



東京大学  
THE UNIVERSITY OF TOKYO



# Improving jet energy reconstruction for ILC Higgs precision measurement with kinematic fits

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On behalf of ILD concept group

October 29, 2019

LCWS 2019

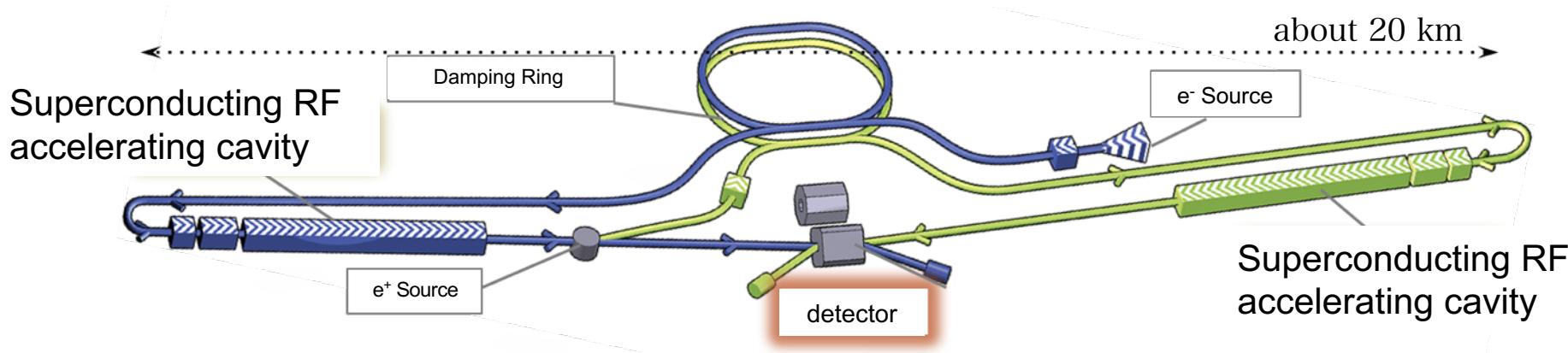
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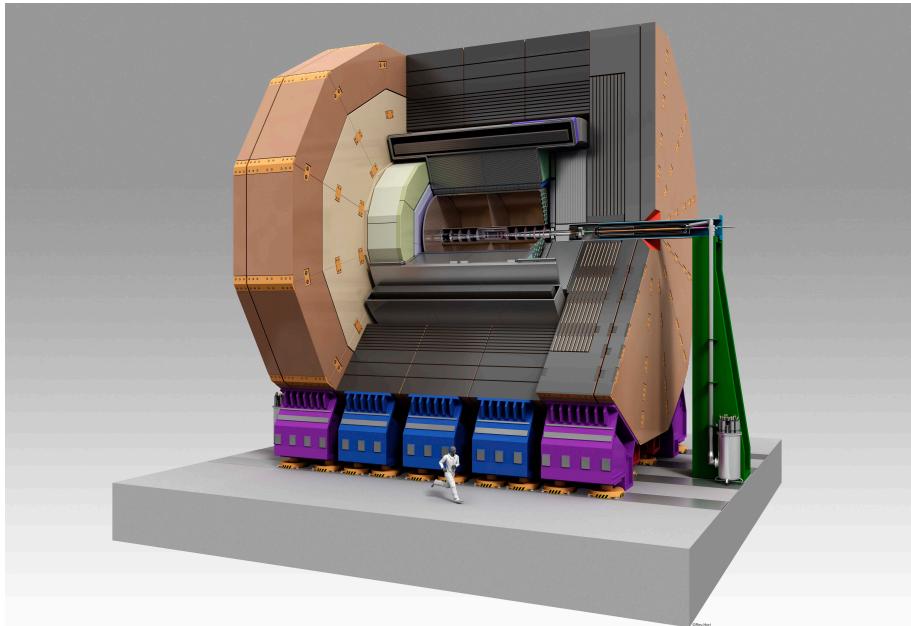
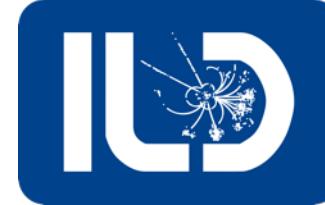
# International Linear Collider Project



- Next-generation  $e^+e^-$  linear collider
  - Start as a Higgs factory at 250 GeV
  - Future upgrade up to 1 TeV and beyond
- Proposed candidate site: Kitakami Mountains, Iwate/Miyagi, Japan
- Physics goals:
  - Precise measurements of the Higgs boson and electroweak observables, searches for dark matter and other new particles, etc.



# International Large Detector



□ ILD: a detector concept for the ILC

- Vertex detector
- Tracking detector (TPC)
- High-resolution calorimeter
- etc.

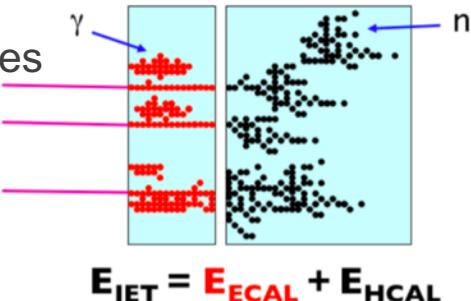
□ Optimized for particle flow for unprecedented jet energy resolution

## Particle Flow Algorithm

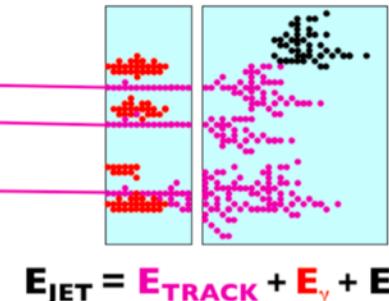
a method using the optimum detector depending on types and energy of particles

charged particles → Tracker  
photons → ECAL  
neutral hadrons → HCAL

jet energy measurement using only calorimeter

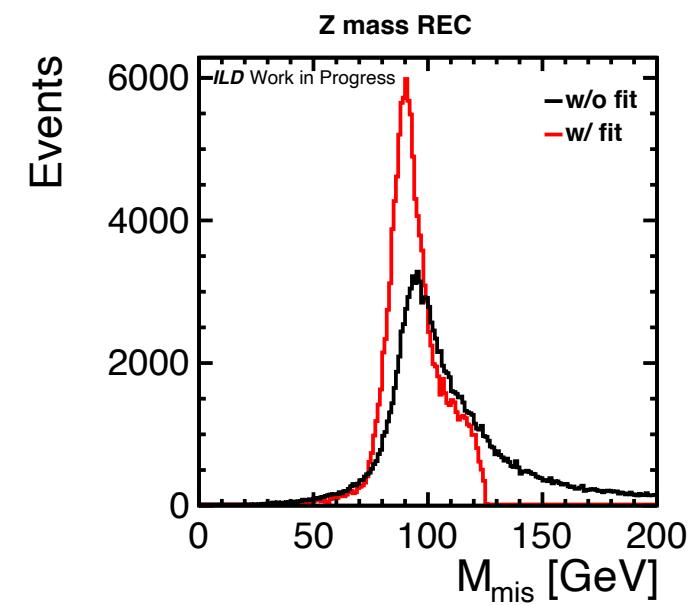
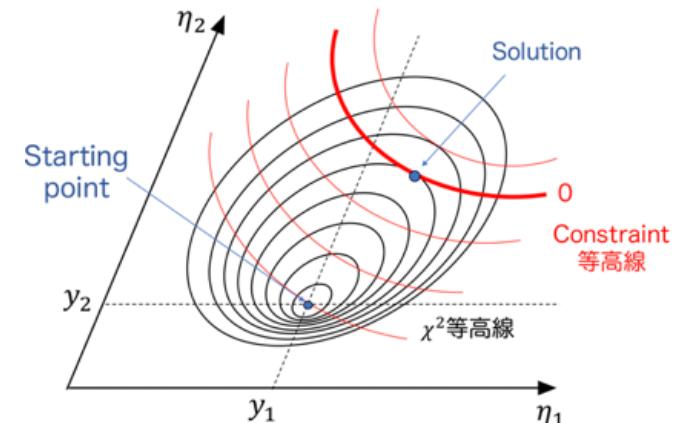


## PFA



# Introduction to Kinematic Fit

- Advantages of kinematic fits:
  - Provides a quantitative estimate of how well a given event matches a signal model.
  - Particularly useful in constraining jet-related kinematics.
  - Can deal with energy loss due to initial-state radiation and beam-beam effects.
- Requirement for using kinematic fits:
  - Intimate knowledge of detector response and underlying kinematic distributions
- Ingredients of kinematic fits:
  - a. Fit objects: represent distribution of measurements (e.g. jet energy/angle)
  - b. Constraints: impose kinematic relations (e.g. energy-momentum conservation and mass constraints)
  - c. Fit engines: numerically solve for the parameters under the constraints

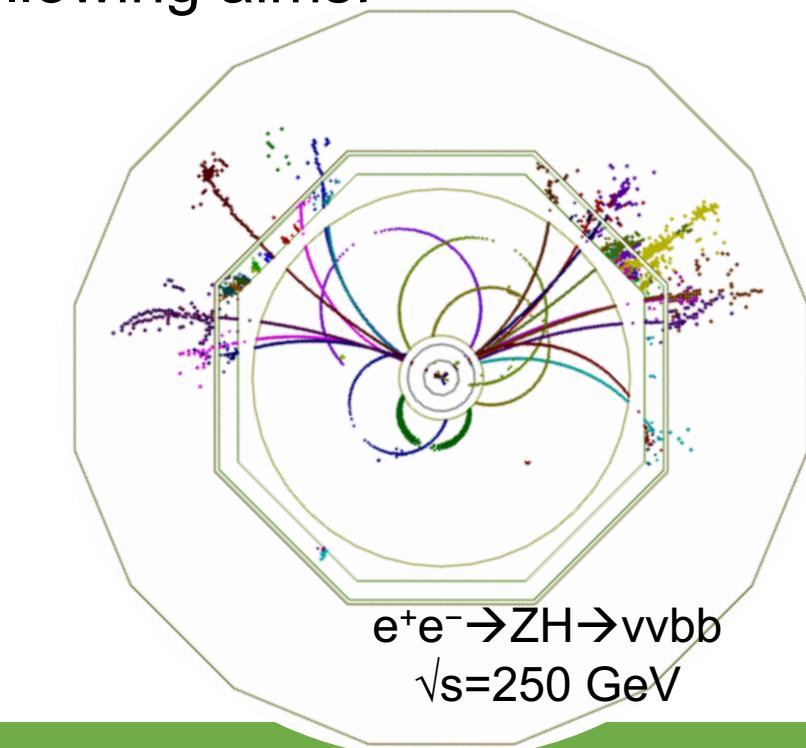
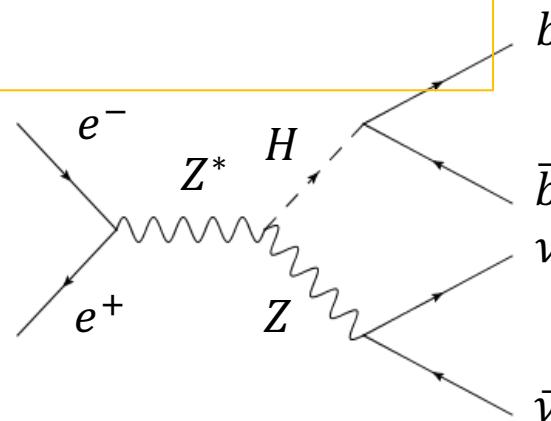


# Notes on Implementation

- Existing tool in iLCSoft: **MarlinKinfit** [LC-TOOL-2009-001]
  - Kinematic fit package with modular design
  - For jet fit objects, Gaussian resolution assumed
  - We have difficulty applying constraints on missing 4-momentum
- For now, we use standalone kinematic fit code with the following aims:

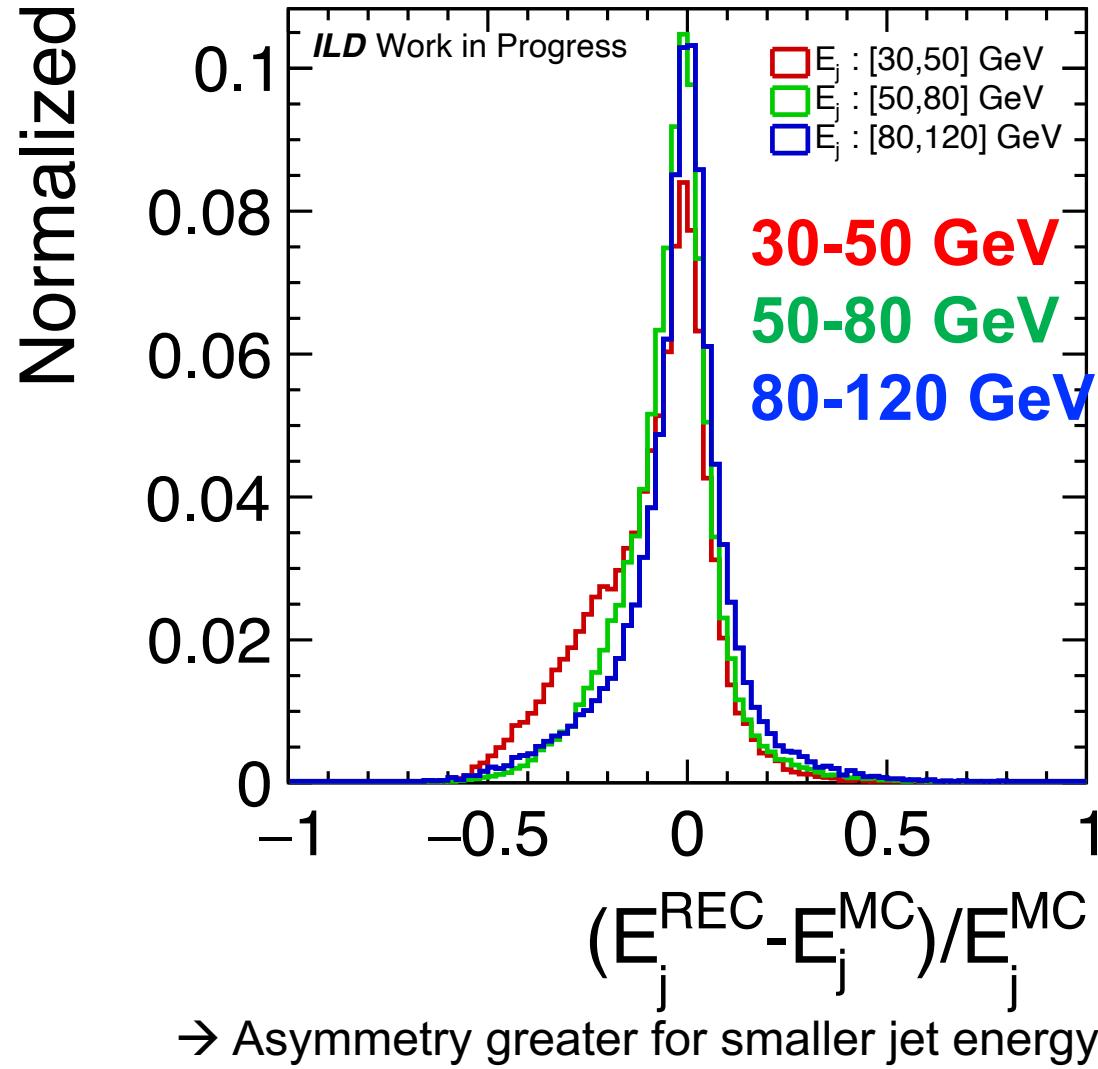
1. Constraints on missing 4-momentum
2. Non-Gaussian fit objects
3. Non-Gaussian constraints

- Example benchmark process:
  - $e^+e^- \rightarrow ZH \rightarrow vvbb$  at 250 GeV

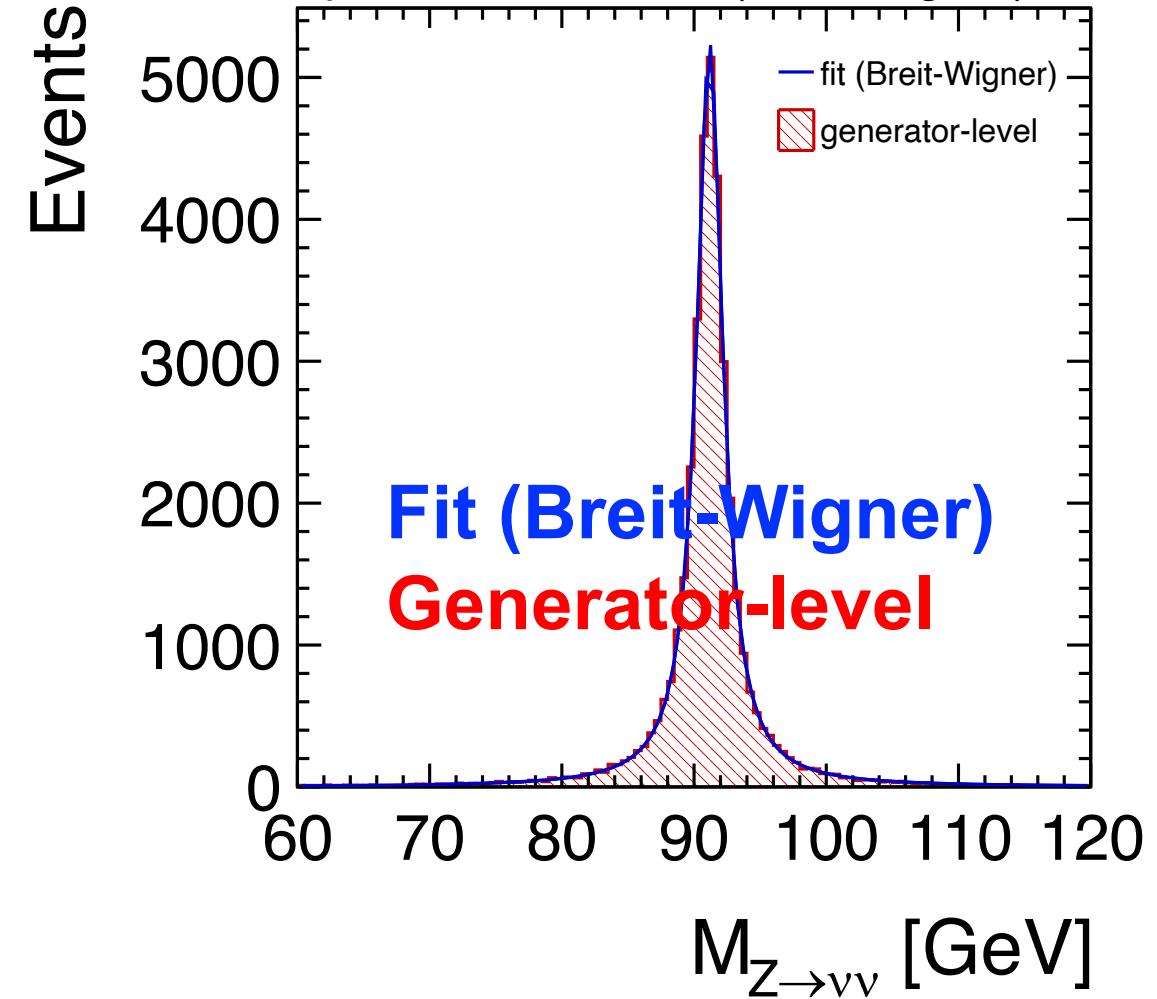


# Non-Gaussian Distributions

- Non-Gaussian Measurements
- Example: b jet energy



- Non-Gaussian Constraints
- Example: Z boson mass (Breit-Wigner)



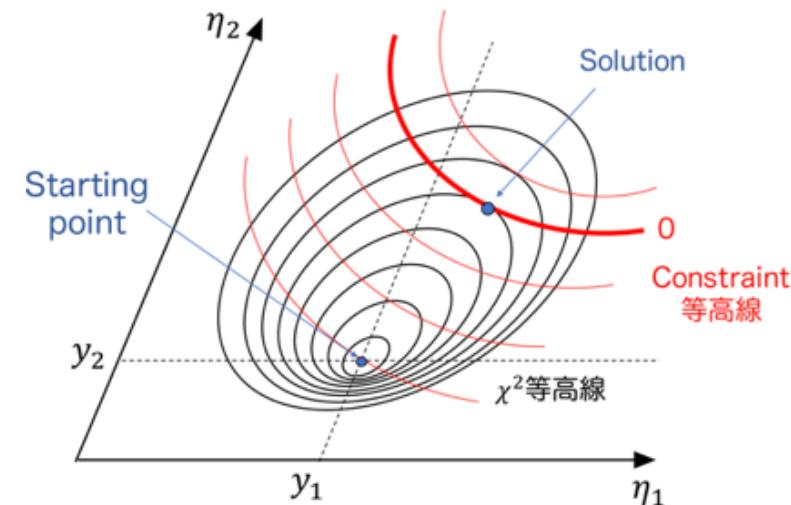
# Kinematic Fit

- Define chi-squared value to be extremized as follows:

e.g. MarlinKinfit implementation:

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = (\vec{y} - \vec{\eta})^T V^{-1} (\vec{y} - \vec{\eta}) + 2 \sum_{k=1}^K \lambda_k h_k(\vec{\eta}, \vec{\xi}) + \sum_{l=1}^L \left( \frac{g_l(\vec{\eta}, \vec{\xi})}{\sigma_{g_l}} \right)^2$$

↔ Fit Objects    
 ↔ Hard Constraints    
 ↔ Soft Constraints



Solve for extremum:  
 $(\nabla_{\vec{a}} \chi_T^2 = \vec{0}, \vec{a} = \vec{\lambda}, \vec{\eta}, \vec{\xi})$   
 using e.g. Newton's method

- Fit Object** terms define the distributions of measured values
  - e.g. jet energy/angle, etc.
- Hard Constraint** terms define fixed relations between parameters (via Lagrange multipliers)
  - e.g. Higgs mass = 125 GeV
- Soft Constraint** terms define the distributions of functions of parameters
  - e.g. Z mass ~ BW distribution with mean 91.2 GeV and width 2.5 GeV

# Kinematic Fit with non-Gaussian distributions

- In order to incorporate non-Gaussian distributions, chi-squared value was implemented using **log-likelihood** functions (can be used for any probability density function):

( $f, s_l$ : probability density functions)

$$L(\vec{\eta}, \vec{\xi}) = f(\vec{y}; \vec{\eta}) \prod_{k=1}^K \delta(h_k(\vec{\eta}, \vec{\xi})) \prod_{l=1}^L s_l(\vec{\eta}, \vec{\xi})$$

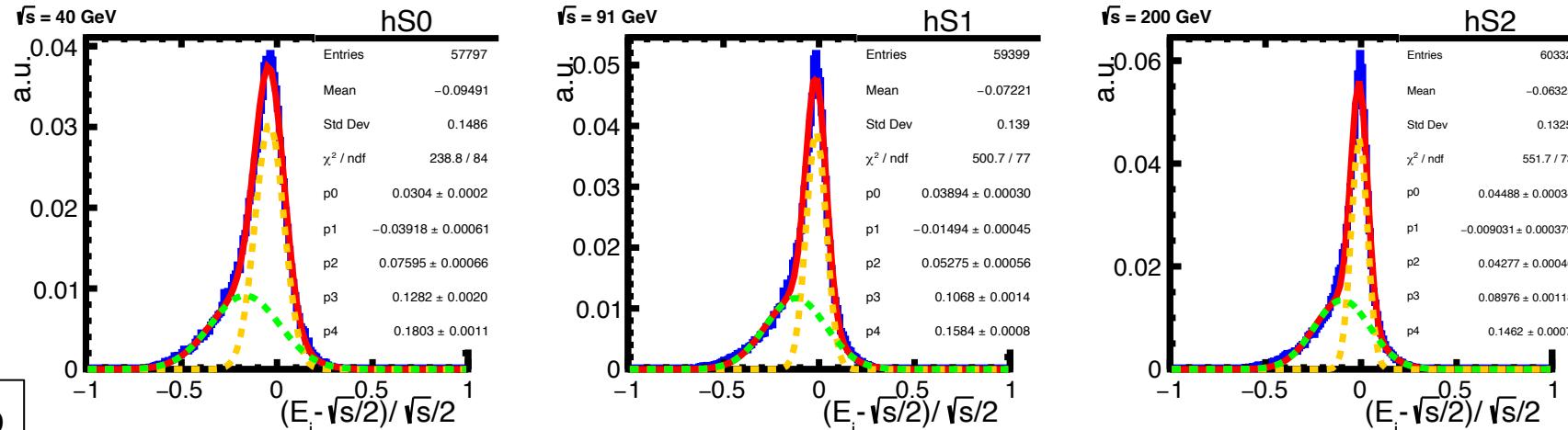
- This likelihood function  $L$  is used to compute  $\chi_T^2$  value as  $-2\ln(L)$ .
- In the case of Gaussian distributions, the formula in previous page is reproduced.

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = -2 \ln f(\vec{y}; \vec{\eta}) + 2 \sum_{k=1}^K \lambda_k h_k(\vec{\eta}, \vec{\xi}) - 2 \sum_{l=1}^L \ln s_l(\vec{\eta}, \vec{\xi})$$



Implemented in code

# Single jet energy resolution



Yu Kato

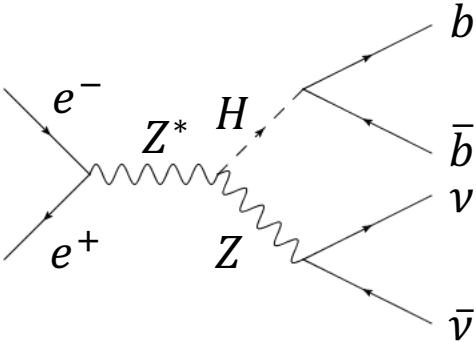
Sample: 2-jet events, I5 detector

Fit function:  $\text{Gaus}(x, \mu_1, \sigma_1) + 0.3 * \text{Gaus}(x, \mu_2 - \mu_1, \sigma_2)$  where  $x = (E^{\text{REC}} - E^{\text{MC}}) / E^{\text{MC}}$

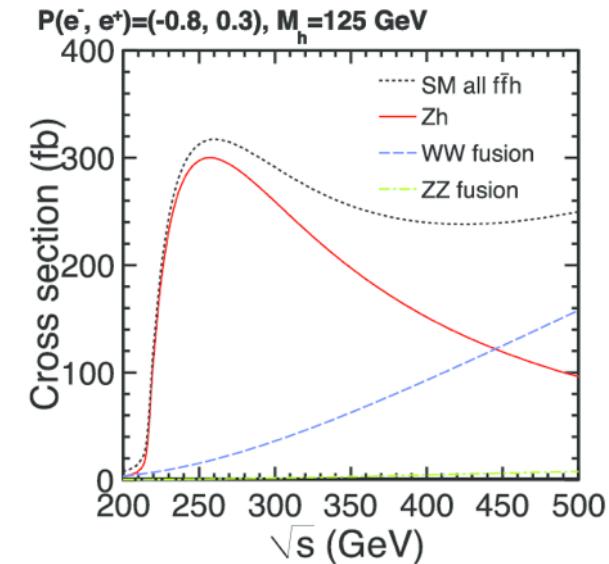
- Jet energy resolution is adjusted on a jet-by-jet basis
  - $\sigma_E = \sigma_E(E, \cos\theta)$ ,  $\sigma_\theta = \text{const.}$ ,  $\sigma_\phi = \text{const.}$
- Resolution table is created using 2-jet samples in bins of  $E_{\text{CM}}$  and  $|\cos\theta|$ 
  - $E_{\text{CM}}$ : 45, 90, 200, 350, 500 [GeV]
  - $|\cos\theta|$ : 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.925, 0.95, 0.975, 1.0
- Given a reconstructed jet, use table to look up the JER (interpolated in energy)

# Signal and Backgrounds

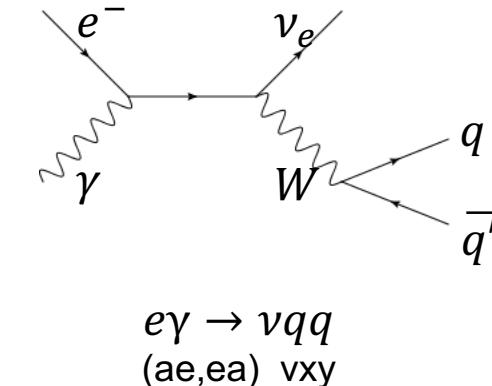
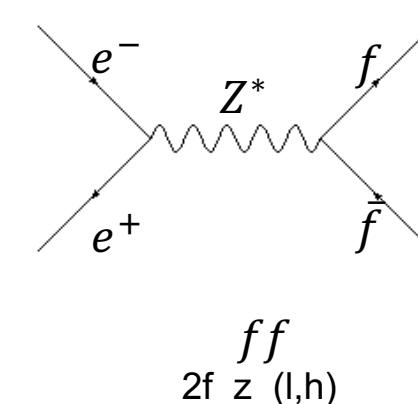
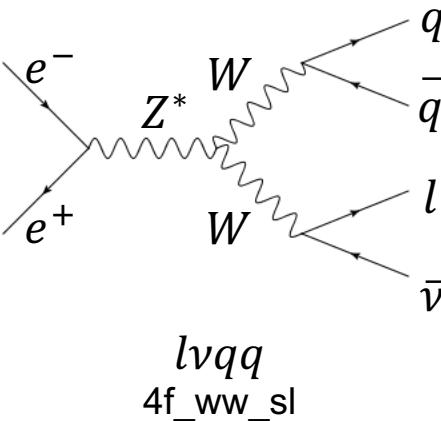
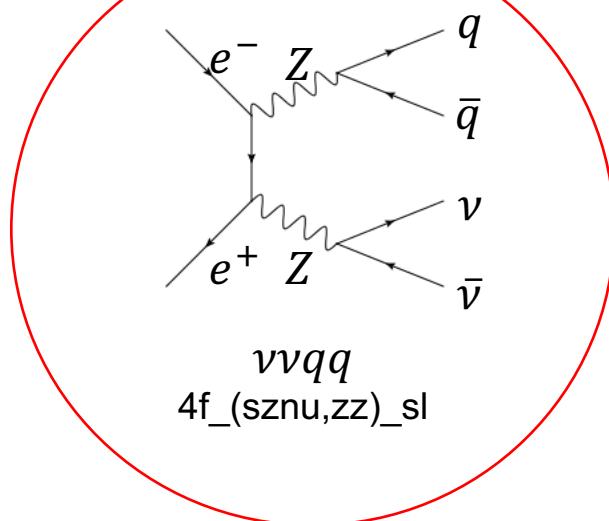
- Signal



- s-channel
- 2-jet mass = Higgs mass
- Missing mass = Z mass
- No isolated lepton



- Main backgrounds



# Software Setup

- Simulation Setup

- Detector Model: DBD (2012)
- Event Generator: WHIZARD 1.95
- CM Energy:  $\sqrt{s} = 250 \text{ GeV}$
- Beam Polarization:  $(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$
- Integrated Luminosity:  $\int \mathcal{L} dt = 900 \text{ fb}^{-1}$

- Event Reconstruction

1. Particle Reconstruction (PandoraPFA)
2. Jet Reconstruction (Durham)
  - forced 2 jets
3. b-tagging (LCFIPlus)

# Event Selection

“pre-selection”



- $80 < M_{\text{mis}} < 140 \text{ [GeV]}$
  - $20 < P_{T,\text{vis}} < 70 \text{ [GeV]}$
  - $P_{L,\text{vis}} < 60 \text{ [GeV]}$
- } Require missing 4-momentum  
Reduce t-channel processes
- 
- $N_{\text{charged}} > 10$
  - $P_{\text{max}} < 30 \text{ [GeV]}$
  - $Y_{23} < 0.02$
  - $0.2 < Y_{12} < 0.8$
  - $105 < M_{jj} < 135 \text{ [GeV]}$
  - $b\text{prob}_1, b\text{prob}_2 > 0.5$
  - $\text{coplanarity} < 3.08 \text{ [rad]}$
  - $\cos j12 < -0.46$
- } Require jets
- } Require 2 jets+missing topology (signal-like)
- } Require b-jets
- } Eliminate 2f bkg.
- } Reduce  $vvqq$  bkg.

\* cut table in backup slide

# Results

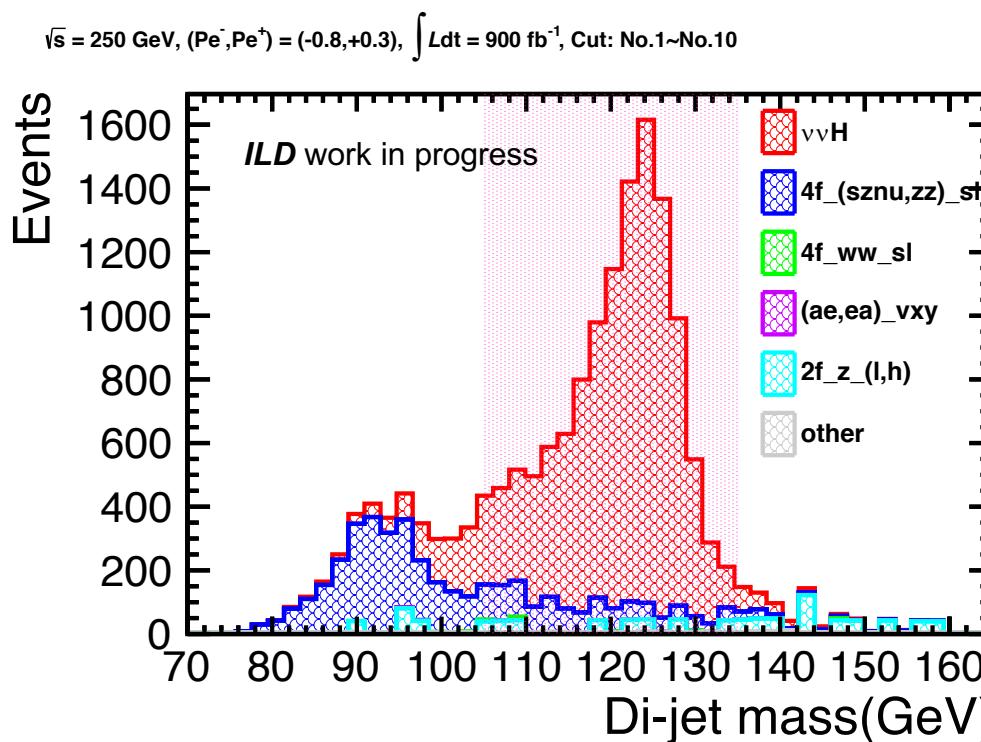
Integrated Luminosity =  $900\text{fb}^{-1}$ , Polarization:  $(e^-, e^+) = (-0.8, +0.3)$

$$\text{Significance} = \frac{N_S}{\sqrt{N_S + N_B}}$$

$N_S$  : # signal events,  $N_B$  : # bkg events

	$N_S$	$N_B$	Signif.	$\frac{\Delta(\sigma \cdot BR)}{\sigma \cdot BR}$
w/o fit	10816	1527	97.4	1.03%
w/ fit	TODO			

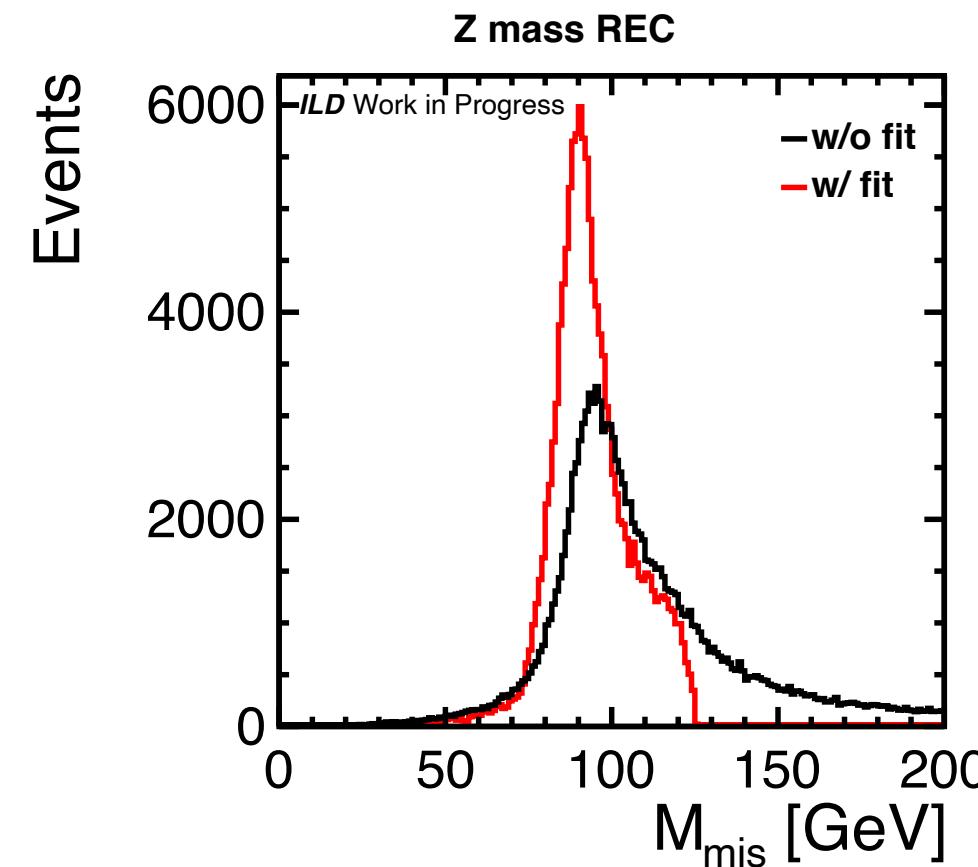
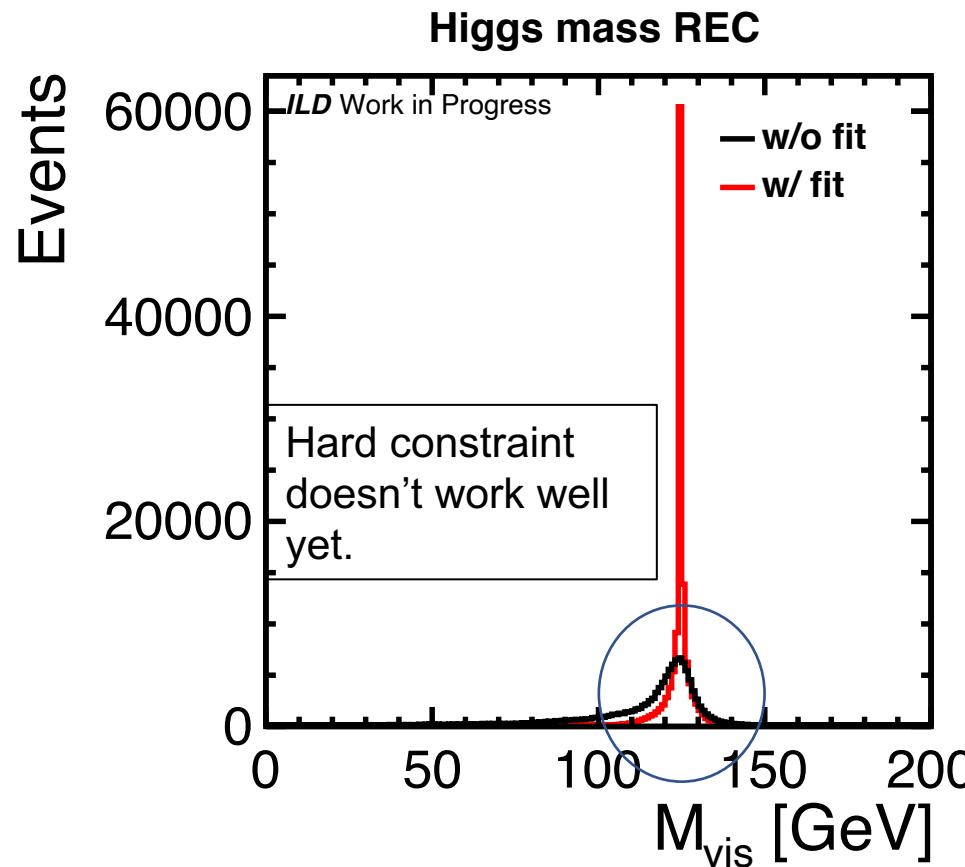
stat. error only



w/o  
kinematic  
fit

# Signal with Kinematic Fit (1/2)

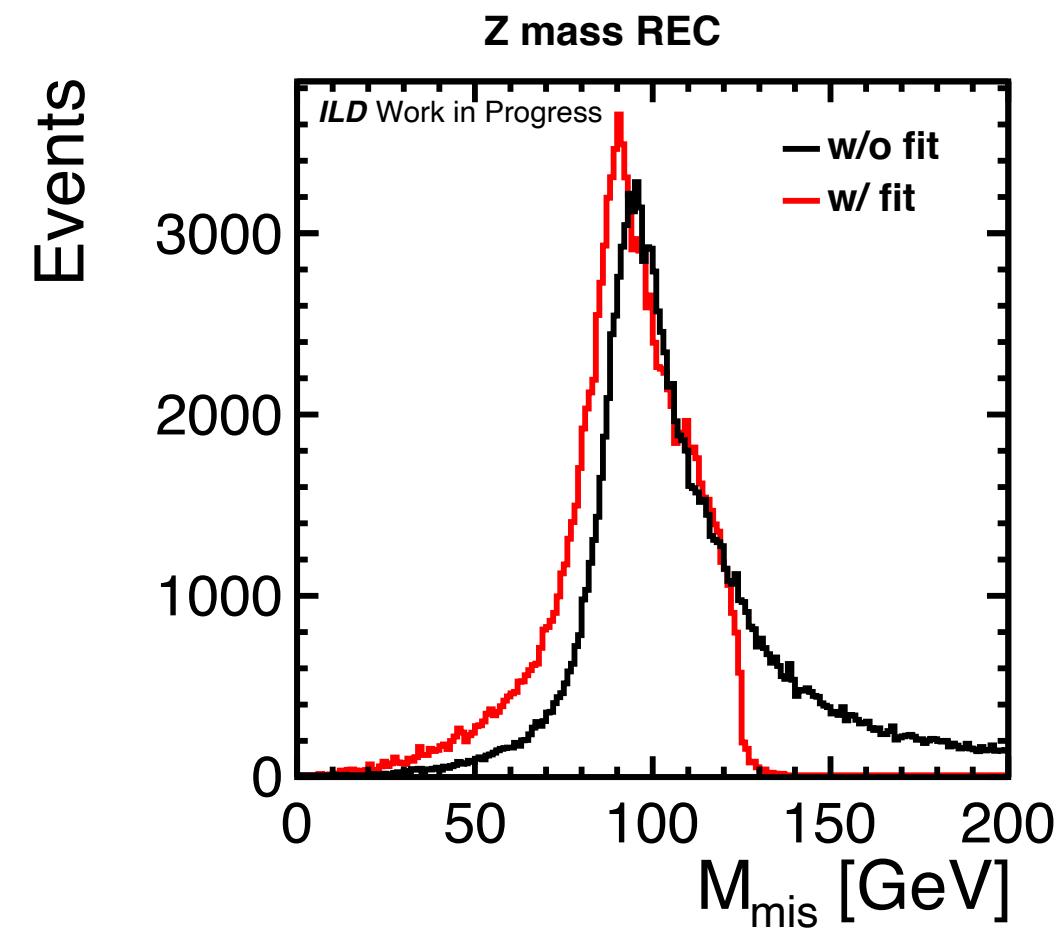
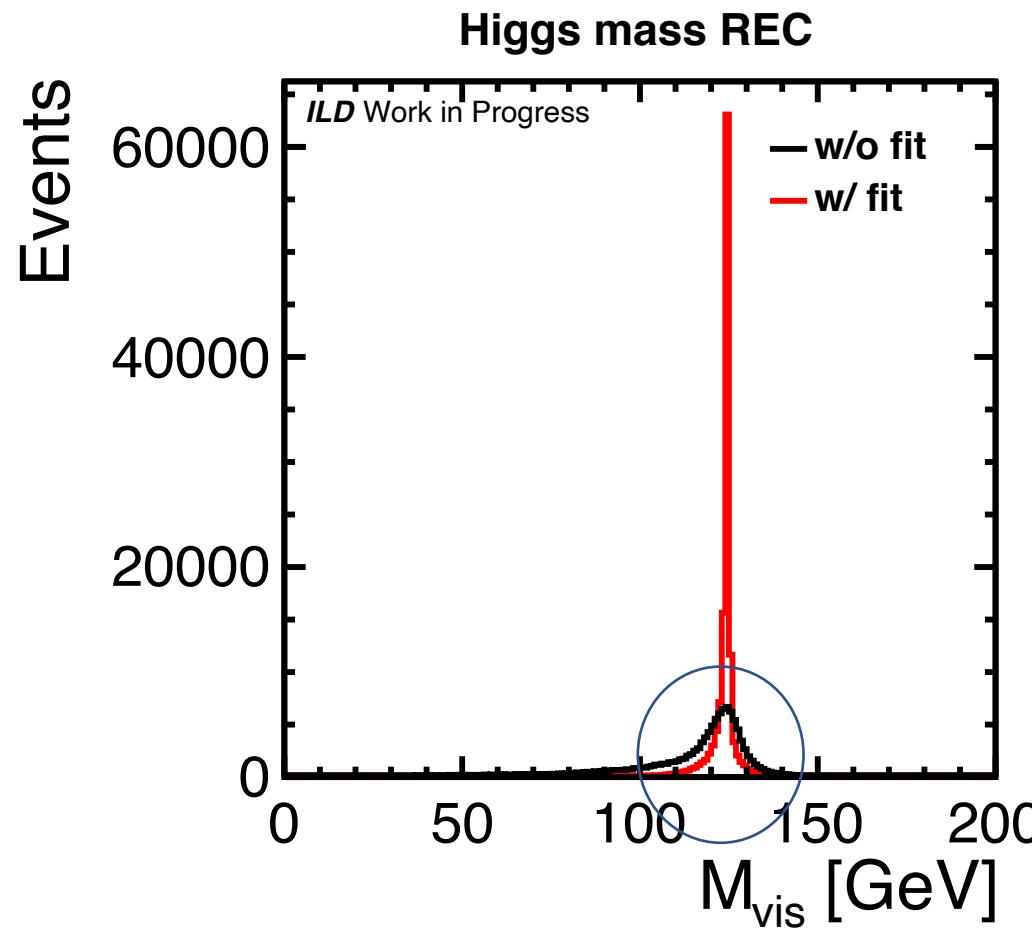
SIGNAL (vvbb)



Fit Object : **Single Gauss** & Soft Constraint : BW

# Signal with Kinematic Fit (2/2)

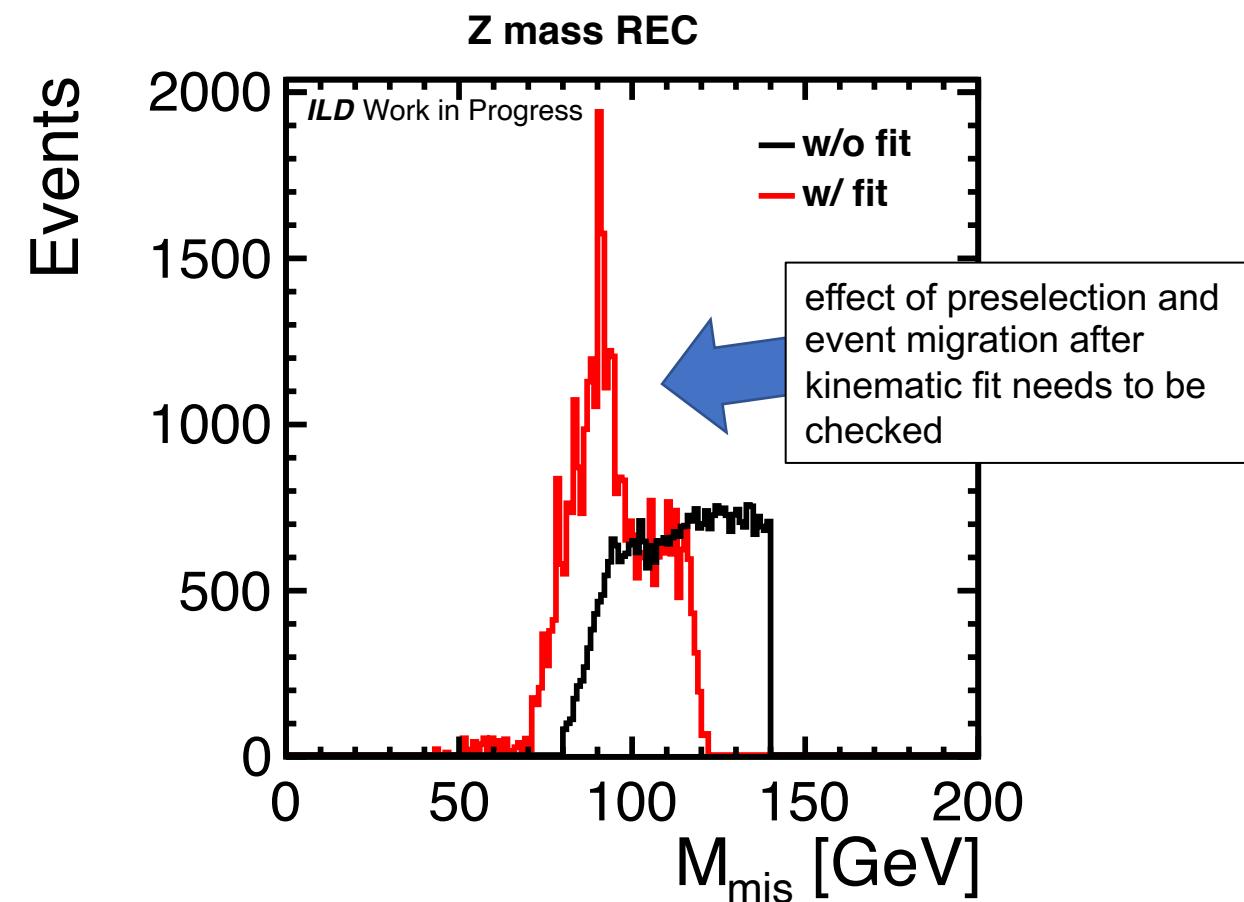
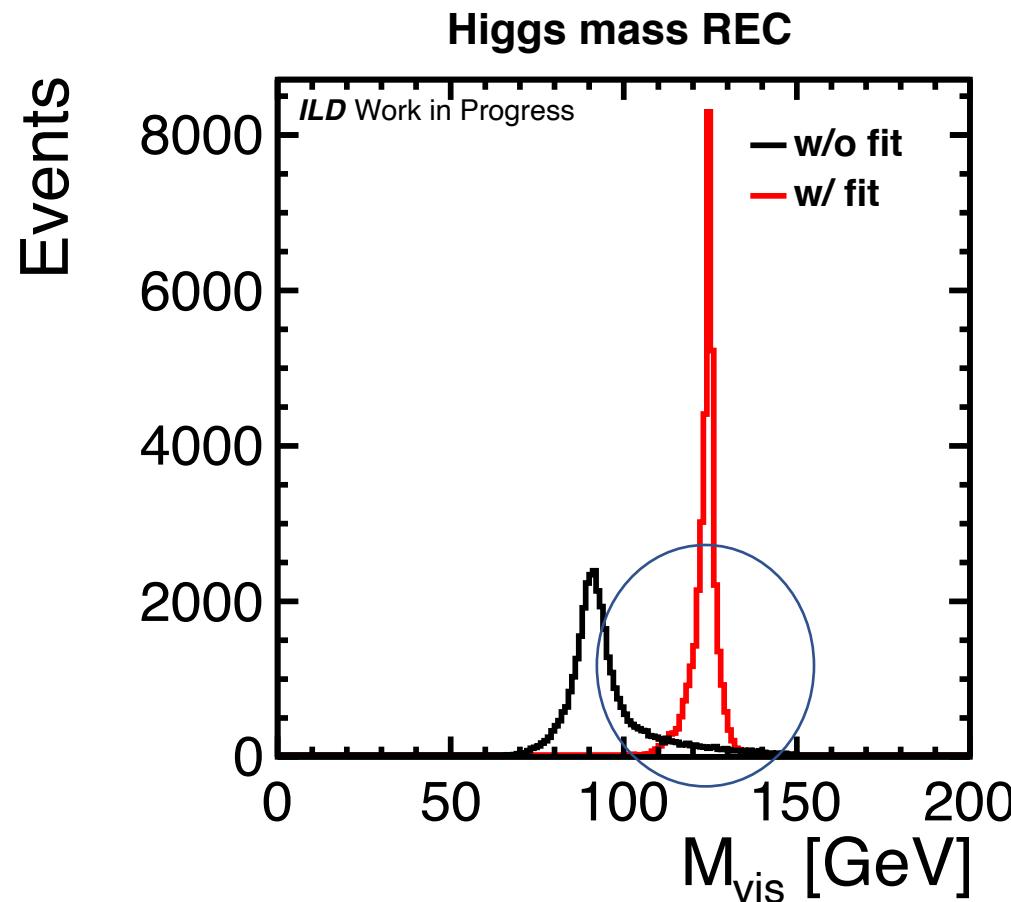
SIGNAL (vvbb)



Fit Object : **Double** Gauss & Soft Constraint : BW

→ Still struggling with numerical issues in chi2 derivative computation, under investigation

# Test with 4f background (pre-selection)



Fit Object : **Single Gauss & Soft Constraint : BW**

Kinematic fit succeeds on “signal-like” events

→ Overall effect on the significance will be checked after re-optimization

# Some Comments about the Implementation

- We rely on numerical differentiation for the capability to deal with non-Gaussian distributions
  - However, for derivatives which are trivially zero by analytic computation (e.g. cross terms), the numerical method can still give non-zero values
    - Dedicated treatment distinguishing these cases is being implemented
- If the Newtonian step diverges, we back up and retry the iteration with slightly different values.
  - Appears to help with convergence for some events
    - The retry criteria still need to be optimized
- Current speed of our kinematic fit code is about ~30 events/s (signal) and ~10 events/s (background) at KEKCC

# Summary and Outlook

- We are developing kinematic fits based on the log-likelihood method, with the following aims:
  1. Apply constraints on missing 4-momentum
  2. Implement non-Gaussian fit objects
  3. Implement non-Gaussian constraints
- Schedule and plans:
  - Started implementation in 1st week of September.
  - Plan to release the standalone fit code by end of the year.
  - Will consult software experts for advice on incorporating our code into MarlinKinfit.

# additional

# Cut Flow

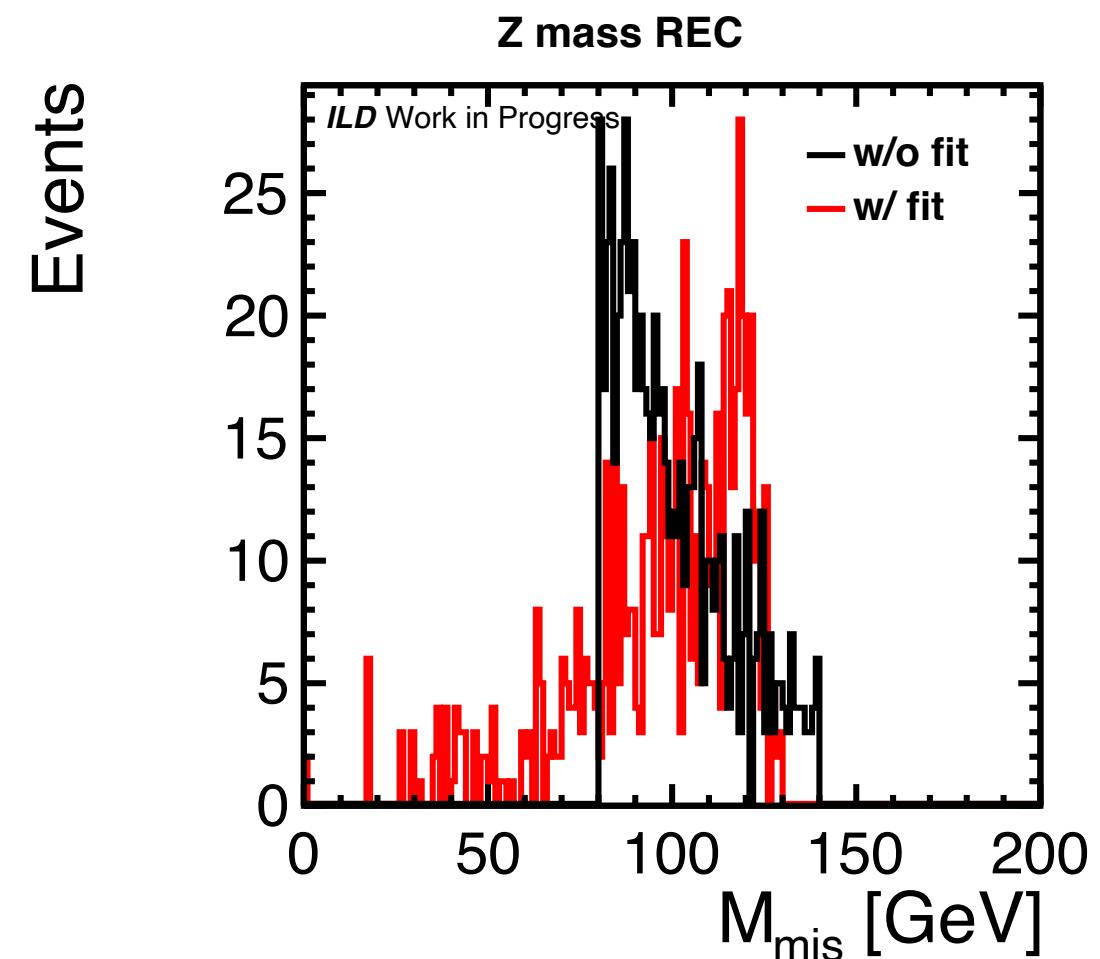
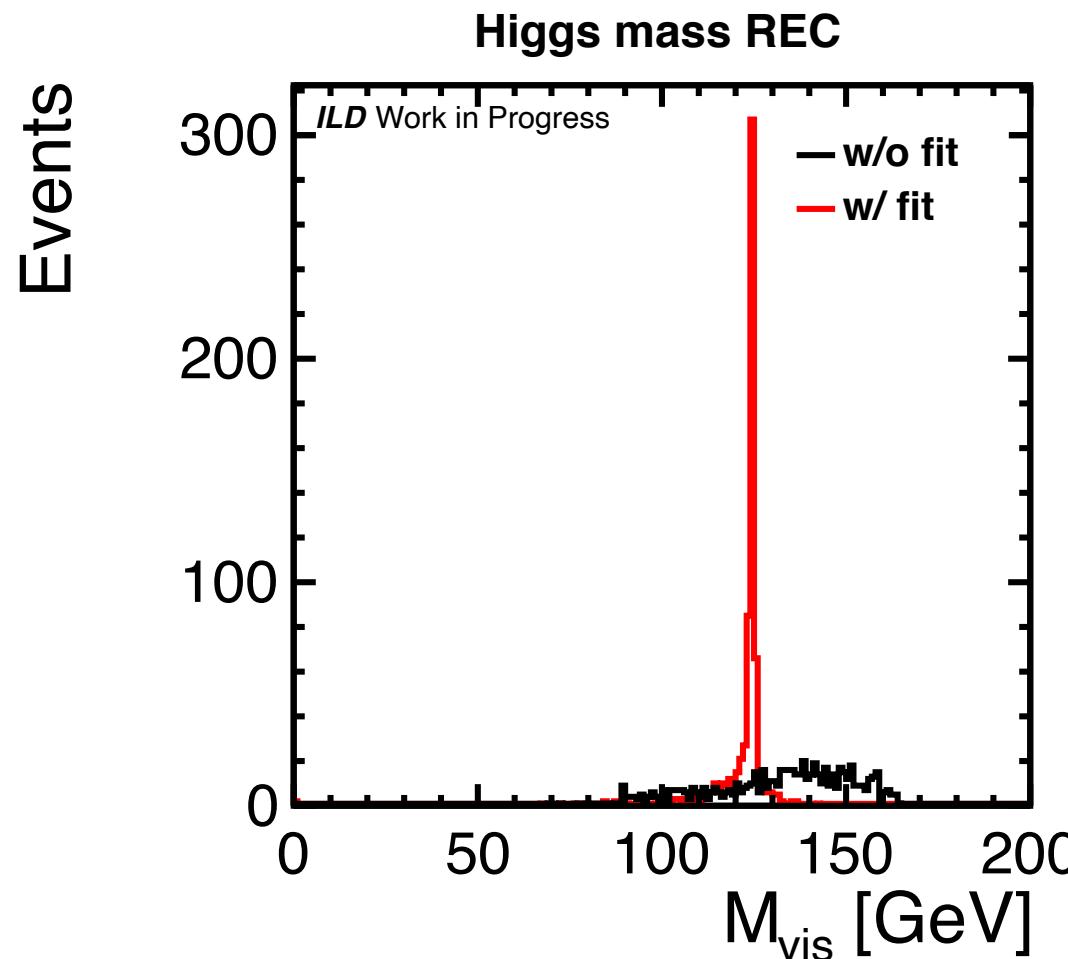
Integrated Luminosity = 900fb<sup>-1</sup>, Polarization: (e-,e+) = (-0.8,+0.3)

Cut	vvH (H→bb)	vvH (H →bb)	qqvv (4f_sznu/zz_sl)	qqlv (4f_ww_sl)	ff (2f_z_l/h)	2f,4f other	eγ→qqv (ae/ea_vxy)	eγ,γγ other
Expected	40333	29447	1.02e+6	9.89e+6	8.19e+7	4.87e+7	1.11e+6	1.82e+8
80<M <sub>miss</sub> <140 [GeV]	35087	18773	500845	1.15e+6	7.73e+6	2.03e+6	431090	1.82e+8
20<P <sub>T,vis</sub> <70 [GeV]	31620	16668	318747	804038	658021	1.27e+6	310855	125534
P <sub>L,vis</sub> <60 [GeV]	31181	16238	173041	563153	460775	987929	192999	27683
N <sub>charged</sub> >10	31181	15447	173007	563138	99090	189194	192978	5895
P <sub>max</sub> <30 [GeV]	29077	13635	152515	368915	45531	44629	173796	1574
Y <sub>23</sub> <0.02	21837	4640	111609	91723	32126	15869	133085	692
0.2<Y <sub>12</sub> <0.8	20164	4397	88250	76468	25126	12799	98468	552
105<M <sub>jj</sub> <135[GeV]	17925	3795	11657	27196	11720	6116	2963	240
bprob <sub>1,2</sub> >0.5	11327	44	1373	21	1976	82	5	7
coplanarity <3.08 [rad]	10902	42	1326	21	315	78	5	0
cosj12<-0.46	10816	40	1072	21	315	74	5	0

cut1: based on existing work

cut2: new cuts for this study

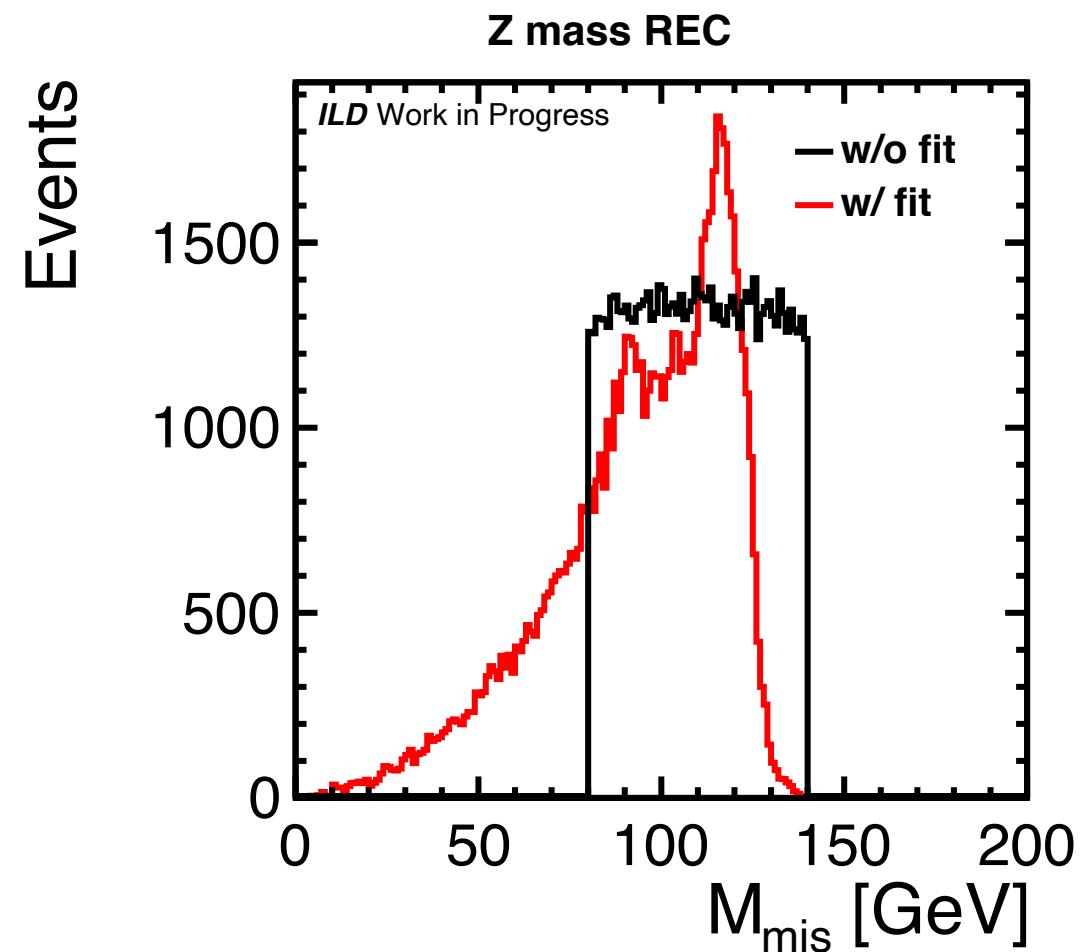
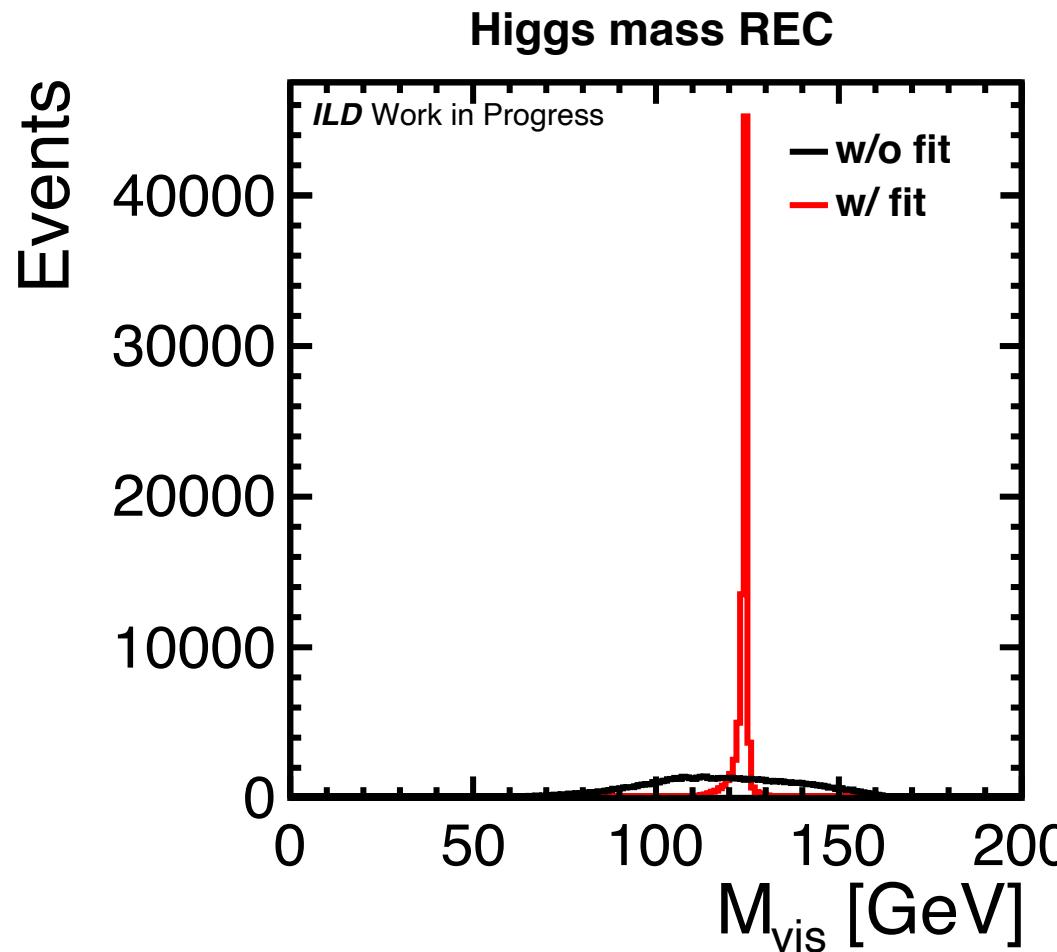
# BG( $e^+e^- \rightarrow Z \rightarrow qq$ ) Reconstruction



Fit Object : double Gauss  
Soft Constraint : BW

# BG( $e^+e^- \rightarrow Z \rightarrow \ell\ell$ ) Reconstruction

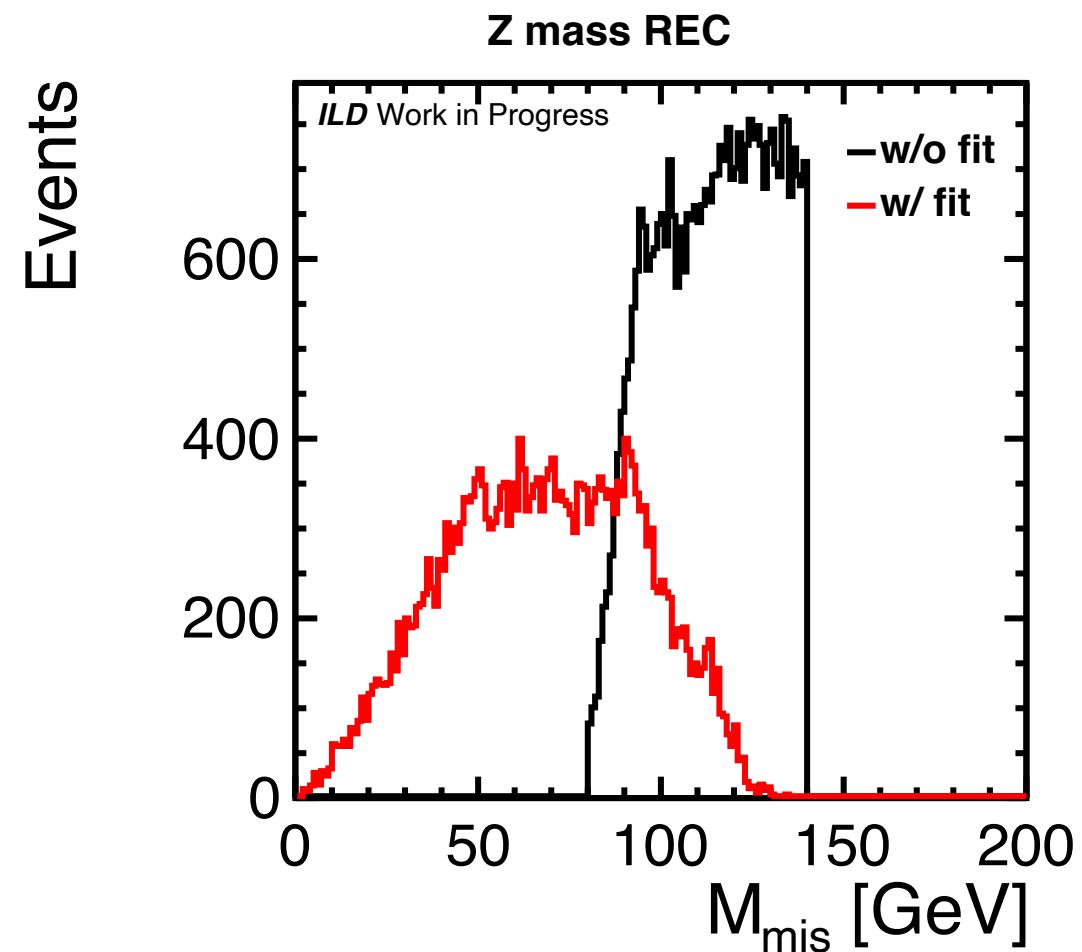
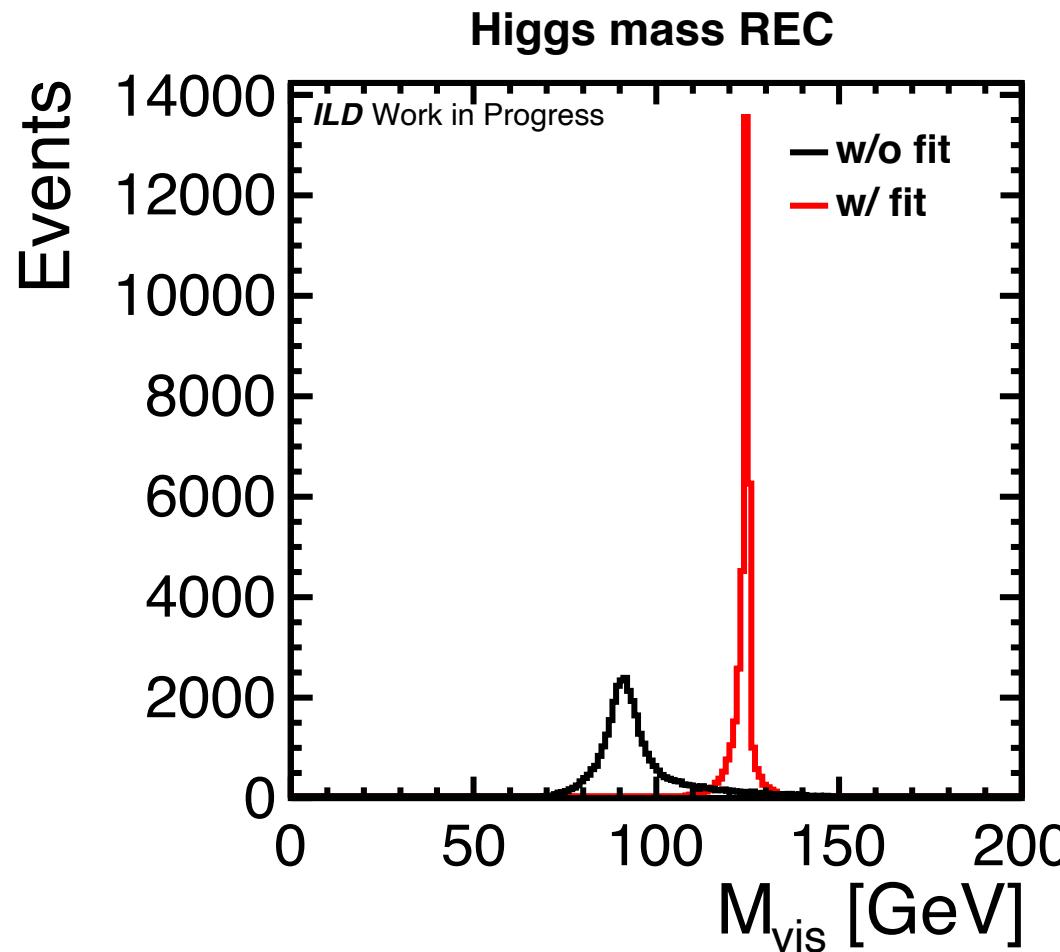
WORK IN PROGRESS



Fit Object : double Gauss  
Soft Constraint : BW

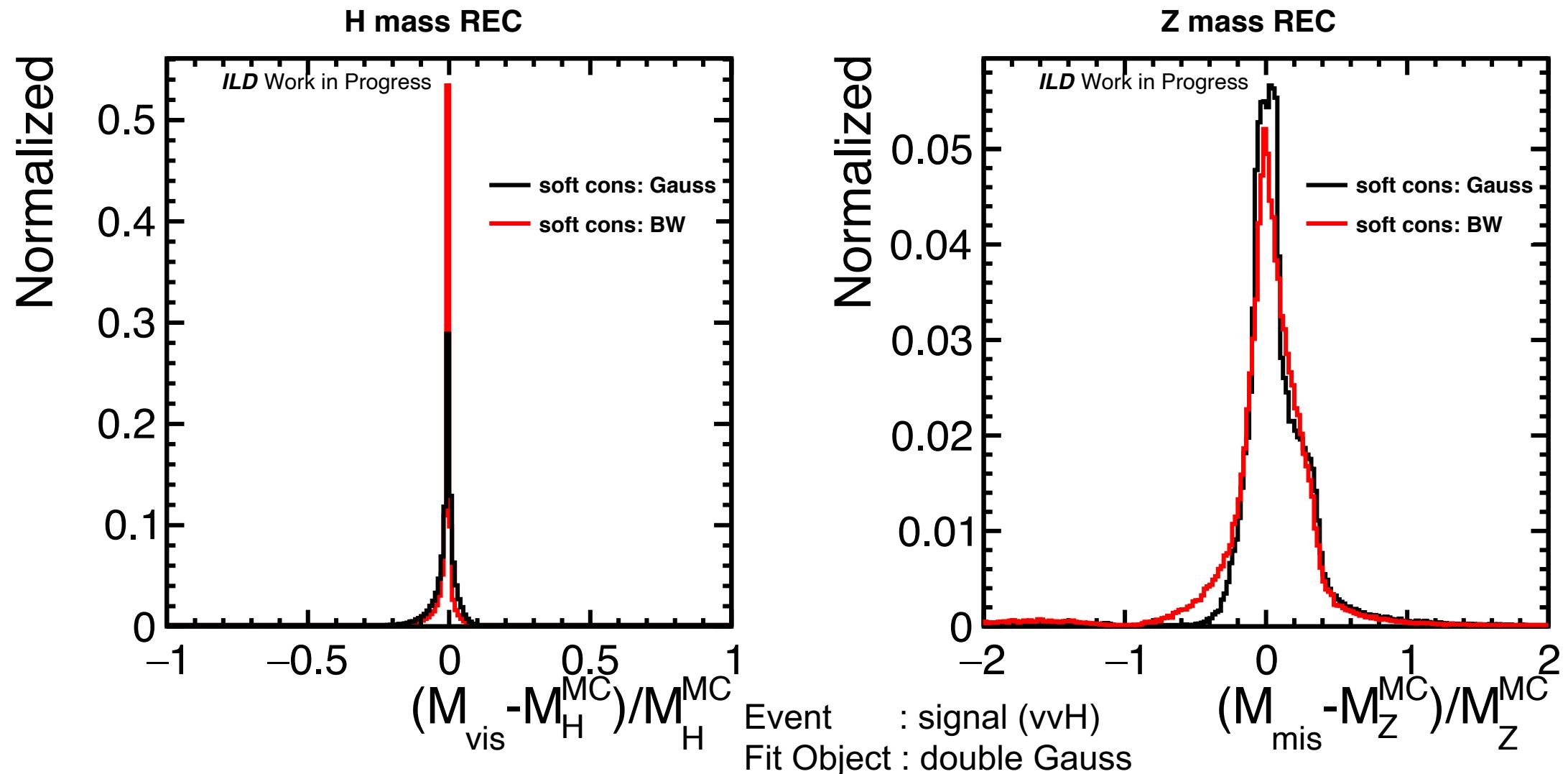
# BG( $e^+e^- \rightarrow ZZ \rightarrow vvqq$ ) Reconstruction

WORK IN PROGRESS



Fit Object : double Gauss  
Soft Constraint : BW

# soft constraint comparison



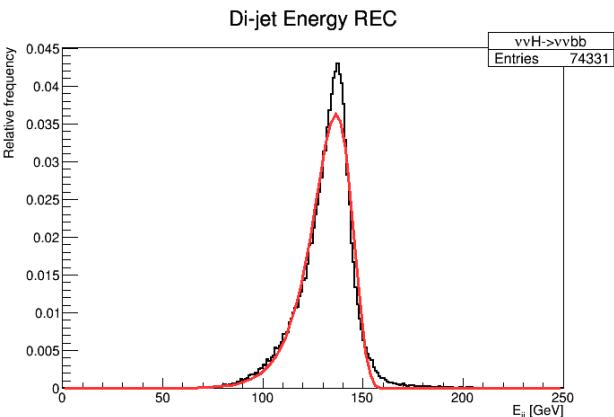
# Example for ee $\rightarrow$ ZH $\rightarrow$ vvbb

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = -2 \ln f(\vec{y}; \vec{\eta}) + 2 \sum_{k=1}^K \lambda_k h_k(\vec{\eta}, \vec{\xi}) - 2 \sum_{l=1}^L \ln s_l(\vec{\eta}, \vec{\xi})$$

$$-2 \ln f(\vec{y}; \vec{\eta}) = \sum_{i=0}^1 \left\{ -2 \ln \frac{e^{\left(\frac{E_i - \widehat{E}_i}{\eta_{E_i}}\right)} - e^{\frac{E_i - \widehat{E}_i}{\eta_{E_i}}}}{\eta_{E_i}} + \left(\frac{\theta_i - \widehat{\theta}_i}{\sigma_{\theta_i}}\right)^2 + \left(\frac{\phi_i - \widehat{\phi}_i}{\sigma_{\phi_i}}\right)^2 \right\} \text{ Fit Object}$$

Gumbel distribution

$$G(x; \mu, \eta) = \frac{1}{\eta} e^{\frac{x-\mu}{\eta}} - e^{\frac{x-\mu}{\eta}}$$



$$\vec{h}(\vec{\eta}, \vec{\xi}) = \begin{bmatrix} \overrightarrow{\widehat{p}_{vis}} + \overrightarrow{\widehat{p}_{mis}} = \overrightarrow{p_{cm}} \\ \widehat{M_{vis}} = M_H \end{bmatrix}$$

Hard Constraint for momentum conservation and Higgs mass

$$-2 \sum_{l=1}^L \ln s_l(\vec{\eta}, \vec{\xi}) = -2 \ln \frac{1}{\pi \Gamma_{m_Z} \left\{ 1 + \left( \frac{\widehat{M_{mis}} - M_Z}{\Gamma_{M_Z}} \right)^2 \right\}}$$

Soft Breit-Wigner constraint

# Outline

purposes

1. to apply Kinematic Fit for measurement values non Gaussian distribution on the analysis for the ILC
2. to develop Kinematic Fit to introduce non Gaussian distribution as Soft Constraint on the analysis for the ILC

reasons

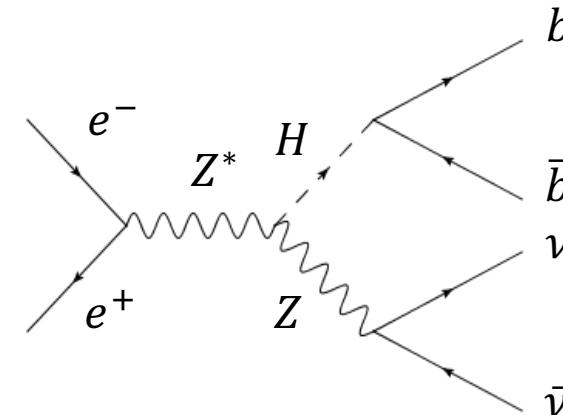
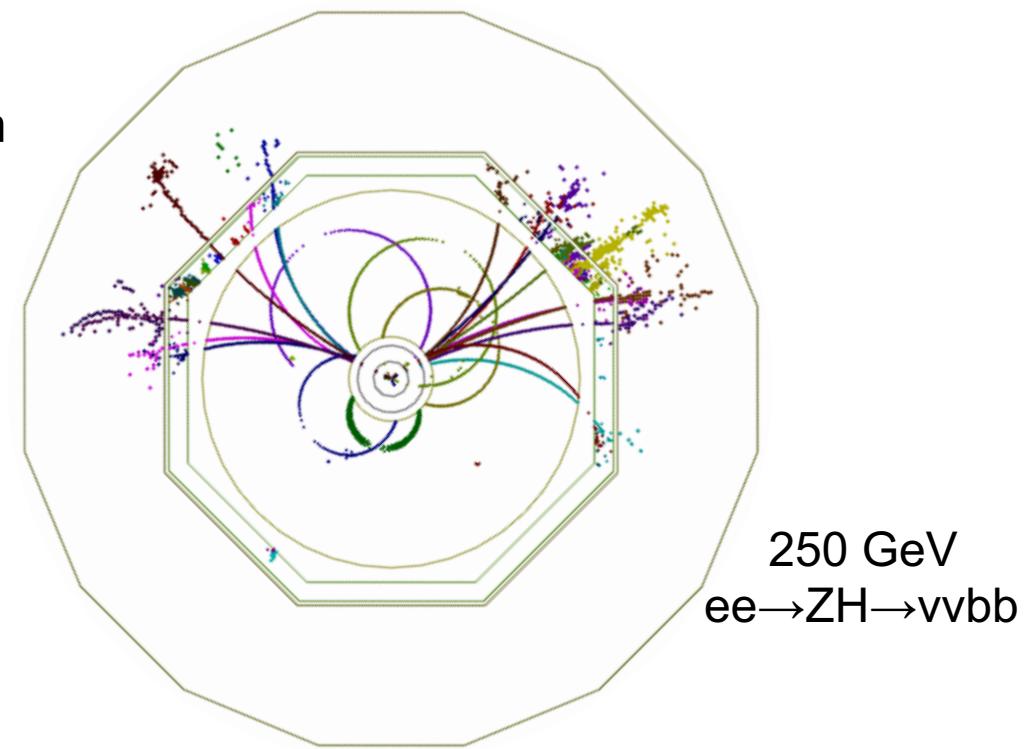
1. example of measurement values with non Gaussian resolution: b jet energy
  2. example of Soft Constraint
    - mass of Z boson (Breit-Wigner distribution)
    - decline in  $E_{cm}$  because of ISR or beam-beam effect
- Conventional Kinematic Fit assumes Gaussian distribution.



to develop Kinematic Fit using general distribution

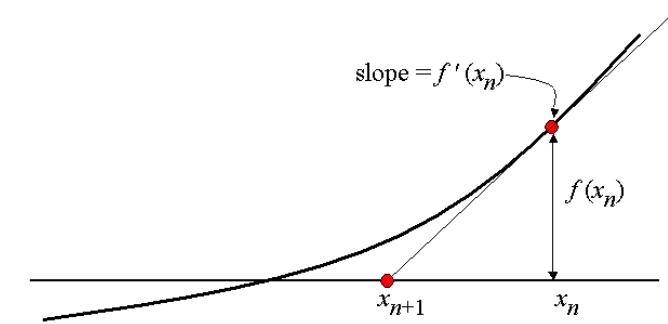
Benchmark process:

$ee \rightarrow ZH \rightarrow vvbb$



# Newton's method

- a library for Kinematic Fit
- uses Newton's method for chi-square minimization  $\chi_T^2$   
$$f(x) = 0 \Rightarrow f'(x^n)(x^n - x^{n+1}) = f(x^n)$$
- Both **Hard Constraint** • **Soft Constraint** are available.
- following problems
  - Fit Object : can use only when measured values follow Gaussian distribution
  - Soft Constraint: assuming Gaussian distribution, not general distribution

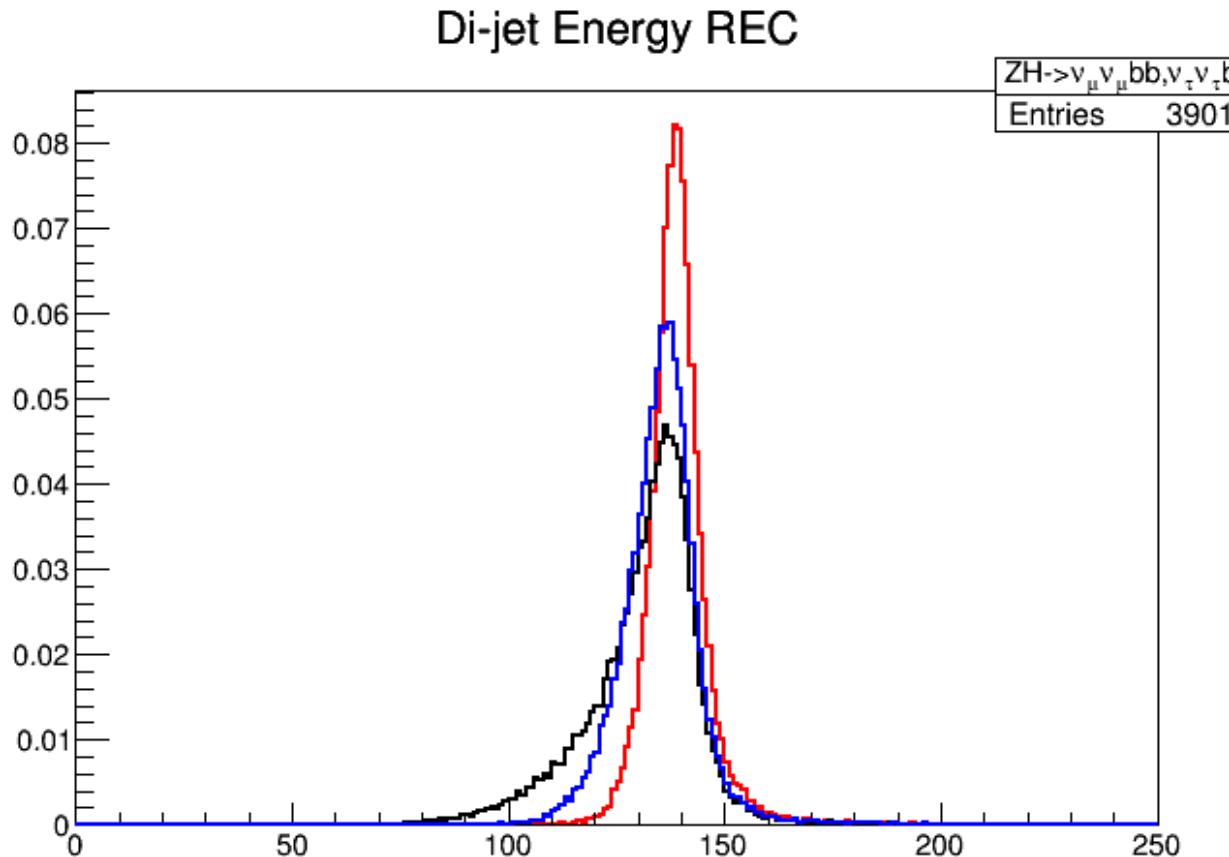


$\vec{y}$ :measured value,  $\vec{\eta}$ :parameter for measured,  $V$ :covariance matrix,  $\vec{\xi}$ : parameter for unmeasured

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = (\vec{y} - \vec{\eta})^T V^{-1} (\vec{y} - \vec{\eta}) + 2 \sum_{k=1}^K \lambda_k h_k(\vec{\eta}, \vec{\xi}) + \sum_{l=1}^L \left( \frac{g_l(\vec{\eta}, \vec{\xi})}{\sigma_{g_l}} \right)^2$$



# Recovering neutrinos in jets: $E_{jj}$



- Di-jet energy:  $E_{jj}$
- confirm recovering energy of neutrinos in jets using Monte Carlo information

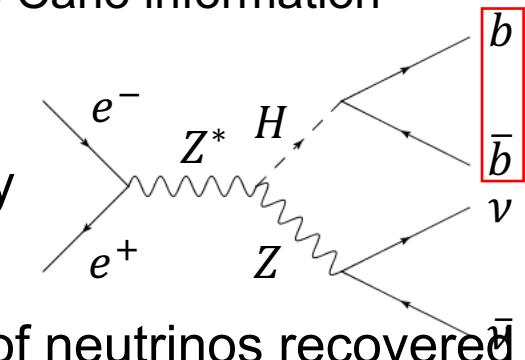
- final state:  $vvbb$

non recovery

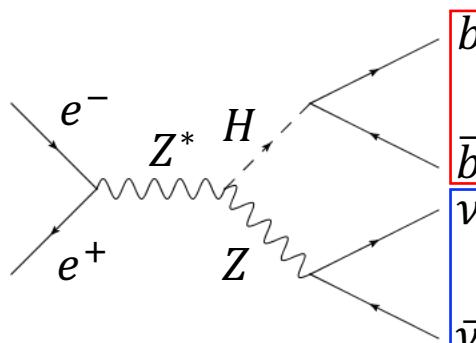
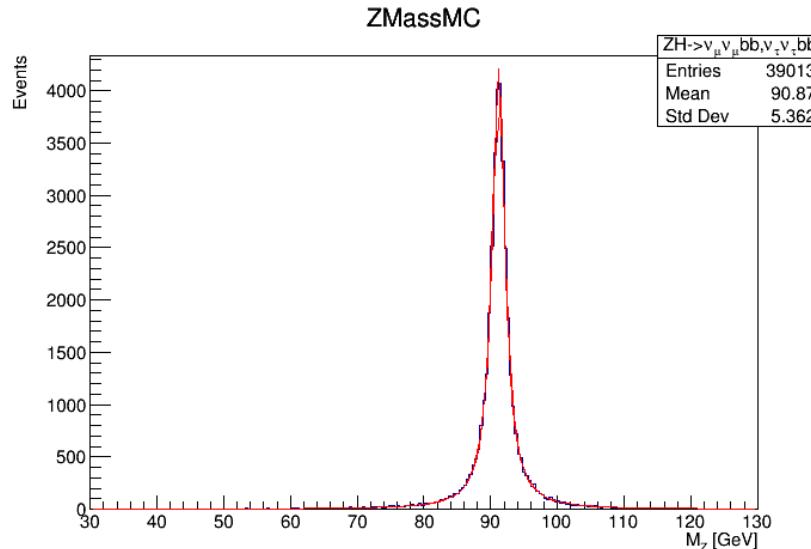
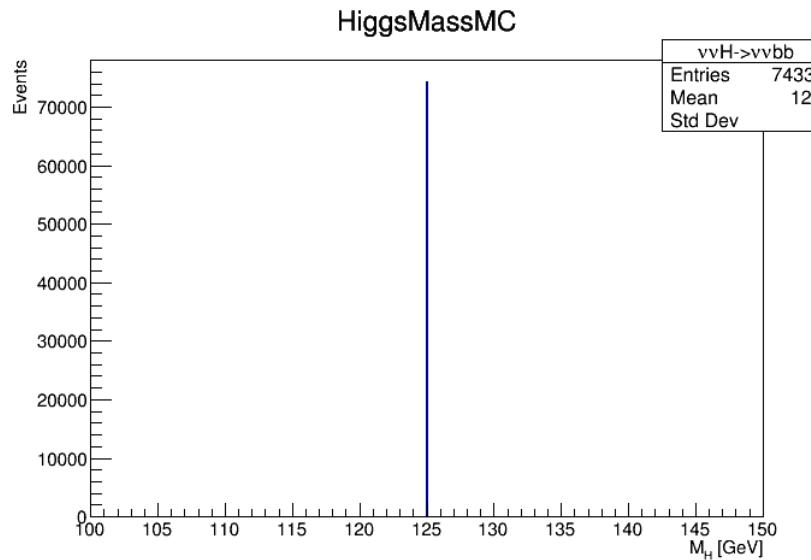
half energy of neutrinos recovered

full energy of neutrinos recovered

- It is difficult to recover all energy of neutrinos in jets.
- Asymmetrical jet energy resolution cannot be applied to conventional Kinematic.  
→new Kinematic Fit



# Constraint



- final state: vvbb

- $\Gamma_H < 0.013 [\text{GeV}]$

- $\Gamma_Z \sim 2.5 [\text{GeV}]$

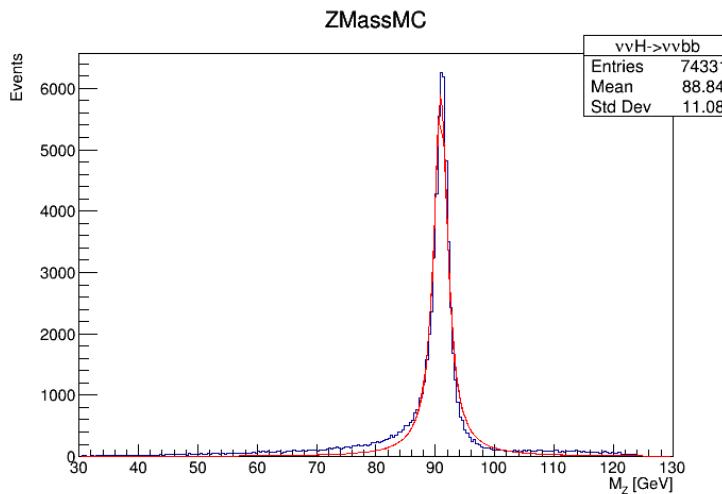


$M_H \rightarrow$  Hard Constraint



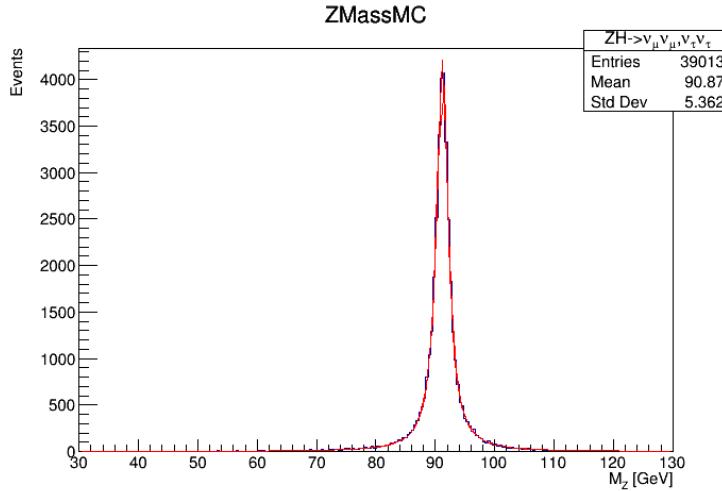
$M_Z \rightarrow$  Soft Breit-Wigner Constraint

# $M_Z \rightarrow$ Soft Constraint



- the flavor of neutrinos :  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$

→ be off BW due to  
WW-fusion



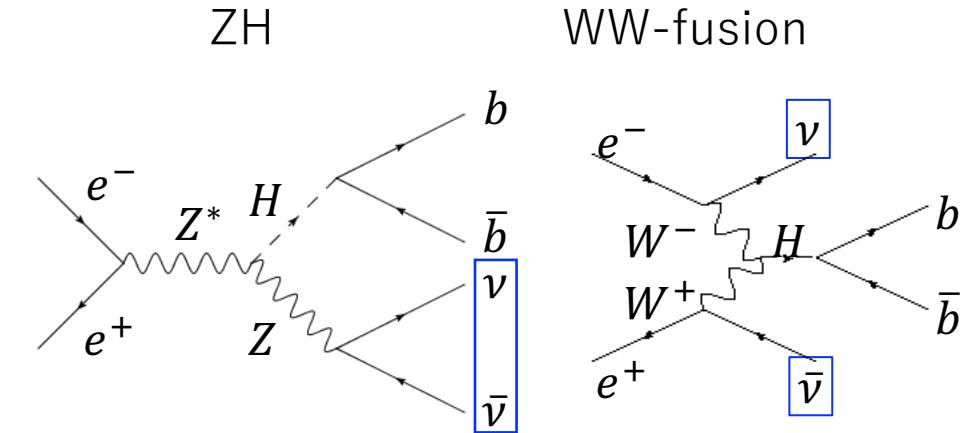
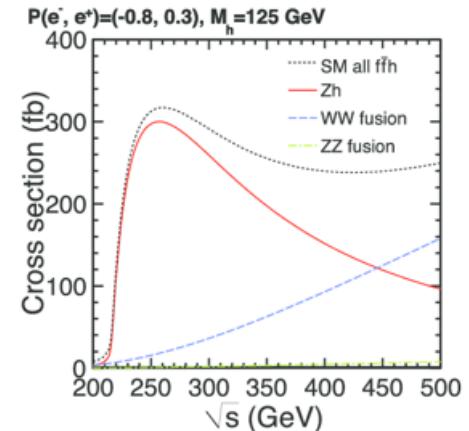
- the flavor of neutrinos :  $\nu_\mu$ ,  $\nu_\tau$
- eliminate WW-fusion

- final state : vvbb

- fit with Breit Wigner

$$\Gamma_Z \sim 2.5 \text{ [GeV]} (\text{PDG2019})$$

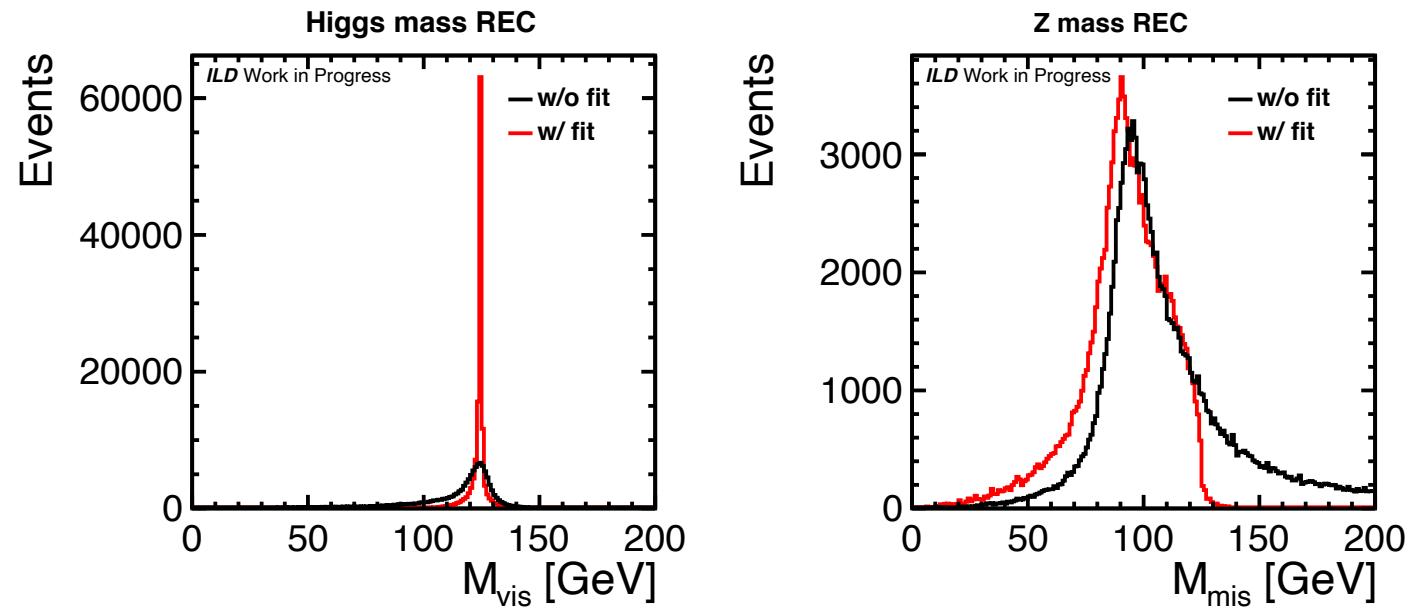
→ Soft Breit-Wigner Constraint



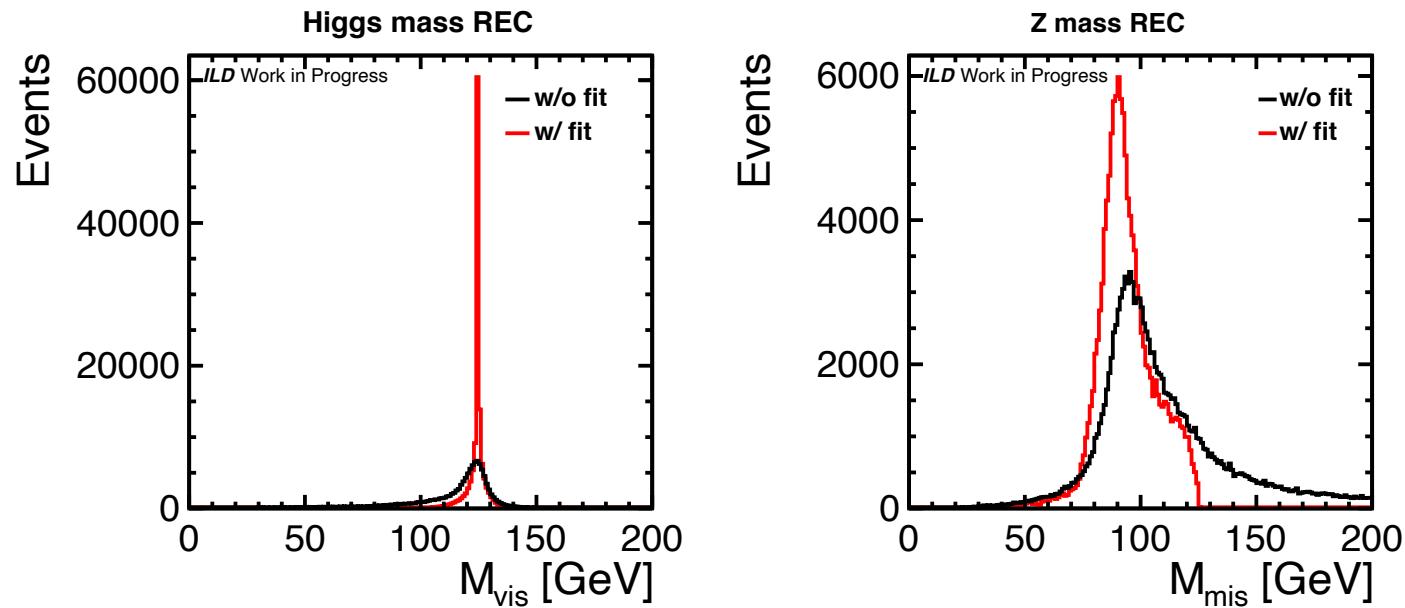
# Current Status

- Fit objects implemented:
  - Jet fit object: energy, theta, phi
    - Energy distributions: Gaussian, double Gaussian (work in progress)
  - Missing 4-momentum fit object:  $E$ ,  $p_x$ ,  $p_y$ ,  $p_z$
- Hard constraints implemented:
  - Total  $E$ ,  $p_x$ ,  $p_y$ ,  $p_z$  conservation constraint
  - Mass constraint (fixed)
- Soft constraints implemented:
  - Mass constraint: Gaussian, Breit-Wigner
- Convergence condition:
  - Change in chi-squared is smaller than a threshold value
  - OR maximum number of iterations (currently 200)

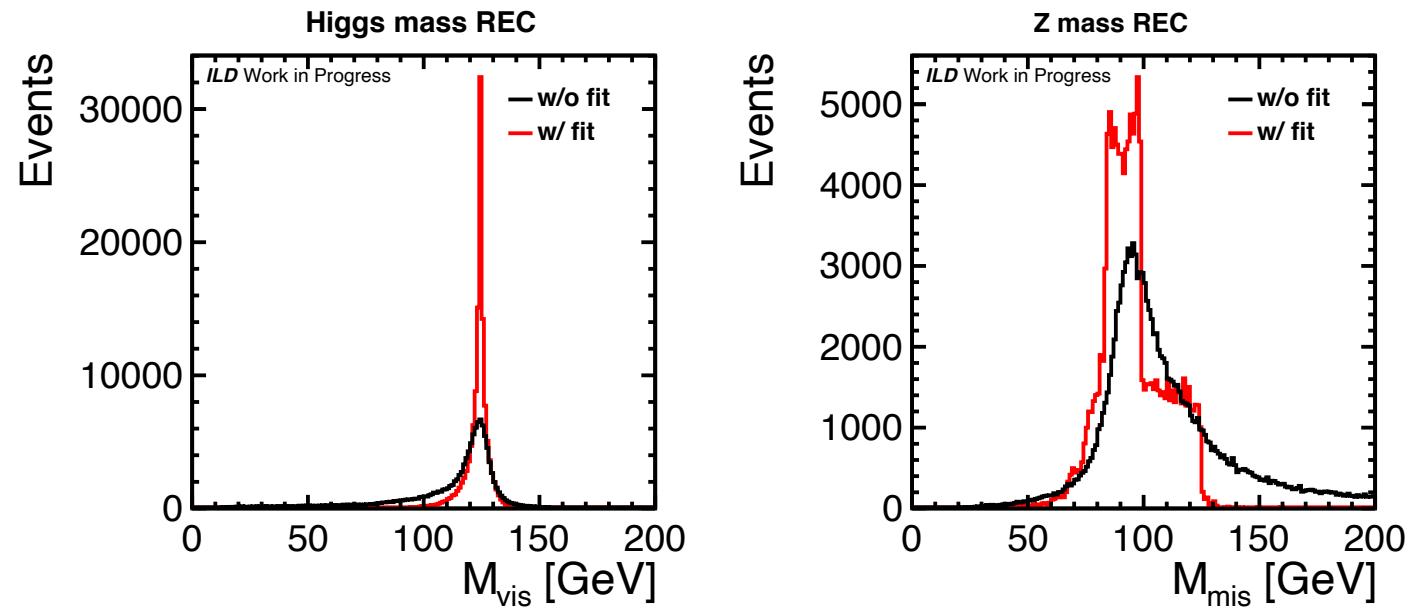
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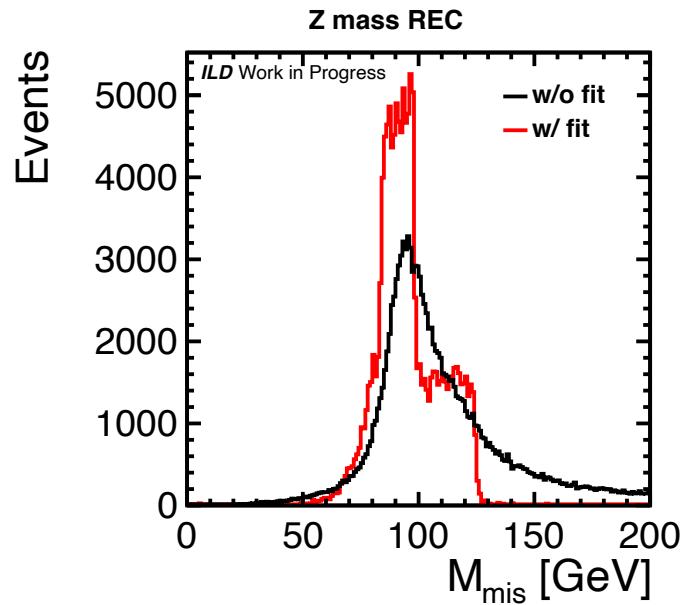
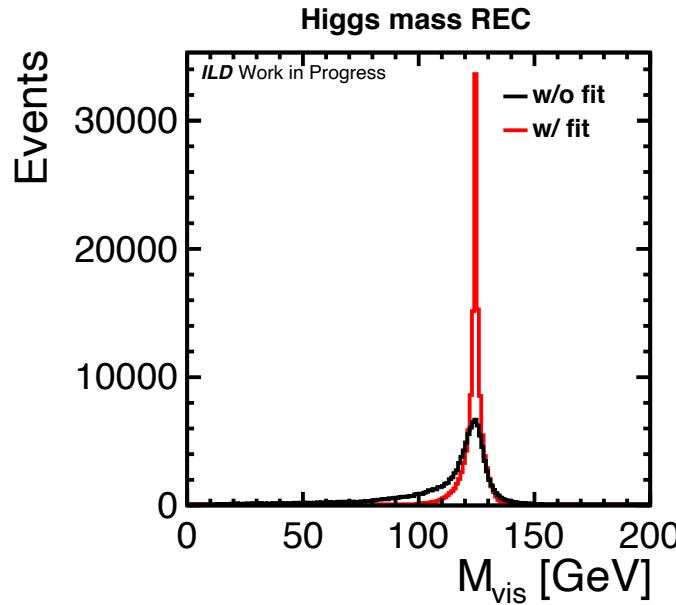
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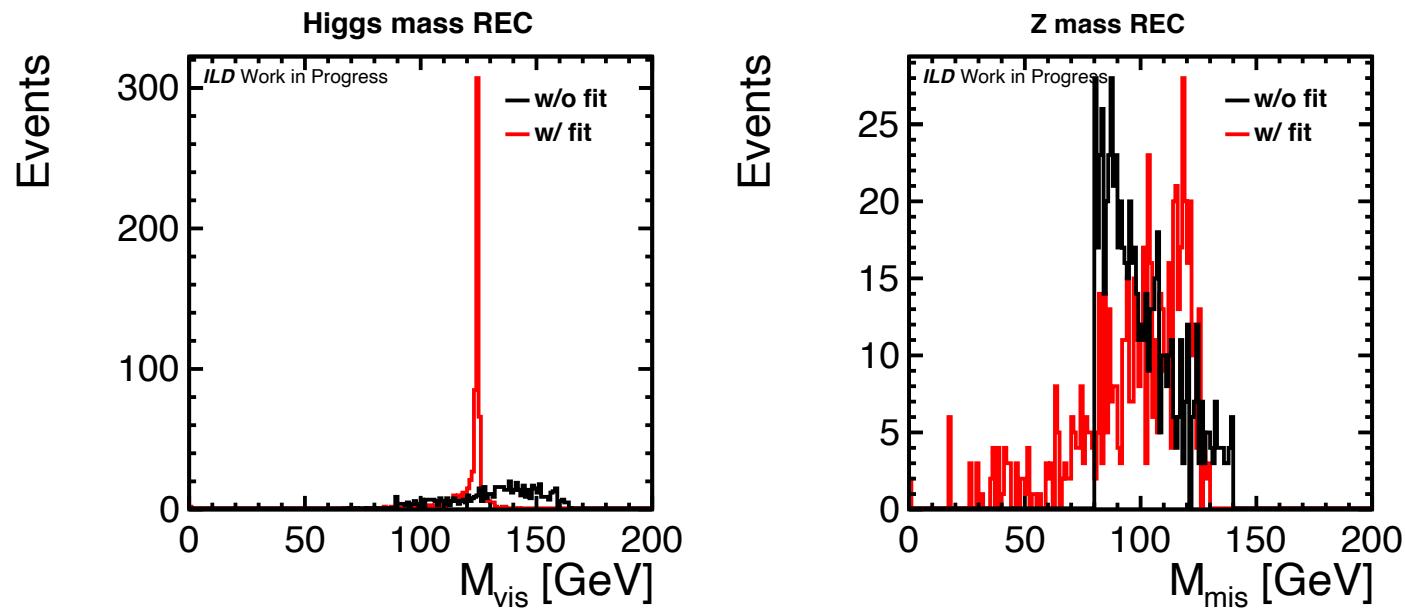
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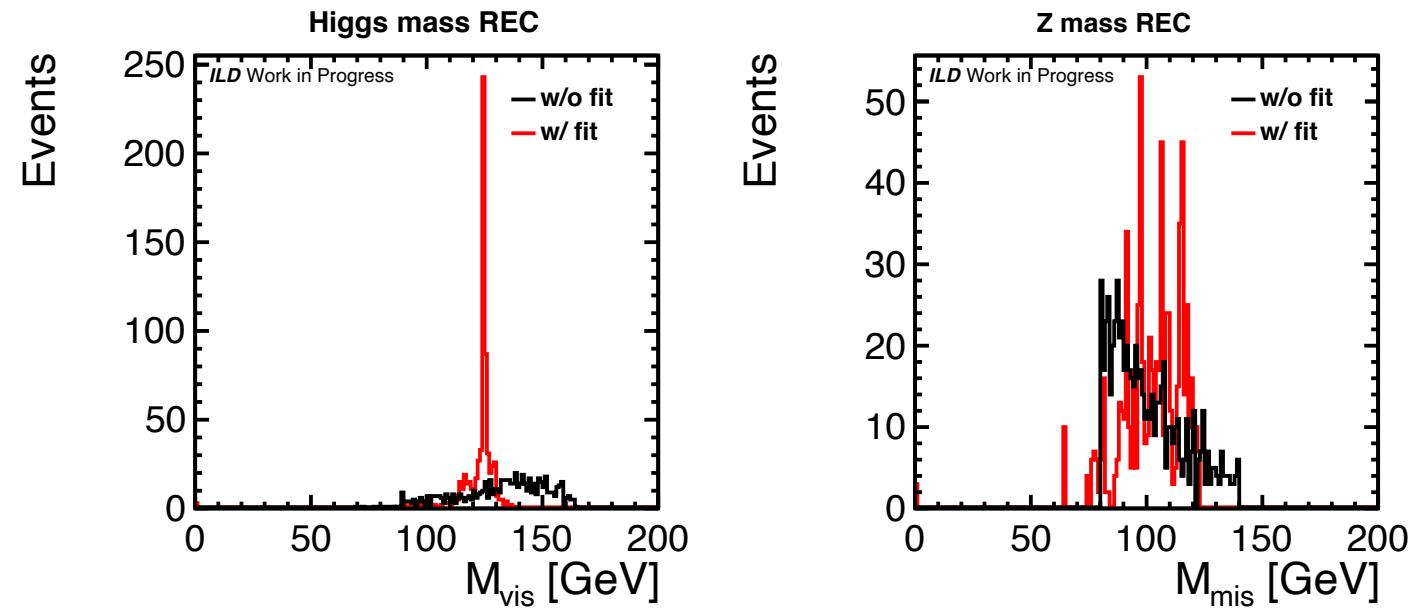
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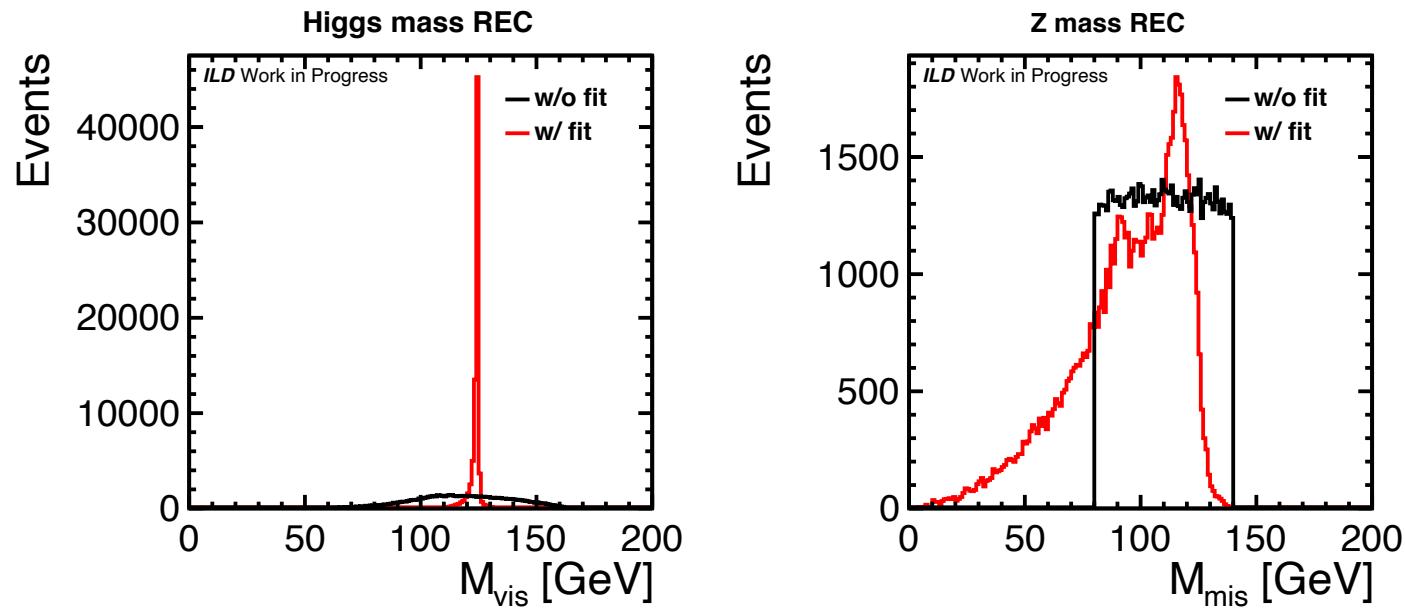
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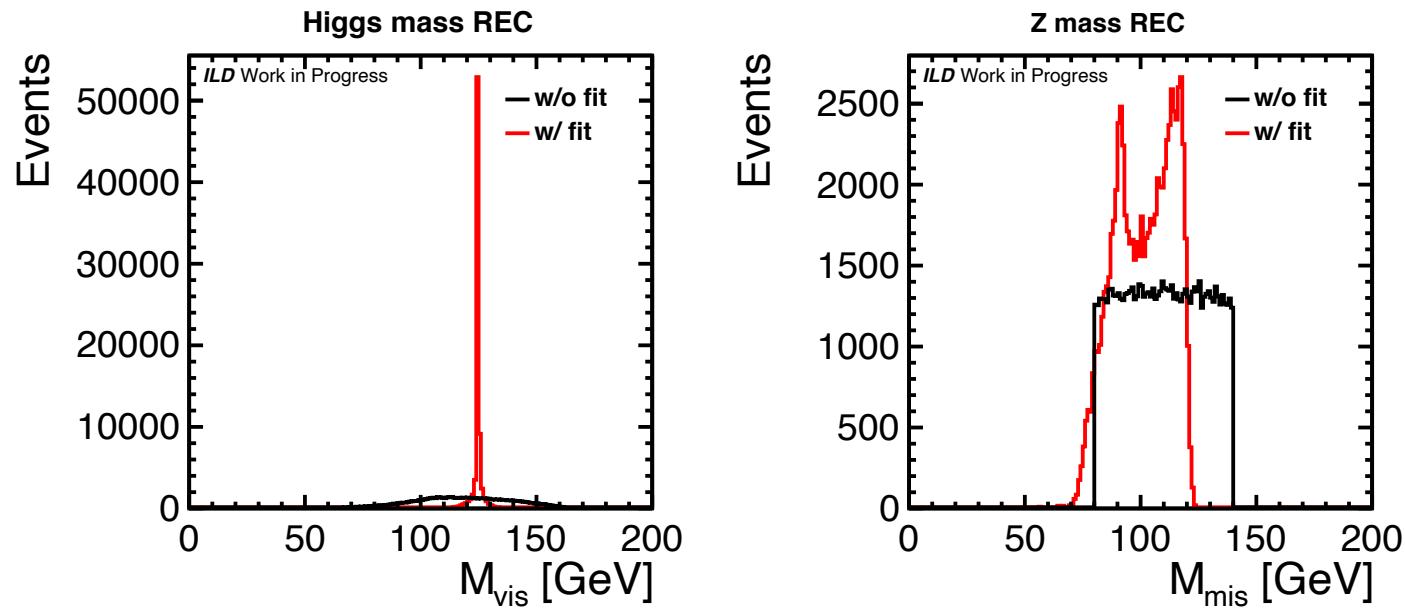
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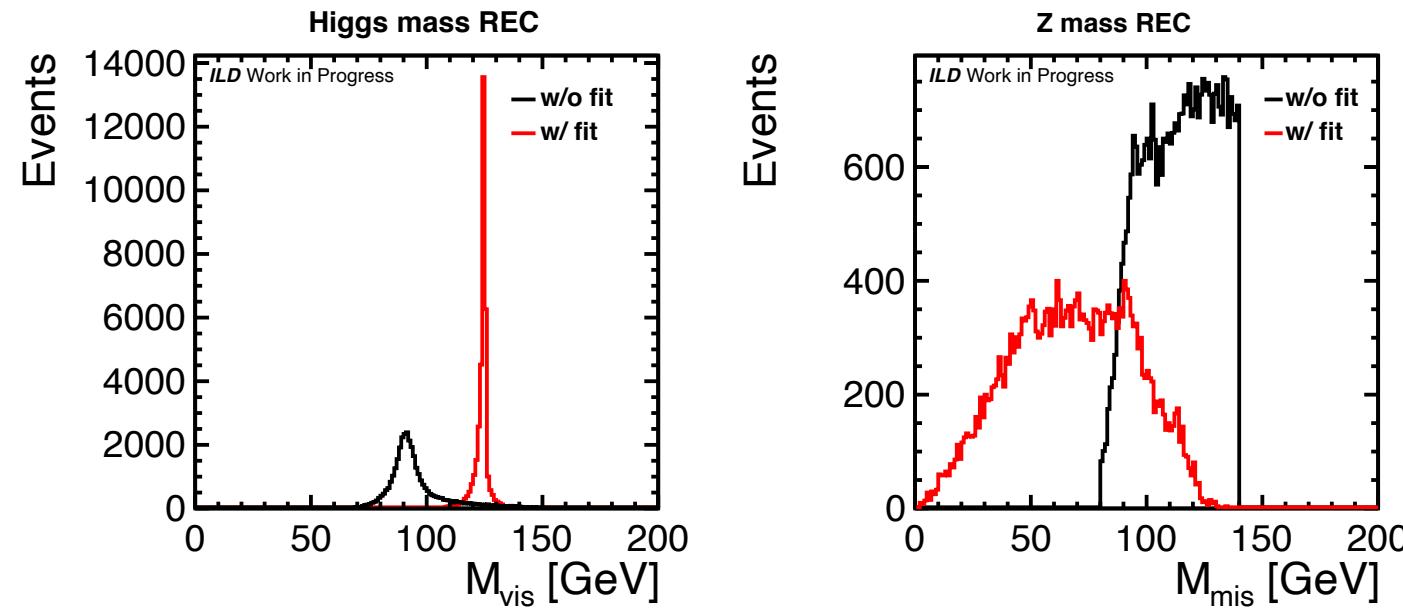
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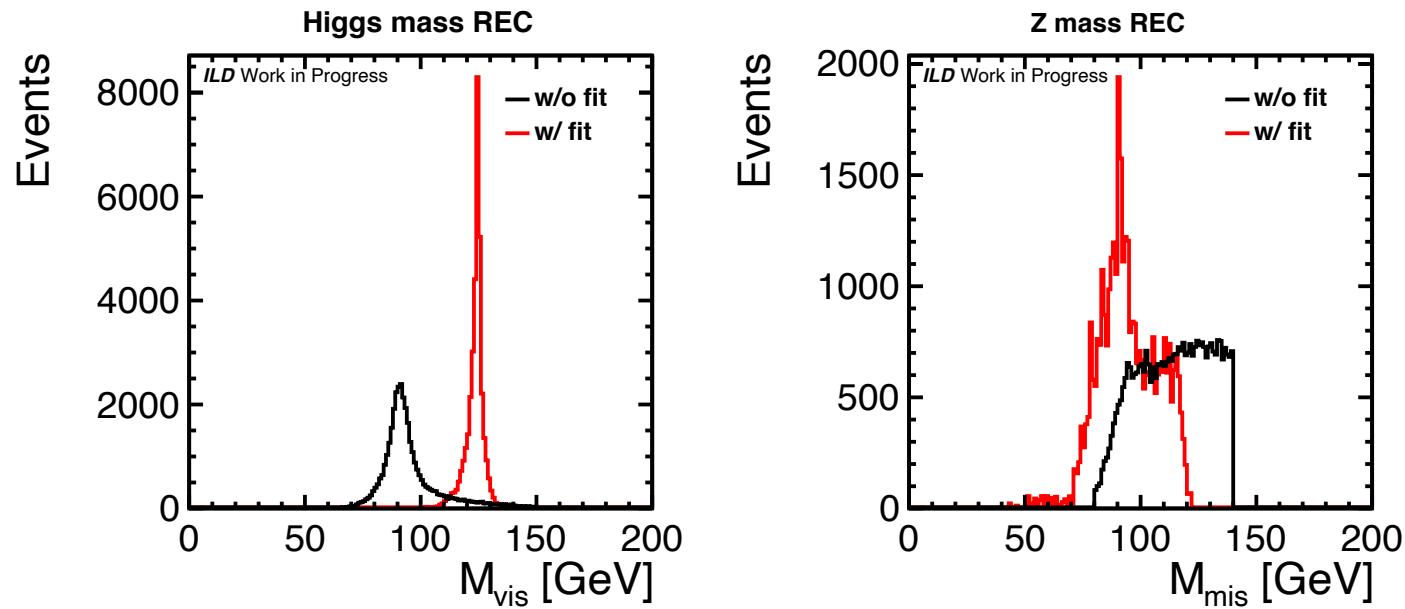
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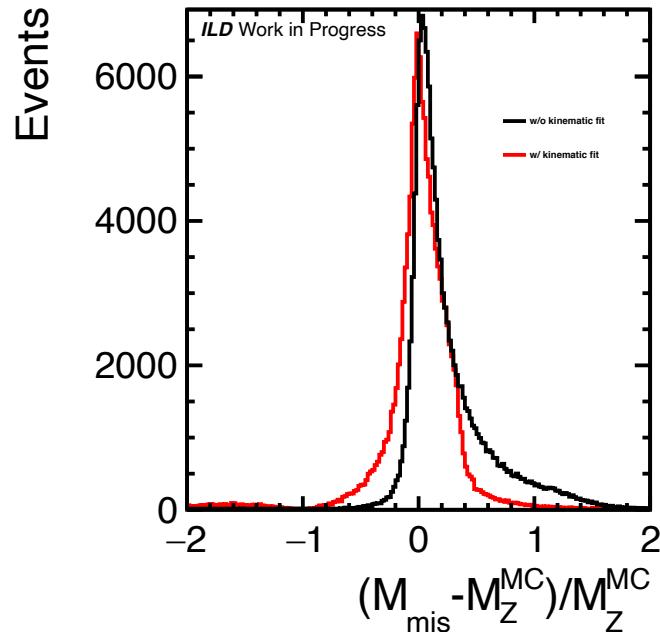
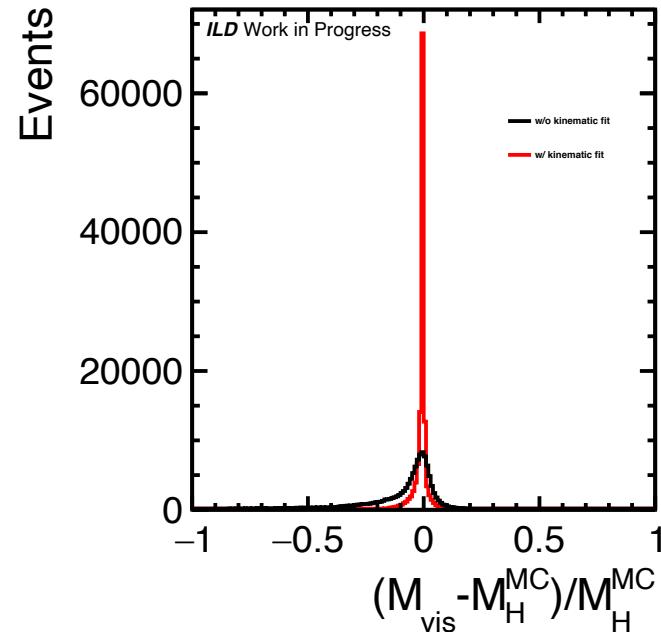
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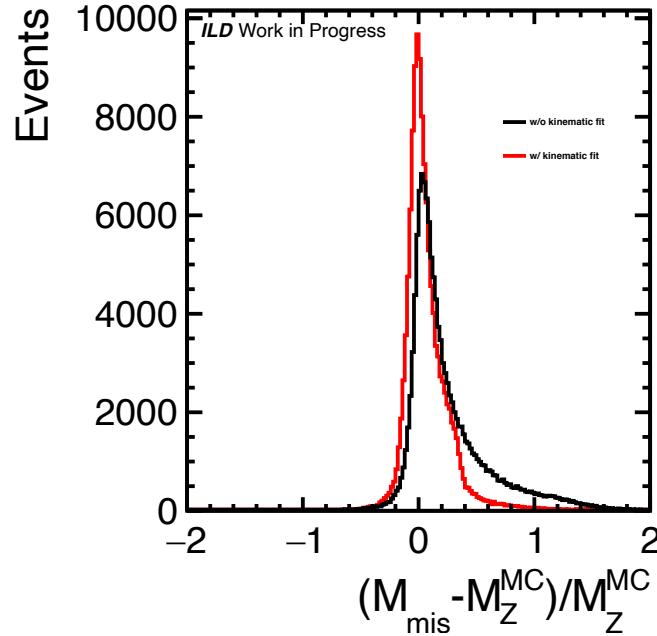
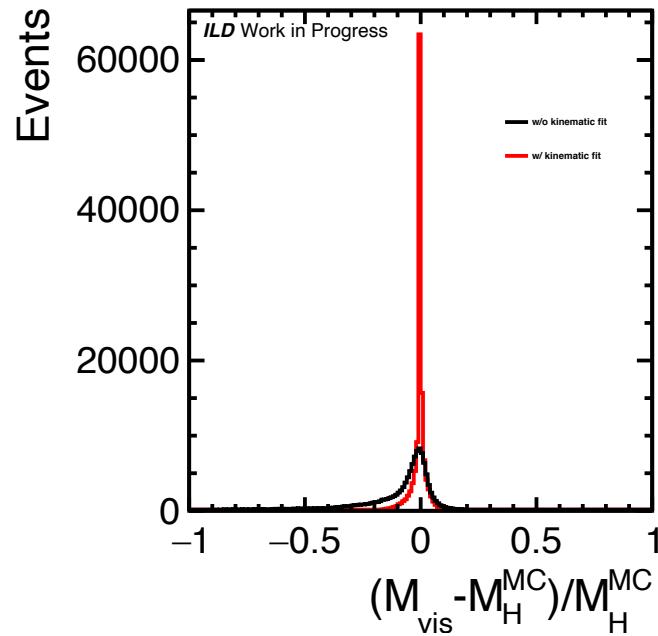
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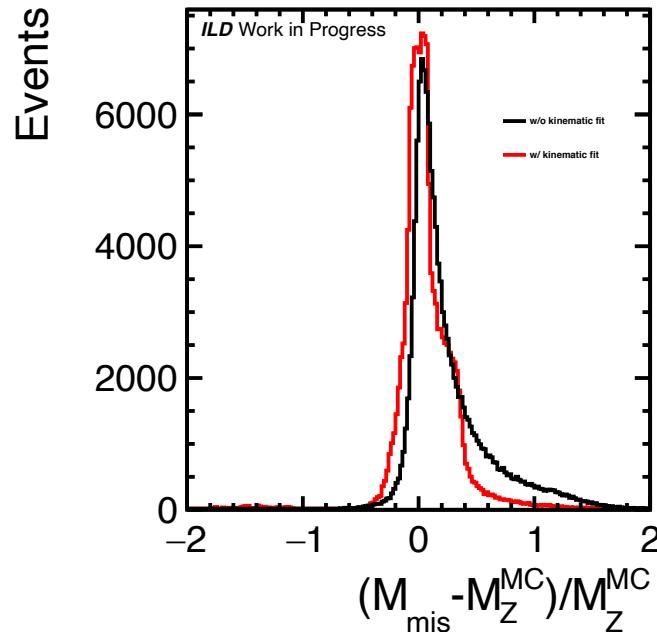
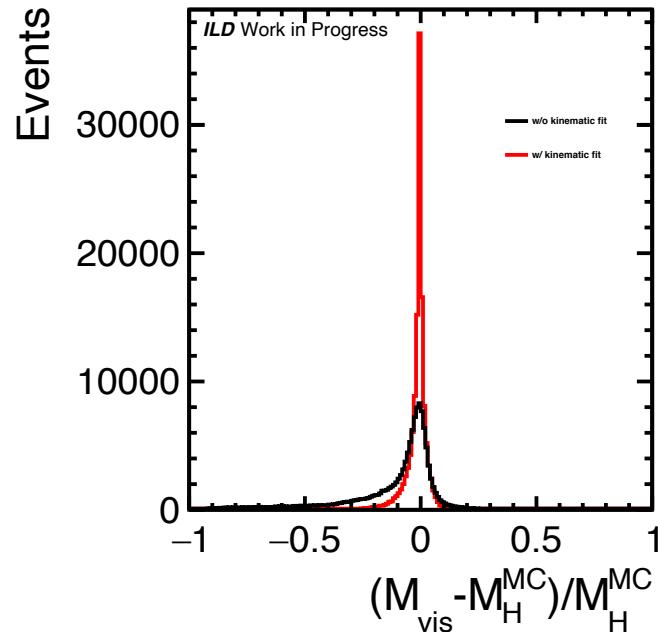
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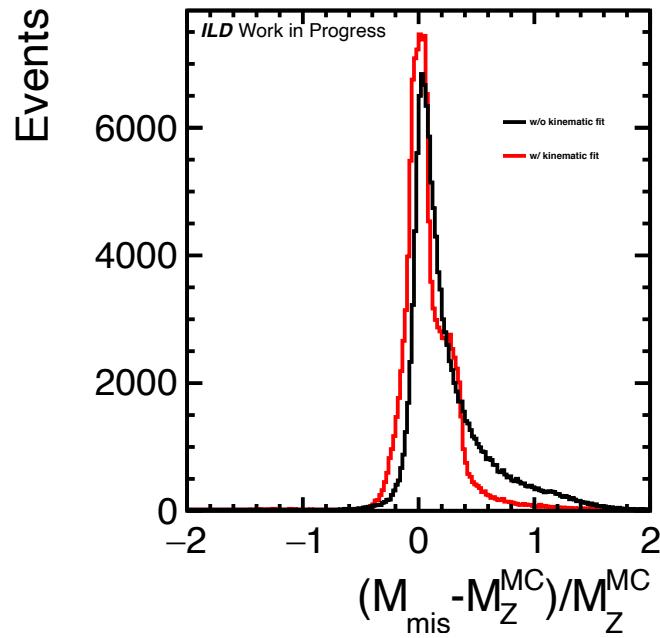
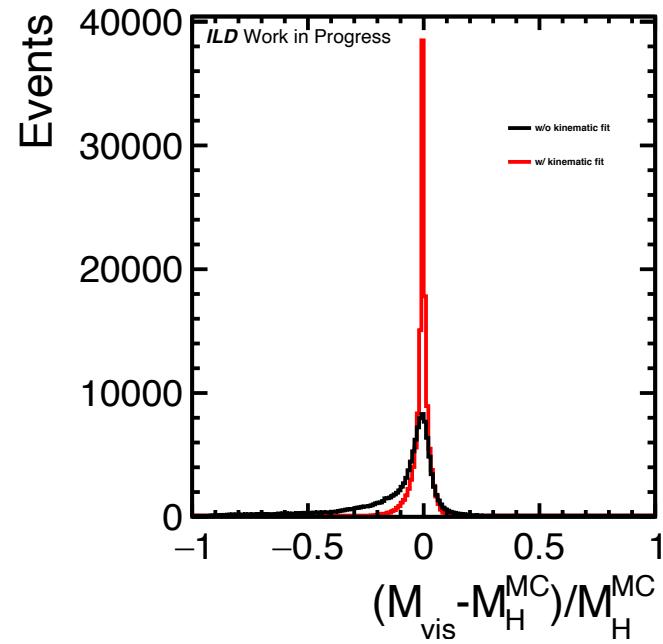
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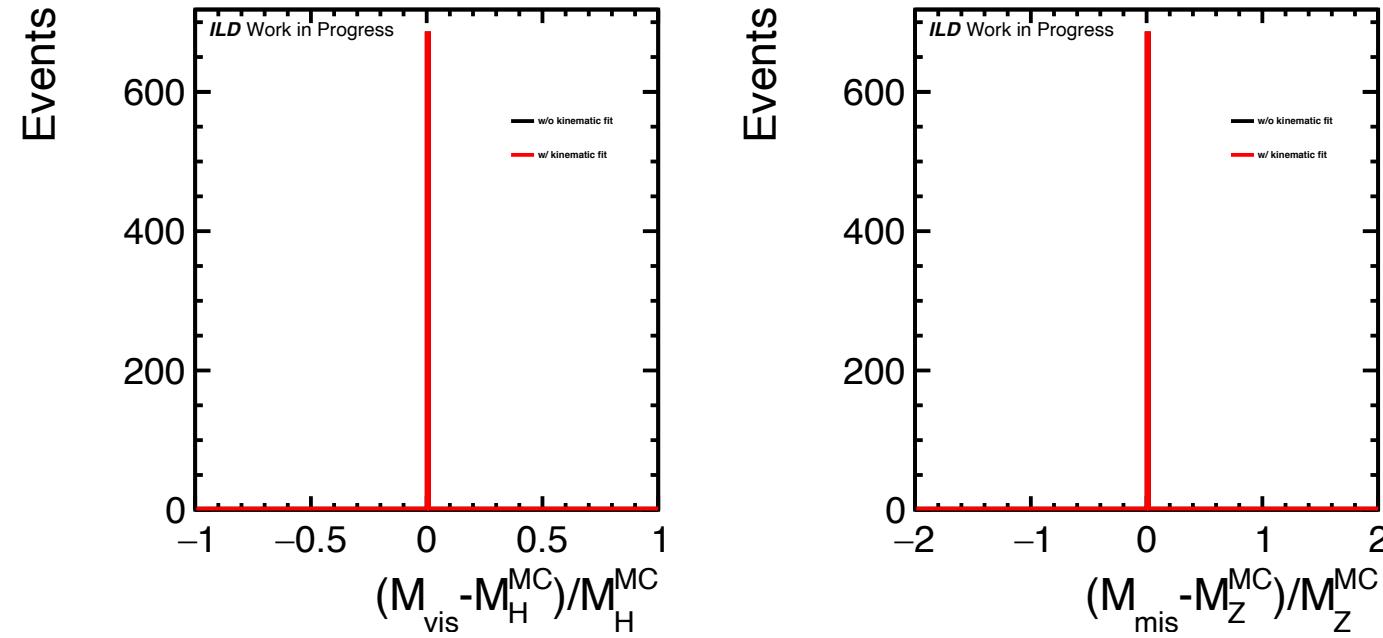
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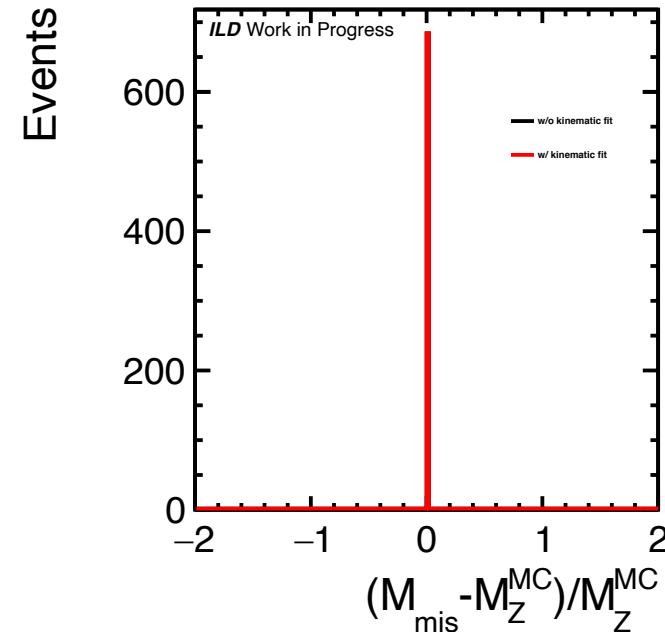
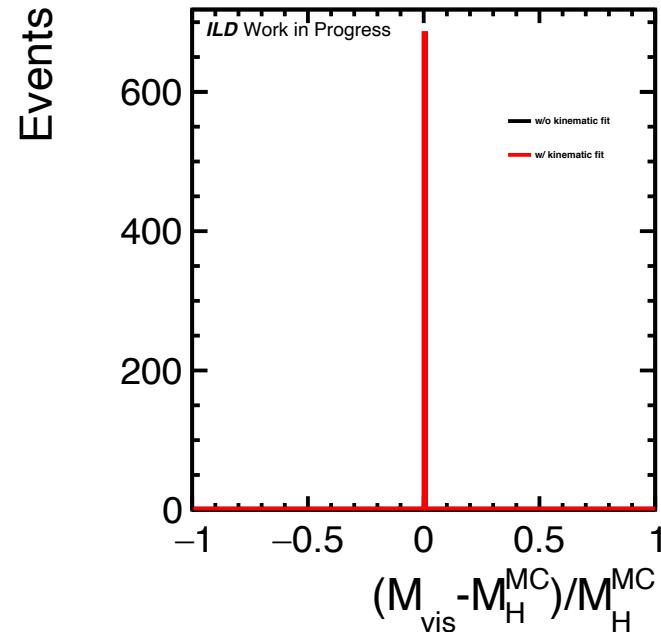
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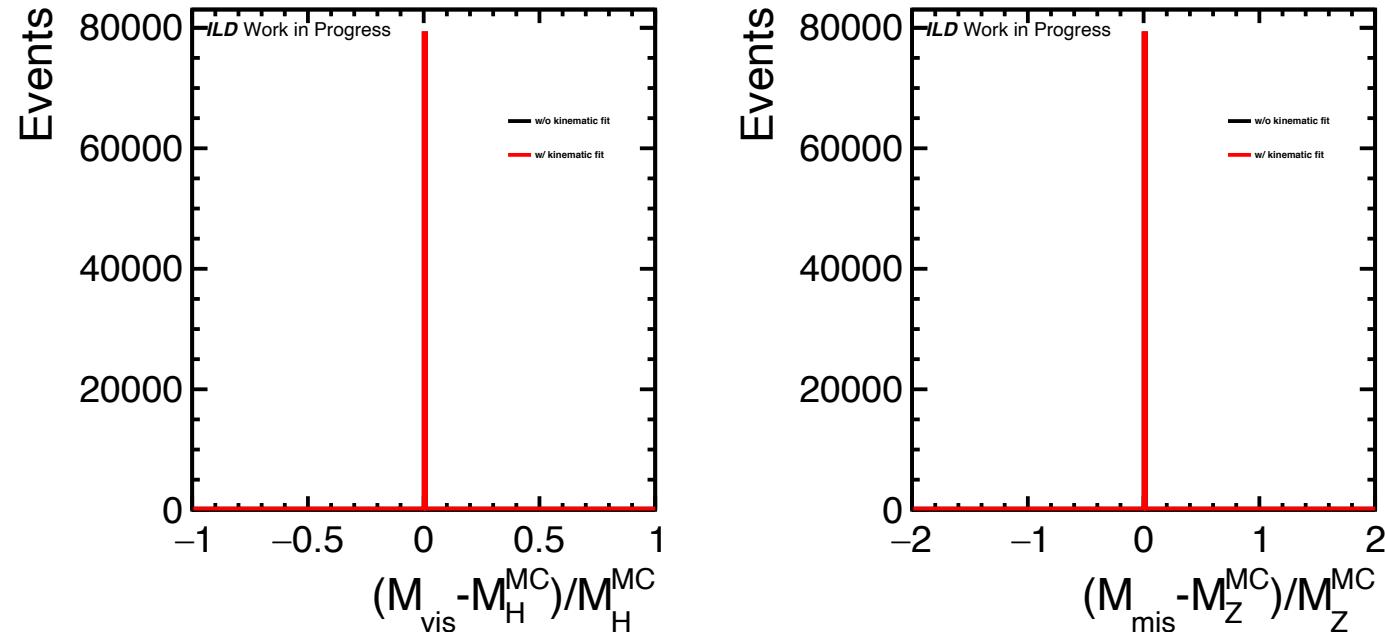
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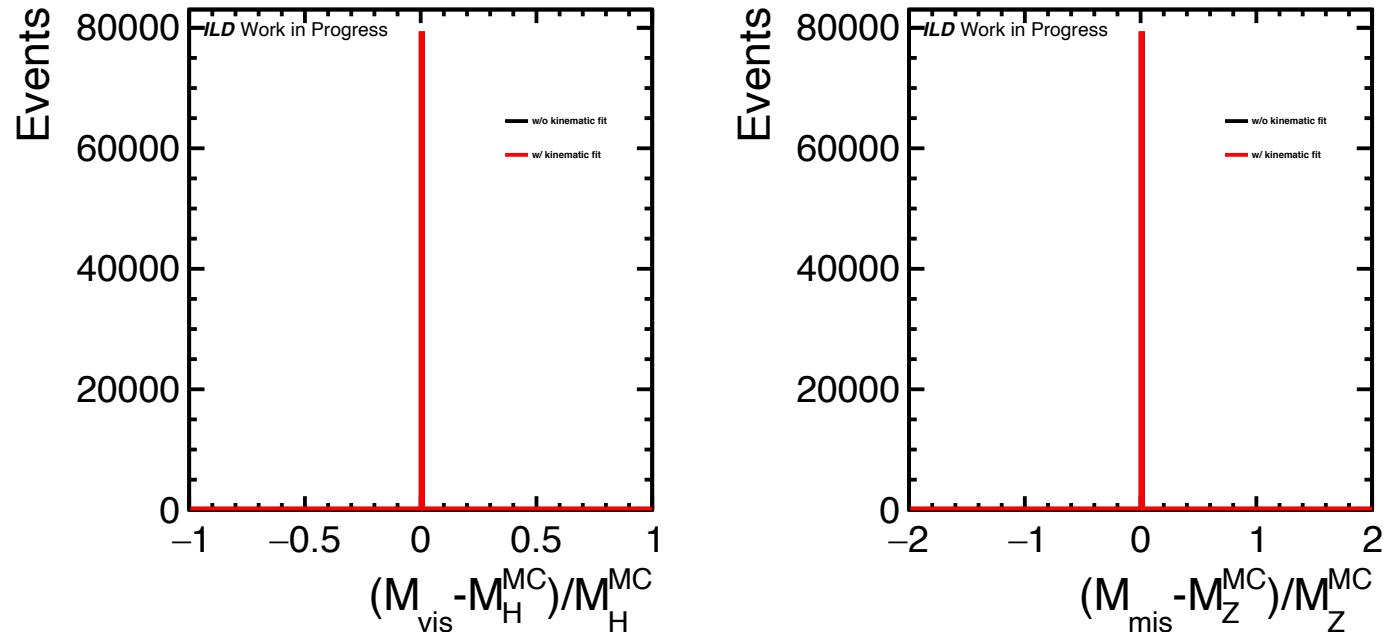
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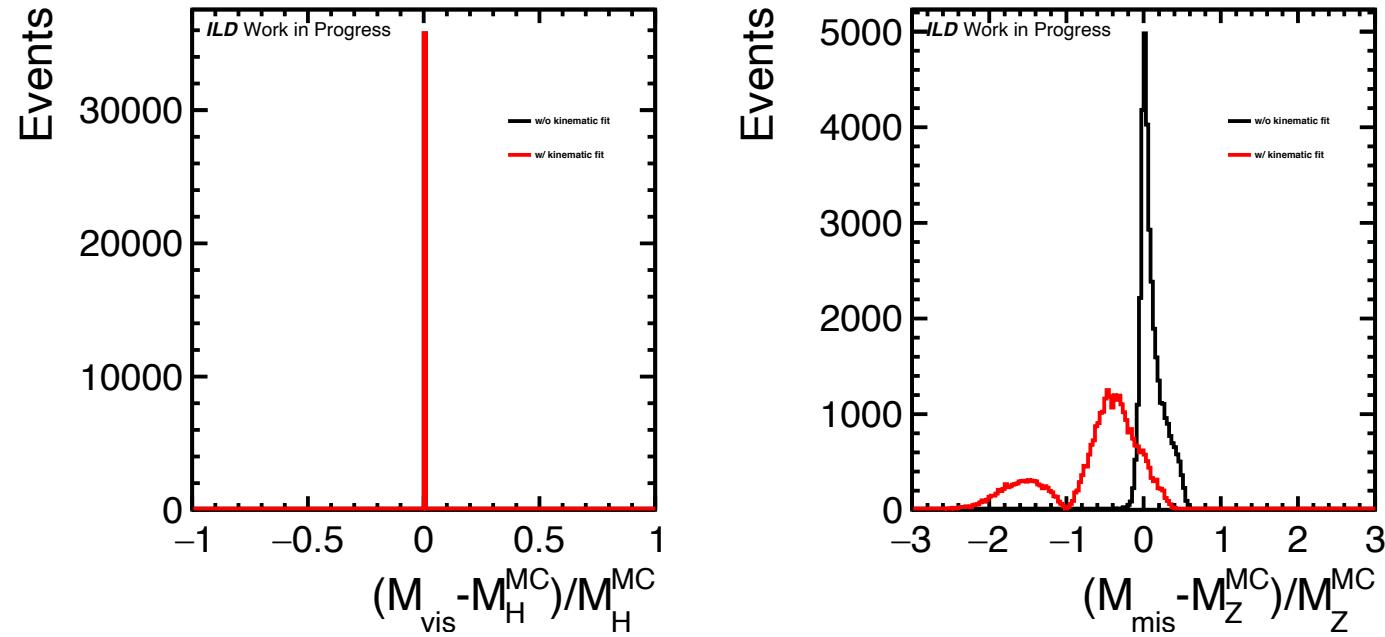
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# skim\_4f\_sznu\_sl.eL.pR\_scBW\_fodoubleGauss.pdf

