# Improving the jet energy reconstruction for Higgs precision measurement at the ILC

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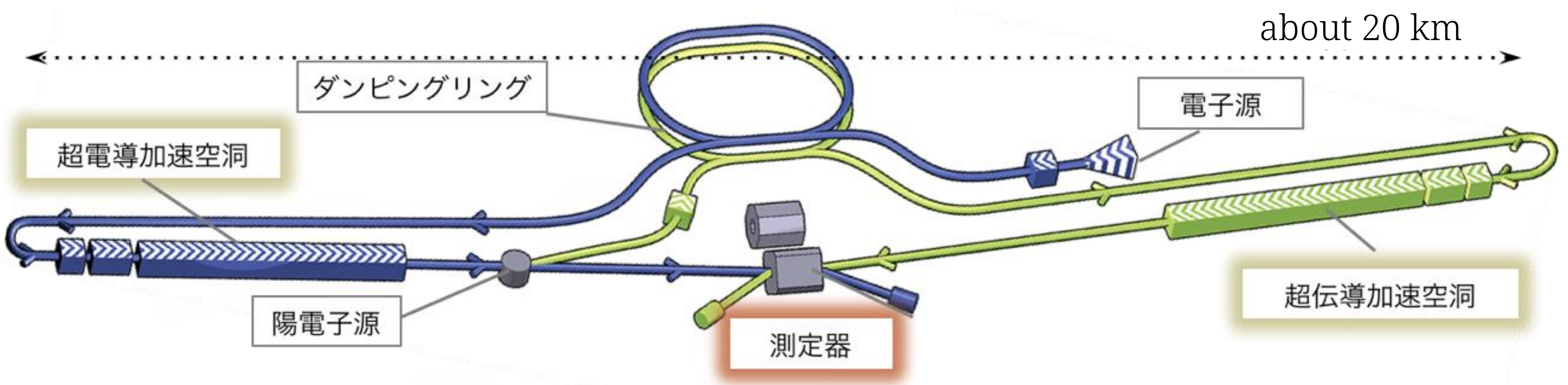
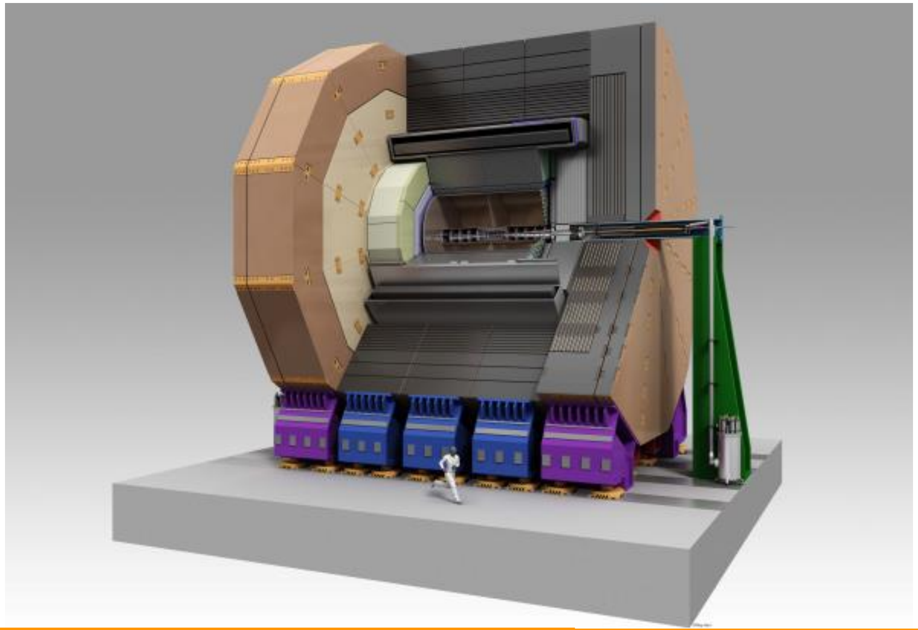
# International Linear Collider Project

- 250 GeV $e^+e^-$ collidor for Higgs factory
- linear collider for next generation
  20km→30km→50km
- precision measurement for Higgs etc.



# International Large Detector

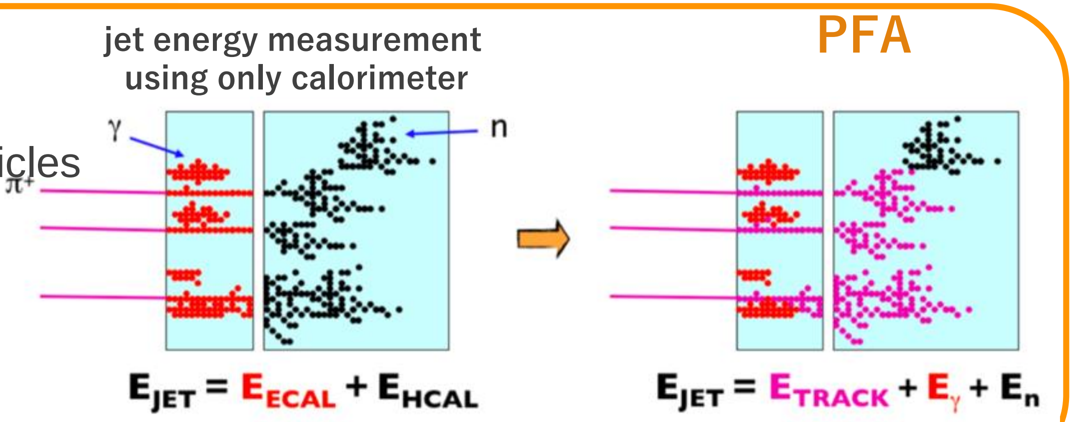


- Large Detector used at the ILC
  - vertex detector
  - tracker(TPC)
  - high-resolution calorimeter etc.
- Particle Flow Algorithm(PFA)

## Particle Flow Algorithm

a method using the optimum detector depending on types and energy of particles

charged particles → Tracker
photons → ECAL
neutral hadrons → HCAL



# Introduction

- Improving Kinematic Fit
  - purposes and reasons
  - about Kinematic Fit
    - definition of chi-square
    - calculation process
    - MarlinKinfit, a library for Kinematic Fit
    - Kinematic Constraint
  - idea of improving Kinematic Fit
    - new definition of chi-square
    - example for my process ($ee \to ZH \to \nu\nu bb$)

# Outline

JPS March,2019 17pK104-4
『Evaluating the Higgs coupling precision using the ee->ZH->vvbb process at the ILC』

- statistical error: $\frac{\Delta(\sigma_{ZH} \cdot BR_{Hbb})}{\sigma_{ZH} \cdot BR_{Hbb}} = 0.0103$
- b quark jet:
  asymmetrical energy resolution
  →kinematic fit for
  asymmetrical energy resolution

purposes

- to apply Kinematic Fit for parameters with non Gaussian resolution on the analysis at the ILC
- to develop Kinematic Fit to introduce non Gaussian distribution as Soft Constraint

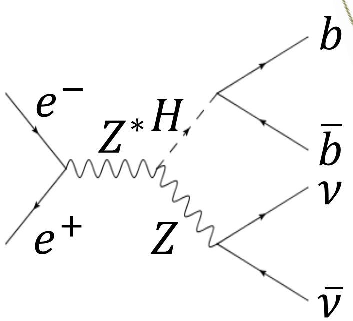
reason

- example of quantity with non Gaussian resolution: b jet energy
- example of Soft Constraint
  - mass of Z boson（Breit-Wigner distribution）
  - decline in $E_{cm}$ because of ISR or beam-beam effect
- Conventional Kinematic Fit assumes only Gaussian distribution.

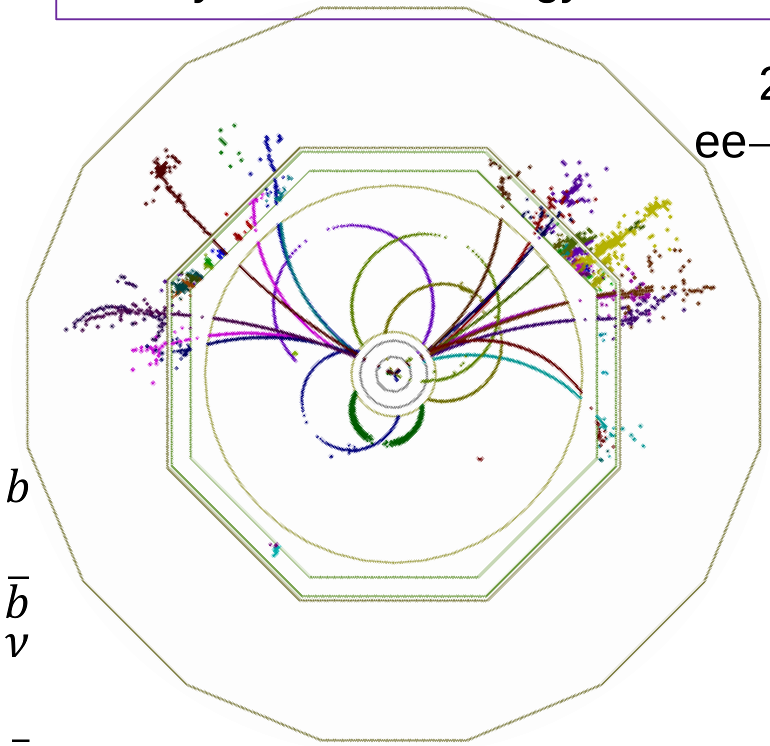
to introduce Kinematic Fit using general distribution

process

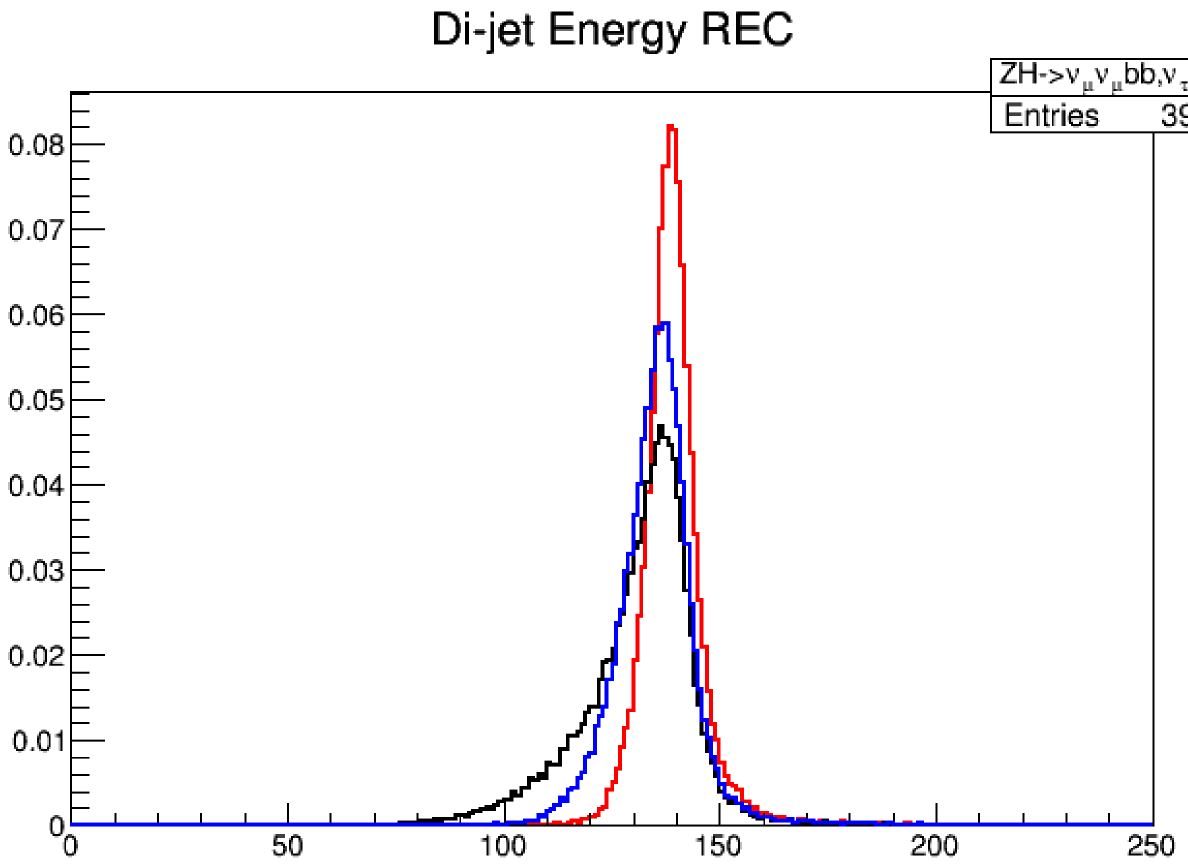
ee→ZH→vvbb: $BR_{Hbb}$




250 GeV
ee→ZH→vvbb

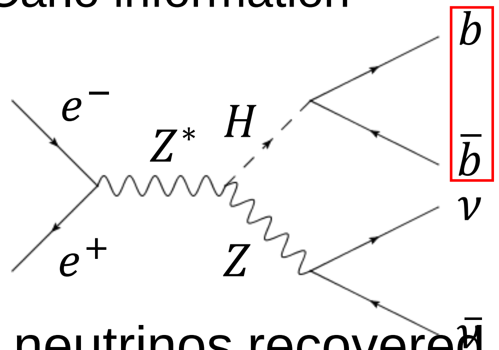


# Recovering neutrinos in jets: $E_{jj}$




- Di-jet energy: $E_{jj}$
- confirm recovering energy of neutrinos in jets using Monte Carlo information

- final state: ν νbb




non recovery

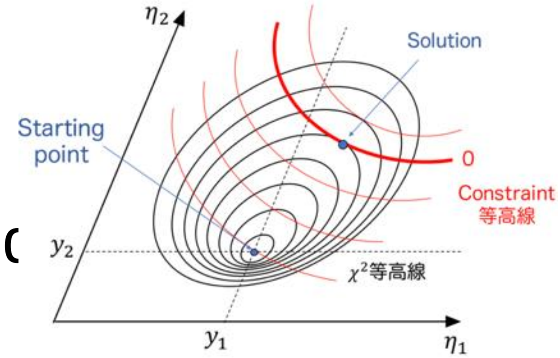
half energy of neutrinos recovered

full energy of neutrinos recovered

- It is difficult to recover all energy of neutrinos in jets.
- Asymmetrical jet energy resolution cannot be applied to conventional Kinematic.

→new Kinematic Fit

# Kinematic Fit




- a means of bringing physical parameters obtained from measured values close to true values
  - impose kinematic constraint ($E^2 = \vec{p}^2 + m^2$ etc.) among parameters when calculating parameters from measured values and their resolution

1. add following constraints to Fit Object($\chi^2$)
   - Hard Constraint: fixed relation among parameters(2nd term)
   - Soft Constraint: assume distribution among parameters(3rd term)
2. decide parameters by calculating numerically extreme problems of total $\chi_T^2$ $\quad (\nabla_{\vec{a}} \chi_T^2 = \vec{0}, \vec{a} = \vec{\lambda}, \vec{\eta}, \vec{\xi})$

$\vec{y}$:measured value, $\vec{\eta}$:parameter for measured, $V$:covariance matrix, $\vec{\xi}$: parameter for unmeasured

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = (\vec{y} - \vec{\eta})^T V^{-1} (\vec{y} - \vec{\eta}) + 2 \sum_{k=1}^{K} \lambda_k h_k(\vec{\eta}, \vec{\xi}) + \sum_{l=1}^{L} \left( \frac{g_l(\vec{\eta}, \vec{\xi})}{\sigma_{g_l}} \right)^2$$

Fit Object($\chi^2$) Hard Constraint Soft Constraint

# MarlinKinfit

- a library for Kinematic Fit
- uses Newton's method for chi-square minimization $\chi_T^2$

$$y(x) = 0 \Rightarrow y'(x^\nu)(x^\nu - x^{\nu+1}) = y(x^\nu)$$

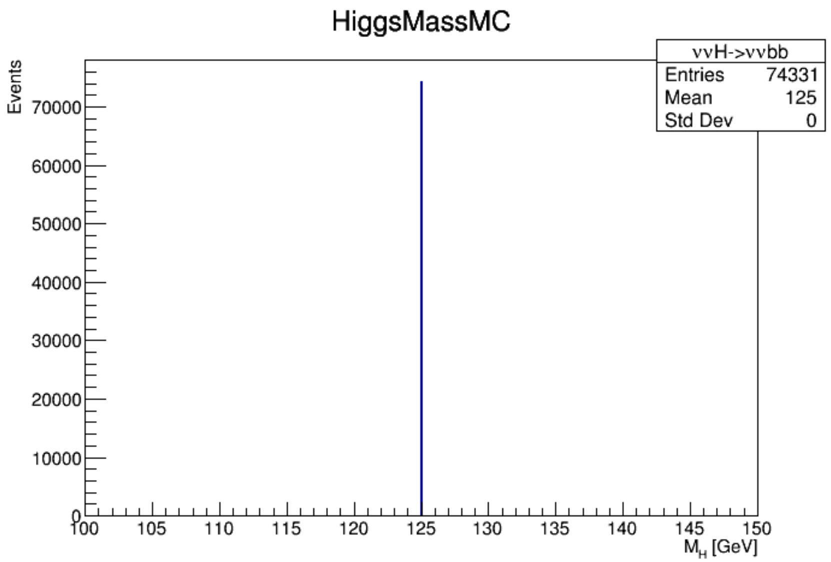
- Both Hard Constraint・Soft Constraint are available.
- following problems
  - Soft Constraint: assuming Gaussian distribution, not general distribution
  - Fit Object : can use only when measured values follow Gaussian distribution

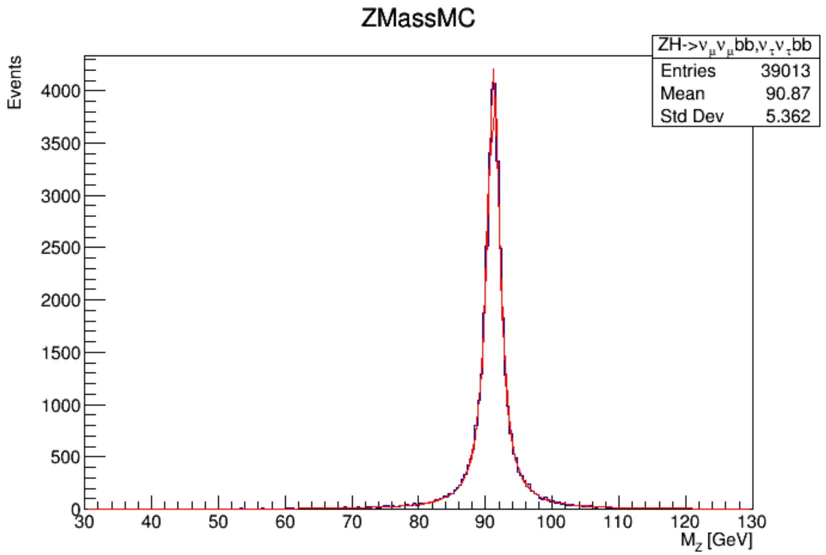
$\vec{y}$:measured value, $\vec{\eta}$:parameter for measured, $V$:covariance matrix, $\vec{\xi}$: parameter for unmeasured

$$\chi_T^2(\vec{\lambda}, \vec{\eta}, \vec{\xi}) = (\vec{y} - \vec{\eta})^T V^{-1} (\vec{y} - \vec{\eta}) + 2 \sum_{k=1}^{K} \lambda_k h_k(\vec{\eta}, \vec{\xi}) + \sum_{l=1}^{L} \left( \frac{g_l(\vec{\eta}, \vec{\xi})}{\sigma_{g_l}} \right)^2$$

Fit Object($\chi^2$) Hard Constraint Soft Constraint

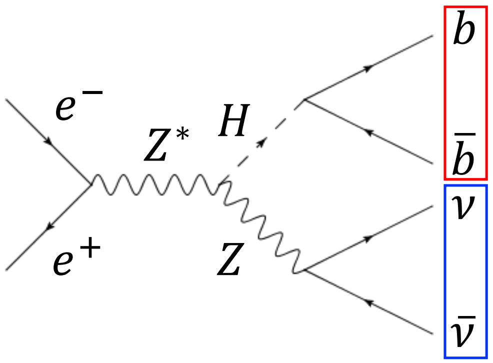
# Constraint




- final state: ν νbb

- $\Gamma_H$<0.013[GeV]




$M_H \rightarrow$ Hard Constraint

- $\Gamma_Z$~ 2.5 [GeV]

$M_Z \rightarrow$ Soft Breit-Wigner Constraint

# Improving Kinematic Fit

## conventional Kinematic Fit

- MarlinKinfit is not correct when measured values doesn't follow Gaussian distribution.

$$\chi^2_T(\vec{\lambda},\vec{\eta},\vec{\xi}) = (\vec{y}-\vec{\eta})^T V^{-1}(\vec{y}-\vec{\eta}) + 2\sum_{k=1}^{K}\lambda_k h_k(\vec{\eta},\vec{\xi}) + \sum_{l=1}^{L}\left(\frac{g_l(\vec{\eta},\vec{\xi})}{\sigma_{g_l}}\right)^2$$

## idea of improving Kinematic Fit

- redefine chi-square to apply following cases using log-likelihood($f$, $s_l$: p.d.f.)
  - Fit Object: measured values doesn't follow Gaussian distribution
  - Soft Constraint: non Gaussian constraint

$$L(\vec{\eta},\vec{\xi}) = f(\vec{y};\vec{\eta})\prod_{k=1}^{K}\delta(h_k(\vec{\eta},\vec{\xi}))\prod_{l=1}^{L}s_l(\vec{\eta},\vec{\xi})$$

deform this formula corresponding to format of conventional $\chi^2_T$

$$\chi^2_T(\vec{\lambda},\vec{\eta},\vec{\xi}) = -2\ln f(\vec{y};\vec{\eta}) + 2\sum_{k=1}^{K}\lambda_k h_k(\vec{\eta},\vec{\xi}) - 2\sum_{l=1}^{L}\ln s_l(\vec{\eta},\vec{\xi})$$

Fit Object | Hard Constraint | Soft Constraint

reproduce conventional $\chi^2_T$ when p.d.f. is Gaussian distribution

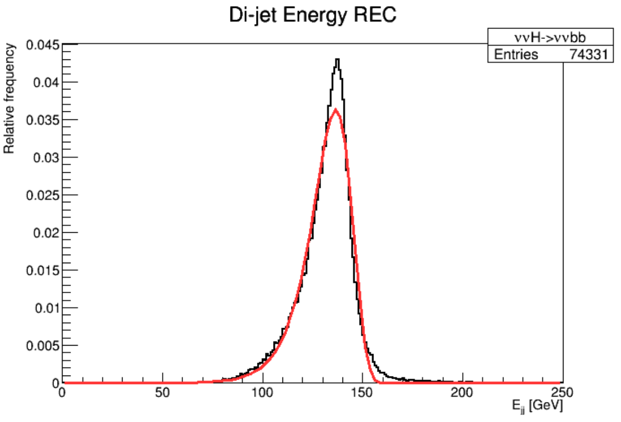
# Example for ee→ZH→vvbb

$$\chi^2_T(\vec{\lambda},\vec{\eta},\vec{\xi}) = -2\ln f(\vec{y};\vec{\eta}) + 2\sum_{k=1}^{K}\lambda_k\, h_k(\vec{\eta},\vec{\xi}) - 2\sum_{l=1}^{L}\ln s_l(\vec{\eta},\vec{\xi}\,)$$

$$-2\ln f(\vec{y};\vec{\eta}) = \sum_{i=0}^{1}\left\{-2\ln\frac{e^{\left(\frac{E_i-\widehat{E_i}}{\eta_{E_i}}\right)-e^{\frac{E_i-\widehat{E_i}}{\eta_{E_i}}}}}{\eta_{E_i}} + \left(\frac{\theta_i-\widehat{\theta_i}}{\sigma_{\theta_i}}\right)^2 + \left(\frac{\phi_i-\widehat{\phi_i}}{\sigma_{\phi_i}}\right)^2\right\}$$ Fit Object

Gumbel distribution

$$G(x;\mu,\eta) = \frac{1}{\eta}e^{\frac{x-\mu}{\eta}-e^{\frac{x-\mu}{\eta}}}$$




$$\vec{h}(\vec{\eta},\vec{\xi}) = \begin{bmatrix}\overrightarrow{\widehat{p_{vis}}} + \overrightarrow{\widehat{p_{mis}}} = \overrightarrow{p_{cm}} \\ \widehat{M_{vis}} = M_H\end{bmatrix}$$

Hard Constraint for momentum conservation and Higgs mass

$$-2\sum_{l=1}^{L}\ln s_l(\vec{\eta},\vec{\xi}\,) = \ln\frac{1}{\pi\Gamma_{m_Z}\left\{1+\left(\frac{\widehat{M_{mis}}-M_Z}{\Gamma_{M_Z}}\right)^2\right\}}$$

Soft Breit-Wigner constraint

# Summary

- to introduce general distribution to Kinematic Fit
    - b jet energy resolution is asymmetric.
    - Conventional Kinematic Fit assume only Gaussian distribution
- Kinematic Fit
    - impose kinematic constraint and $\nabla_{\vec{a}}\chi^2_T = \vec{0}$
- Conventional Kinematic Fit cannot be applied to following cases.
    - Fit Object: measured values doesn't follow Gaussian distribution
    - Soft Constraint: non Gaussian constraint
- redefine as log-likelihood

$$\chi^2_T(\vec{\lambda},\vec{\eta},\vec{\xi}) = -2\ln f(\vec{y};\vec{\eta}) + 2\sum_{k=1}^{K}\lambda_k h_k(\vec{\eta},\vec{\xi}) - 2\sum_{l=1}^{L}\ln s_l(\vec{\eta},\vec{\xi})$$

Fit Object / Hard Constraint / Soft Constraint

- solve $\nabla_{\vec{a}}\chi^2_T = \vec{0}$ using Newton method and improve kinematic fit to introduce non Gaussian distribution

# additional

# 偏極率の定義

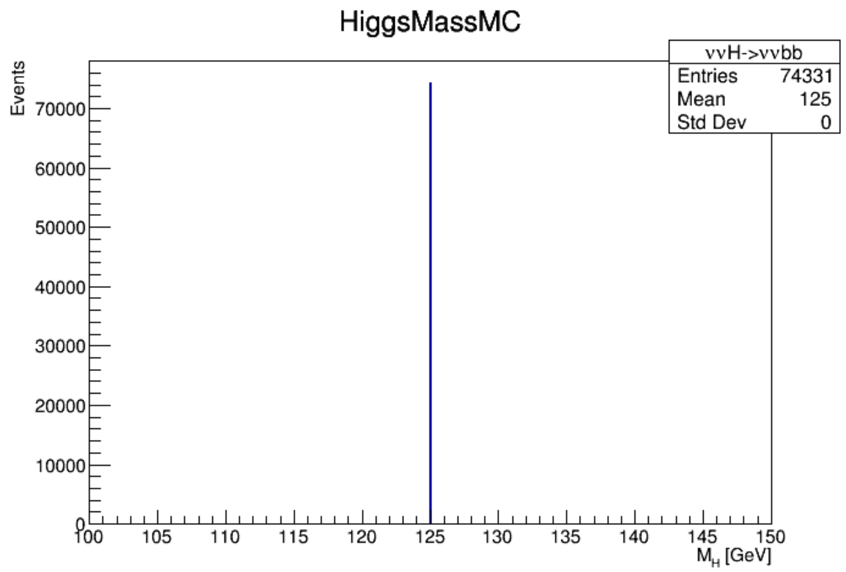
スピン偏極度: $P$

$$= \frac{N_R - N_L}{N_R + N_L}$$

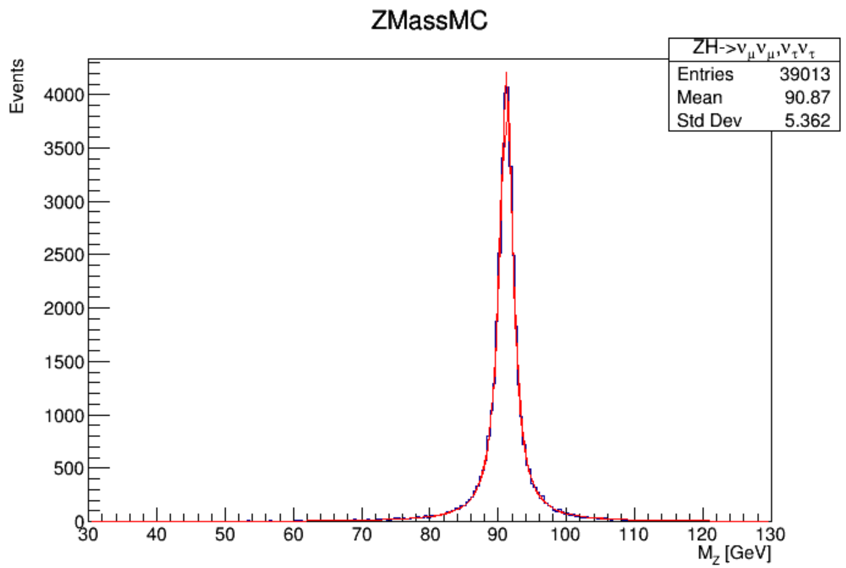
($N_R$: ヘリシティー正粒子数、$N_L$:ヘリシティー負粒子数)

# Kinematic Constraint

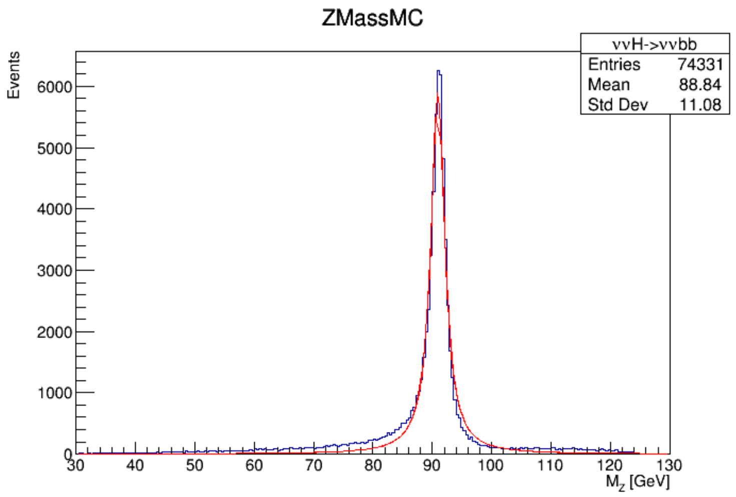
- $M_H$ →Hard Constraint ($M_H$=125 [GeV])

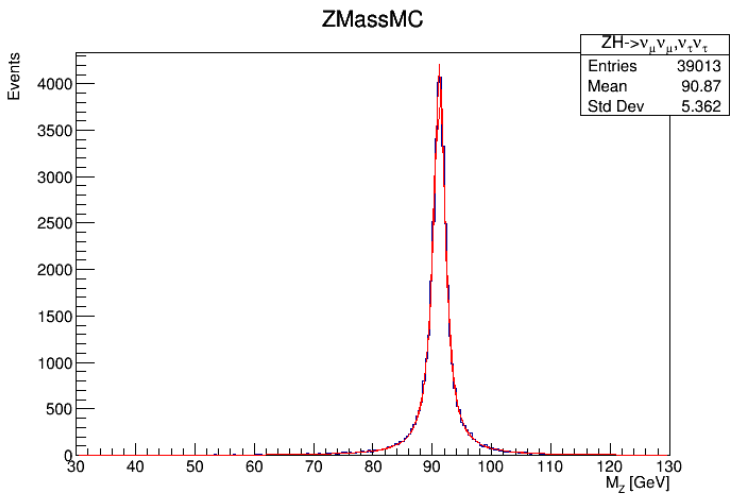



- $M_Z$ →Breit-Wigner Soft Constraint



# $M_Z$ →Soft Constraint




- ニュートリノのフレーバーは$\nu_e$、$\nu_\mu$、$\nu_\tau$

→WW-fusionにより、BWからずれ

- ニュートリノのフレーバーは$\nu_\mu$、$\nu_\tau$に限定
→WW-fusionを除くため

- 終状態はν νbbに限定
- $\Gamma_Z$~ 2.5 [GeV](PDG2019)
- Breit-Wigner分布→ Soft Breit-Wigner Constraint