma^{\text{LO}}[\text{fb}]$	MG5_AMC $\sigma^{\text{NLO}}[\text{fb}]$	$K$	$\sigma^{\text{LO}}[\text{fb}]$	WHIZARD $\sigma^{\text{NLO}}[\text{fb}]$	$K$
$e^+e^- \rightarrow t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \rightarrow t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \rightarrow t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \rightarrow t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \rightarrow t\bar{t}tt\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \rightarrow t\bar{t}tt\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \rightarrow t\bar{t}bb\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \rightarrow t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \rightarrow t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \rightarrow t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \rightarrow t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \rightarrow t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \rightarrow t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \rightarrow t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \rightarrow t\bar{t}W^\pm jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \rightarrow t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \rightarrow t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \rightarrow t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \rightarrow t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \rightarrow t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	
$e^+e^- \rightarrow t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	
$e^+e^- \rightarrow t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	



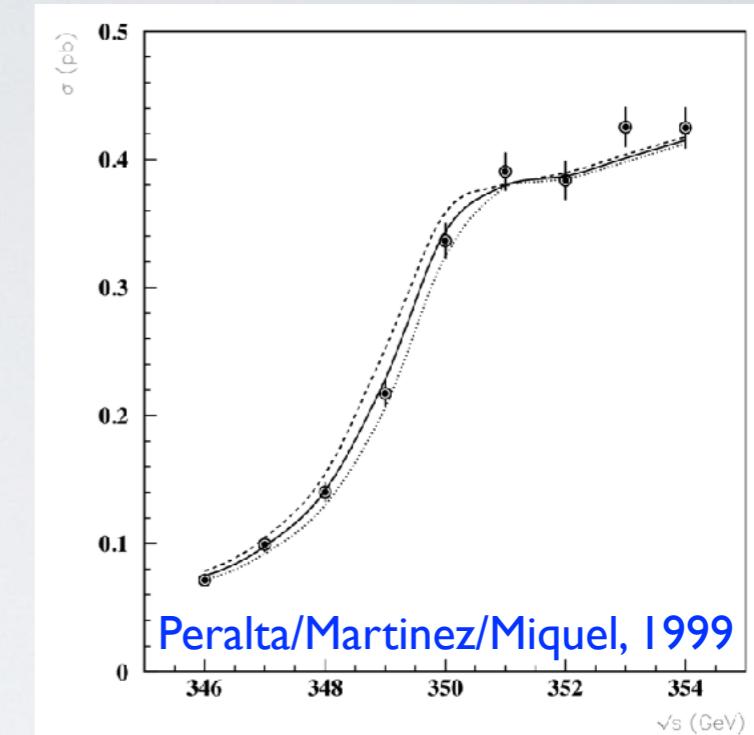
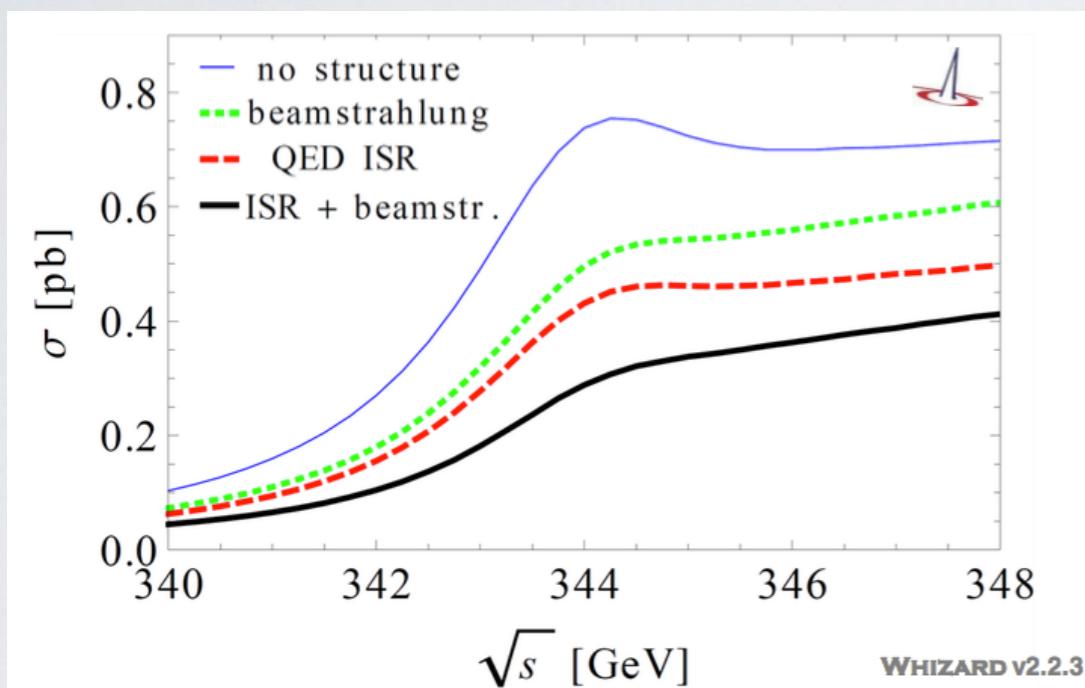
# TOP THRESHOLD SCAN: THE “TOP” TOP MASS MEASUREMENT



# Top Mass Measurement: Threshold

ILC top threshold scan best-known method to measure top quark mass,  $\Delta M \sim 30\text{-}70 \text{ MeV}$

- ▶ Close to threshold: top quarks non-relativistic
- ▶ Very strong QCD attraction due to “Coulomb” gluon exchange
- ▶ Leads to a remnant IS toponium (quasi-) bound state



error source	$\Delta m_t^{\text{PS}} [\text{MeV}]$
stat. error ( $200 \text{ fb}^{-1}$ )	13
theory (NNNLO scale variations, PS scheme)	40
parametric ( $\alpha_s$ , current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 – 50
total (stat. + syst.)	40 – 75

from 1702.05333



# Top Mass Definitions

- ▶ On-shell mass  $M$ : inverse quark propagator has zero at on-shell mass
- ▶  $\overline{\text{MS}}$ mass  $m$ : just divergent part is subtracted to achieve a finite quark propagator
- ▶ Short-distance masses: PS [potential subtr.], IS, RS [renormalon subtr.] masses

$$m^0 = Z_m^{\overline{\text{MS}}} m$$

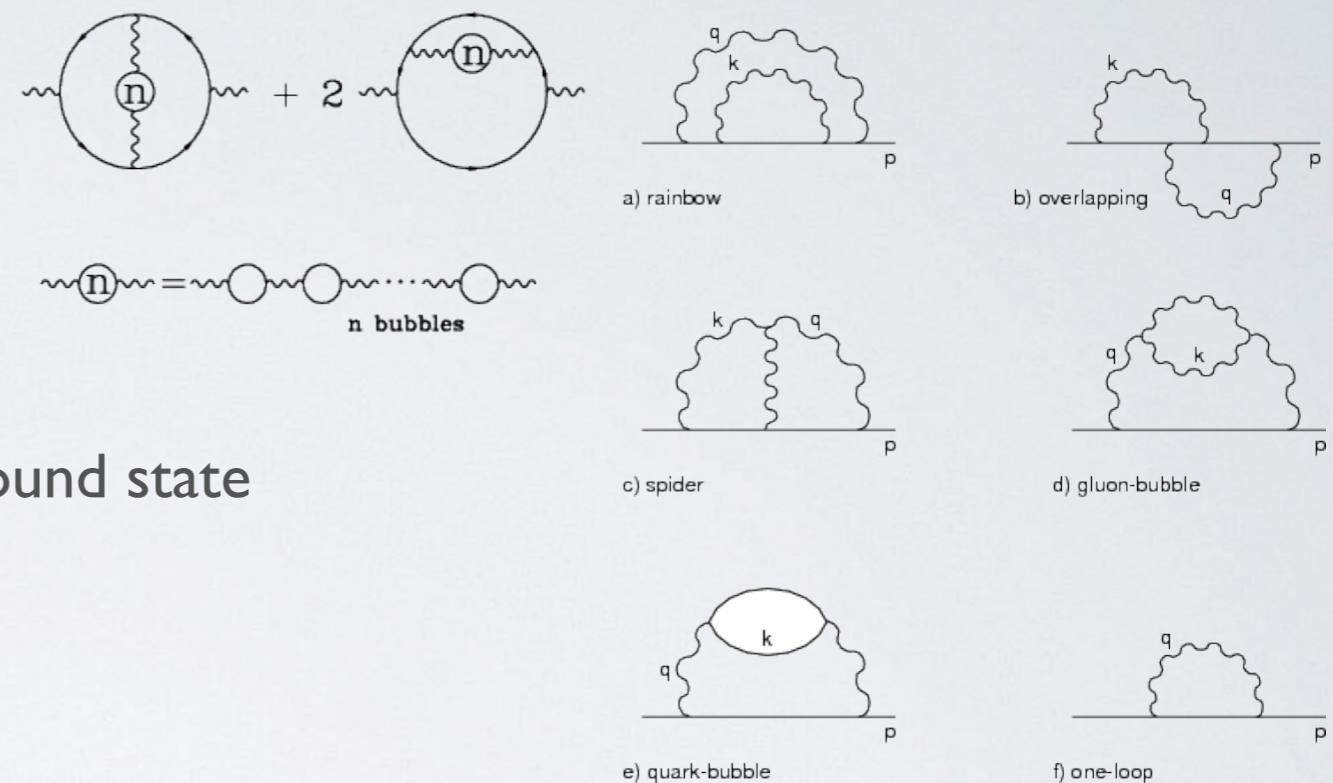
$$m^0 = Z_m^{\text{OS}} M$$

$$Z_m^{\text{OS}} = 1 + \Sigma_V(q^2 = M^2) + \Sigma_S(q^2 = M^2)$$

$$m^{\text{PS}} = M - \frac{1}{2} \int_{|\vec{q}| < \mu_f} \frac{d^3 q}{(2\pi)^3} V(\vec{q})$$

- IS mass is half the (pert.) mass of the  $1^3S_1$  bound state

$$m^{1S} = M + \frac{1}{2} E_1^{\text{pt}} \Big|_{\alpha_s^n \rightarrow \alpha_s^n \epsilon^{n-1}}$$



- Relation between pole and  $\overline{\text{MS}}$  mass @ 4-loop

[Marquard/Smirnov/Smirnov/Steinhauser, 1502.01030](#)

$$\begin{aligned} M_t &= m_t (1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.375\alpha_s^3 + (8.49 \pm 0.25)\alpha_s^4) \\ &= 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm .005 \text{ GeV} \end{aligned}$$

Final uncertainties from top mass conversions:

$$\Delta M_{PS} = 23 \text{ MeV}$$

$$\Delta M_{1S} = 7 \text{ MeV}$$

$$\Delta M_{RS} = 11 \text{ MeV}$$

# Top Threshold: a demanding theory calculation

21 / 29

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate  $M \cdot v$  and  $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f. Hoang et al. '99-'01; Beneke et al., '13-'14
- Resummation of singular terms close to threshold ( $v = 0$ ), NNNLO/NNLL available (!)

Phase space of two massive particles      (p/v)NRQCD EFT w/ RG improvement

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left( \frac{\alpha_s}{v} \right)^k \sum_i (\alpha_s \ln v)^i \times \\ \times \{ 1 \text{ (LL)}; \alpha_s, v \text{ (NLL)}; \alpha_s^2, \alpha_s v, v^2 \text{ (NNLL)} \}$$

# Top Threshold: a demanding theory calculation

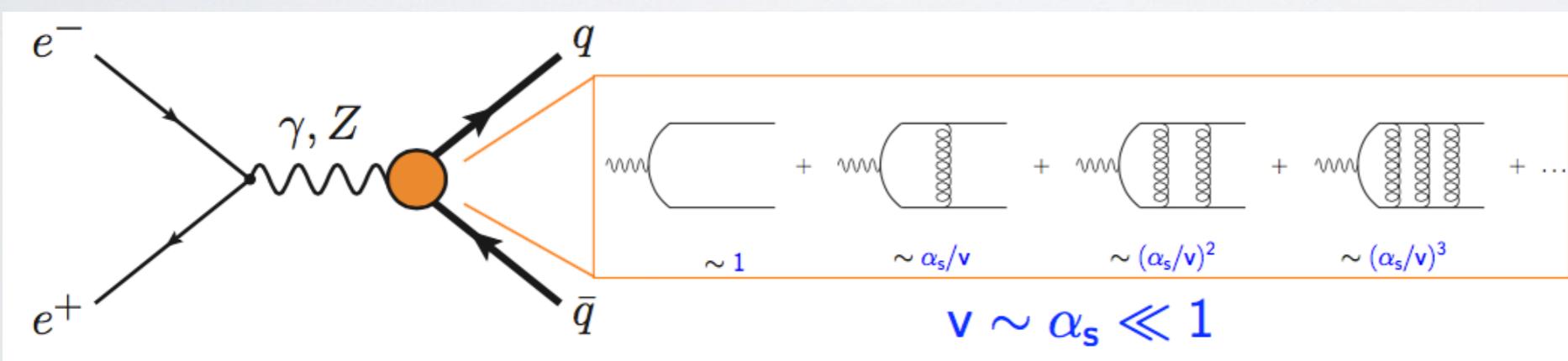
21 / 29

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate  $M \cdot v$  and  $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f. Hoang et al. '99-'01; Beneke et al., '13-'14
- Resummation of singular terms close to threshold ( $v = 0$ ), NNNLO/NNLL available (!)

Phase space of two massive particles      (p/v)NRQCD EFT w/ RG improvement

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left( \frac{\alpha_s}{v} \right)^k \sum_i (\alpha_s \ln v)^i \times$$
$$\times \{ 1 (\text{LL}); \alpha_s, v (\text{NLL}); \alpha_s^2, \alpha_s v, v^2 (\text{NNLL}) \}$$

Threshold region: top velocity  $v \sim \alpha_s \ll 1$       non-relativistic EFT: (v)NRQCD



Continuum region: “standard” fixed-order QCD

# Top Threshold: a demanding theory calculation

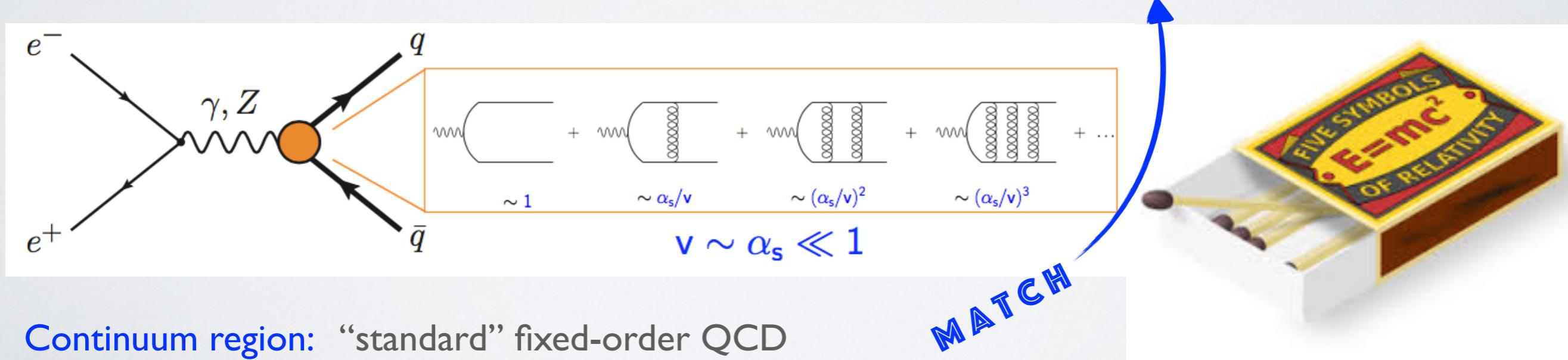
21 / 29

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate  $M \cdot v$  and  $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f. Hoang et al. '99-'01; Beneke et al., '13-'14
- Resummation of singular terms close to threshold ( $v = 0$ ), NNNLO/NNLL available (!)

Phase space of two massive particles      (p/v)NRQCD EFT w/ RG improvement

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left( \frac{\alpha_s}{v} \right)^k \sum_i (\alpha_s \ln v)^i \times$$
$$\times \{ 1 (\text{LL}); \alpha_s, v (\text{NLL}); \alpha_s^2, \alpha_s v, v^2 (\text{NNLL}) \}$$

Threshold region: top velocity  $v \sim \alpha_s \ll 1$       non-relativistic EFT: (v)NRQCD

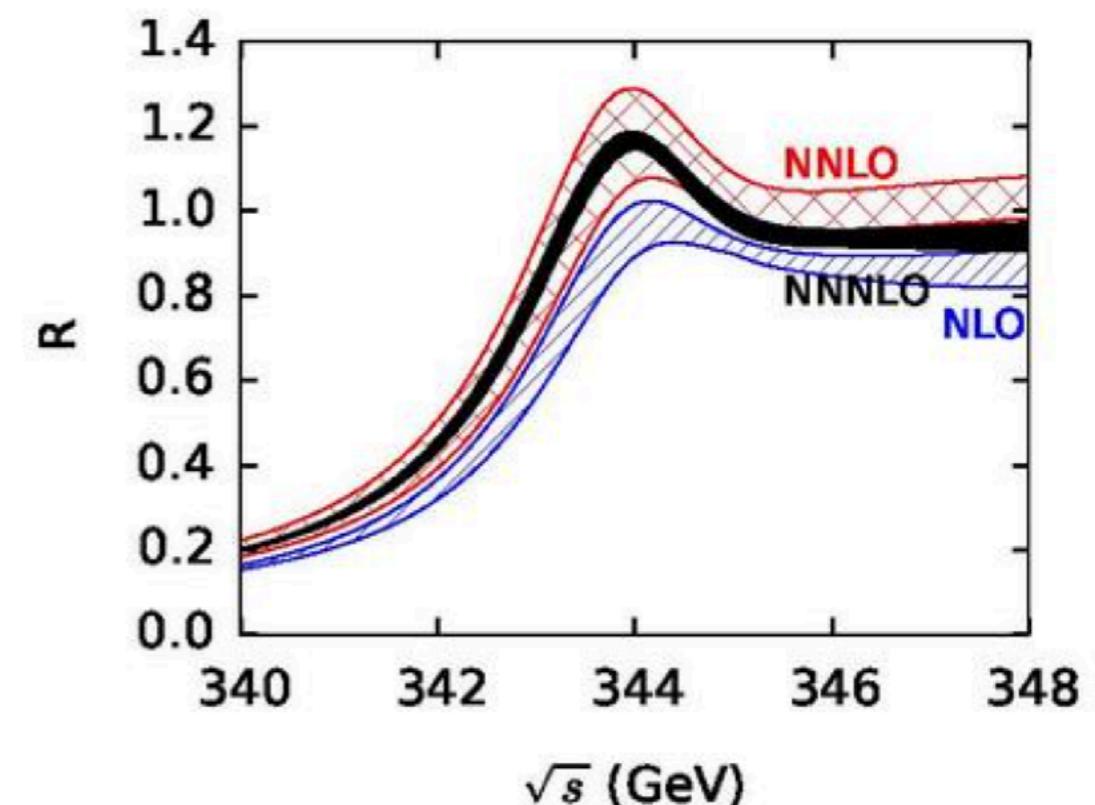


Continuum region: “standard” fixed-order QCD

# Fixed-order vs. resummation uncertainties

NRQCD NNNLO fixed order  
+  $\alpha_s$  logarithms

Kiyo et al., 2005; Beneke et al., 2008-2015



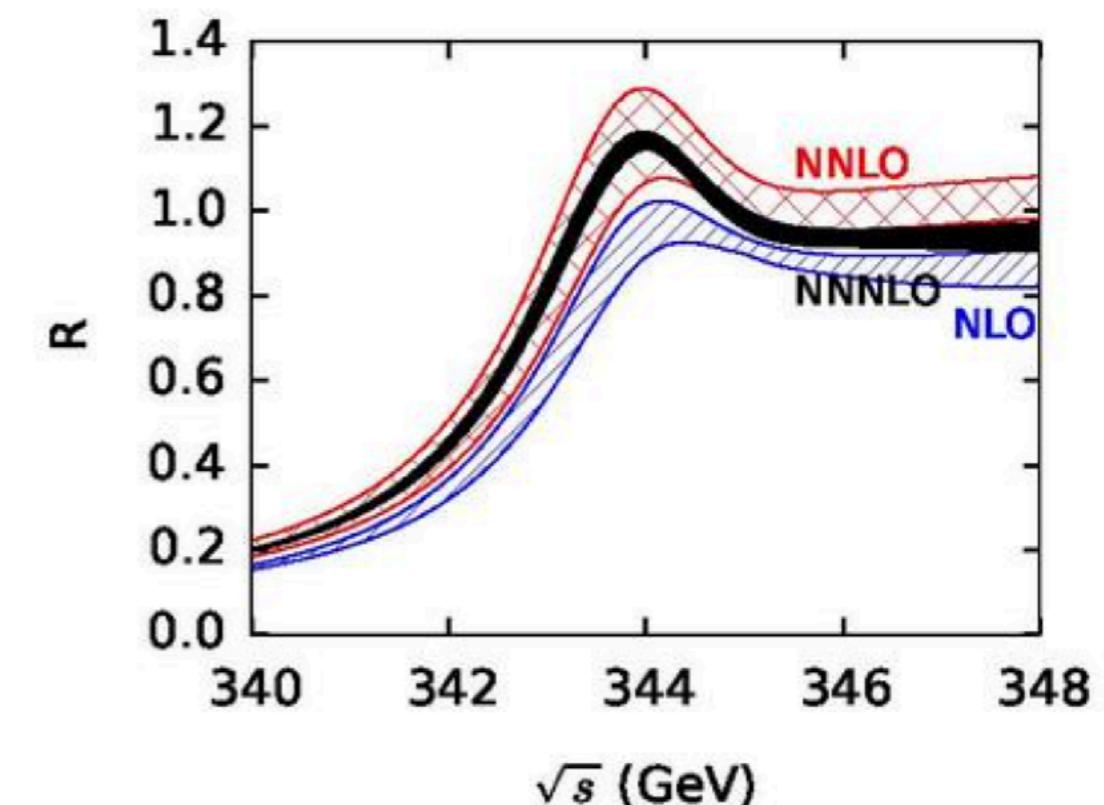
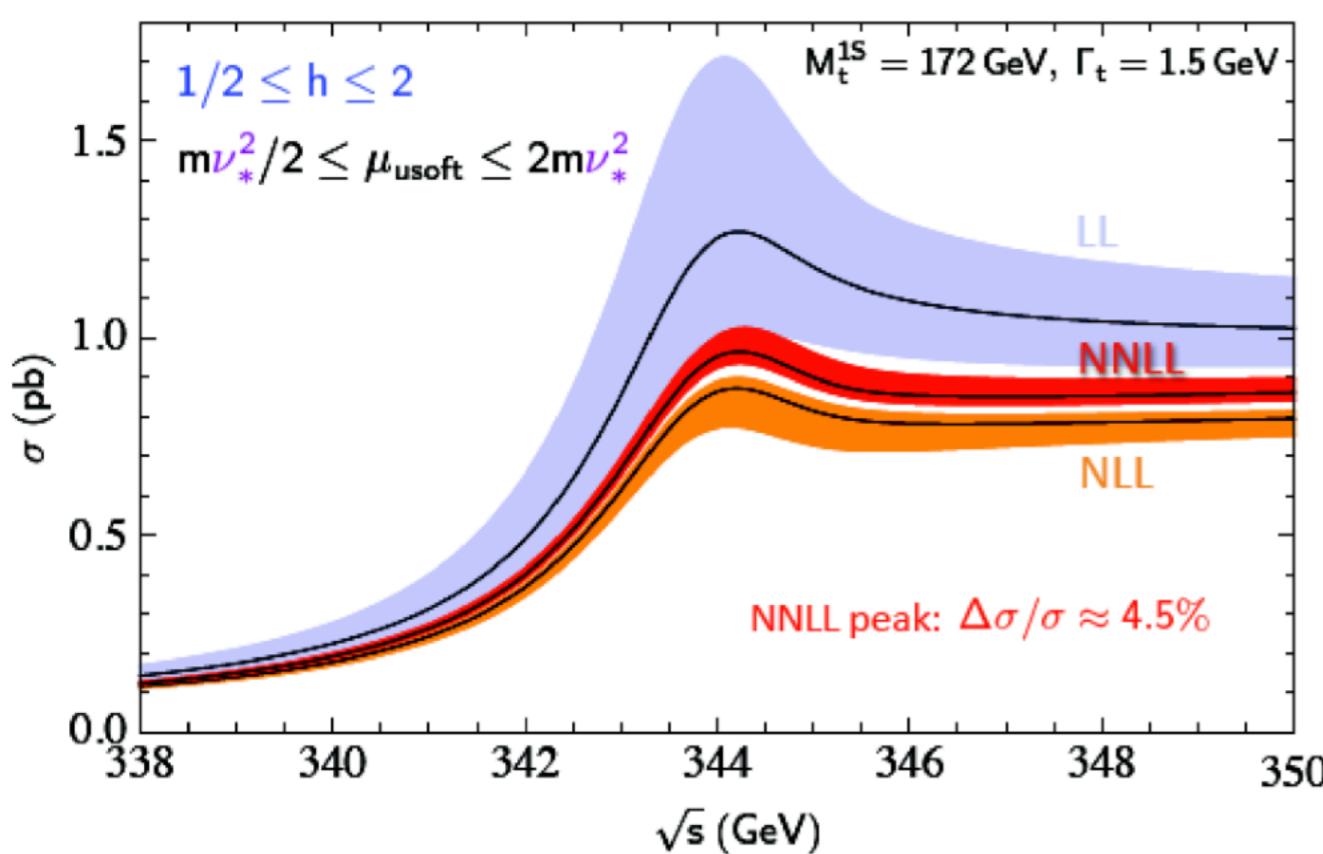
# Fixed-order vs. resummation uncertainties

NRQCD NNNLO fixed order  
+  $\alpha_s$  logarithms

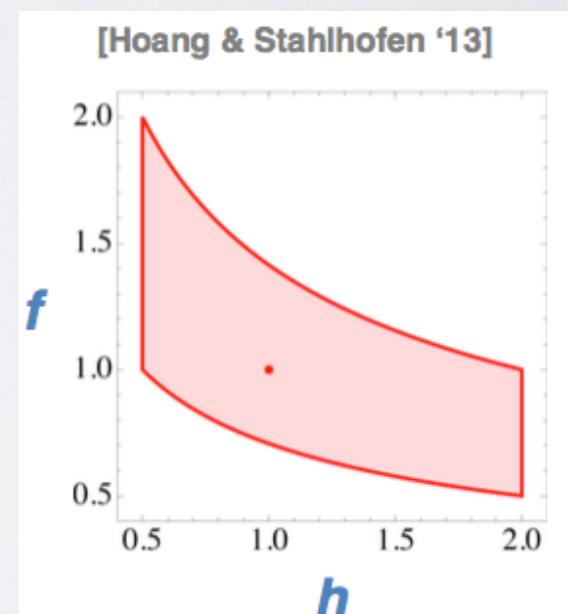
Kiyo et al., 2005; Beneke et al., 2008-2015

Resummation of  
velocity logarithms

Hoang/Stahlhofen, 2012



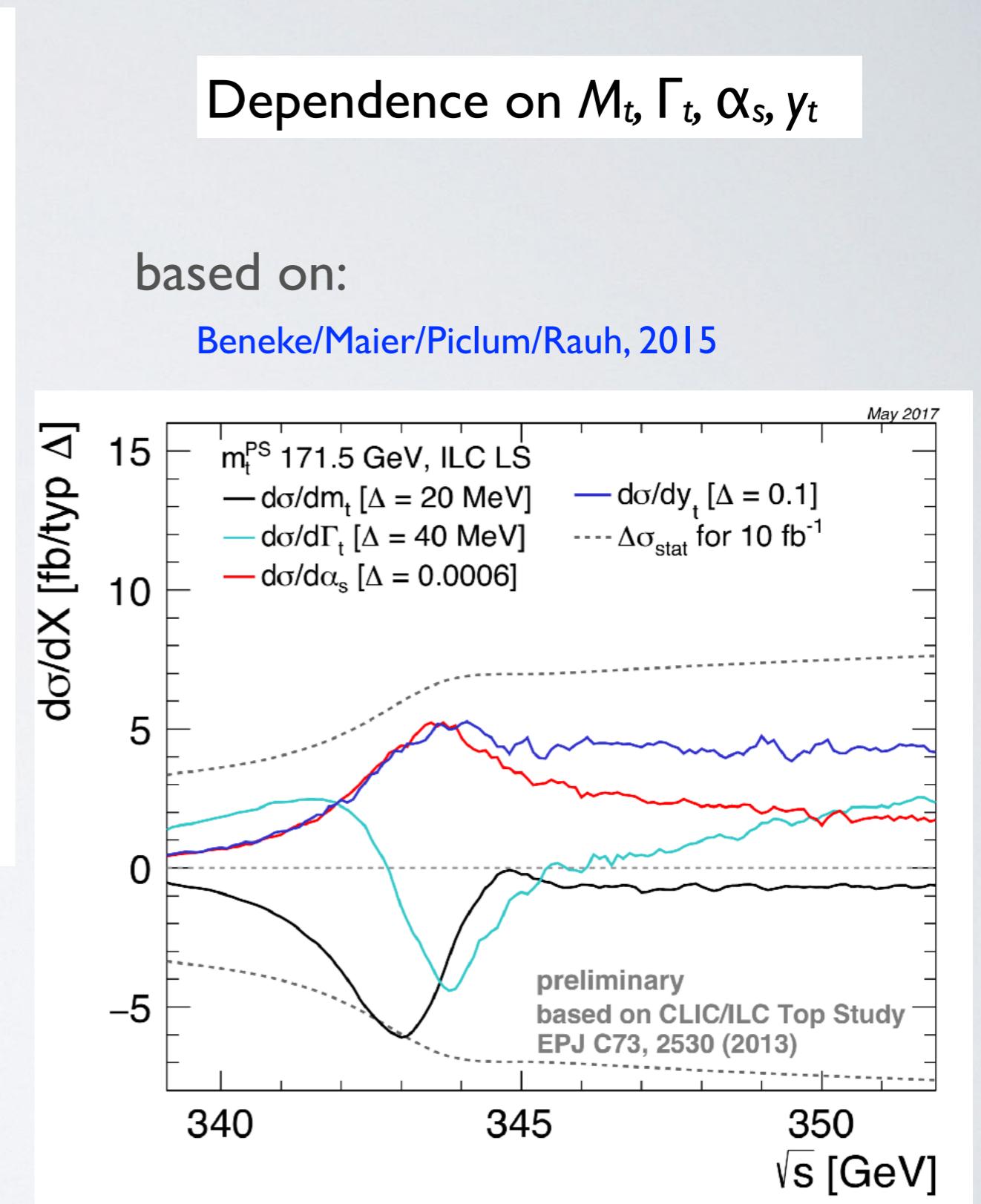
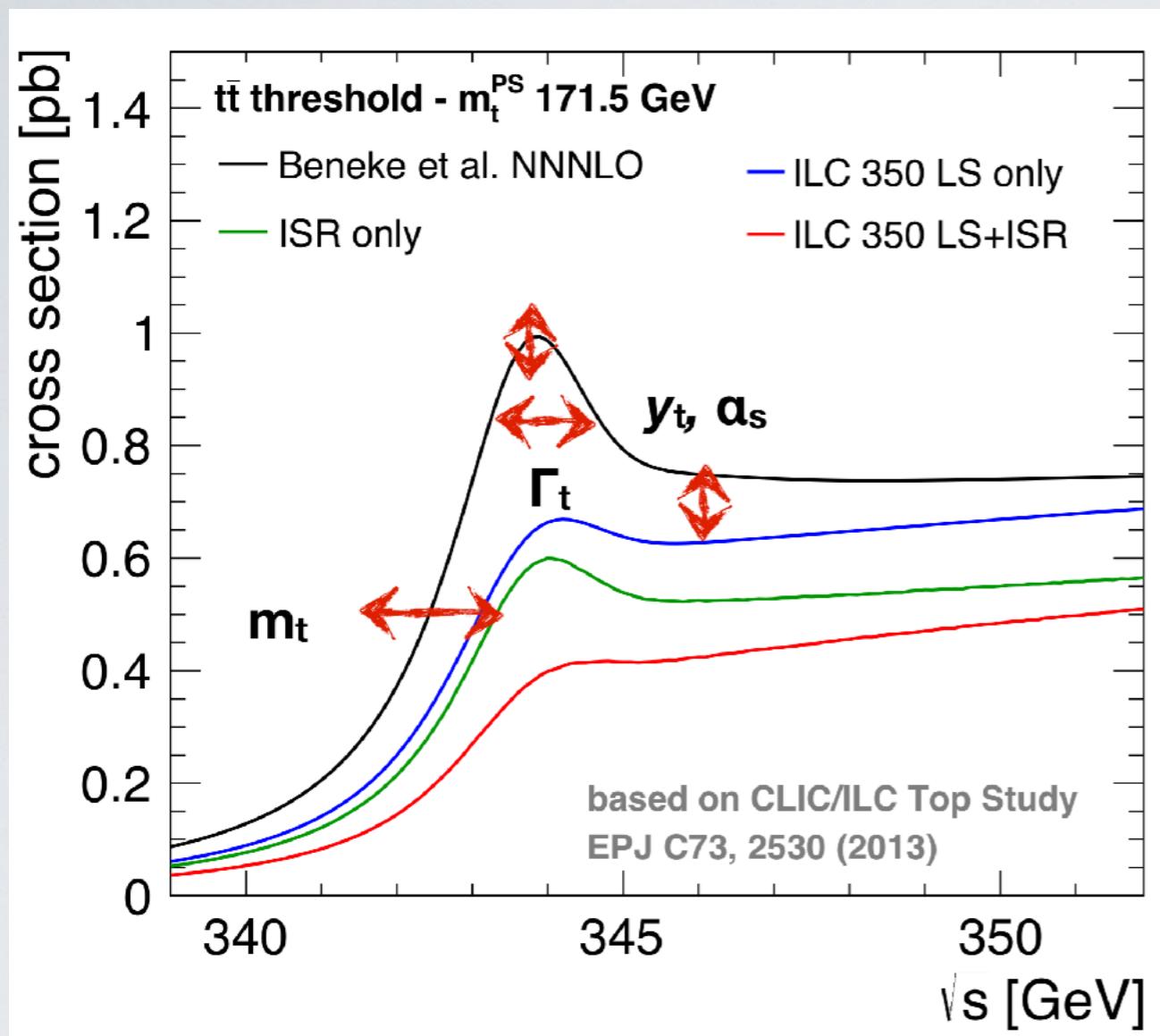
Theory uncertainties  
from scale variations:  
hard and soft scale



$$\mu_h = h \cdot m_t \quad \mu_s = f \cdot m_t v$$

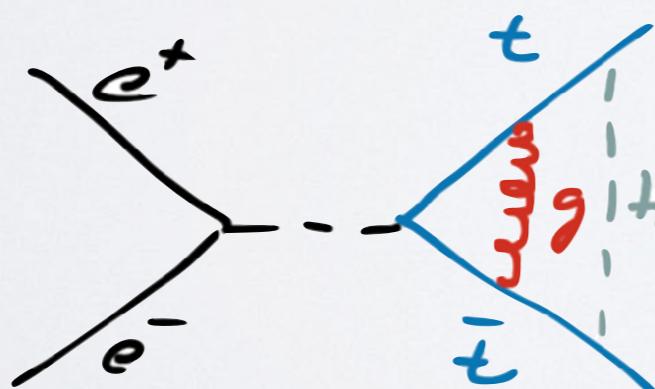
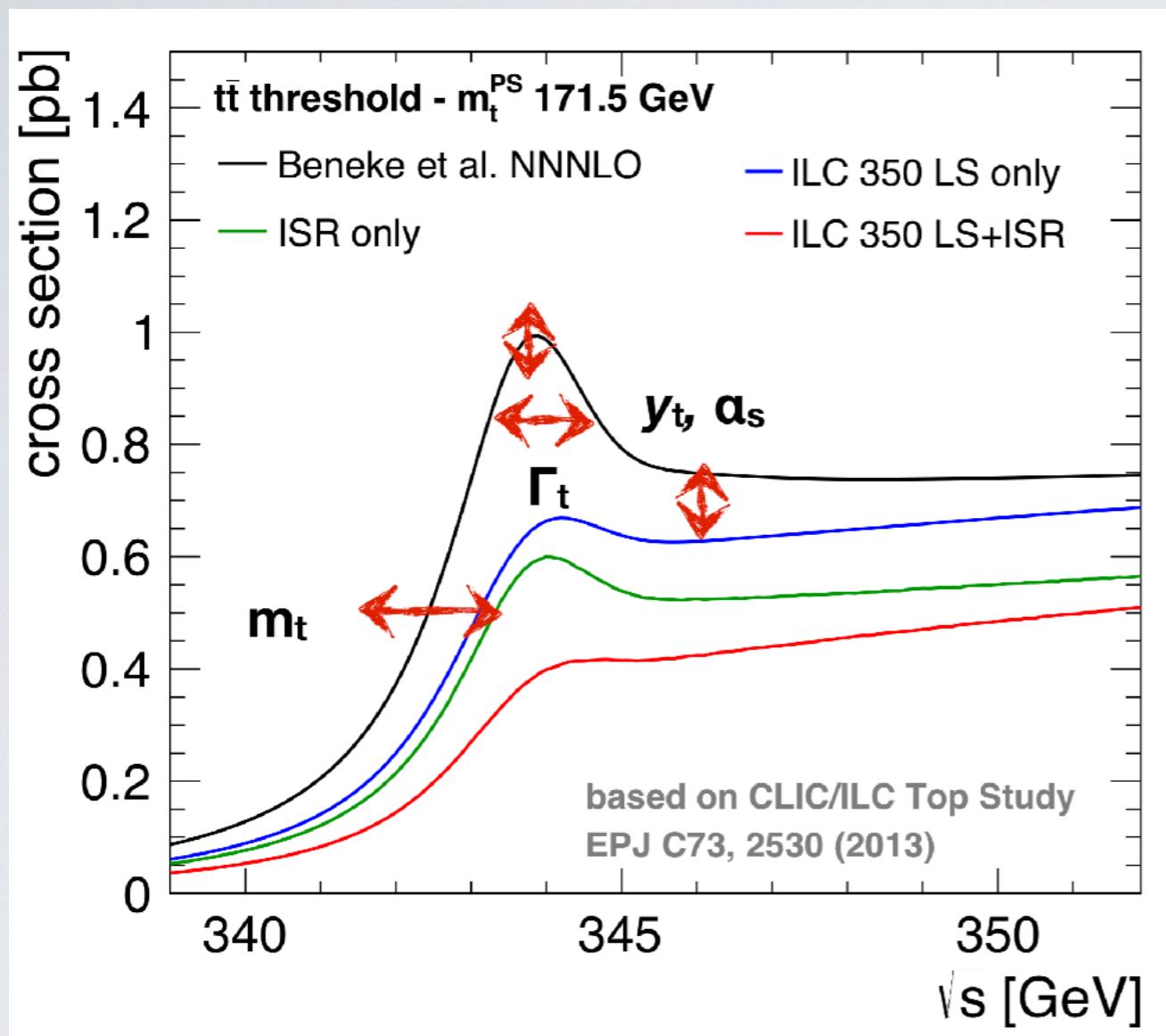
# Top Threshold: parametric dependencies

23 / 29



# Top Threshold: parametric dependencies

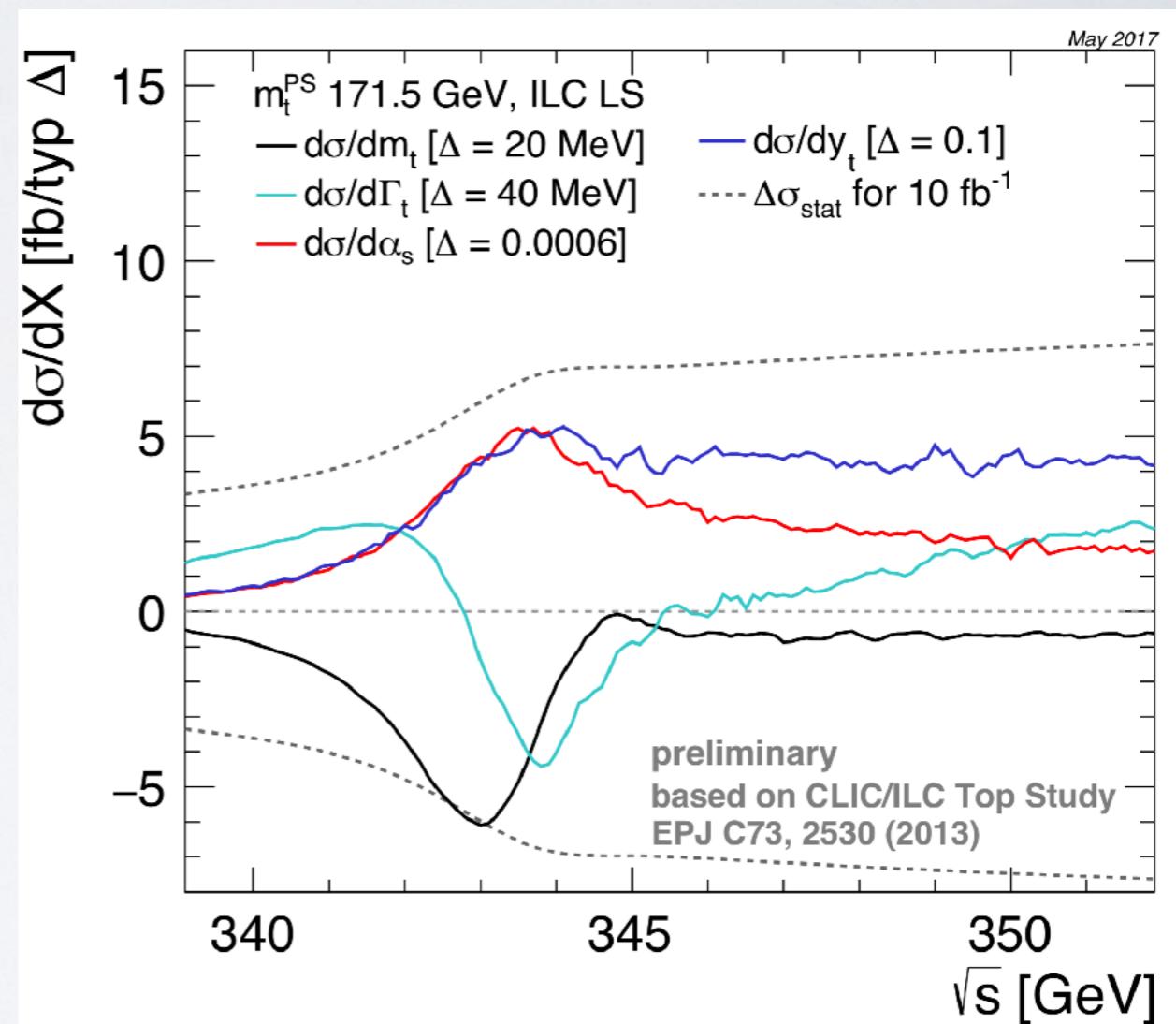
23 / 29



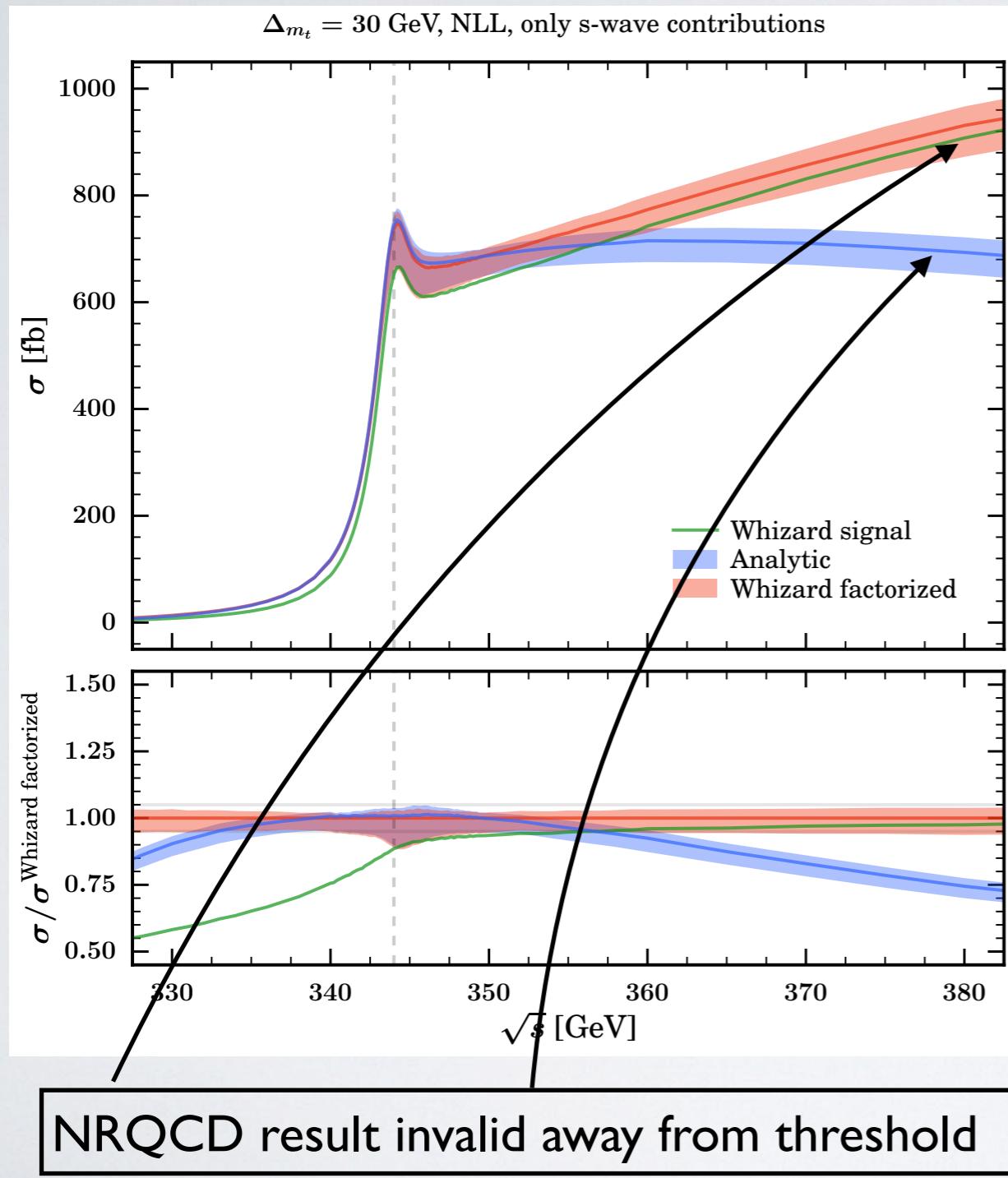
Dependence on  $M_t, \Gamma_t, \alpha_s, y_t$

based on:

Beneke/Maier/Piclum/Rauh, 2015



## Fully Exclusive Events: assess selection uncertainties



NRQCD result invalid away from threshold

Bach/Chokouf  /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220



## Fully Exclusive Events: assess selection uncertainties

$$\sigma_{\text{NLO+NLL}} = \sigma_{\text{NLO}} + \left( (\tilde{F}_{\text{NLL}} - \tilde{F}_{\text{NLL}}^{\text{exp}}) \right) \left( \begin{array}{c} e^- \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left\{ \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \tilde{F}_{\text{NLL}} \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) \right\} + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ g \\ W^+ \\ W^- \end{array} \right) + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ g \\ W^+ \\ W^- \end{array} \right),$$

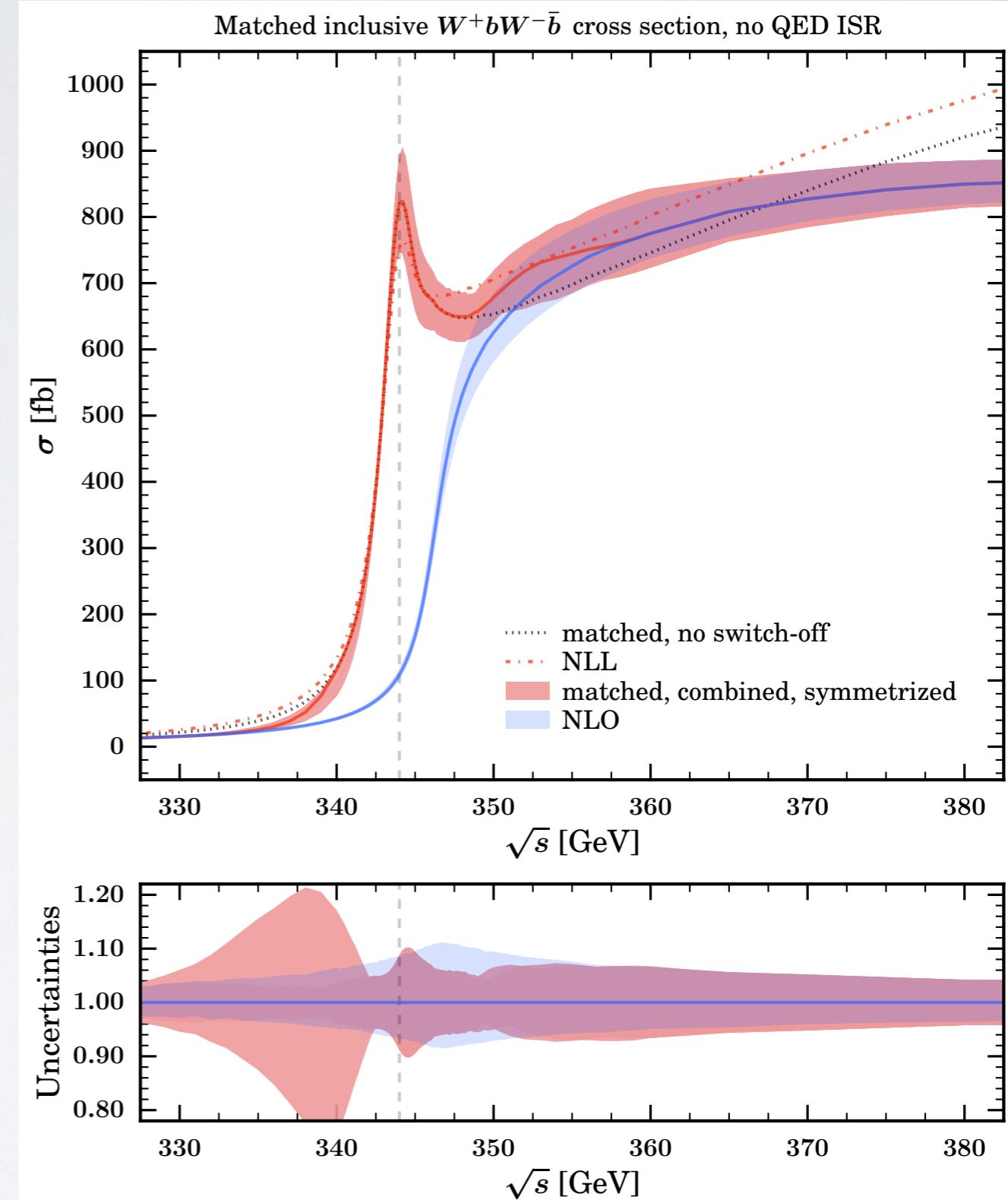
- Only factorized calculation feasible
- NRQCD result technically demanding
- Remove double-counting between NRQCD-NLL & QCD-NLO

Bach/Chokouf  /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

## Fully Exclusive Events: assess selection uncertainties

$$\sigma_{\text{NLO+NLL}} = \sigma_{\text{NLO}} + \left( (\tilde{F}_{\text{NLL}} - \tilde{F}_{\text{NLL}}^{\text{exp}}) \right) \left( \begin{array}{c} e^- \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^- \\ W^- \end{array} \right) + \left\{ \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) \tilde{F}_{\text{NLL}} \right\} \left( \begin{array}{c} b \\ b \\ W^+ \\ W^- \end{array} \right) + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ g \\ W^+ \\ W^- \end{array} \right) + \left| \tilde{F}_{\text{NLL}} \right|^2 \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \\ g \\ W^- \\ W^+ \end{array} \right),$$

- Only factorized calculation feasible
- NRQCD result technically demanding
- Remove double-counting between NRQCD-NLL & QCD-NLO



Bach/Chokouf  /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

## Fully Exclusive Events: assess selection uncertainties

$$\sigma_{\text{NLO+NLL}} = \sigma_{\text{NLO}} + \left( \tilde{F}_{\text{NLL}} - \tilde{F}_{\text{NLL}}^{\text{exp}} \right) \left[ \begin{array}{c} e^- \\ e^- \end{array} \right] \left[ \begin{array}{c} b \\ b \end{array} \right] \left[ \begin{array}{c} W^+ \\ W^- \end{array} \right] \left[ \begin{array}{c} W^+ \\ W^- \end{array} \right] \left[ \begin{array}{c} e^+ \\ e^- \end{array} \right]$$

+  $\left| \tilde{F}_{\text{NLL}} \right|^2$

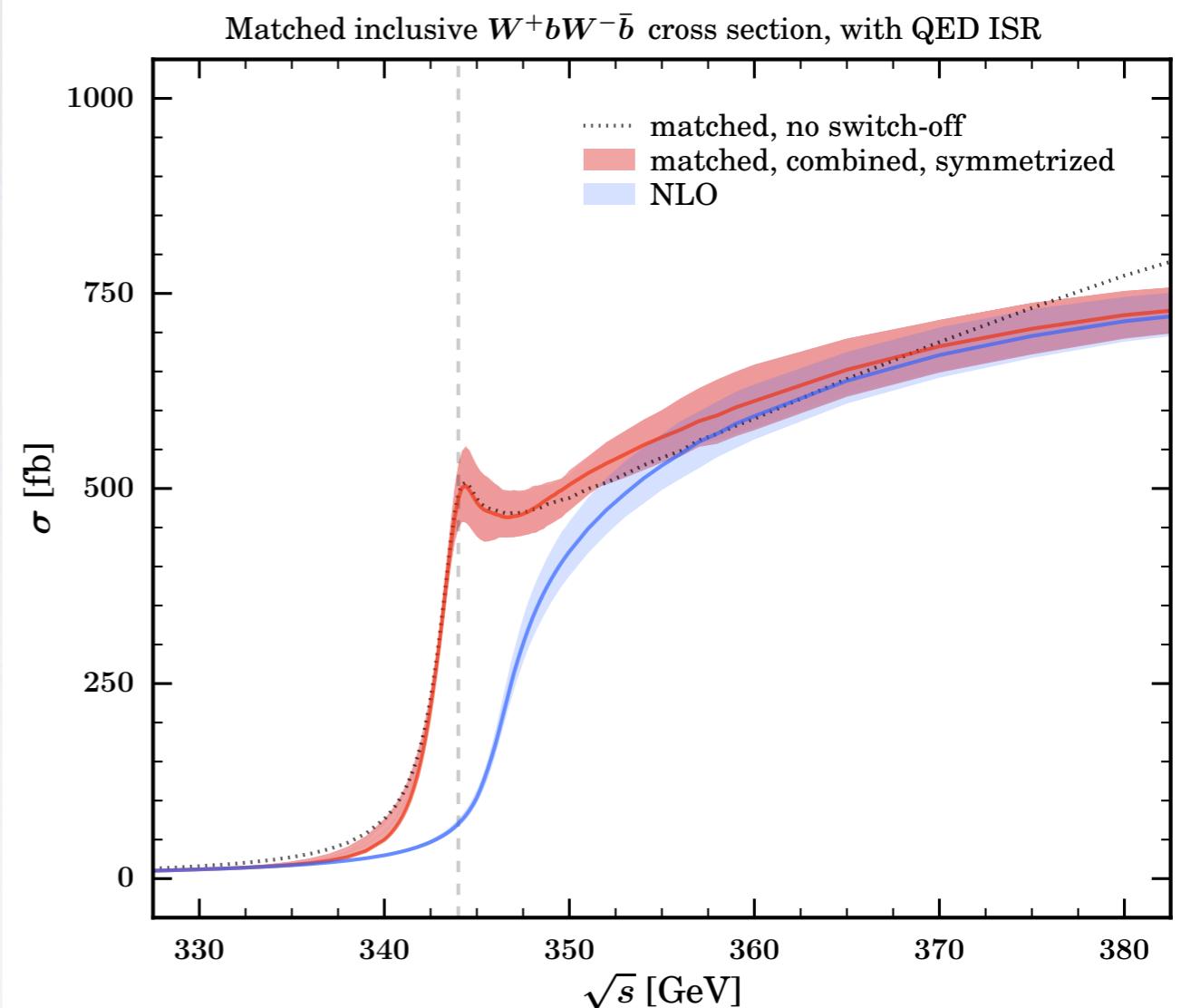
$$+ \left\{ \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} W^+ \\ W^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \right\}$$

+  $\left| \tilde{F}_{\text{NLL}} \right|^2$

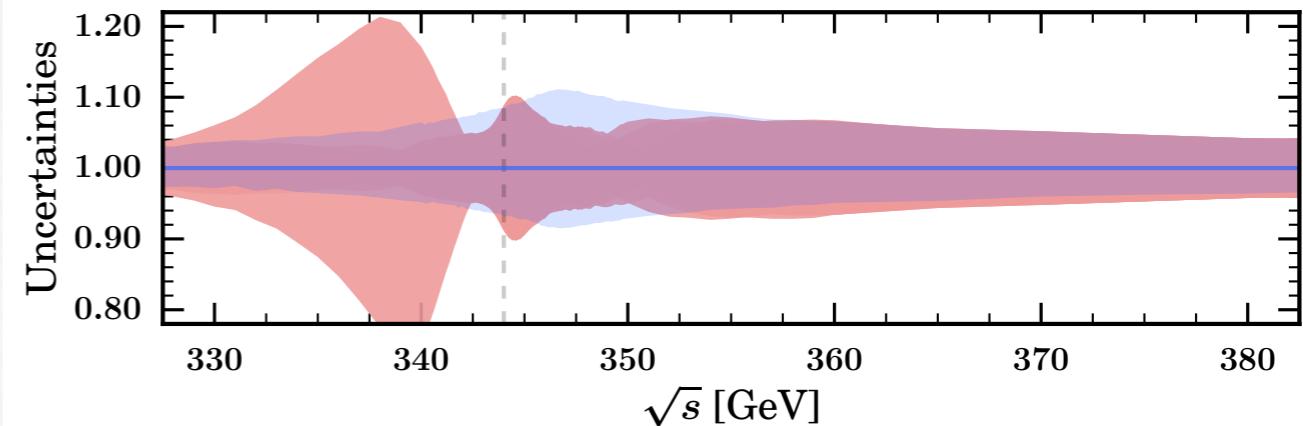
$$+ \left| \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} W^+ \\ W^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \right|^2$$

+  $\left| \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} W^+ \\ W^- \end{array} \right) \left( \begin{array}{c} g \\ g \end{array} \right) \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \right|^2$ 

$$+ \left| \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \left( \begin{array}{c} b \\ b \end{array} \right) \left( \begin{array}{c} W^+ \\ W^- \end{array} \right) \left( \begin{array}{c} g \\ g \end{array} \right) \left( \begin{array}{c} e^+ \\ e^- \end{array} \right) \right|^2,$$



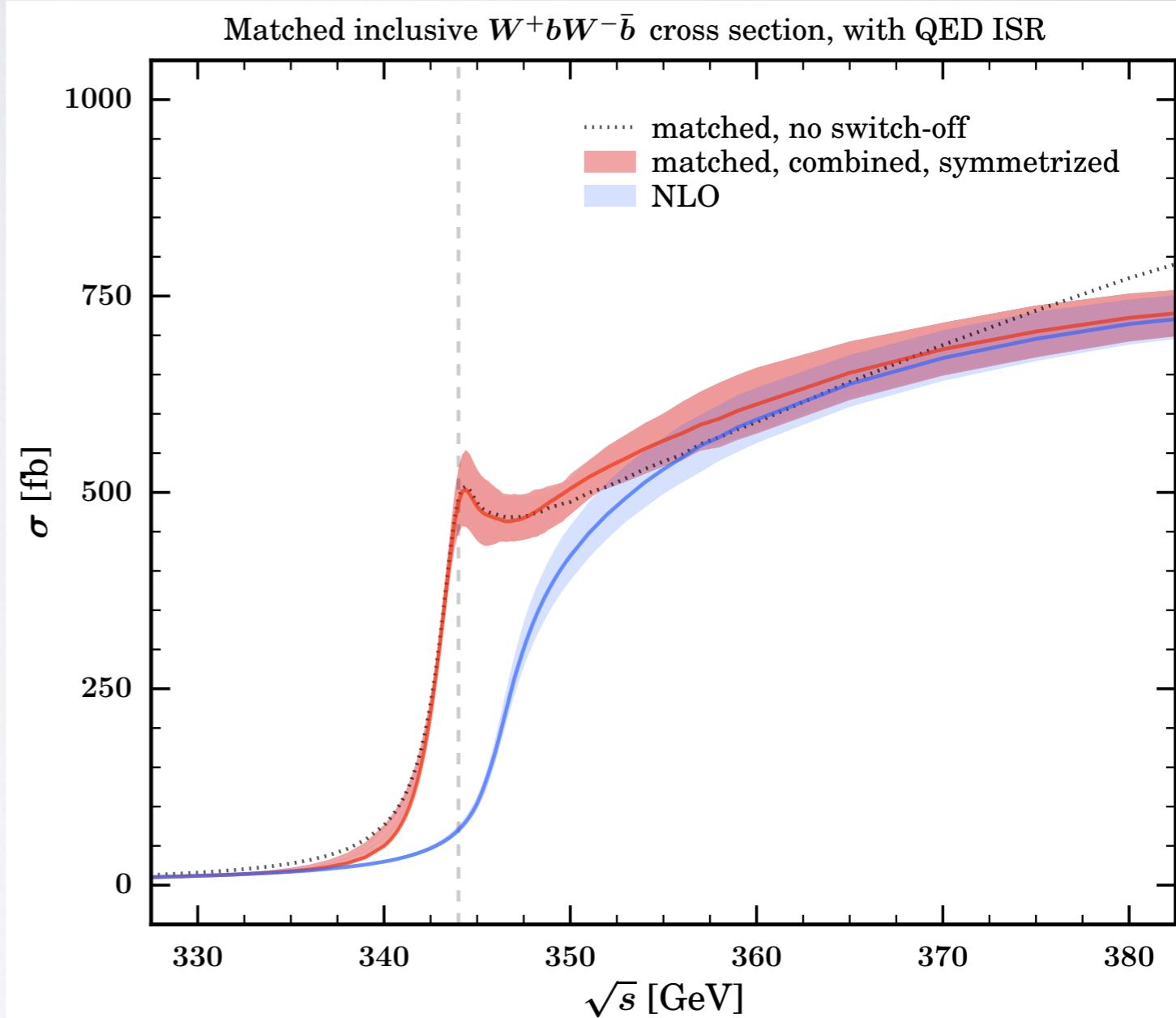
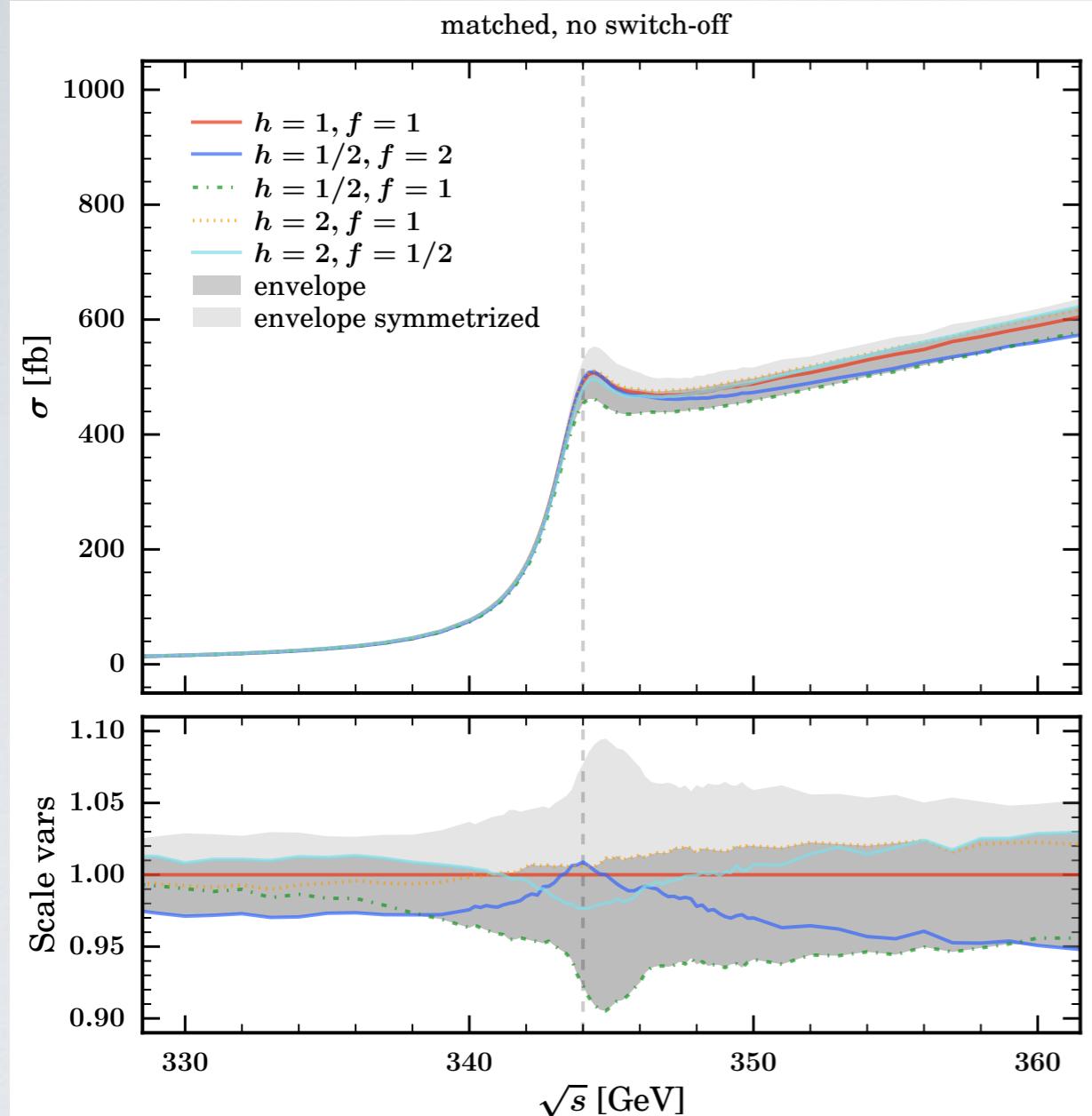
- Only factorized calculation feasible
- NRQCD result technically demanding
- Remove double-counting between NRQCD-NLL & QCD-NLO



Bach/Chokouf /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

# Threshold matching with QED ISR

25 / 29



Bach/Chokouf  /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, I7I2.02220



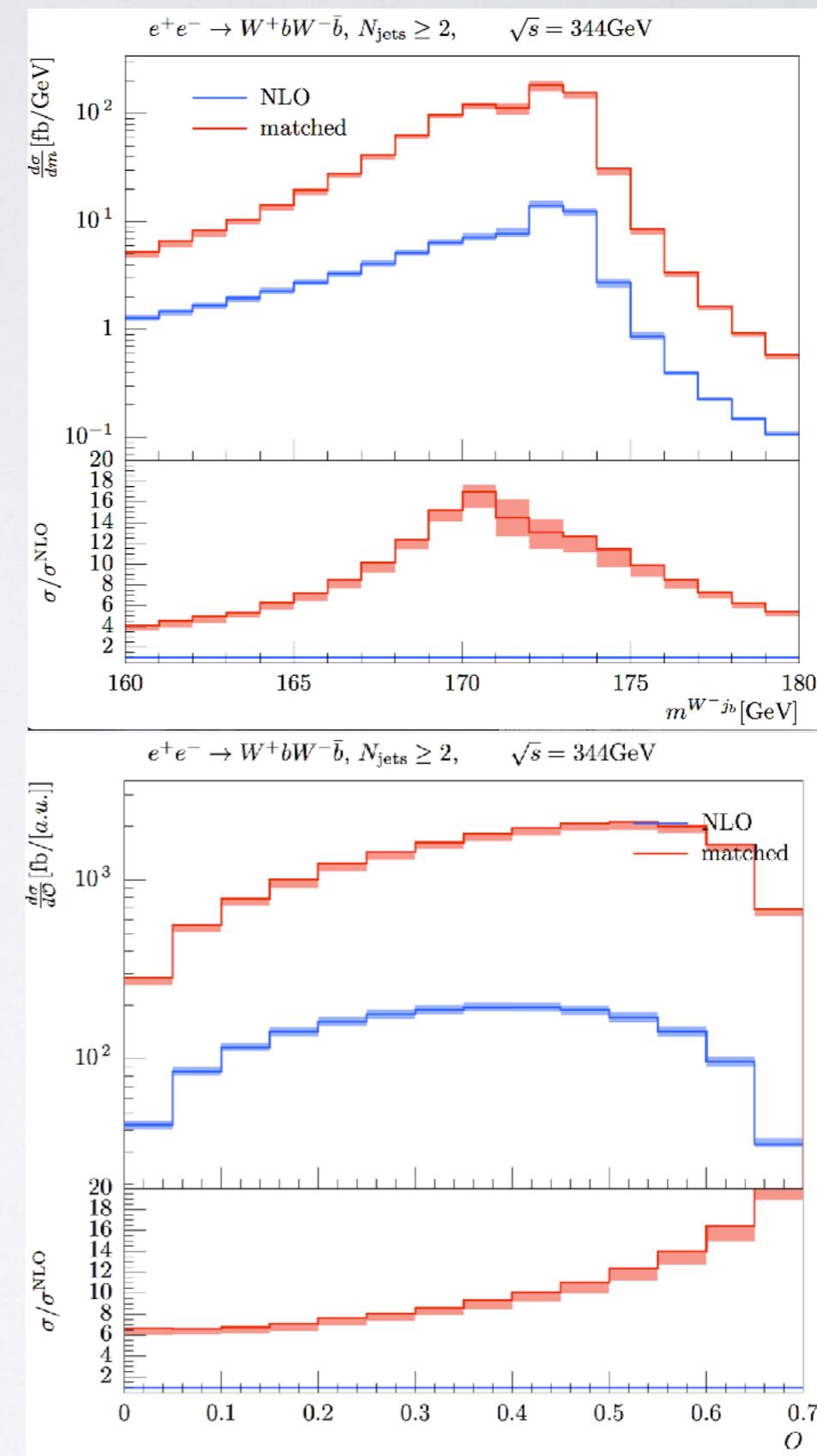
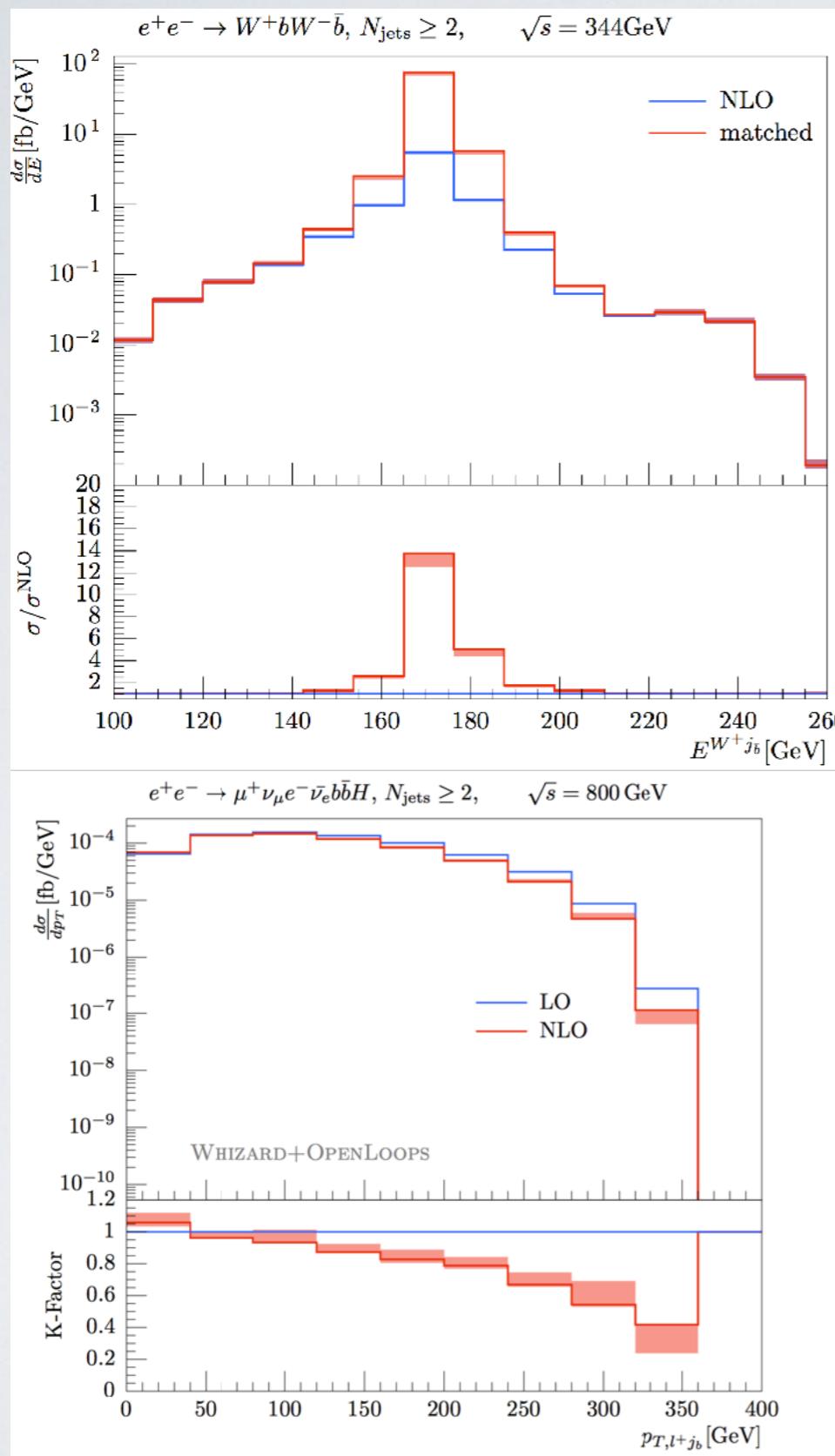
J.R.Reuter

Precision prediction for top physics

LCWS 2019, Sendai, 31.10.10.19

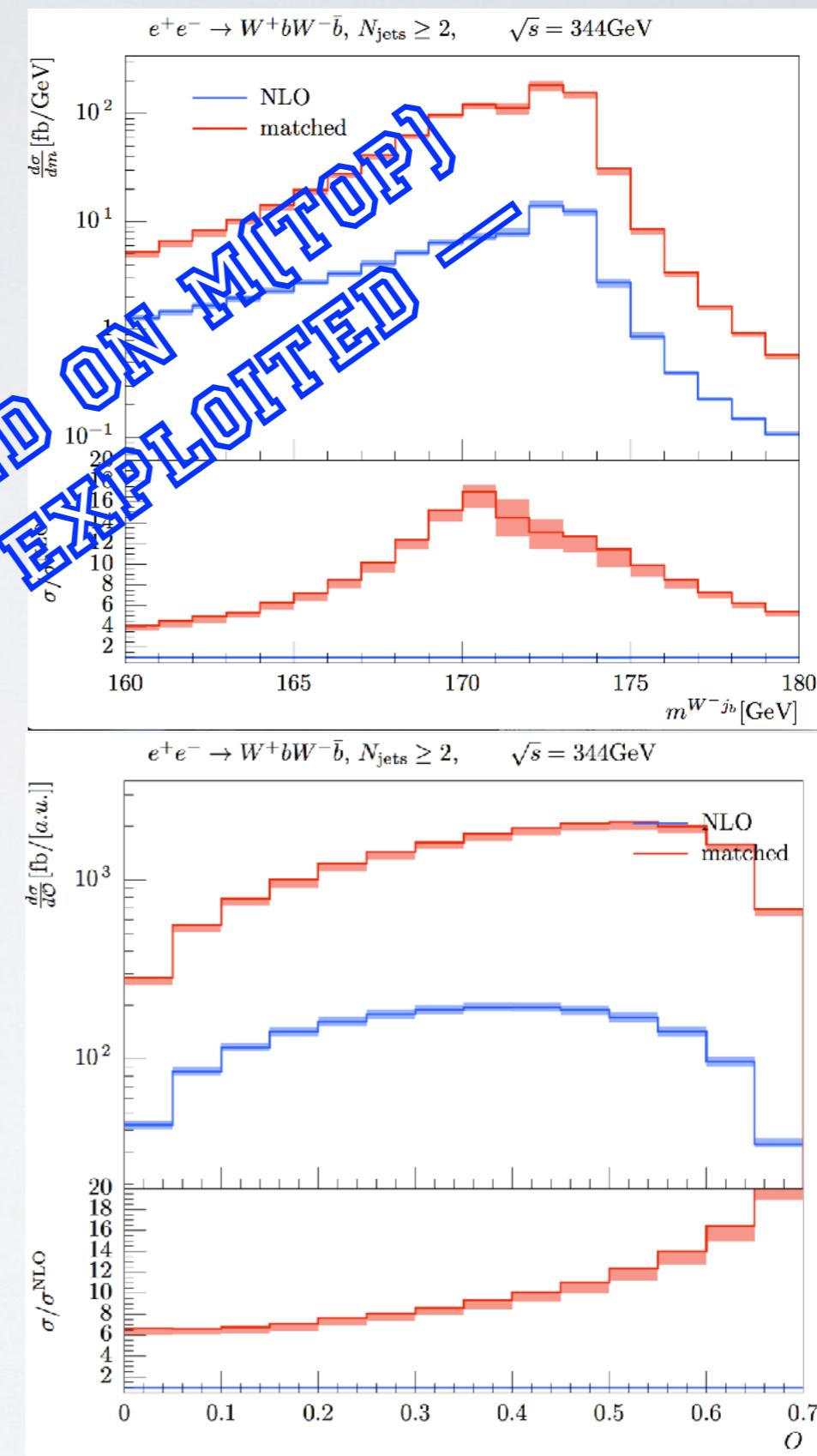
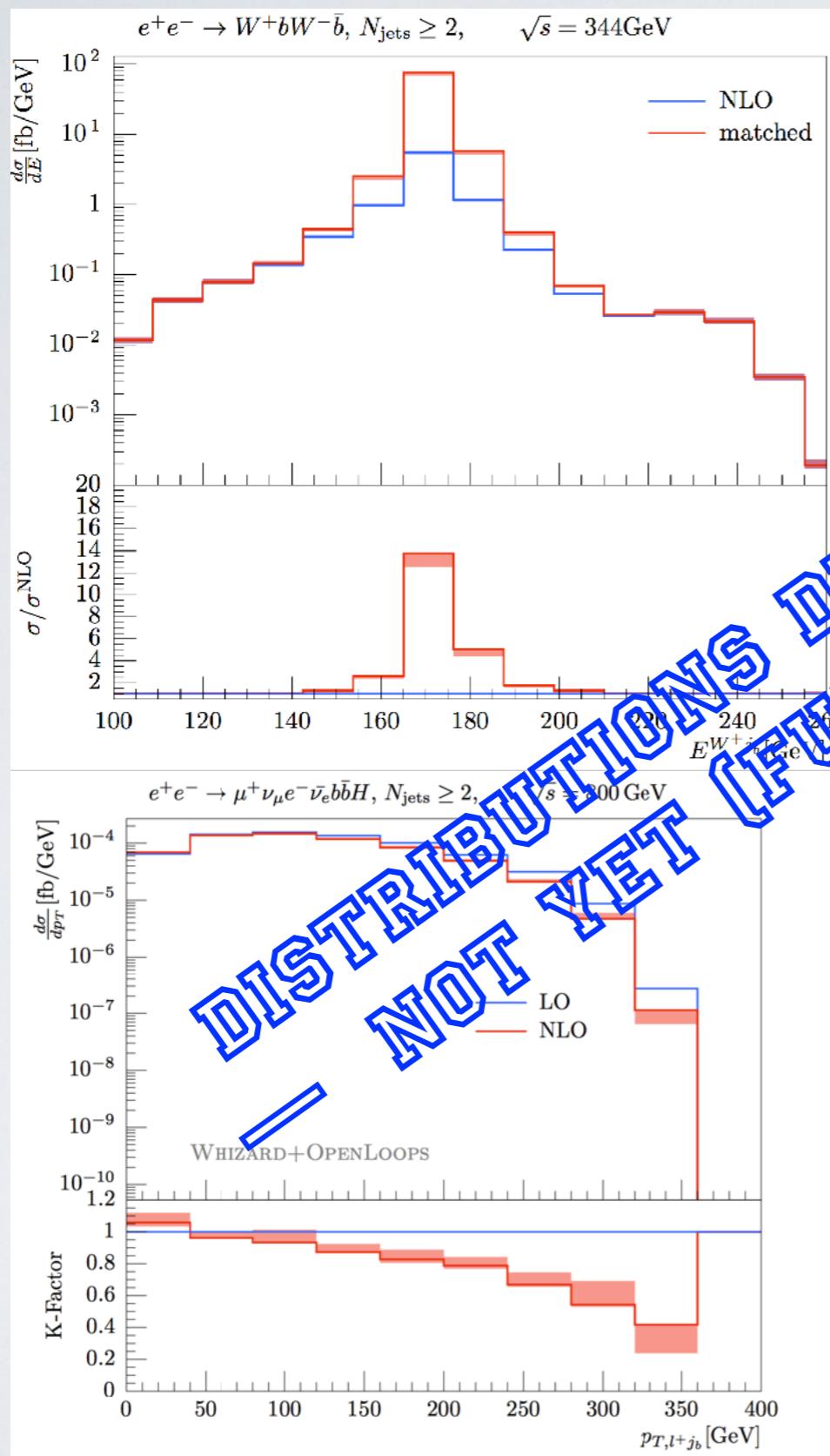
# Matched threshold differential distributions

26 / 29



# Matched threshold differential distributions

26 / 29

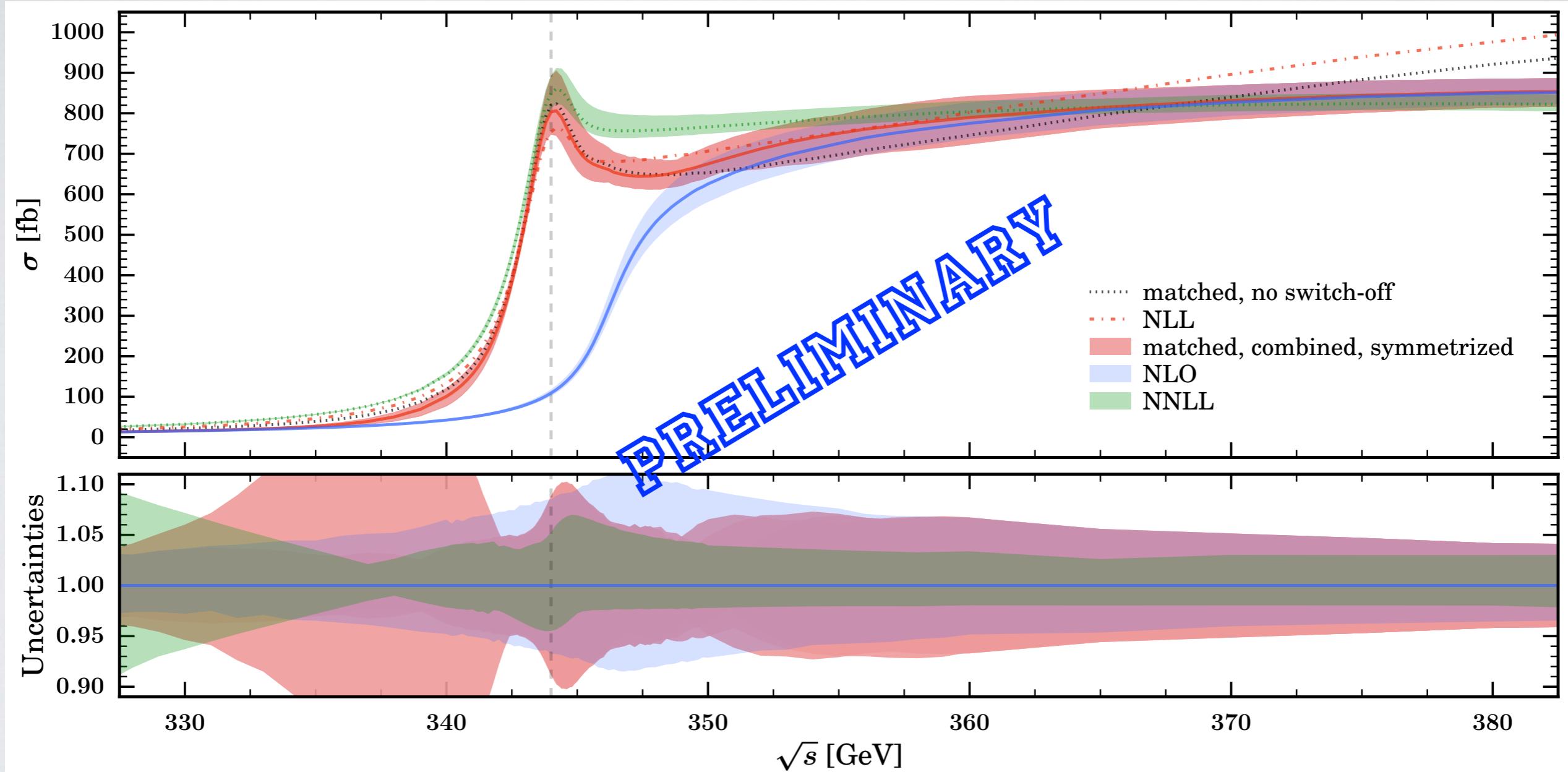


**DISTRIBUTIONS DEPEND ON MC TOP  
NOT YET (FULLY) EXPLOITED**



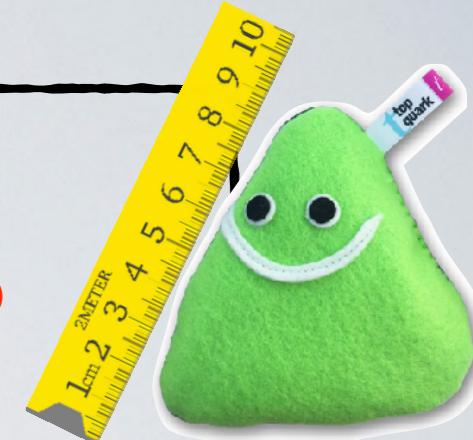
Next steps: higher QCD order, EW corrections (ISR matching!!), soft gluons . . . . .

$$e^+ e^- \rightarrow W^+ b W^- \bar{b}$$



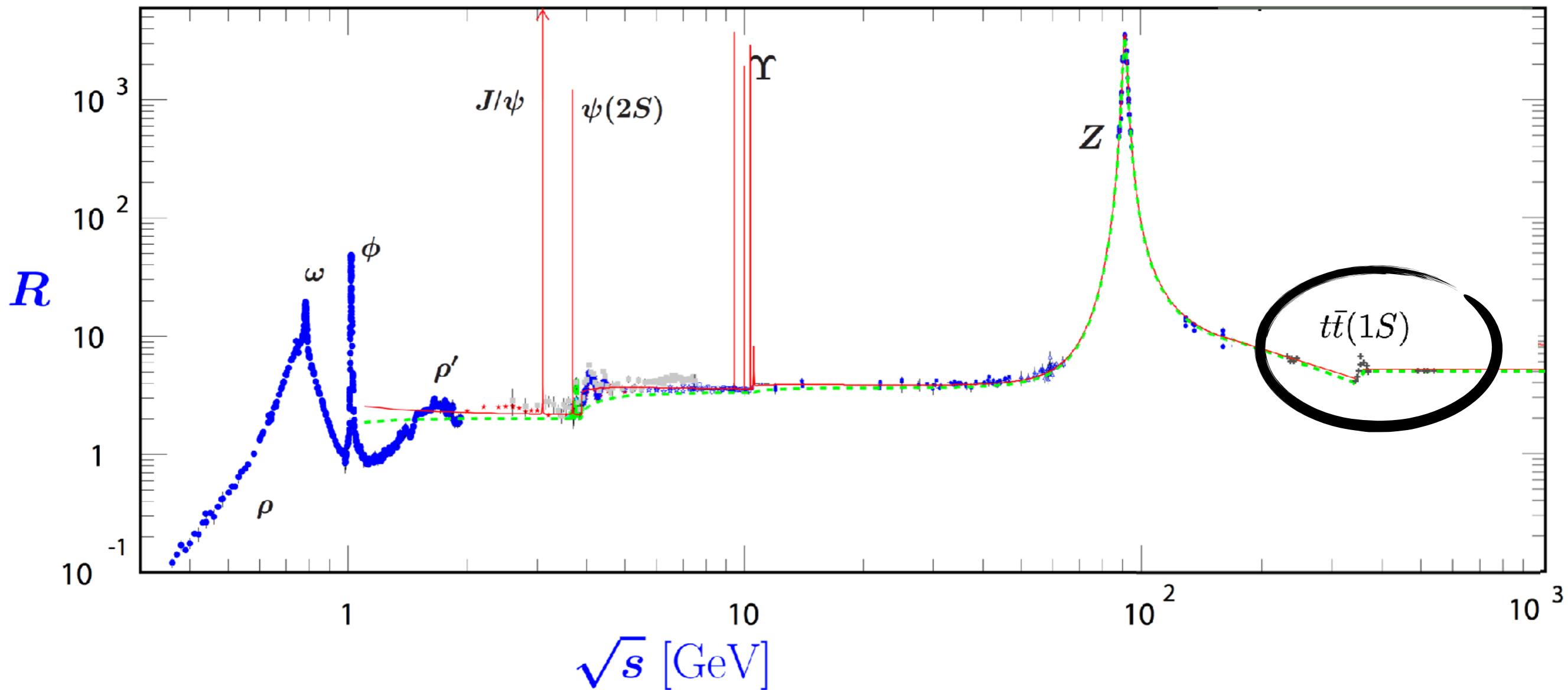
Bach/Chokouf /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

- Top physics is cornerstone of future lepton collider program
- Leptonic top and associated Higgs fully off-shell at NLO QCD
- Inclusive processes: off-shell background grows with energy
- Top Yukawa extraction @NLO QCD: stronger interference effects
- Complete NRQCD threshold / NLO continuum matching
- Can be reweighted to NNNLO QCD accuracy at threshold
- Offers framework for new differential top mass measurements



## Top Theorists To-Do List

- EW corrections (off-shell), effects of QED FSR
- Top threshold matched with EW corrections
- $t\bar{t}H$  threshold matching
- Systematic study of parton shower, logs, soft QCD effects
- ... unforeseen to-dos ...



# BACKUP

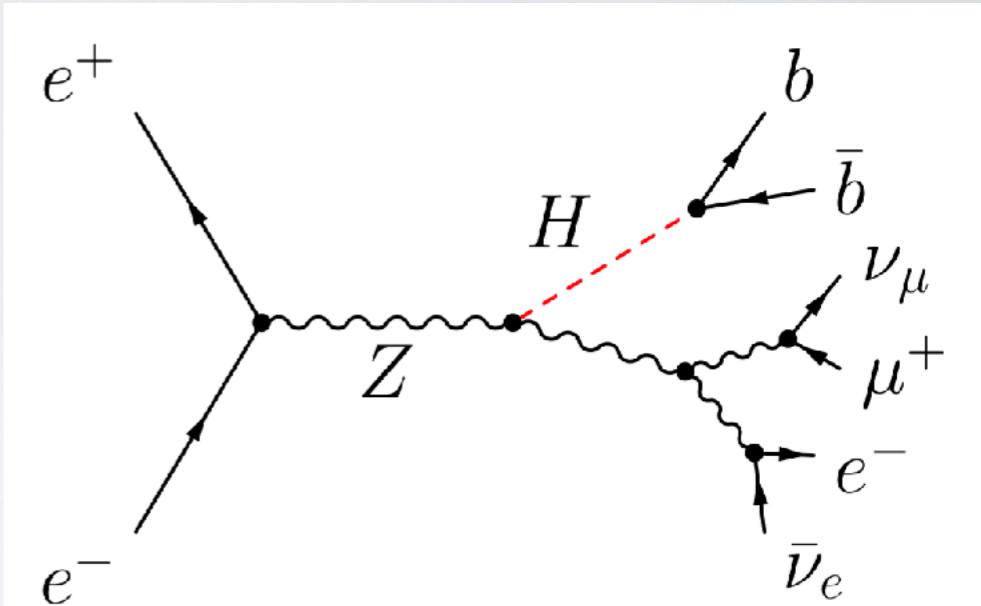
- Amplitudes (except for pure QCD/QED) contain **resonances ( $Z, W, H, t$ )**
  - In general: resonance masses *not respected by modified kinematics of subtraction terms*
  - Algorithm to include resonance histories**
- [Ježo/Nason, I509.0907I]
- Most important for narrow resonances ( $H \rightarrow b\bar{b}$ )
  - Additional soft mismatch integration component

$$\triangleright D_H^{\text{Born}} = \left[ (\bar{p}_{bb}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1},$$

$$\triangleright D_H^{\text{Real}} = \left[ \left( p_{bbg}^2 - m_H^2 \right)^2 + m_H^2 \Gamma_H^2 \right]^{-1}$$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2$$

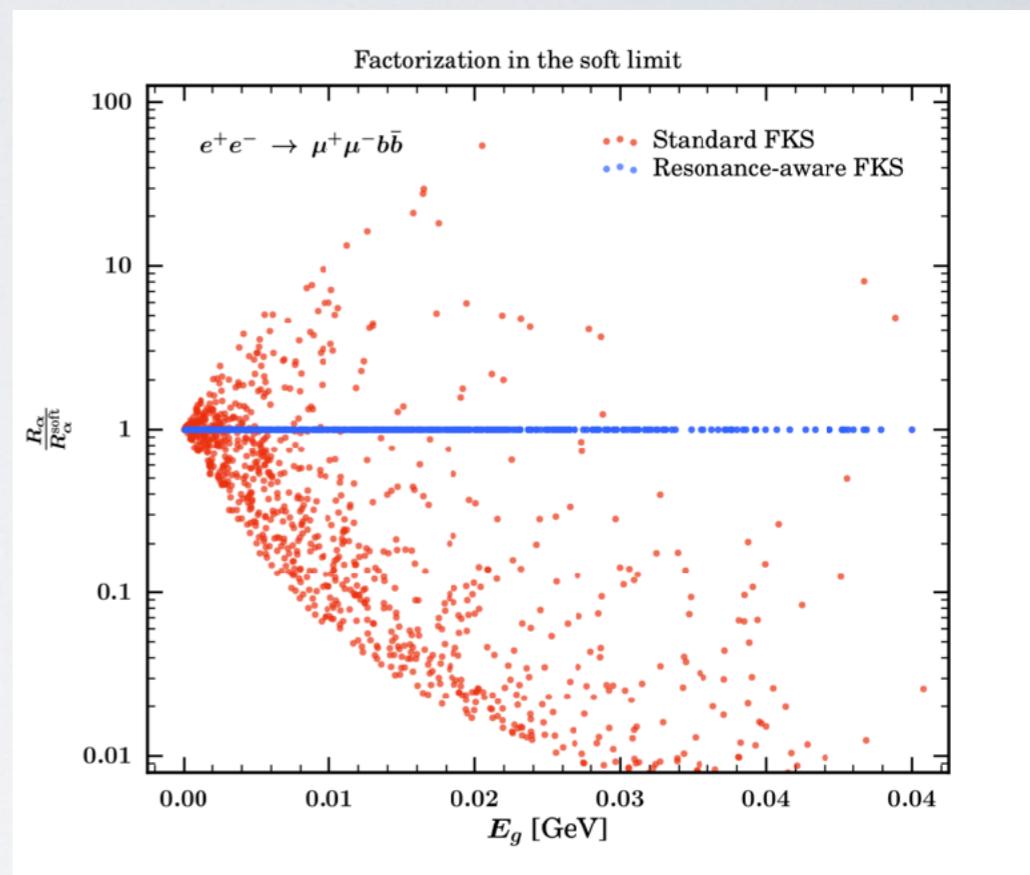
$$\frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \xrightarrow{\bar{p}_{bb}^2 \rightarrow m_H^2} 1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}$$



- Amplitudes (except for pure QCD/QED) contain **resonances ( $Z, W, H, t$ )**
- In general: resonance masses *not respected by modified kinematics of subtraction terms*
- Algorithm to include resonance histories**
- [Ježo/Nason, [I509.09071](#)]
- Most important for narrow resonances ( $H \rightarrow b\bar{b}$ )
- Additional soft mismatch integration component

$$\begin{aligned} \triangleright D_H^{\text{Born}} &= \left[ (\bar{p}_{bb}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1}, \\ \triangleright D_H^{\text{Real}} &= \left[ (p_{bbg}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1} \end{aligned}$$

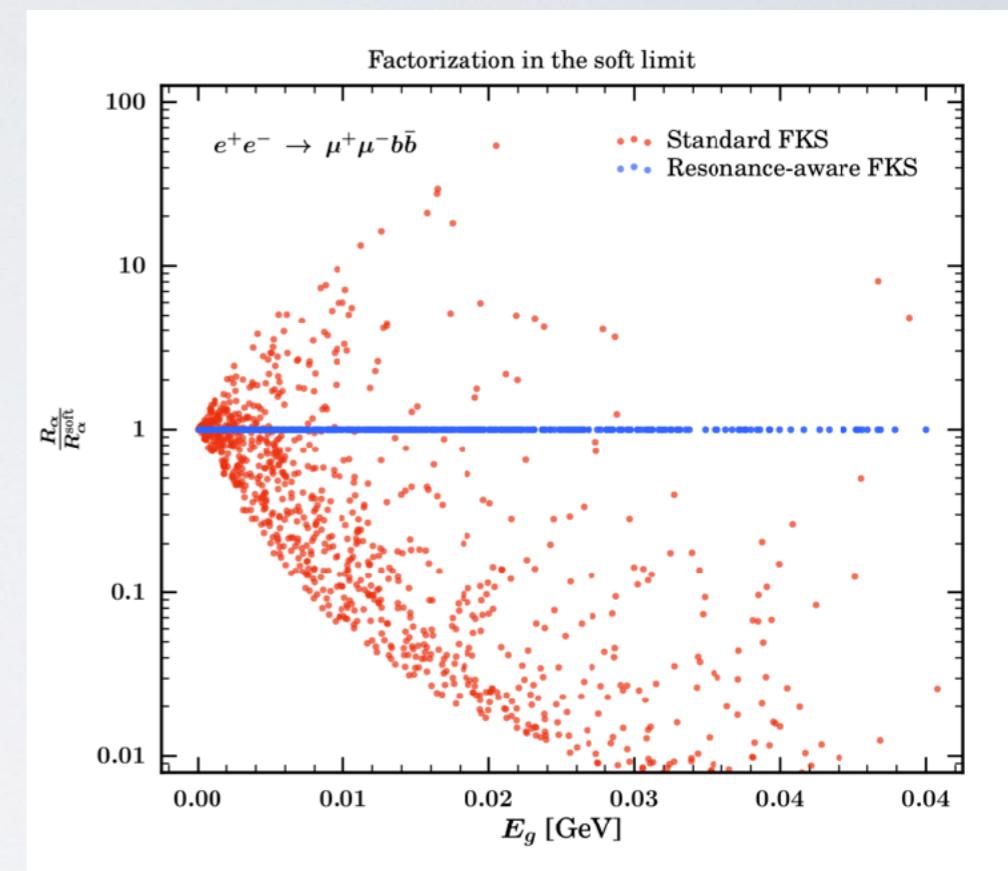
$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2 \quad \frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \frac{\bar{p}_{bb}^2 \rightarrow m_H^2}{1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}}$$



- Amplitudes (except for pure QCD/QED) contain **resonances ( $Z, W, H, t$ )**
- In general: resonance masses *not respected by modified kinematics of subtraction terms*
- Algorithm to include resonance histories**
- [Ježo/Nason, I509.09071]
- Most important for narrow resonances ( $H \rightarrow b\bar{b}$ )
- Additional soft mismatch integration component

$$\begin{aligned} \triangleright D_H^{\text{Born}} &= \left[ (\bar{p}_{bb}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1}, \\ \triangleright D_H^{\text{Real}} &= \left[ (p_{bbg}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1} \end{aligned}$$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2 \quad \frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \frac{\bar{p}_{bb}^2 \rightarrow m_H^2}{1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}}$$



- WHIZARD complete automatic implementation: example  $e^+ e^- \rightarrow \mu\mu b\bar{b}$  (ZZ, ZH histories)

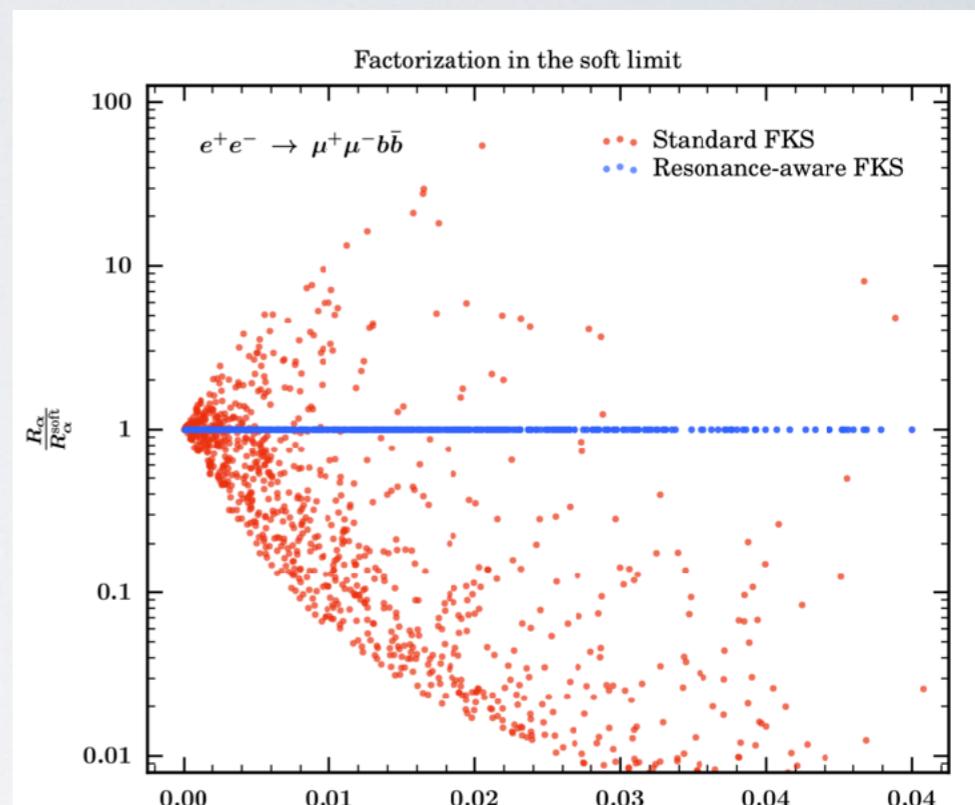
It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS

- Amplitudes (except for pure QCD/QED) contain **resonances ( $Z, W, H, t$ )**
- In general: resonance masses *not respected by modified kinematics of subtraction terms*
- Algorithm to include resonance histories**
- [Ježo/Nason, I509.09071]
- Most important for narrow resonances ( $H \rightarrow bb$ )
- Additional soft mismatch integration component

$$\begin{aligned} \triangleright D_H^{\text{Born}} &= \left[ (\bar{p}_{bb}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1}, \\ \triangleright D_H^{\text{Real}} &= \left[ (p_{bbg}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1} \end{aligned}$$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2 \quad \frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \frac{\bar{p}_{bb}^2 \rightarrow m_H^2}{1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}}$$



- WHIZARD complete automatic implementation: example  $e^+ e^- \rightarrow \mu\mu bb$  (ZZ, ZH histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	2.9057032E+00	8.35E-02	2.87	3.15*	7.90		
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

FKS with resonance mappings