Precise calculations of Higgs decays in various extended Higgs models

[Nucl. Phys B949 (2019) 114791]
[1910.12769]
[Phys.Lett. B783 (2018) 140-149]

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LCWS2019, Sendai, 30.10.19

Motivation

- The structure of Higgs sector remains unknown, whereas the Higgs boson was discovered.
  - Negative mass term in the SM $V_{SM} = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$
  - Number of Higgs fields, its representation $\Phi + X + \ldots$
  - Symmetries of the Higgs potential

- There are many possibility of extended Higgs sectors.
  - Singlet Extension of the SM (HSM) $\Phi + S$
  - Two Higgs doublet models (THDMs) $\Phi_1 + \Phi_2$
  - …

- Relation between BSM phenomena and extended Higgs sectors

→ **Testing extended Higgs sectors is essential for exploring NP.**

Our approach

Our approaches to the determination of the shape of the Higgs sector is following:

Precise calculations

$$\Phi + X + \dots$$

Precise measurements

HL-LHC, ILC, CLIC

FCC-ee, CEPC, etc.

Observables for the Higgs boson :

cross section, branching ratio, coupling, etc

Determination of the shape of the Higgs sector

→ We studied on radiative corrections to branching ratios in 6 different extended Higgs models.

Setup

We treat 6 simple extended Higgs models.

- **Higgs singlet models (HSM)**

The potential has shift invariance → $<S> = 0$
[$S \to S + v_s'$]

$$V(\Phi, S) = m_\Phi^2|\Phi|^2 + \lambda|\Phi|^4 + \mu_{\Phi S}|\Phi|^2 S + \lambda_{\Phi S}|\Phi|^2 S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4$$

Physical state : *h, H*   Free parameter : $m_H,\ \cos\alpha,\ m_s^2,\ \mu_S,\ \lambda_S$

- **Two Higgs doublet models (THDMs)**

[softly broken Z_2 symmetry]
→ 4types of Yukawa int.

$$V = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_1^\dagger\Phi_2 + \text{h.c.}) + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}]$$

Physical state : $h,H,A,H^\pm$ Free parameter : $m_H, m_A, m_{H^\pm},\ s_{\beta-\alpha},\ t_\beta,\ M^2(=m_3^2/(s_\beta c_\beta))$

- **Inert doublet model (IDM)**

[exact Z_2 symmetry]
→ Dark matte candidate

$$V_{IDM} = V_{THDM}\ \ (\ m_3^2 \to 0,\ \ <\Phi_2> \to 0\ )$$

Physical state : $h,H,A,H^\pm$ Free parameter : $m_H, m_A, m_{H^\pm},\ \lambda_2,\ m_2$

Higgs couplings

$$\kappa_X = \frac{g(hXX)^{EX.}}{g(hXX)^{SM}}$$

HSM : $\kappa_V = \kappa_f = \cos\alpha$

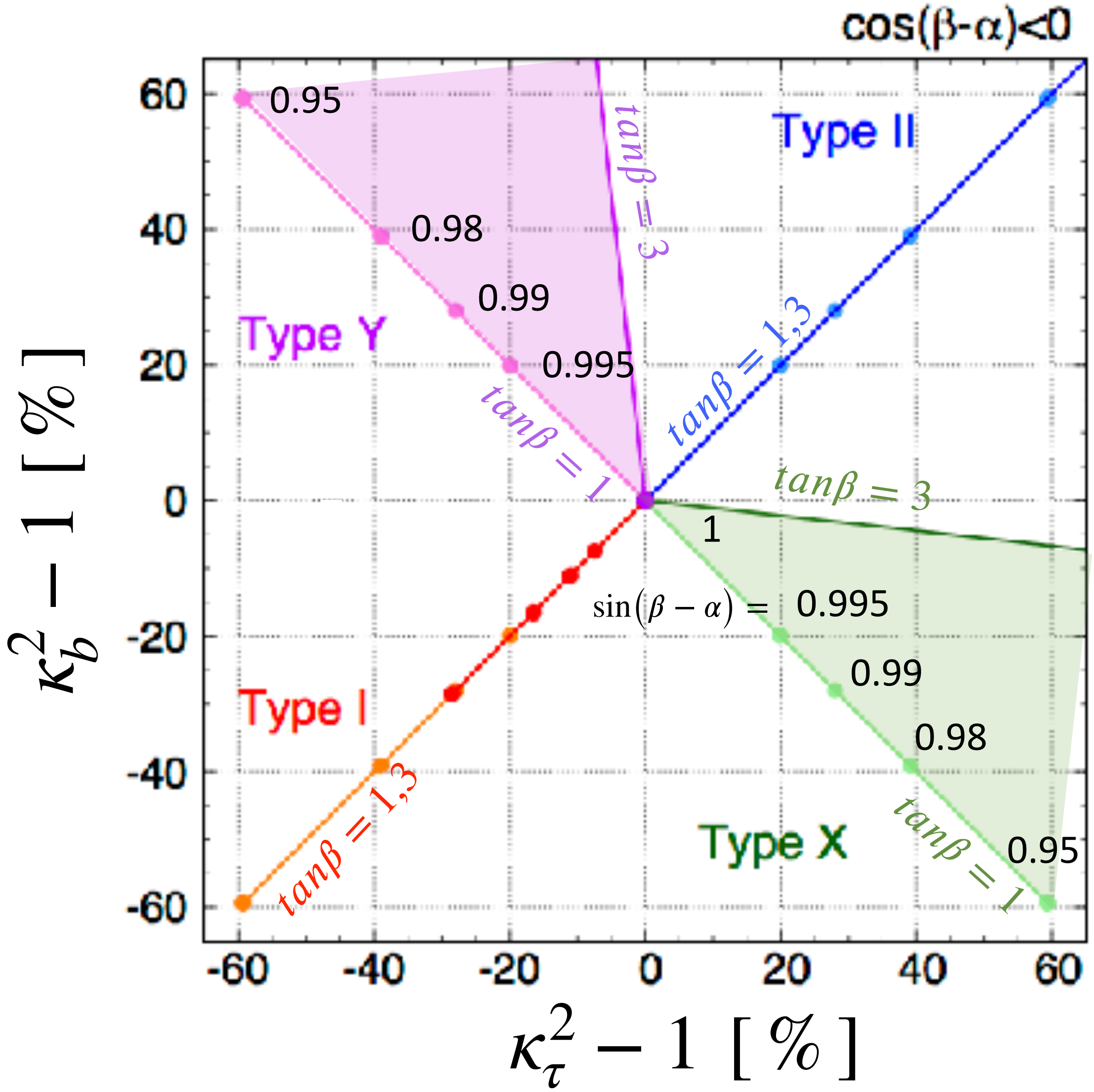
THDMs : $\kappa_V = \sin(\beta - \alpha)$, $\kappa_f = \sin(\beta - \alpha) + \xi_f \cos(\beta - \alpha)$

	ξ_u	ξ_d	ξ_e
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$

IDM : $\kappa_V = \kappa_f = 1$

Model discrimination by couplings

Ex.) THDM Type I, II, X, and Y



[Kanemura, Tsumura, Yagyu, Yokoya, PRD90 (2014) 075001]

$$\kappa_f = \sin(\beta-\alpha) - \xi_f \cos(\beta-\alpha)$$

	ξ_u	ξ_d	ξ_e
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$

Each extended Higgs model can give different pattern of deviations

Nondecoupling effects

In the alignment limit, the heavy Higgs masses can be commonly expressed as:

$$m_\Phi^2 \simeq M^2 + \lambda_i v^2 \qquad (\Phi = H, A, H^\pm)$$

$$M^2 = \begin{cases} 2m_s & \text{(HSM)} \\ m_3^2/s_\beta c_\beta & \text{(THDMs)} \\ \mu_2^2 & \text{(IDM)} \end{cases}$$

There are two cases for large m_Φ^2 :

(1) $M^2 \gg \lambda_i v^2 \longrightarrow m_\Phi^2 \simeq M^2$

Loop contributions of Φ decouple, obeying decoupling theorem.

[T. Appelquist, J. Carazzone, PRD 11 (1975) 2856]

$$\kappa_X - 1 \simeq -\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \frac{1}{m_\Phi^2} + \ldots$$

(2) $M^2 \ll \lambda_i v^2 \longrightarrow m_\Phi^2 \simeq \lambda_i v^2$

Nondecoupling effects can be obtained.

$$\kappa_X - 1 \simeq -\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \frac{m_\Phi^2}{v^2} + \ldots$$

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Nondecoupling effects can be obtained.

$$\kappa_X - 1 \simeq -\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \frac{m_\Phi^2}{v^2}$$

In the case of THDMs, maximally

$\sim$ 2.5% for *hVV* [Kanemura, Okada, Senaha, Yuan, PRD70,115002]

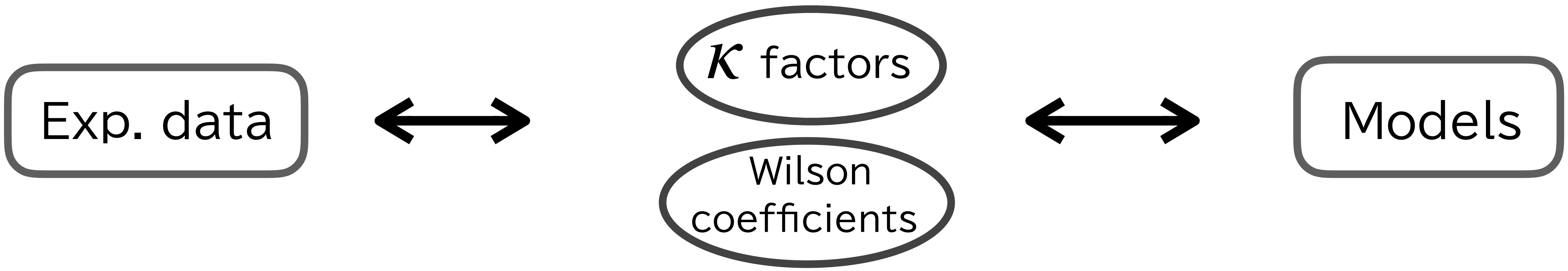
$\sim$ 5% for *hff* [Kanemura, Kikuchi, Yagyu, PLB731, 27]

→ comparable with sensitivity of future experiments

Couplings → branching ratios

While the experimental data of Higgs couplings are not directly compared with predictions of models, the branching ratios can be done :

- Higgs couplings



- Branching ratios

Open questions:

- how is decoupling property of additional Higgs bosons for BRs?
- What is pattern of deviations from the SM for BRs for each model?

▶ We show size of additional Higgs boson loop cont. for BRs.

▶ We discuss if 6 extended Higgs models are discriminated with precise measurements of Higgs BRs.

H-COUP

→ Kentarou Mawatari's talk
[Thu. 16:40 in Track1/Track2]

In order to calculate Higgs branching rations including higher order corrections, we have developed H-COUP program.

H-COUP

[Kanemura, Kikuchi, KS, Yagyu, CPC 233 (2018) 134]
[Kanemura, Kikuchi, KS, Mawatari, Yagyu, 1910.12769]

Fortran program to evaluate loop-corrected Higgs observables in the improved on-shell scheme.

Model

- Higgs Singlet model
- Two Higgs doublet models
  Type I, II, X, Y
- Inert doublet model

Observables

✓ *hff, hVV, hhh* vertex functions (v1.0)

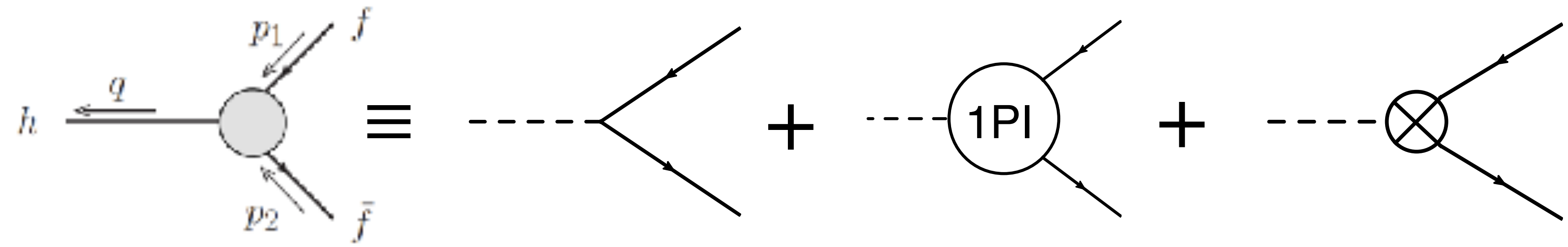
✓ $\mathrm{BR}(h \to ff), \mathrm{BR}(h \to VV^*),$ (v2.0)
$\mathrm{BR}(h \to \gamma\gamma), \mathrm{BR}(h \to Z\gamma), \mathrm{BR}(h \to gg)$

✓ total width of *h*

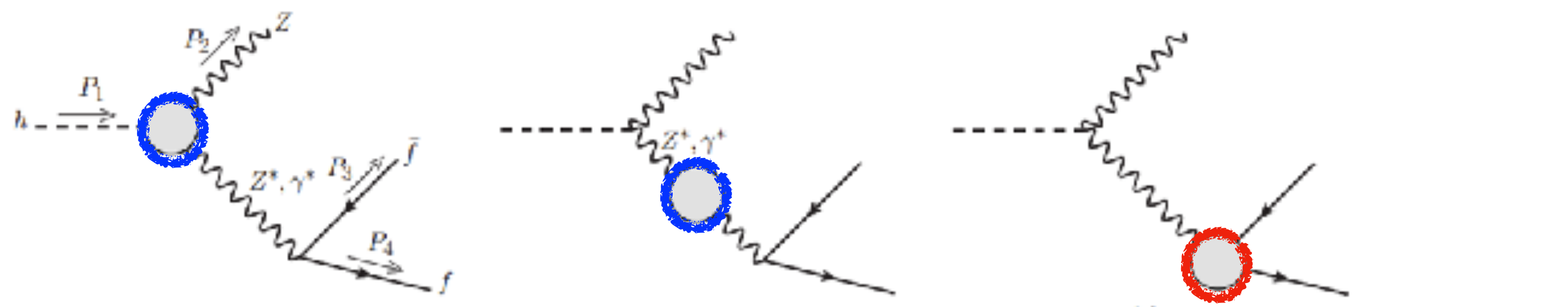
Predictions for each model are evaluated in the same scheme

One-loop calculation of Higgs decay rates

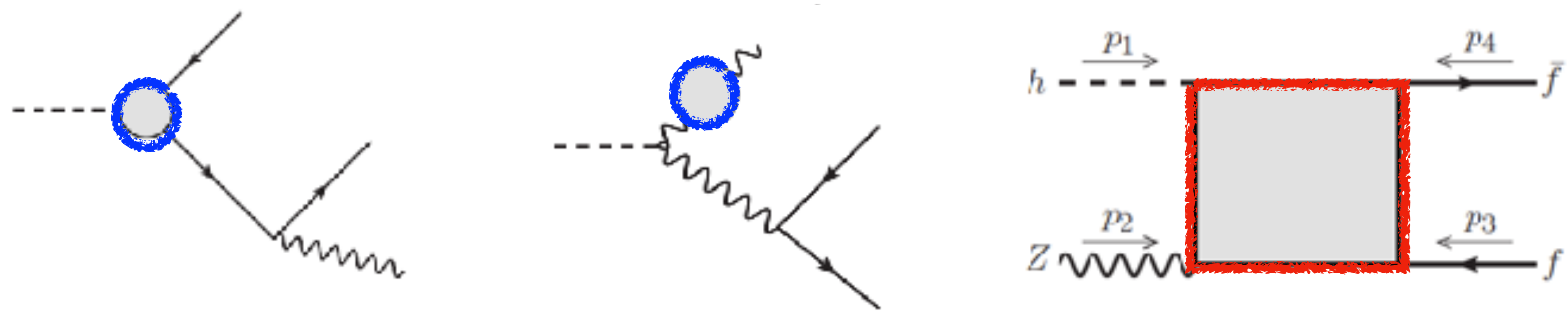
$h \to ff$:



$h \to VV^*$:



hVV vertex correction Self-energy correction *Vff* vertex correction



hff vertex correction Wave function Box diagram correction

Higgs vertices and self -energy are computed by H-COUP ver.1.0.

New ingredients of ver. 2.0

Decay rates for $h\to ff$, $h\to VV^*$, $h\to gg$, $h\to\gamma Z$, $h\to\gamma\gamma$ are calculated at NLO EW and NNLO QCD in H-COUP.

Higgs branching ratios at the 1-loop

HSM, IDM

$$\mathrm{BR}(h \to XX) \simeq \frac{\kappa_X^2 \Gamma(h \to XX)^{\mathrm{LO}}(1 + \Delta_{SM}^{\mathrm{EW,QCD}} + \Delta_{NP}^{\mathrm{EW}})}{\kappa_X^2 \Gamma_h (1 + \Delta_{SM}^{\mathrm{EW,QCD}} + \Delta_{NP}^{\mathrm{EW}})} \simeq \mathrm{BR}(h \to XX)^{\mathrm{SM}}$$

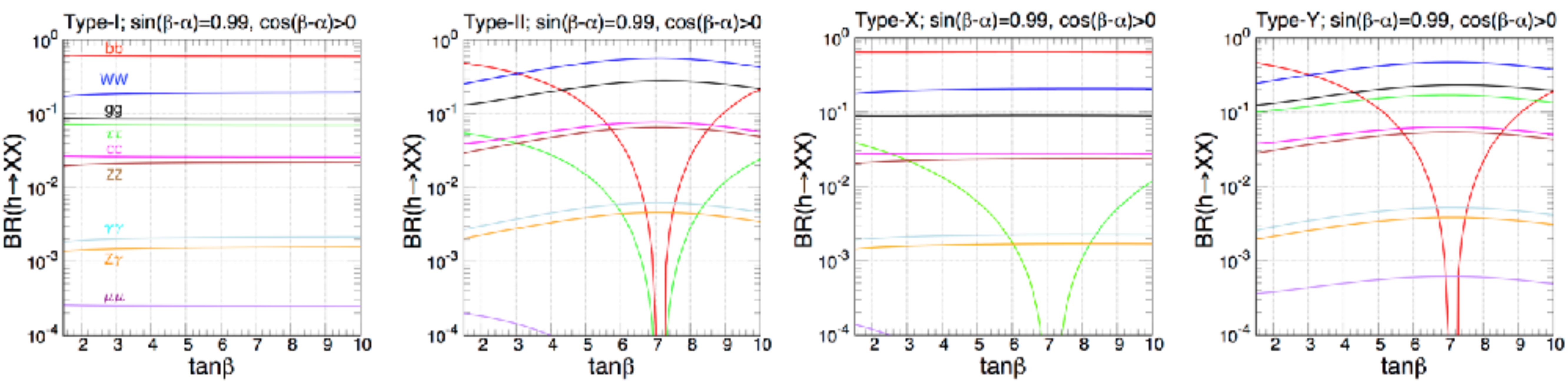
κ_X : Scaling factor

Δ_{EW}^{NP} : Loop contributions of additional Higgs bosons

Typical values of deviation from the SM is about 0.5%.

But, total decay rates deviate with few %.

THDMs [Kanemura, Kikuchi, Mawatari ,KS, Yagyu]



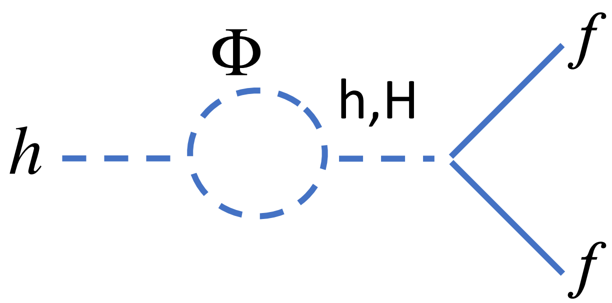
Cause of deviations from SM : ① Mixing, loop effect of additional Higgs

② Correlation of each mode

Higgs branching ratios at the 1-loop

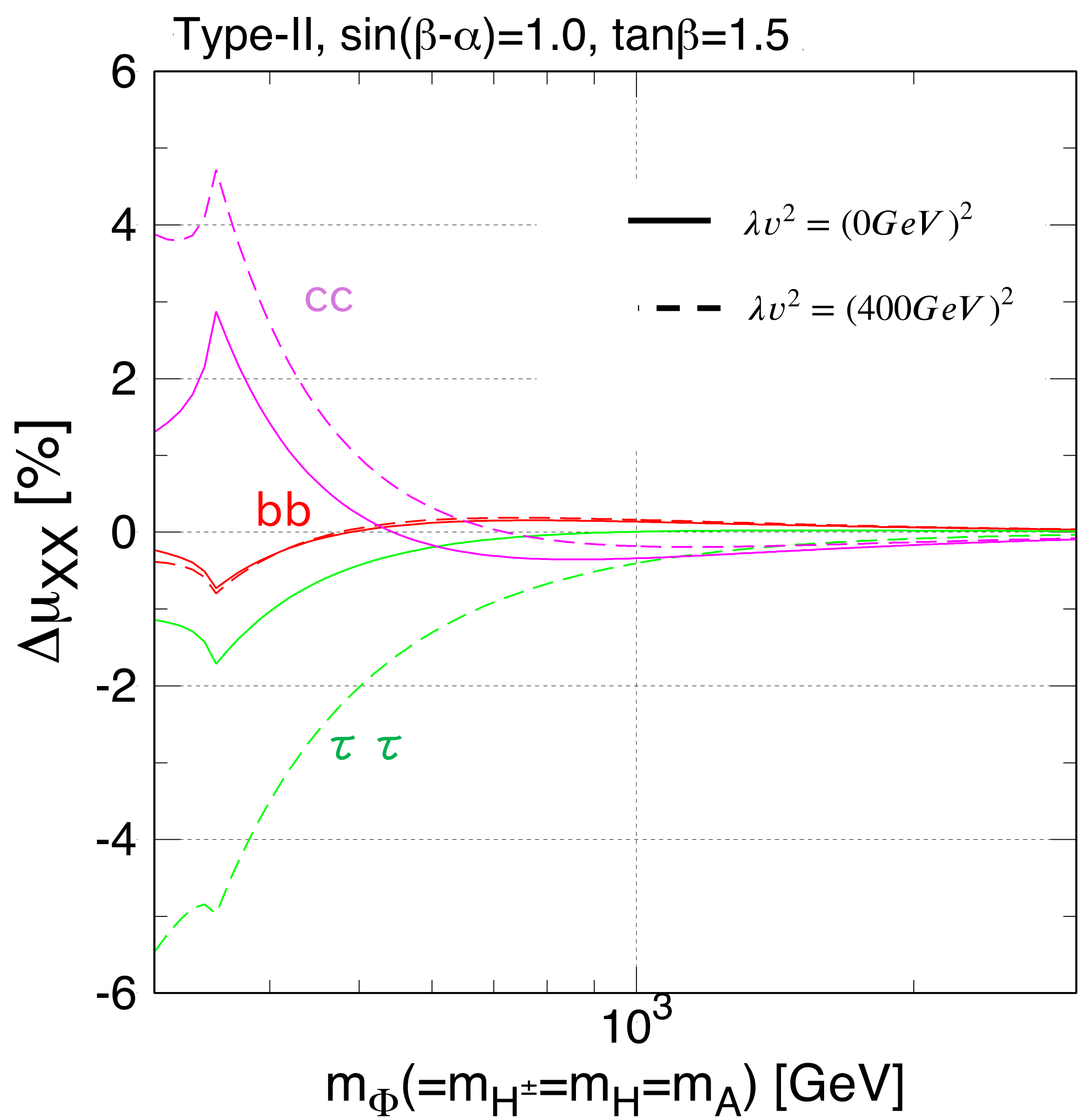
In order to look size of additional Higgs loop contributions, we evaluated deviations from the SM for the branching ratios.

Typical graph :



$$\sim -\frac{1}{16\pi^2}\frac{1}{6}\sum_{\Phi}\frac{m_{\Phi^2}}{v^2}\left(1-\frac{M^2}{m_\Phi^2}\right)^2$$

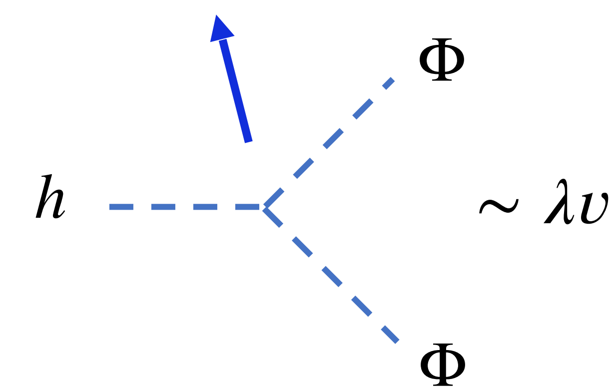
[Kanemura, Kikuchi, Mawatari ,KS, Yagyu]



Deviations from SM :

$$\Delta\mu_{XX} = \frac{\mathrm{BR}(h\to XX)_{NP}}{\mathrm{BR}(h\to XX)_{SM}} - 1$$

$$\lambda v^2 \equiv m_\Phi^2 - M^2$$



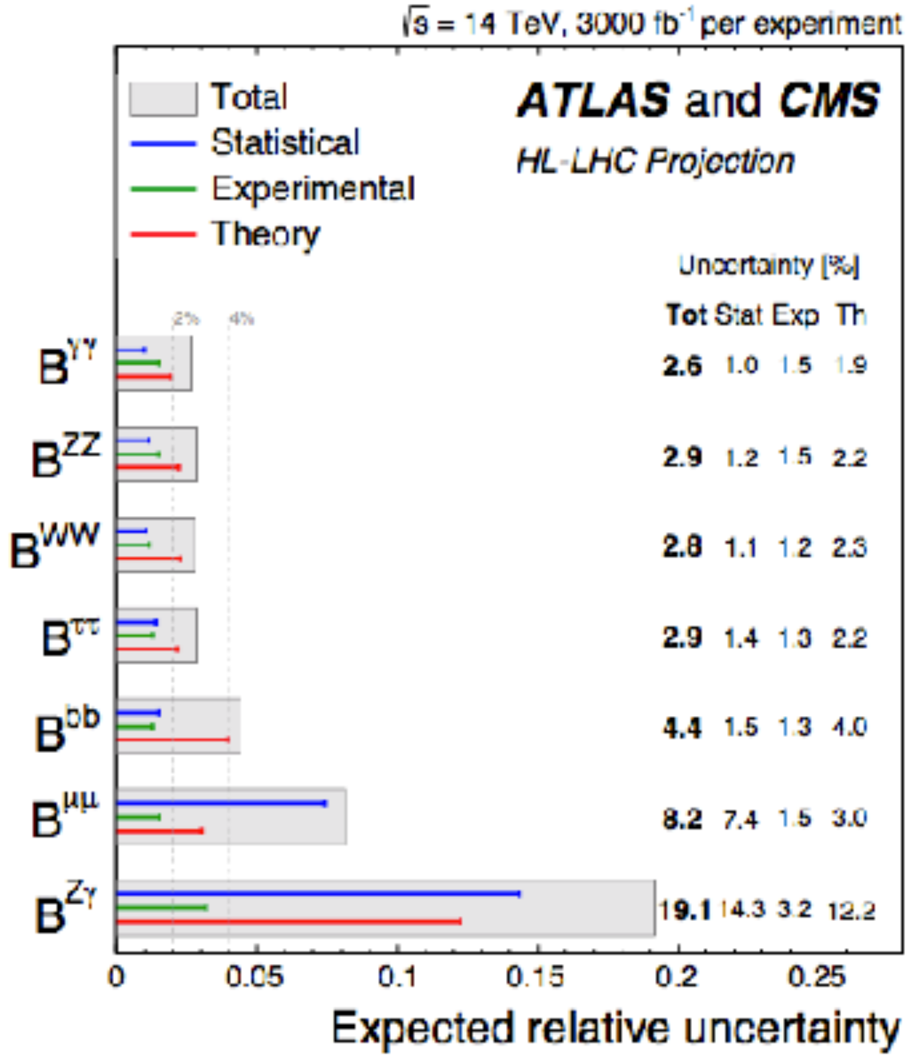
$m_\Phi \gg v$: Additional Higgs loop contributions decouple.

$m_\Phi \sim v$: Non-decoupling effect can be appeared at few %.

Discrimination of the models

We discuss whether 6 different models are discriminated by precise measurements of Higgs branching ratios.

HL-LHC [1902.00134]



ILC [1710.07621]

	1σ	2σ
$B^{\gamma\gamma}$	13%	26%
B^{ZZ}	6.7%	13.4%
B^{WW}	1.9%	3.8%
$B^{\tau\tau}$	1.4%	2.8%
B^{bb}	0.89%	1.78%
$B^{\mu\mu}$	27%	54%

We consider situations that B^{WW} are measured with few % accuracy at the ILC.

→We studied three cases:

①: $\Delta\mu_{WW} = 0 \pm 4\,\%$ ②: $\Delta\mu_{WW} = 5 \pm 4\,\%$ ③: $\Delta\mu_{WW} = -5 \pm 4\,\%$

Case ① : $\Delta\mu_{WW} = 0 \pm 4\,\%$

$$\Delta\mu_{XX} = \frac{\mathrm{BR}(h \to XX)_{NP}}{\mathrm{BR}(h \to XX)_{SM}} - 1$$

- Plot of color :
  Predictions of each model

- Brightness of color :
  Value of m_Φ

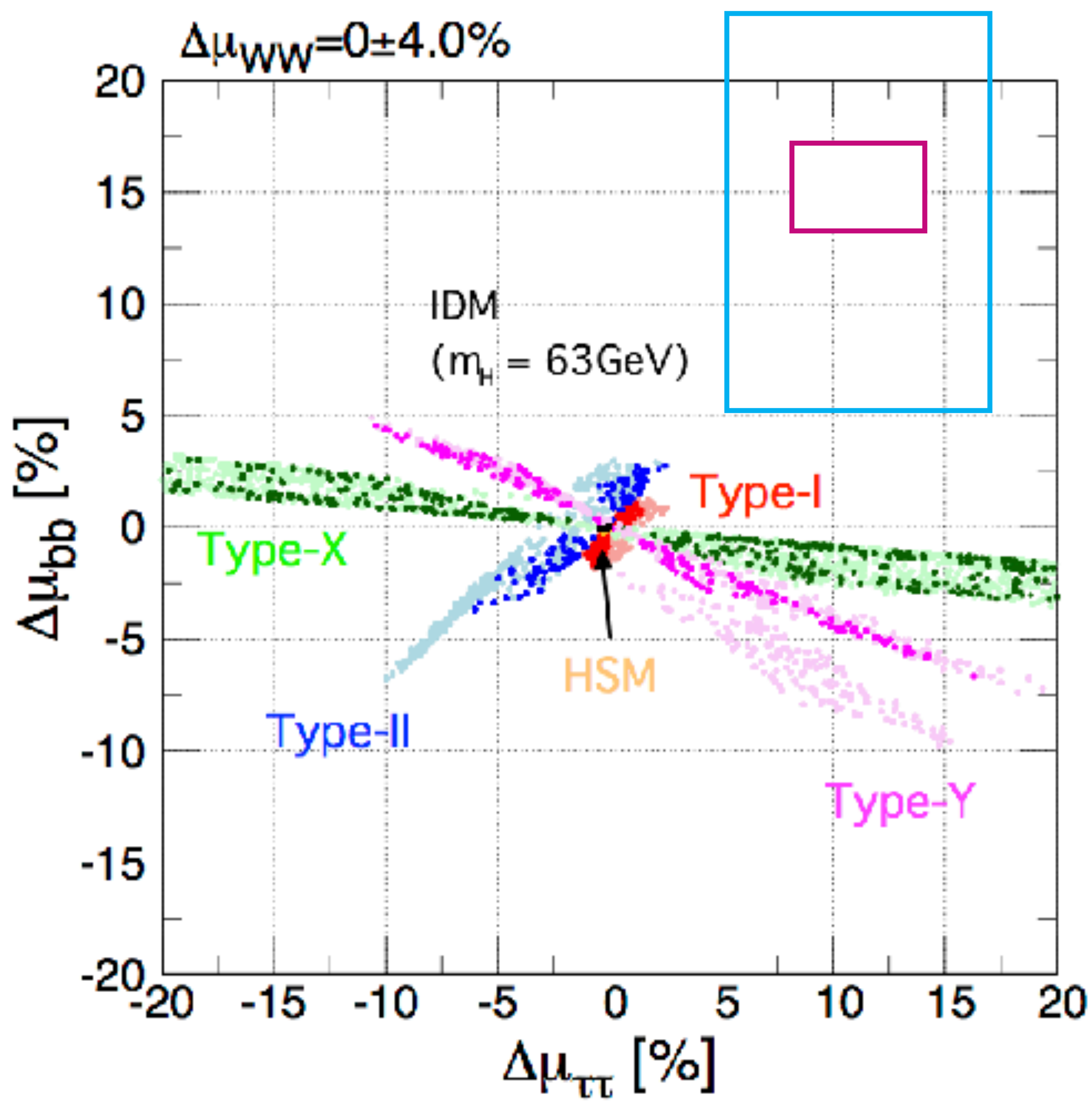
  - Lighter colors: $m_\Phi < 600\mathrm{GeV}$
  - Darker colors: $m_\Phi > 600\mathrm{GeV}$

Lower bound from $b \to s\gamma$
(for Type-II,Y)

HL-LHC(2σ):
[ATLAS, CMS,1902.00134]

ILC(2σ):
[T. Barlow et al. 1710.07621]

[Kanemura, Kikuchi, Mawatari ,KS, Yagyu]

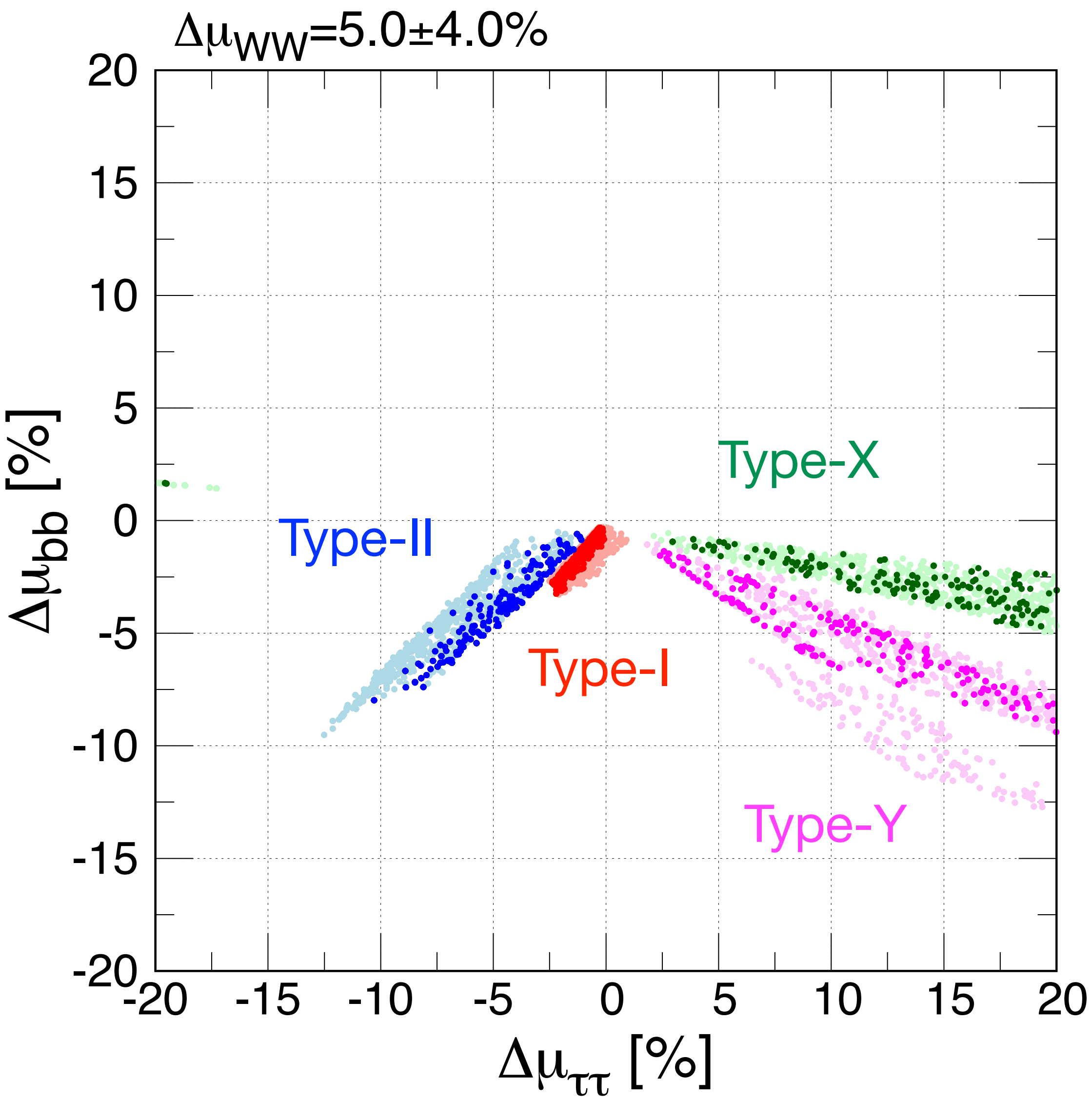


If $|\Delta\mu_{\tau\tau}| \gtrsim 5\,\%$, 4 types of THDMs can be separated.

Case ② : $\Delta\mu_{WW} = 5 \pm 4\,\%$

Case ③ : $\Delta\mu_{WW} = -5 \pm 4\,\%$

[Kanemura, Kikuchi, Mawatari ,KS, Yagyu]

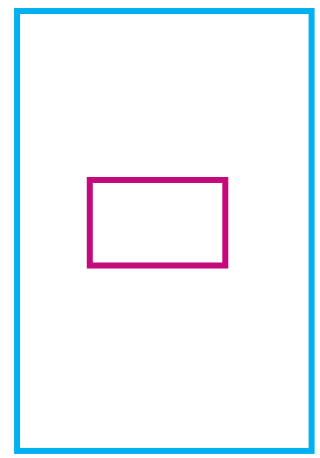


HL-LHC(2σ):

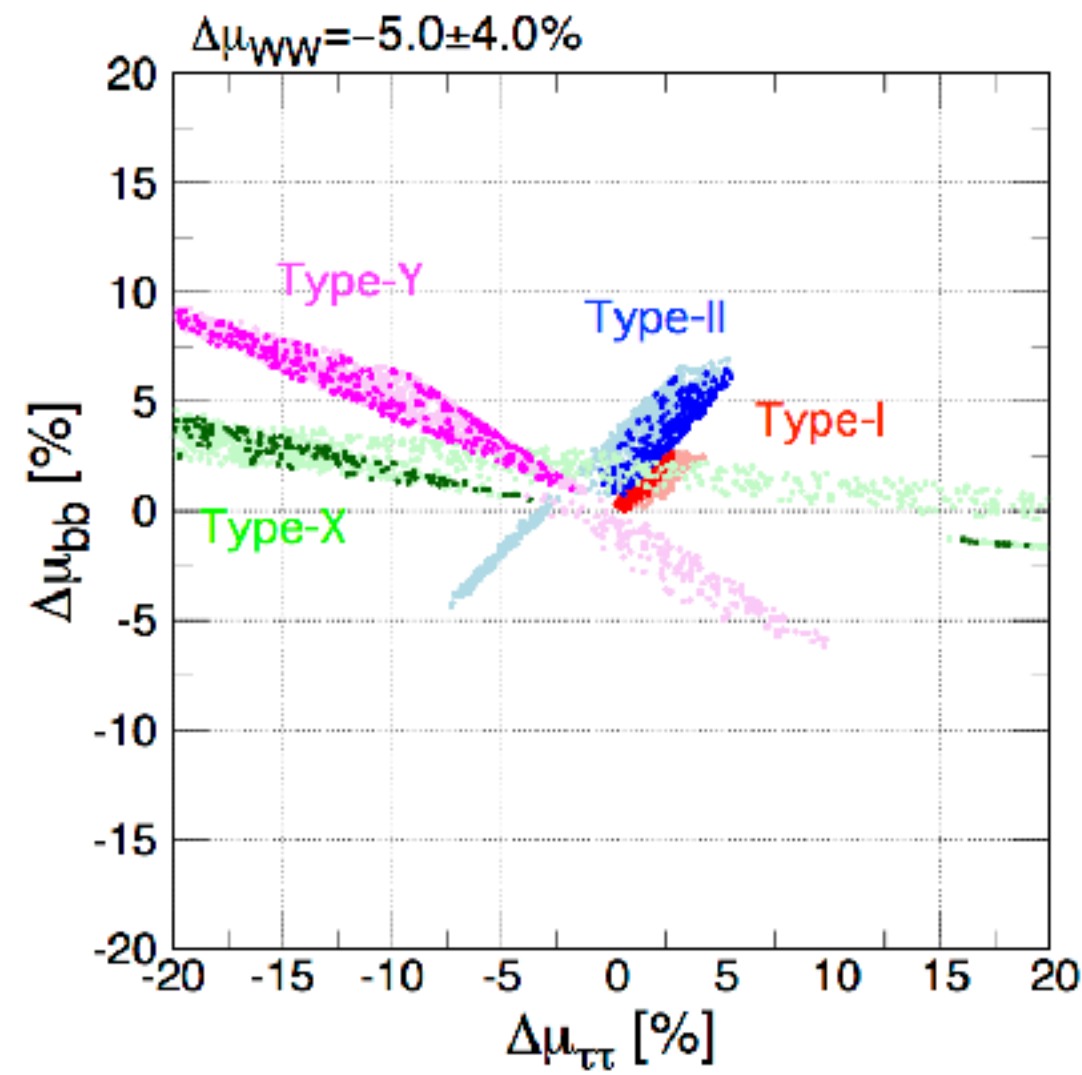
[ATLAS, CMS,1902.00134]

ILC(2σ):

[T. Barlow et al. 1710.07621]



[Kanemura, Kikuchi, Mawatari ,KS, Yagyu]



- In both case, HSM and IDM are already excluded.
- In case② all models predictions are completely separated.
- In case③, if m_Φ>600 GeV, we can distinguish all models

Summary

- **H-COUP2.0** can evaluate Higgs branching ratios at NLO EW and NNLO QCD in various extended Higgs models.
- Few % deviations due to loop effect of additional Higgs bosons can be obtained in context of branching ratios.
- We investigated the deviations from the SM in the 3 cases:

	Constraint for $\Delta\mu_{WW}$	Discriminations of models
①	$\Delta\mu_{WW} = 0 \pm 4\,\%$	Possible (if $\lvert\Delta\mu_{\tau\tau}\rvert \gtrsim 5\,\%$)
②	$\Delta\mu_{WW} = 5 \pm 4\,\%$	Possible
③	$\Delta\mu_{WW} = -5 \pm 4\,\%$	Possible (if m_Φ>600 GeV)

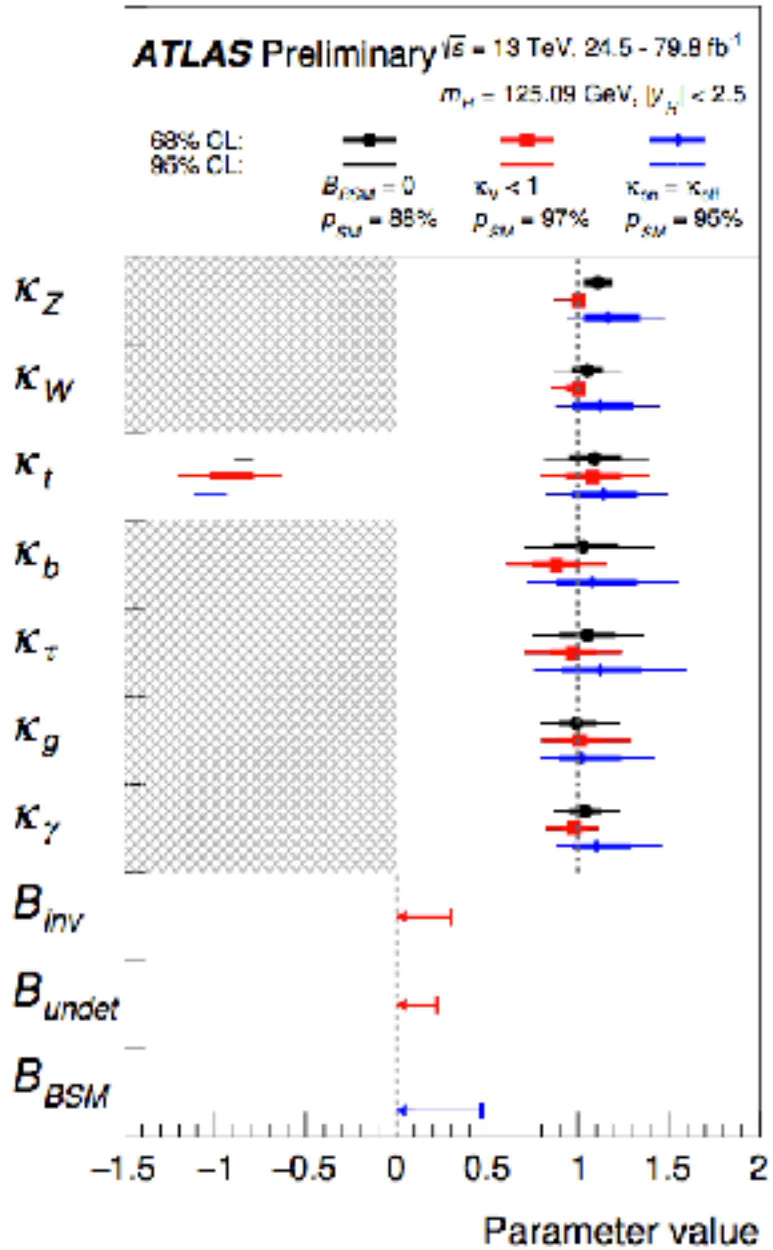
→In any case, there are situations all models can be discriminated.

ILC is needed, in order to compare with precise measurements, accurate calculations also necessary.

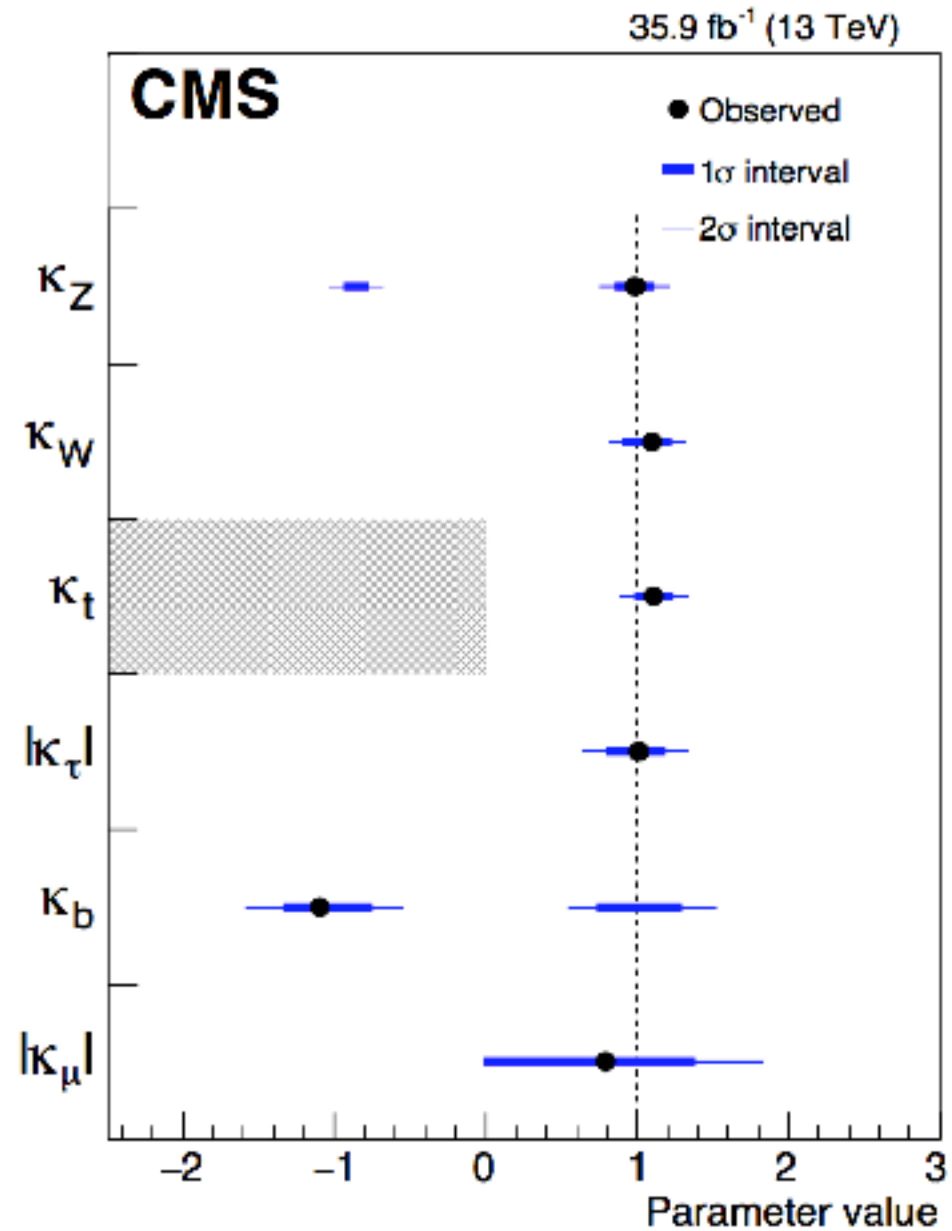
Buck up slide

current measurements (Higgs couplings)

[ATLAS-CONF-2019-005]



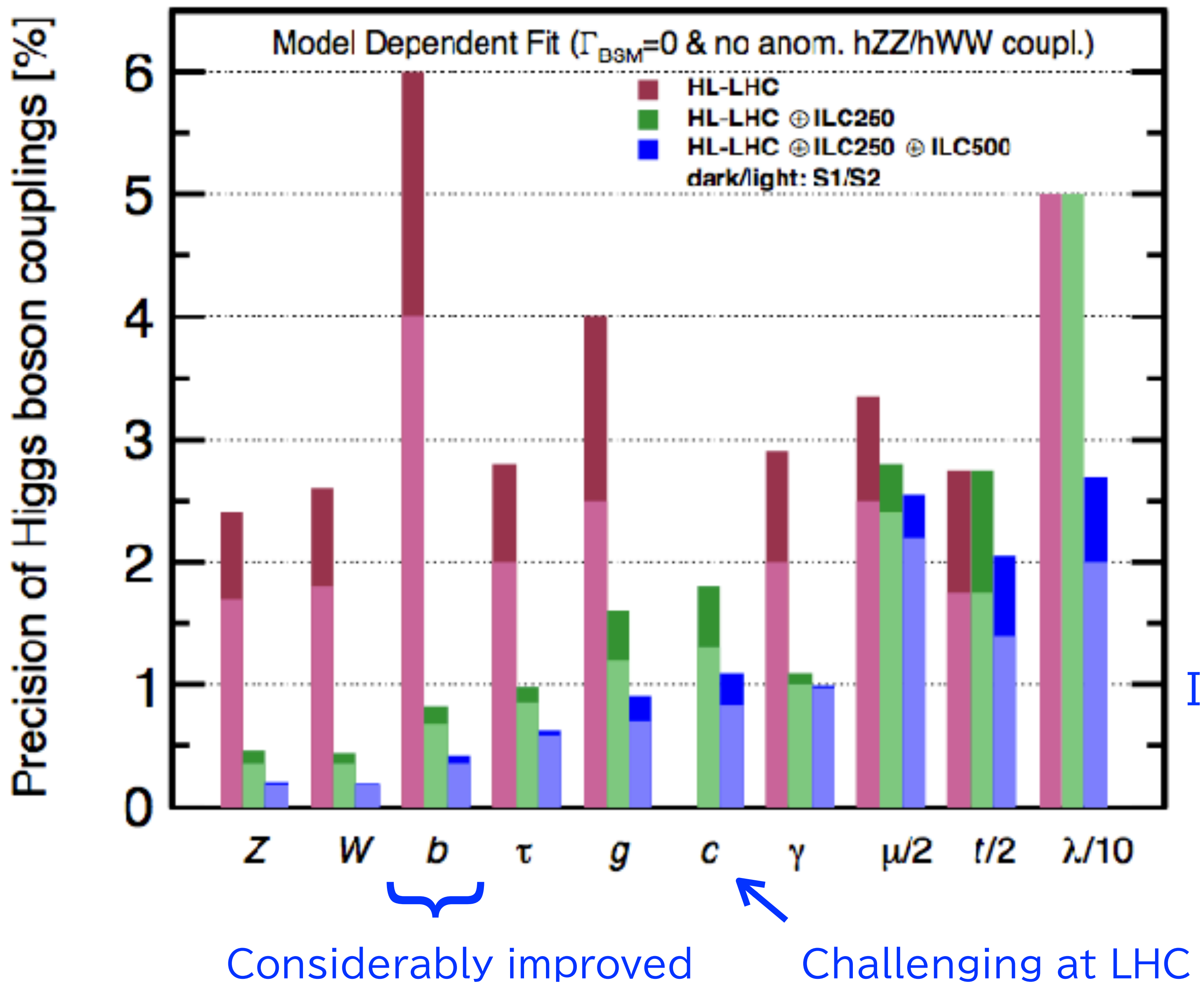
[CMS, Eur.Phys. J. C79 (2019), 421]



Current measurements are consistent with predictions of the SM within O(10)% uncertainty .

Measurements accuracy of the Higgs couplings (prospect)

[arXiv:1901.09829]



HL -LHC: $\mathcal{O}(1)\,\%$

ILC: $\mathcal{O}(1) \sim \mathcal{O}(0.1)\,\%$

- Sensitivity of most of couplings are improved by the ILC.
- In oder to compare with such precise measurements, we should evaluate theoretical predictions with radiative corrections.

Deviations

$$\Delta\mu_{XX} = \frac{\mathrm{BR}(h \to XX)_{NP}}{\mathrm{BR}(h \to XX)_{SM}} - 1$$

$$\Delta\mu_{XX} \simeq \overline{\Delta}^{X}_{\mathrm{EW}} - \sum_f \mathrm{BR}^0(h \to ff)\overline{\Delta}^{f}_{\mathrm{EW}} - \sum_V \mathrm{BR}^0(h \to VV^*)\overline{\Delta}^{V}_{\mathrm{EW}}$$

Two Higgs doublet model(THDM)

- Higgs potential

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + i z_i) \end{pmatrix} \quad \text{with} \quad i = 1, 2$$

$$\begin{aligned} V =& m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_1^\dagger\Phi_2 + \text{h.c.}) \\ &+ \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}] \end{aligned}$$

- Mass eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha)\begin{pmatrix} H \\ h \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = R(\beta)\begin{pmatrix} z \\ A \end{pmatrix}, \quad \begin{pmatrix} w_1^+ \\ w_2^+ \end{pmatrix} = R(\beta)\begin{pmatrix} w^+ \\ H^+ \end{pmatrix}$$

Physical state :$h, H, A, H^\pm$

- Physical parameters

$$\mathrm{v}, \mathrm{m_h}, \mathrm{m_H}, \mathrm{m_A}, \mathrm{m_{H^\pm}}, \alpha, \beta, M^2$$

30

Higgs singlet model(HSM)

- Higgs potential

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi + iG^0) \end{pmatrix}, \quad S = v_S + s,$$

$$V(\Phi, S) = m_\Phi^2|\Phi|^2 + \lambda|\Phi|^4 + \mu_{\Phi S}|\Phi|^2 S + \lambda_{\Phi S}|\Phi|^2 S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4$$

- Mass eigenstates

$$\begin{pmatrix} s \\ \phi \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix} \quad \text{with} \quad R(\alpha) = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

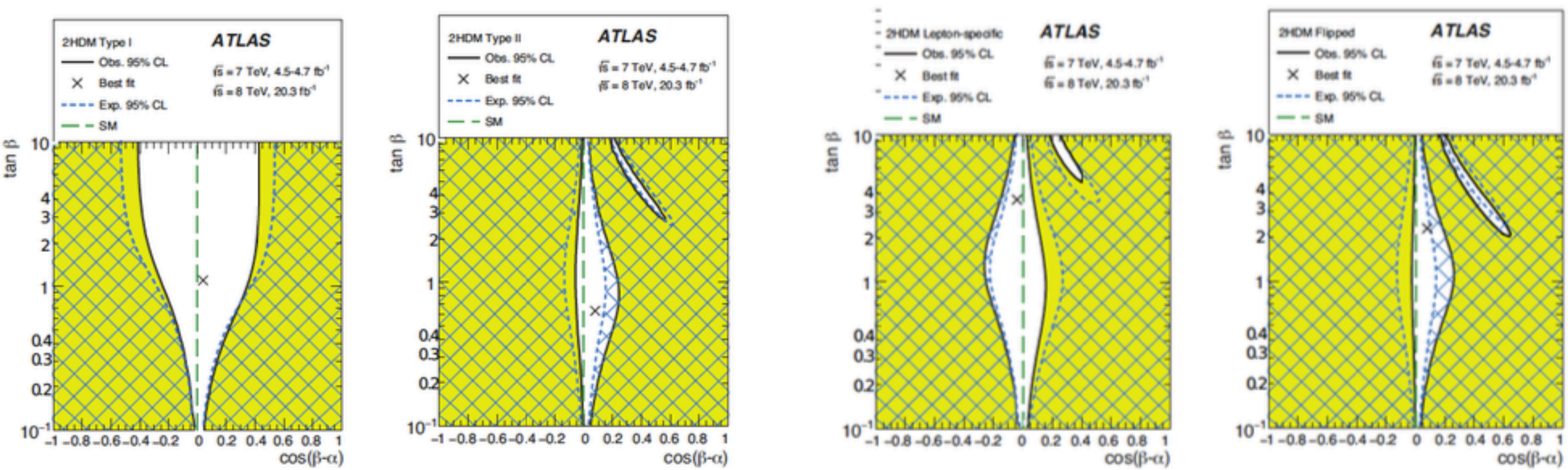
Physical state :*h, H*

- Physical parameters

$$\mathrm{v}, \mathrm{m_h}, \mathrm{m_H}, , \alpha, m_S^2, \lambda_S, \mu_{\Phi S}$$

constraint for THDMs (Higgs signal strength)

[ATLAS, JHEP1511(2015)206]



(a) Type I (b) Type II (c) Lepton-specific (d) Flipped

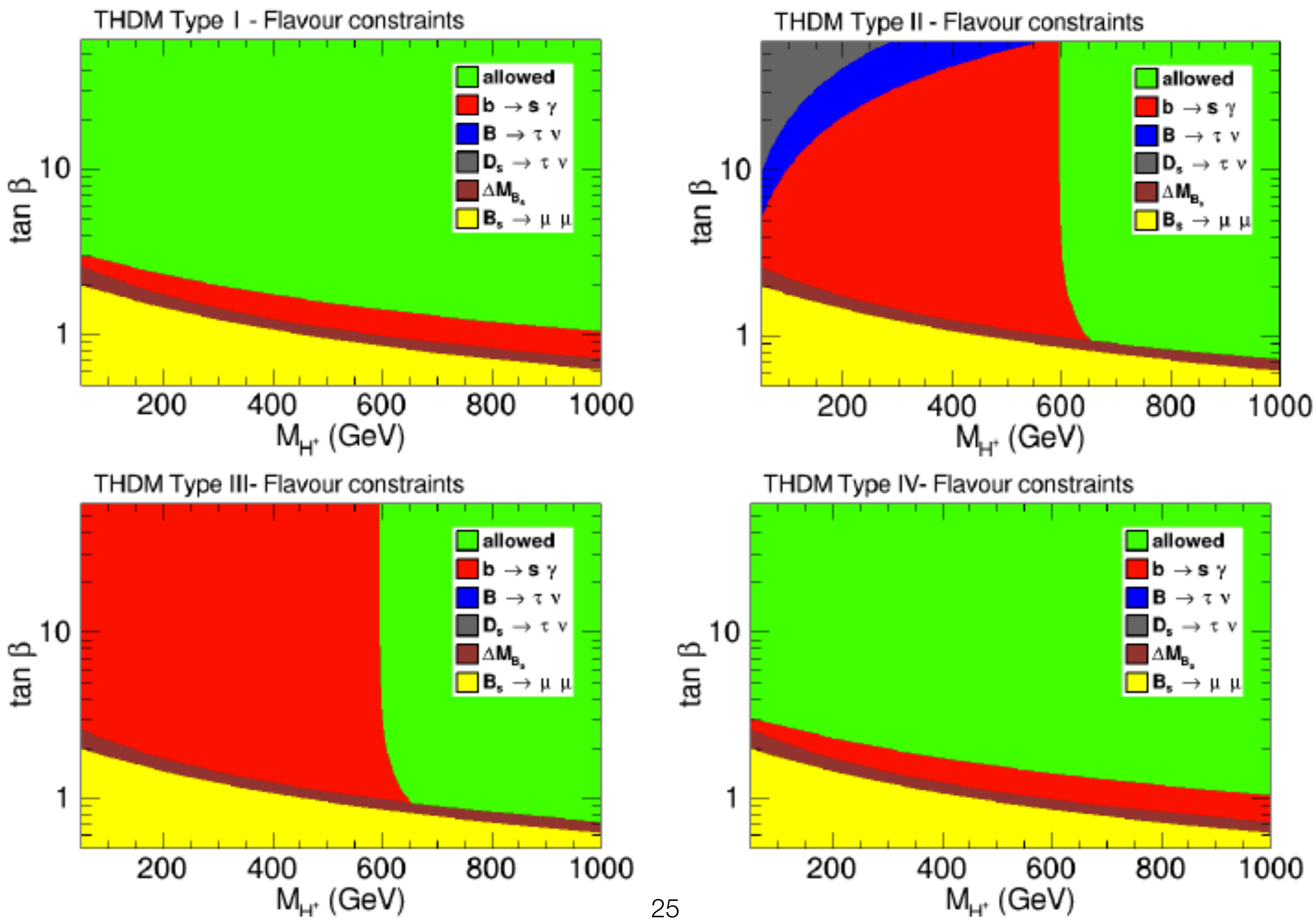
$c_{\beta-\alpha} = 0.1 \rightarrow s_{\beta-\alpha} = 0.99$

$c_{\beta-\alpha} = 0.2 \rightarrow s_{\beta-\alpha} = 0.98$

$c_{\beta-\alpha} = 0.3 \rightarrow s_{\beta-\alpha} = 0.95$

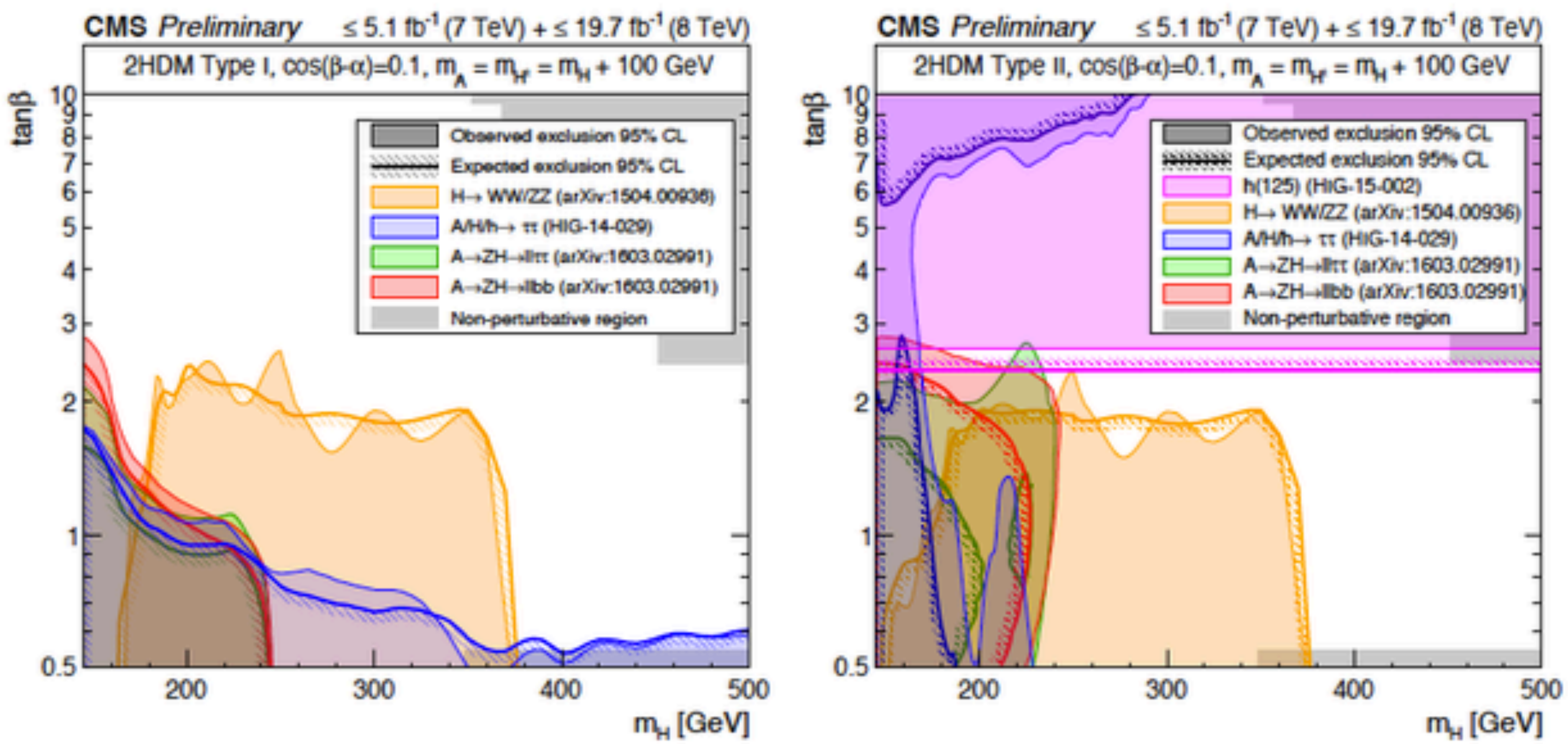
Constraint from flavor experiments

A. Arbey, F. Mahmoudi,O. Stal T. Stefaniak arXiv:1706.07414v1



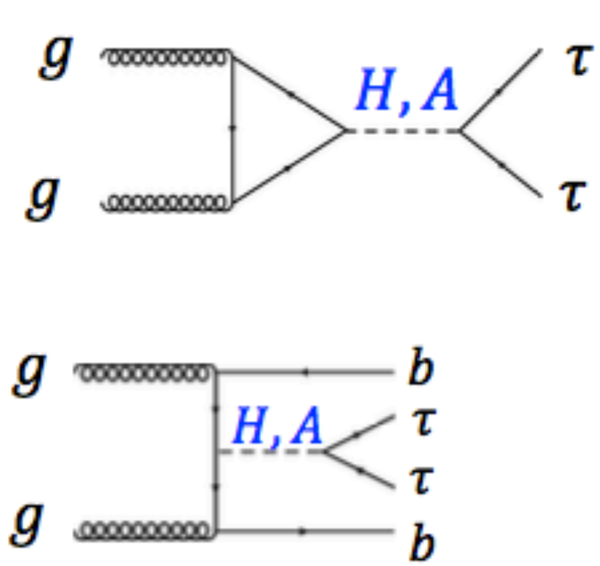
Status of direct search of extra Higgs

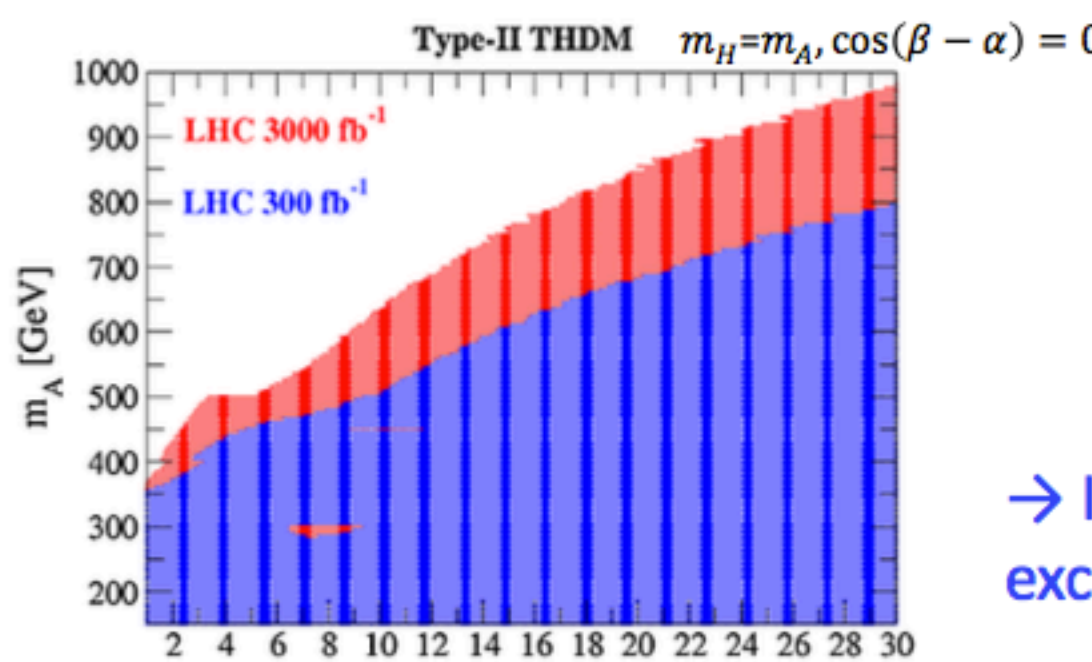
- constraint for THDMs (LHC Run I, Summary plots by CMS) [CMS PAS HIG-16-007]



→ Basically, for Type II $tan\beta < 2$, $m_H < 380$ GeV are excluded.

- Future prospect of excluded regions [Kanemura, Tsumura, Yagyu, Yokoya, PRD90(2014)075001]





→ In the future exp. excluded regions are spread.

Definition of form factors for *hVV* and hff

$$\hat{\Gamma}^{\mu\nu}_{hVV} = \hat{\Gamma}^{1}_{hVV} g^{\mu\nu} + \hat{\Gamma}^{2}_{hVV} \frac{p_1^{\mu} p_2^{\nu}}{m_V^2} + i \hat{\Gamma}^{3}_{hVV} \epsilon^{\mu\nu\rho\sigma} \frac{p_{1\rho} p_{2\sigma}}{m_V^2} ,$$

$$\hat{\Gamma}_{hff} = \hat{\Gamma}^{S}_{hff} + \gamma_5 \hat{\Gamma}^{P}_{hff} + \not{p}_1 \hat{\Gamma}^{V1}_{hff} + \not{p}_2 \hat{\Gamma}^{V2}_{hff} + \not{p}_1 \gamma_5 \hat{\Gamma}^{A1}_{hff} + \not{p}_2 \gamma_5 \hat{\Gamma}^{A2}_{hff} + \not{p}_1 \not{p}_2 \hat{\Gamma}^{T}_{hff} + \not{p}_1 \not{p}_2 \gamma_5 \hat{\Gamma}^{PT}_{hff} ,$$

hVV: 7 form factors

hff: 3 form factors

Constraint of direct search (HSM)

[T. Robens, T. Stefaniak, Eur. Phys. J. C (2016) 76,268]

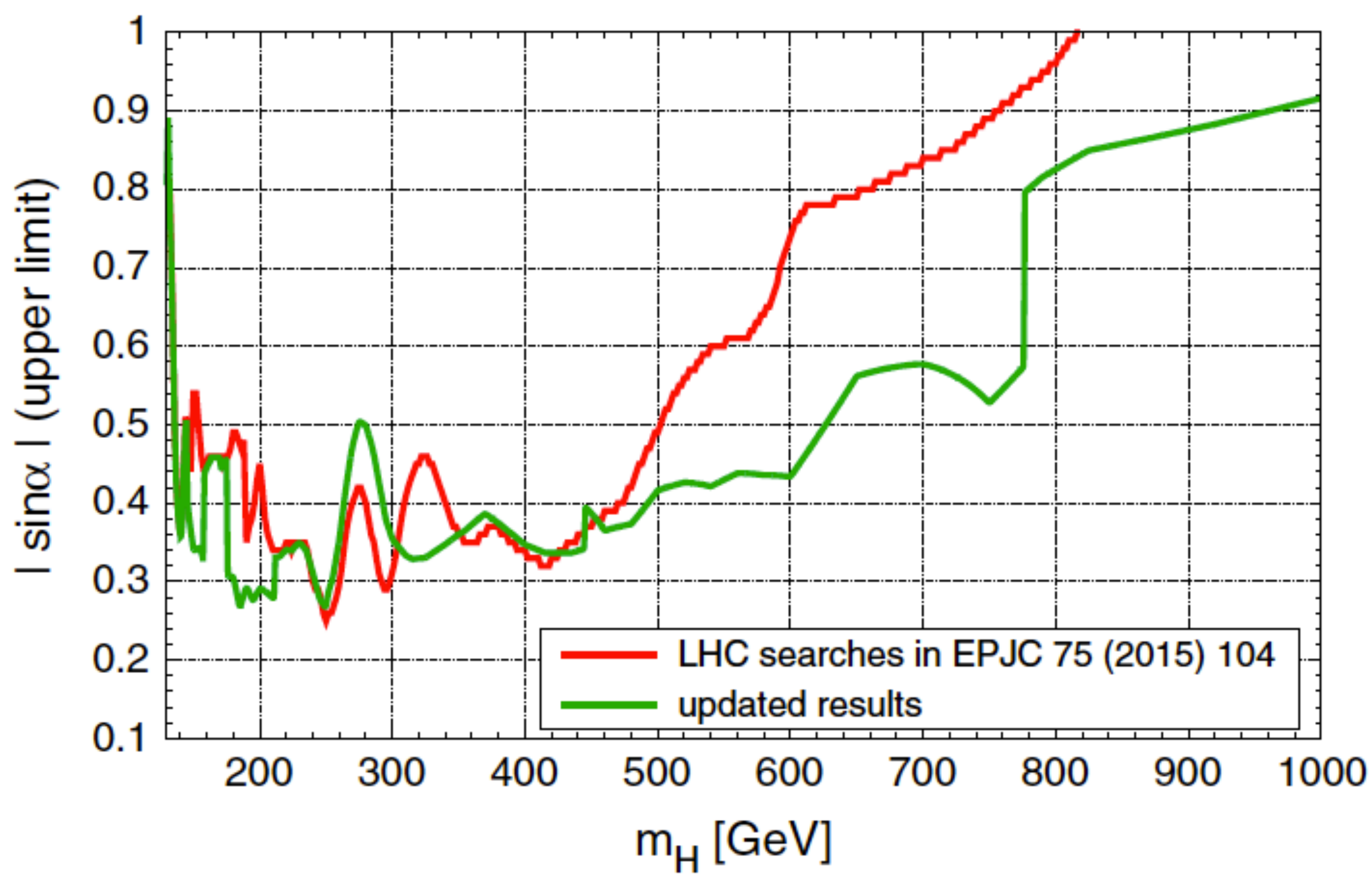


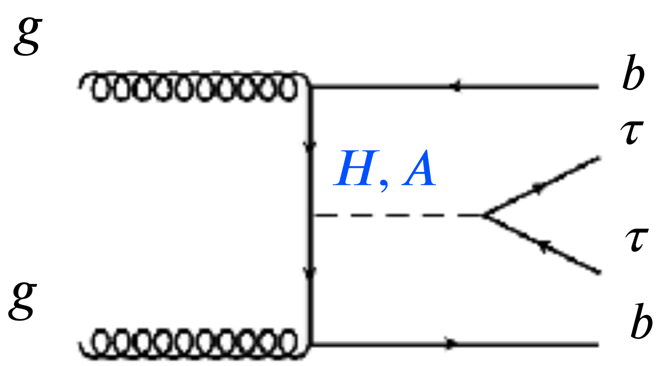
Table 1 List of LHC Higgs search channels that are applied by `HiggsBounds` in the high-mass region, yielding the upper limit on $|\sin\alpha|$ shown in Figs. 1 and 2

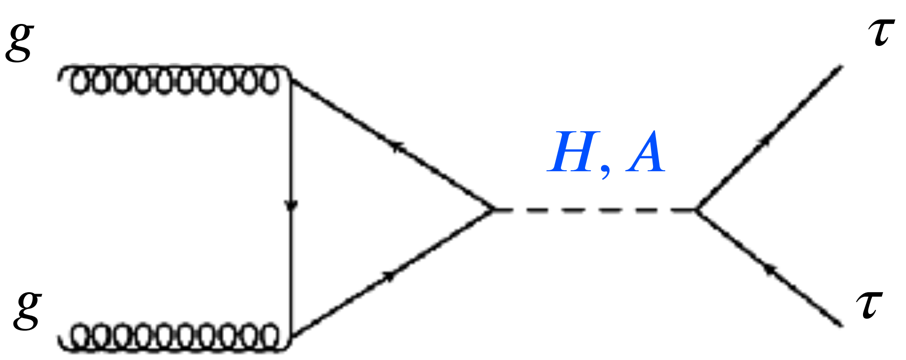
Range of m_H [GeV]	Search channel	Reference
130–145	H→ZZ→4l	[94] (CMS)
145–158	H→VV (V=W,Z)	[66] (CMS)
158–163	SM comb.	[95] (CMS)
163–170	H→WW	[96] (CMS)
170–176	SM comb.	[95] (CMS)
176–211	H→VV (V=W,Z)	[66] (CMS)
211–225	H→ZZ→4l	[94] (CMS)
225–445	H→VV (V=W,Z)	[66] (CMS)
445–776	H→ZZ	[70] (ATLAS)
776–1000	H→VV (V=W,Z)	[66] (CMS)

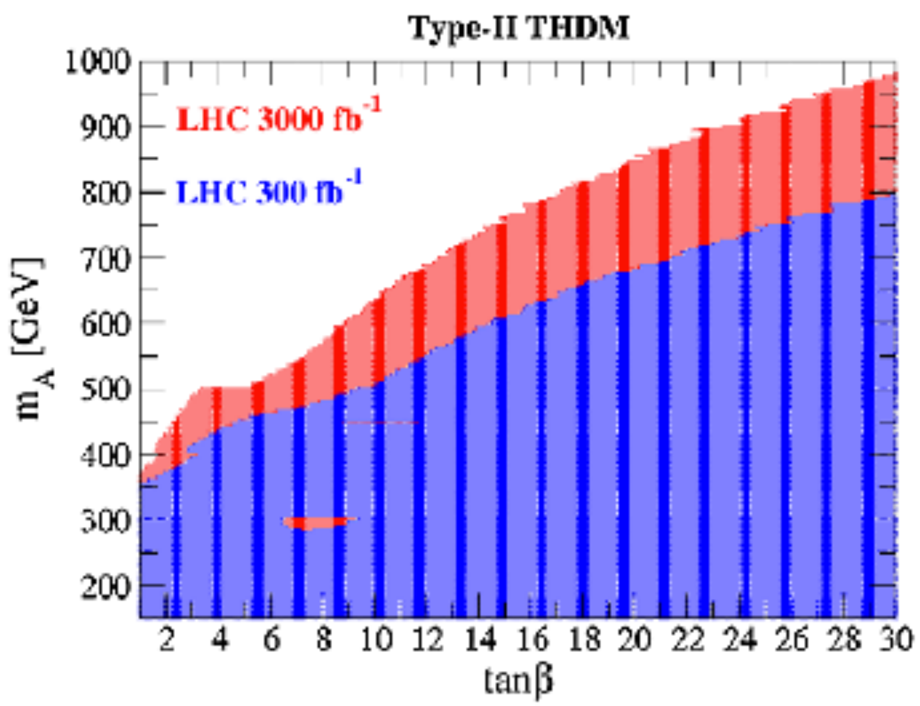
Status of direct search of extra Higgs (Future)

- Future prospect of excluded regions

[Kanemura, Tsumura, Yagyu, Yokoya, PRD90(2014)075001]







→ In the future exp.
excluded regions are spread.

Parameter scan

THDMs:

$$0.95 < \sin(\beta - \alpha) < 1,\ 1.5 \tan\beta < 10$$

$$m_\Phi = m_H = m_A = m_{H^\pm},\ 300 < m_\Phi < 1000\text{GeV},\ 0 < M < m_\Phi$$

HSM:

$$0.95 < \cos\alpha < 1,$$

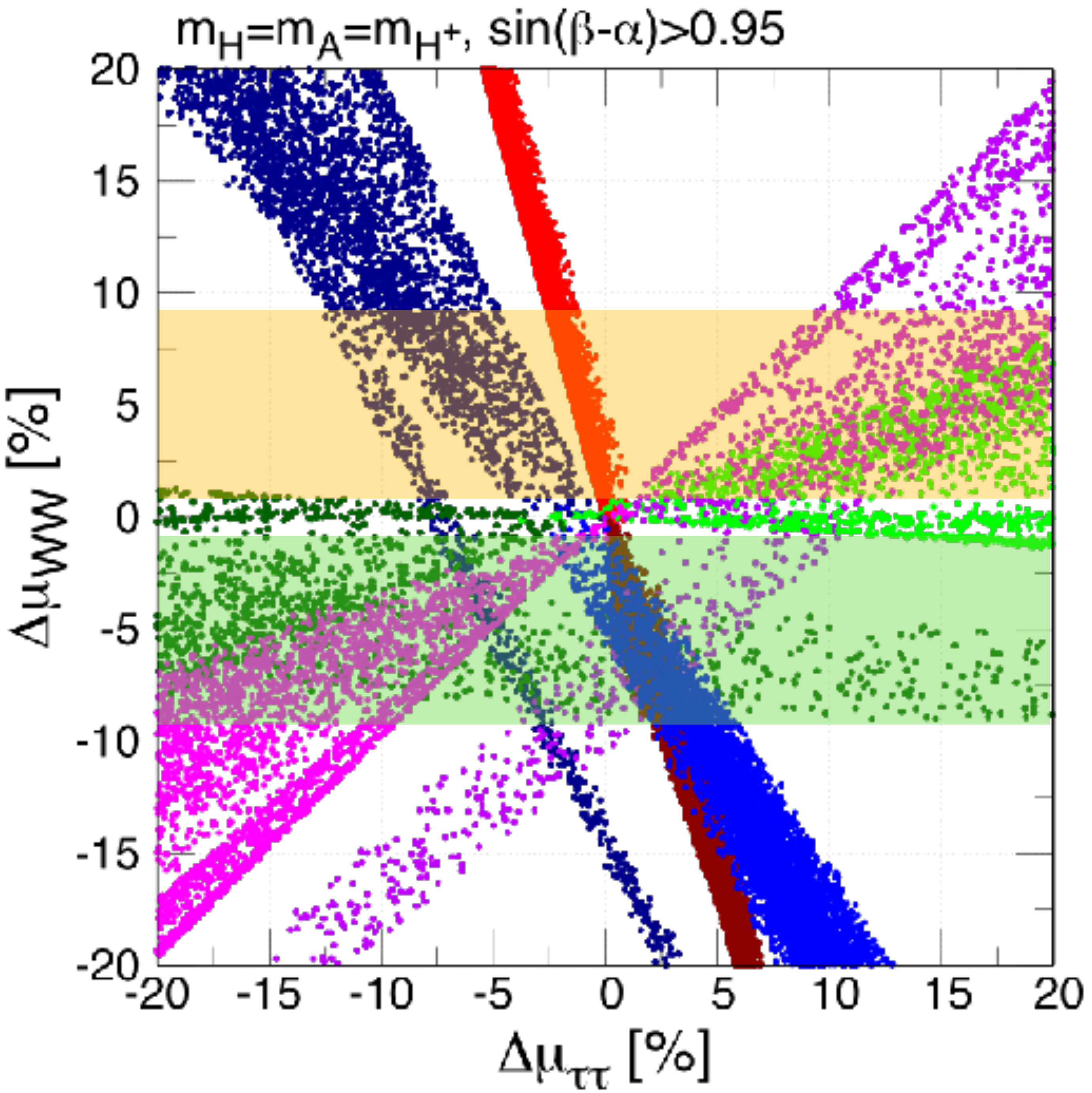
$$300 < m_H < 5000\text{GeV},\ 0 < M < m_\Phi,\ \mu_s = 0, \lambda_s = 0.1$$

IDM:

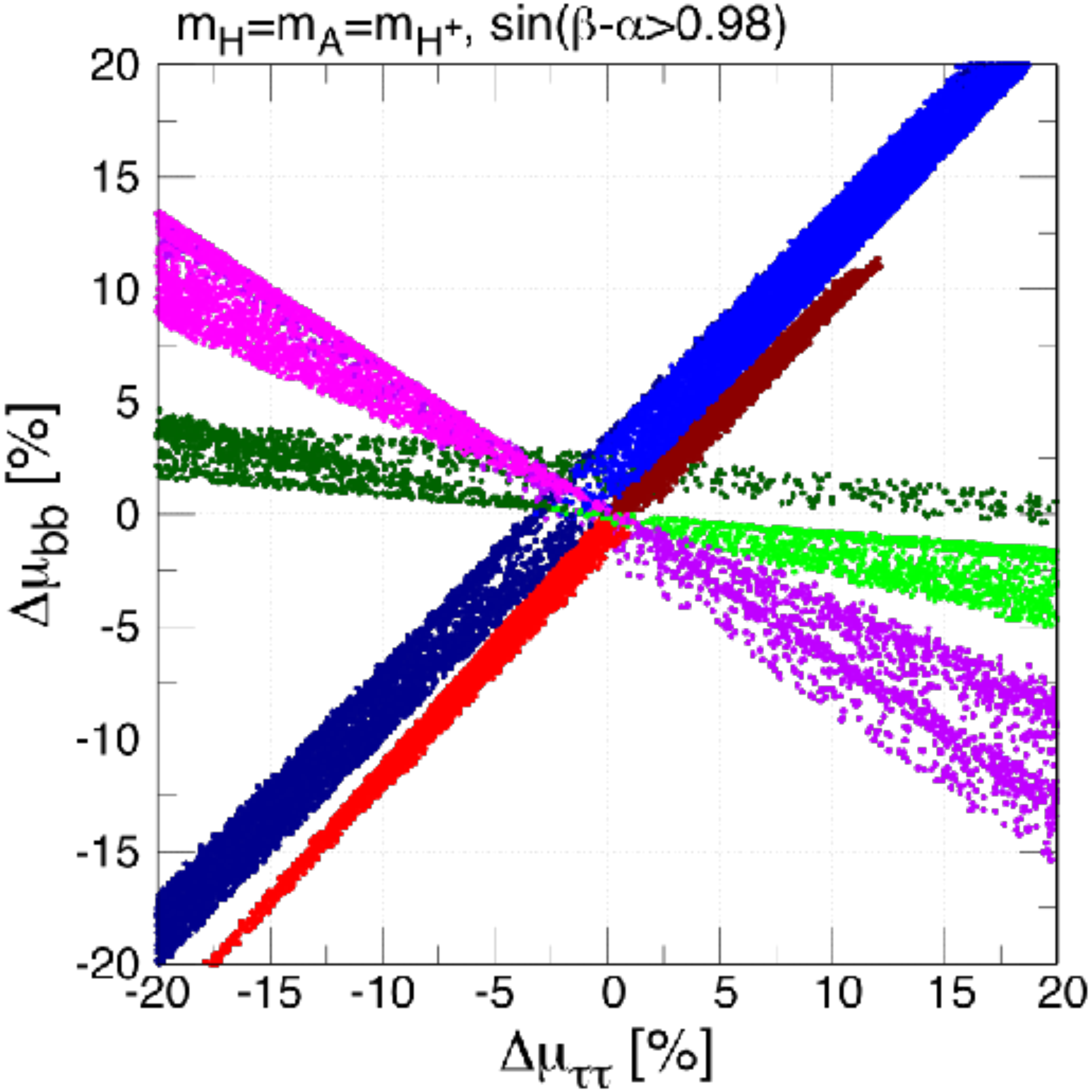
$$m_A = 63\text{GeV},\ 100 < m_H(= m_{H^\pm}) < 1000\text{GeV},\ 0 < M < m_\Phi,\ \mu_2 \simeq m_A, \lambda_2 = 0.1$$

Constraints: perturbative unitarity, vacuum stability, S and T parameters,
values for $\Delta\mu$(WW)

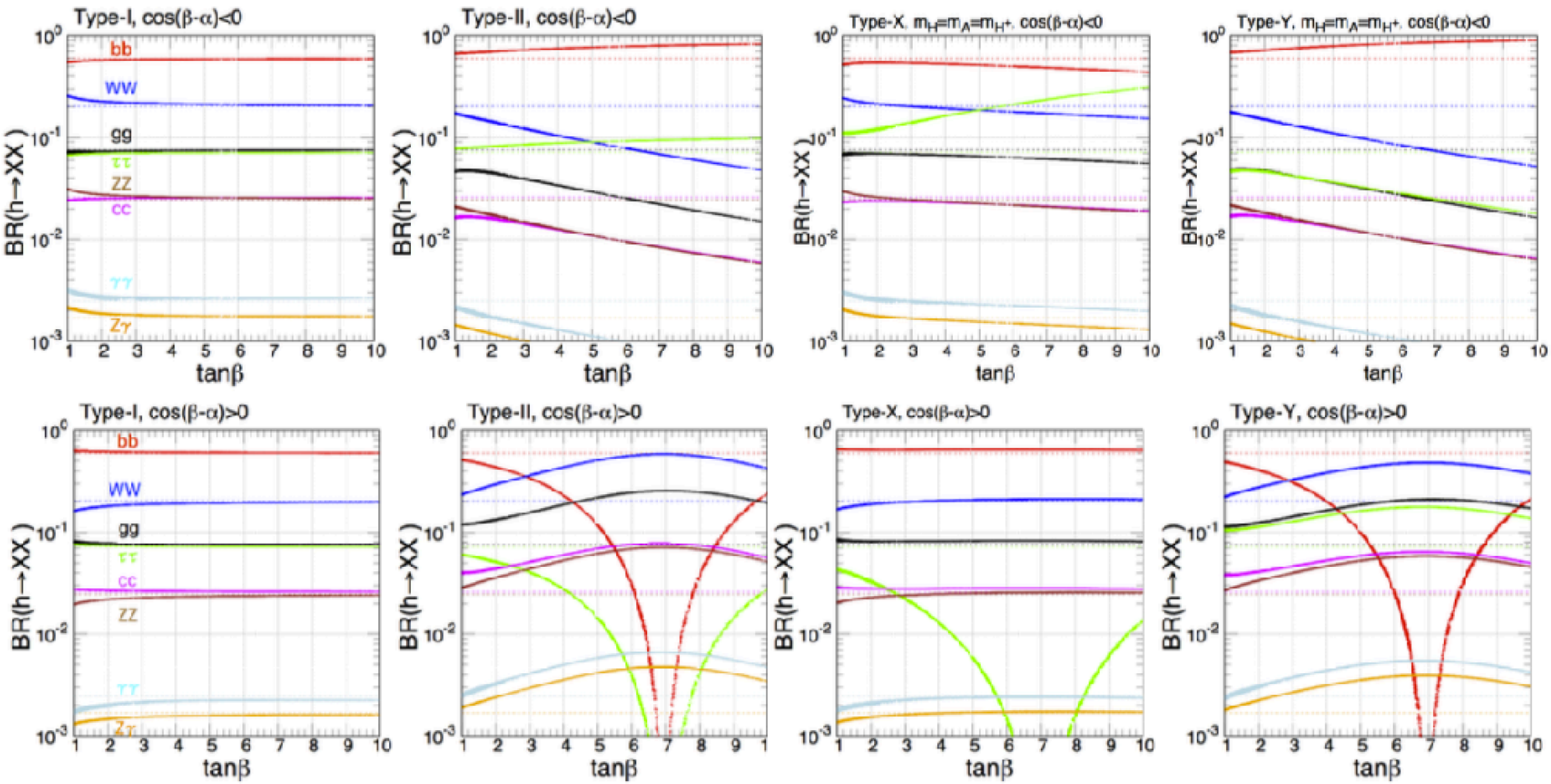
Scan plot without constraint of $\Delta\,\mu$(WW)



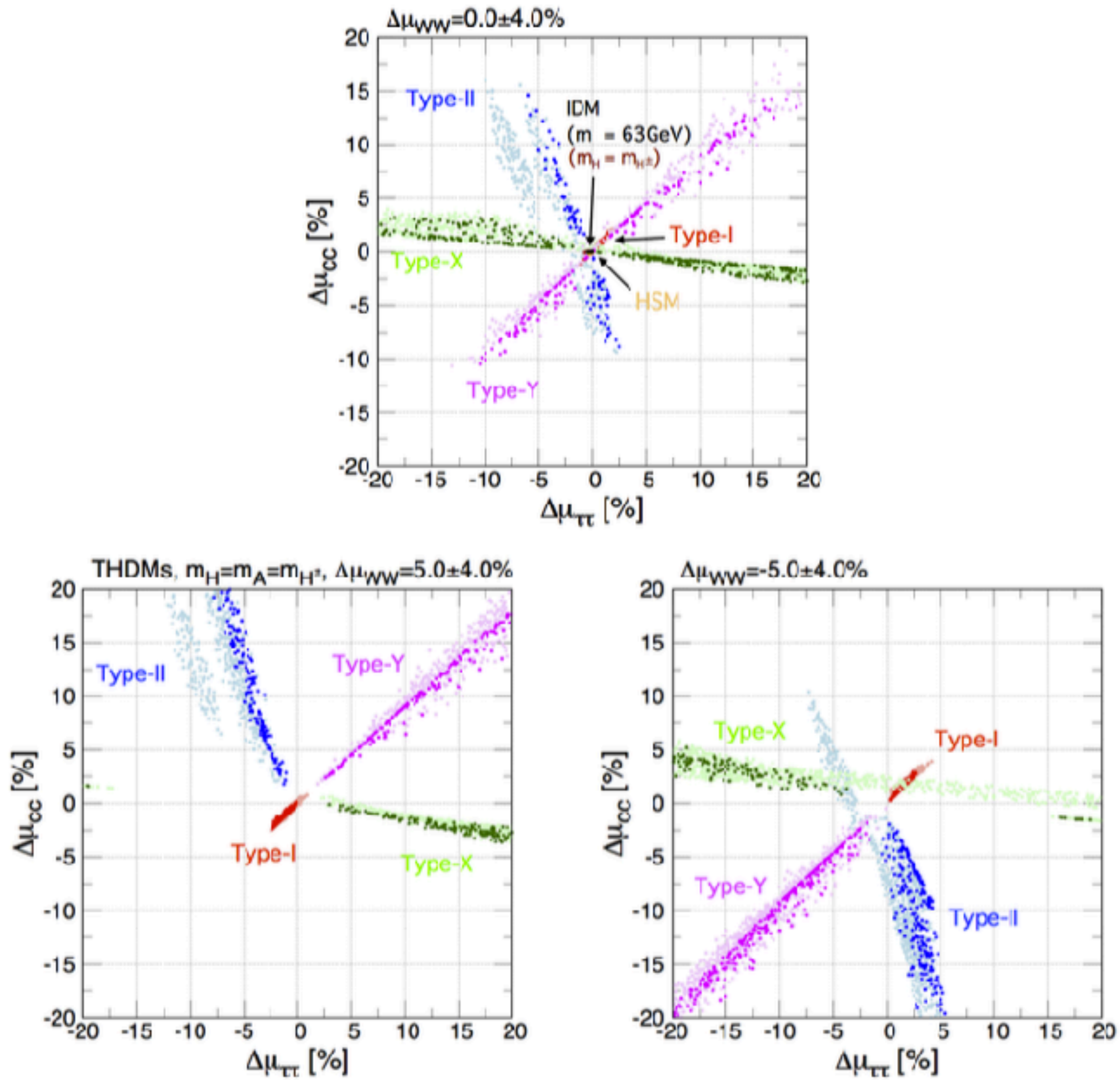
Scan plot without constraint of $\Delta\mu$(WW)



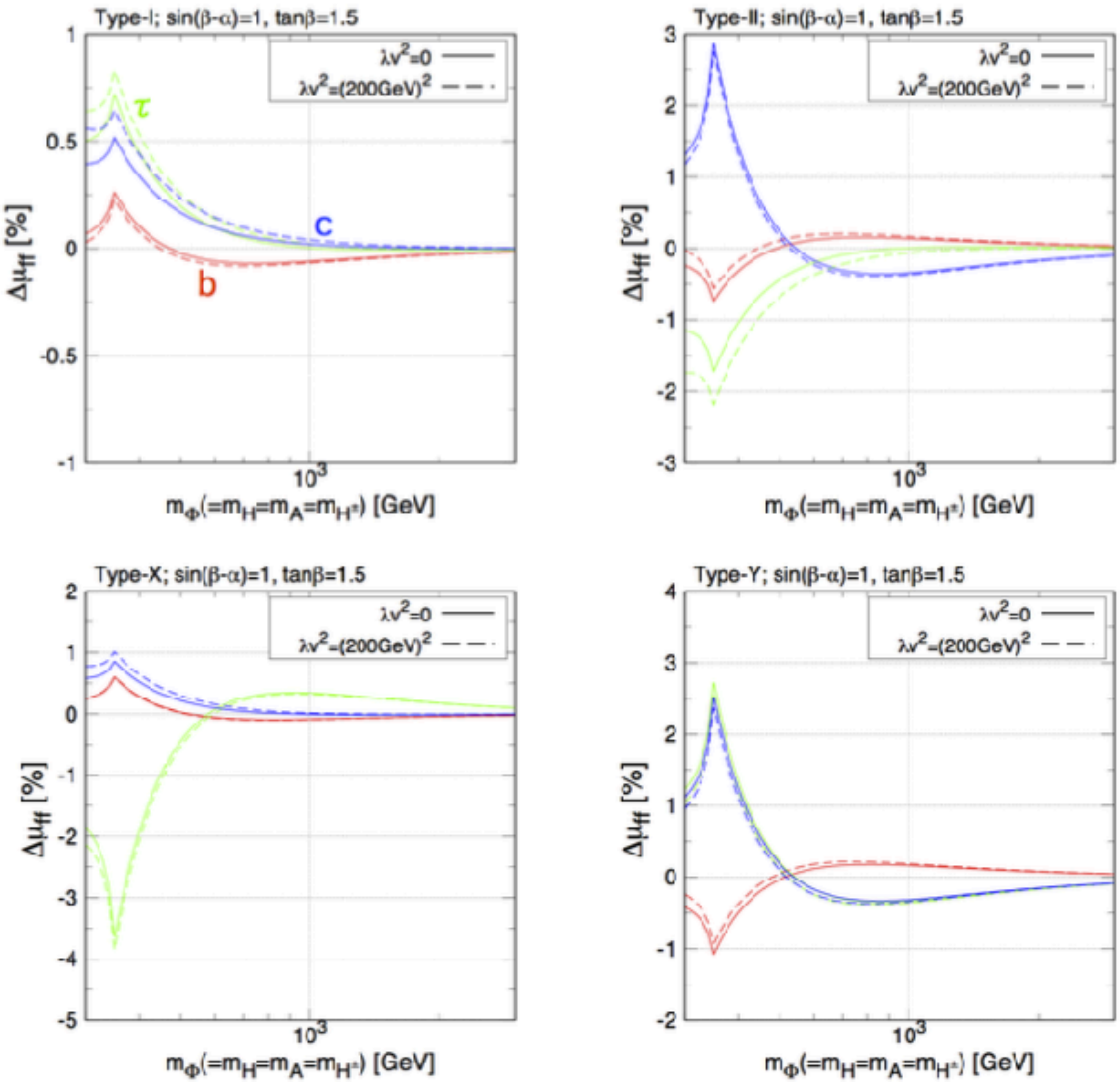
Branching ratios (THDMs)



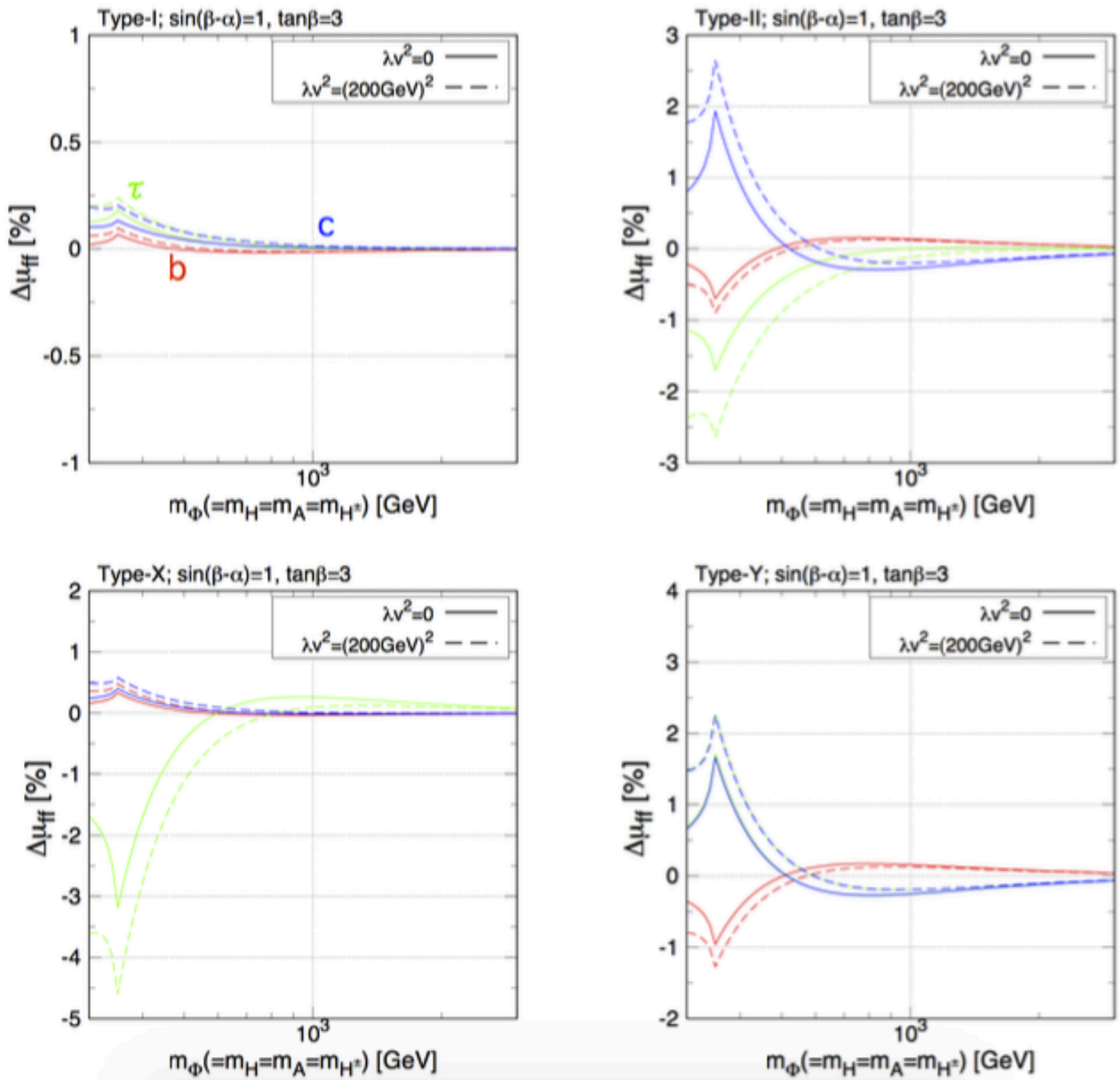
Other correlations



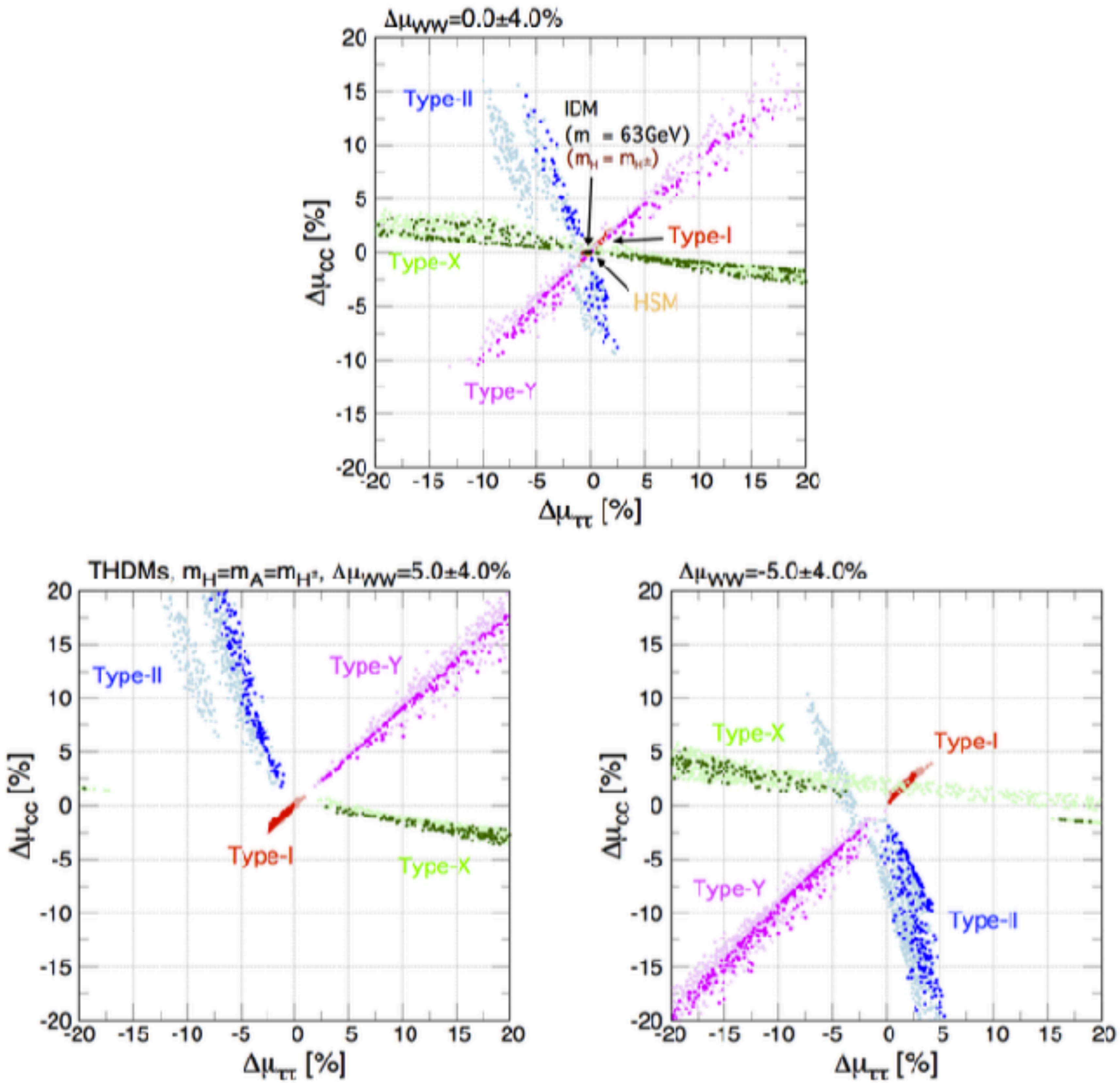
$\Delta\,\mu_{ff}$ vs m_Φ



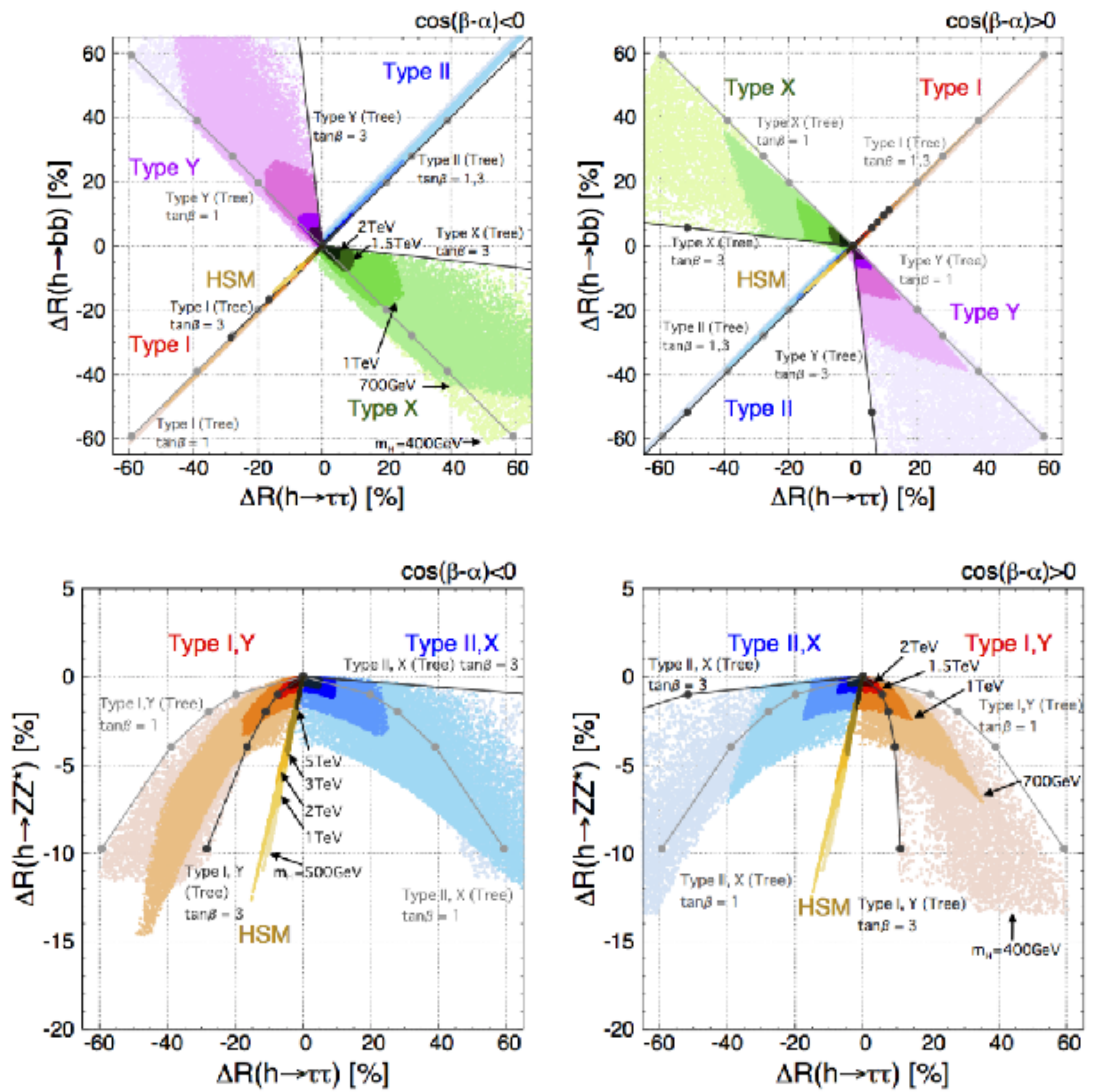
$\Delta\,\mu_{ff}$ vs m_Φ



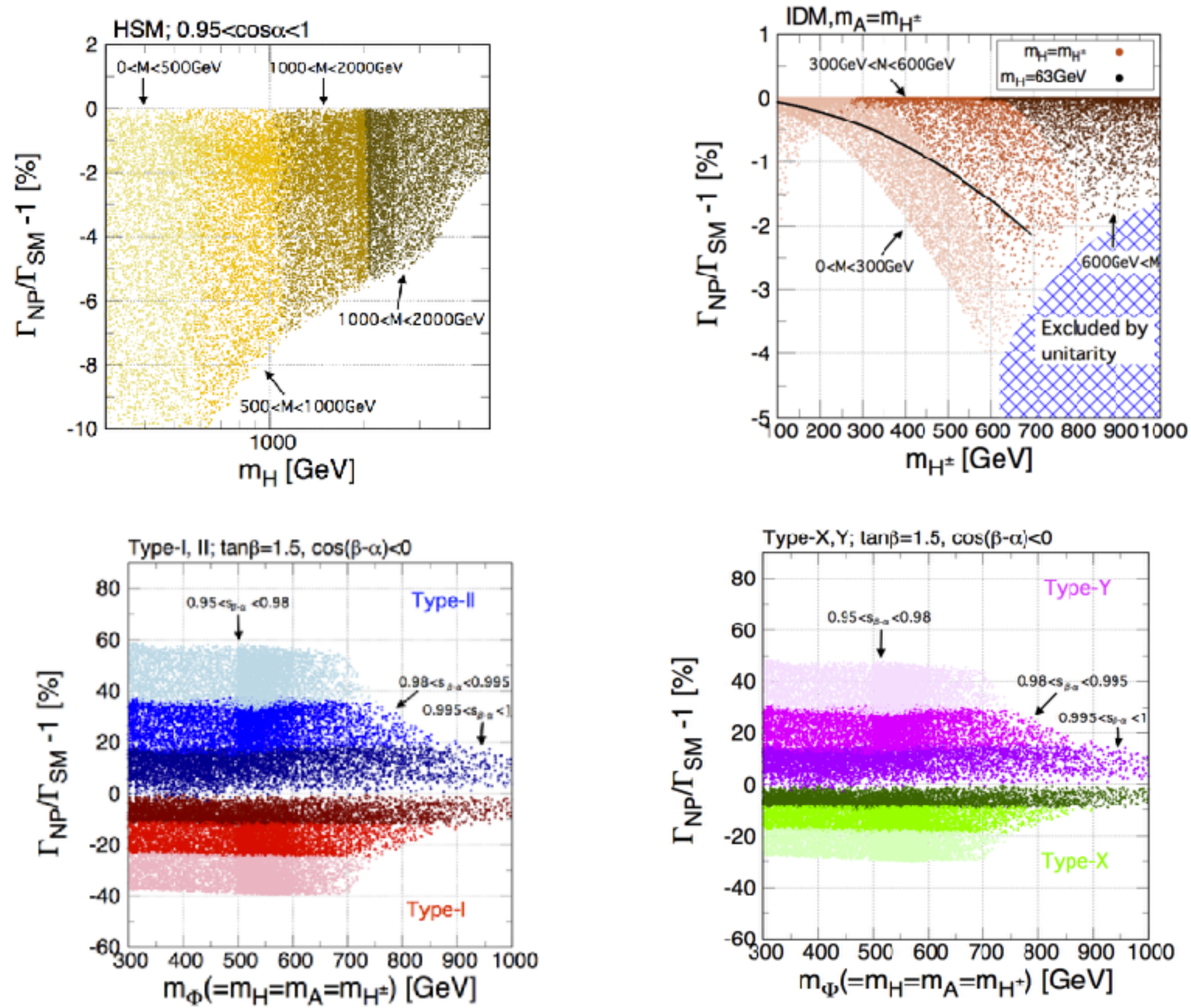
Other correlations



Correlation of the decay rates

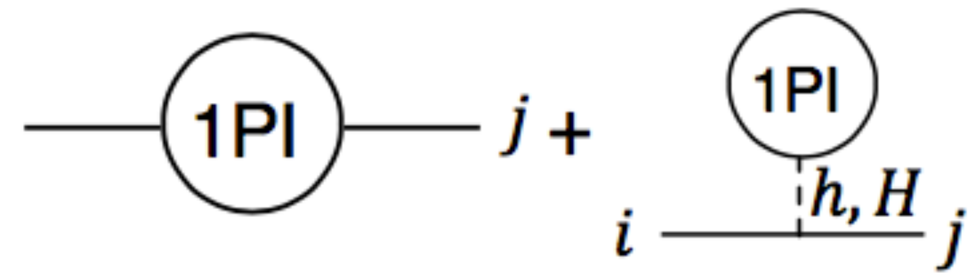


Total decay rate



Gauge dependence on the counter terms

$\Pi_{ij}(q^2) \equiv$



Nielsen identity : [N. K. Nielsen, NPB101 (1975) 173, Y. Yamada, PRD64(2001)036008]

$$\partial_\xi \Pi_{ij}(q^2) = (q^2 - m_i^2)\Lambda_i(q^2) + (q^2 - m_j^2)\Lambda_j(q^2)$$

$\Lambda_i(q^2), \Lambda_j(q^2)$: sum of loop function

ex.2) $\delta\beta$ (the counter term of the mixing angle β)

$$\delta\beta = -\frac{1}{2m_A^2}[\Pi_{AG^0}(m_A^2) + \Pi_{AG^0}(0)]$$

Appling to the Nielsen identity

$$\partial_\xi(\delta\beta) = -\frac{1}{2m_A^2}[(m_A^2 - 0)\Lambda_G(0) + (0 - m_A^2)\Lambda_A(0)]$$

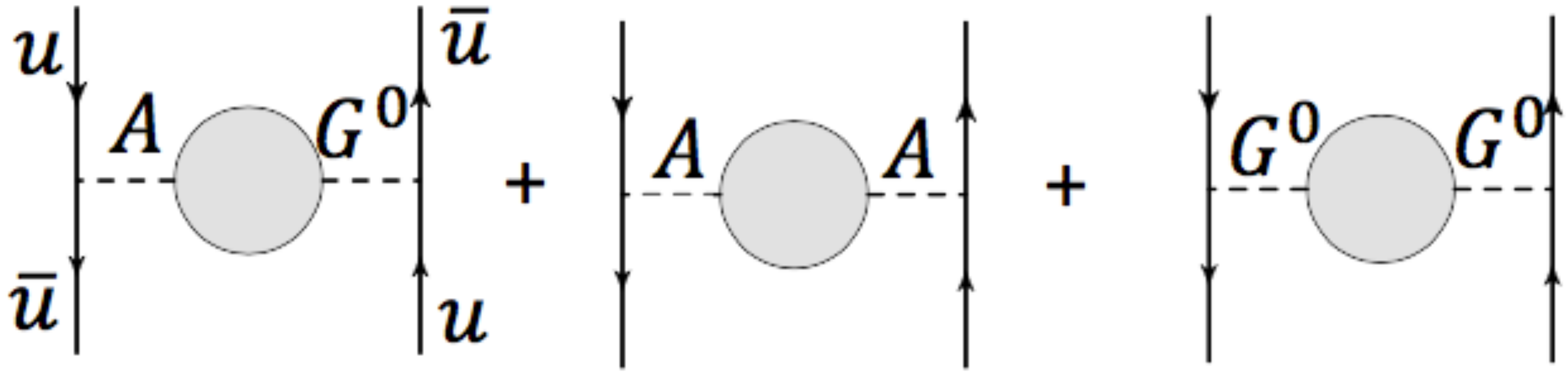
➡ $\partial_\xi(\delta\beta) \neq 0$

With the same argument, we can also find $\partial_\xi(\delta\alpha) \neq 0$

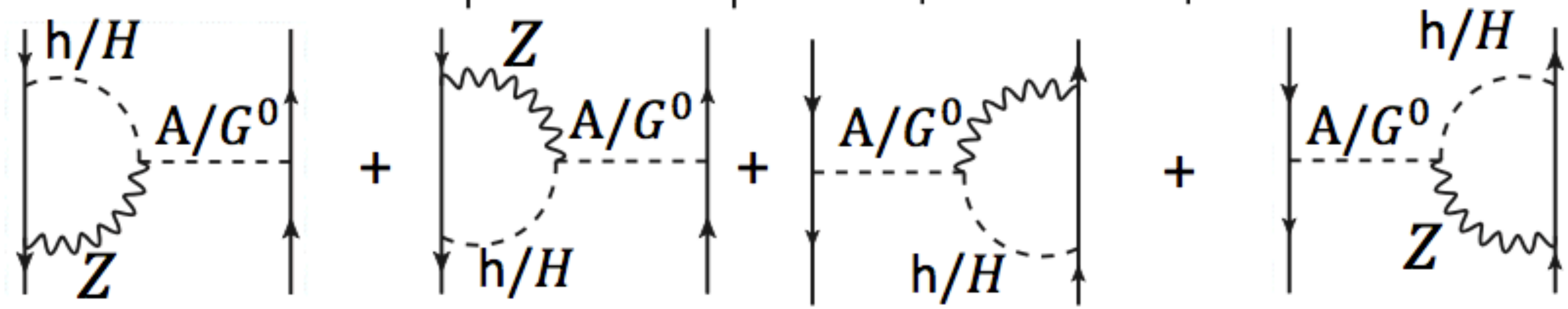
Pinch technique

I demonstrate that ξ_Z dependence for $\Pi_{AG}(q^2)$ are removed.

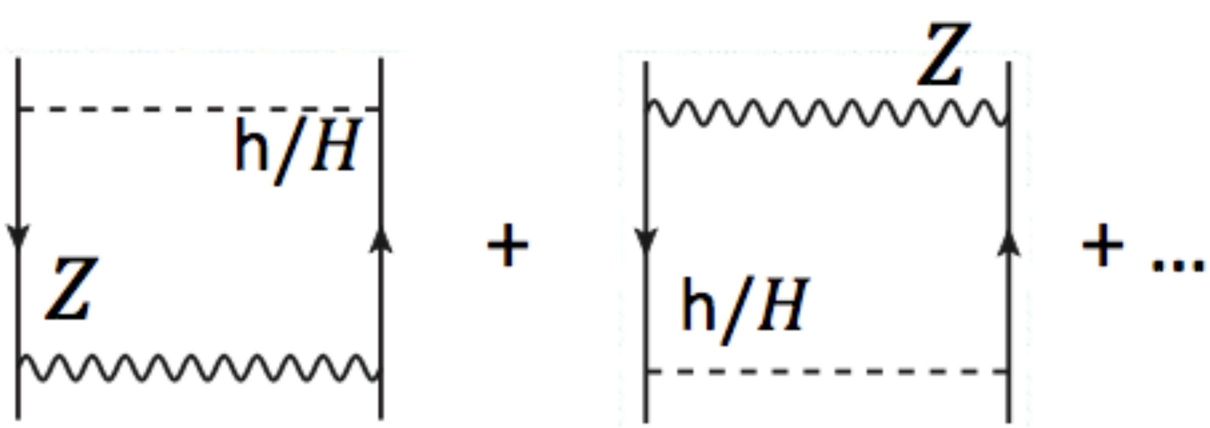
$\mathcal{M}_{self} =$



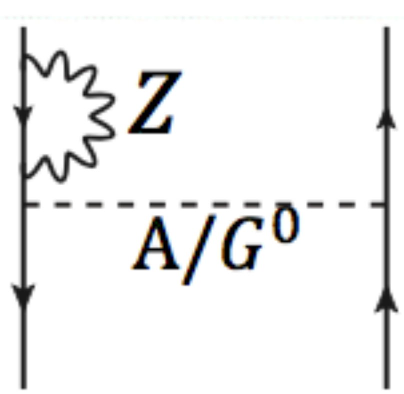
$\mathcal{M}_{vert}^{pinch} \ni$



$\mathcal{M}_{box}^{pinch} \ni$



$\mathcal{M}_{wave}^{pinch} \ni$



Pinch technique

Toy process : $u\bar{u} \rightarrow u\bar{u}$

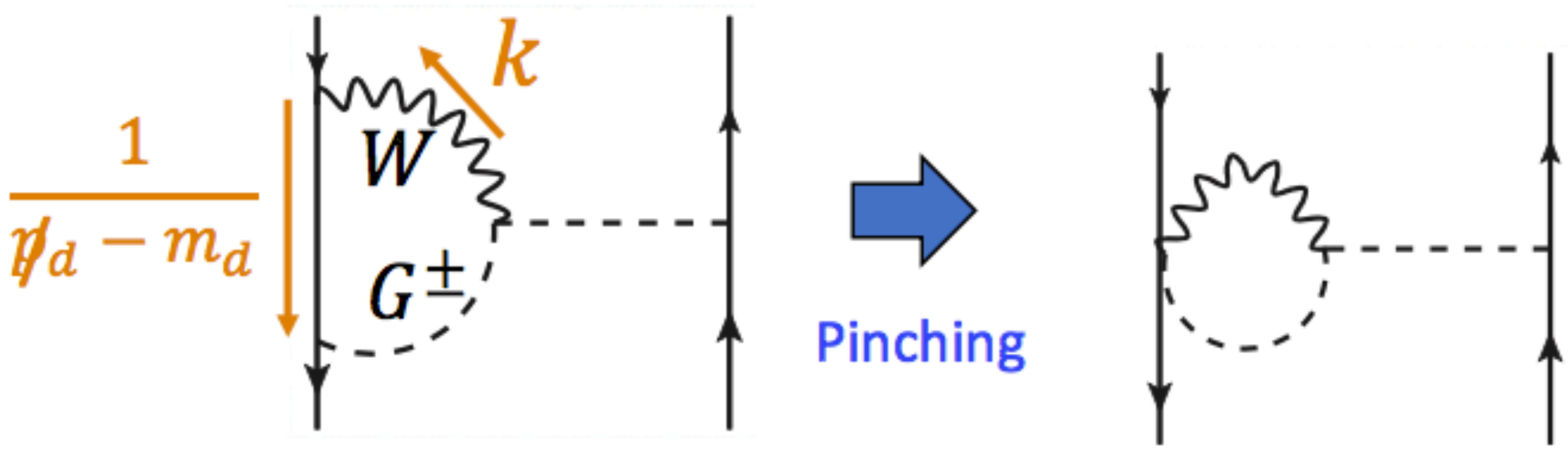
$\mathcal{M}^{PT}_{vert}, \mathcal{M}^{PT}_{box}, \mathcal{M}^{PT}_{wave}$: Pinch term

$\mathcal{M}_{sel}$

$\mathcal{M}^{PT}_{ver}$

$\mathcal{M}^{PT}_{bo}$

$\mathcal{M}^{PT}_{wa}$



Original diagram

Pinch term

Wff coupling

$$\gamma^{\mu} k_{\mu} = (\not{p}_d - m_d) - (\not{p}_d - \not{k} - m_d)$$

Derivative coupling or W boson propagator

$^{T}_{ave}$) =0

We obtained the gauge invariant Π_{hH} , adding the pinch terms