# Jet energy calibration on toy MC

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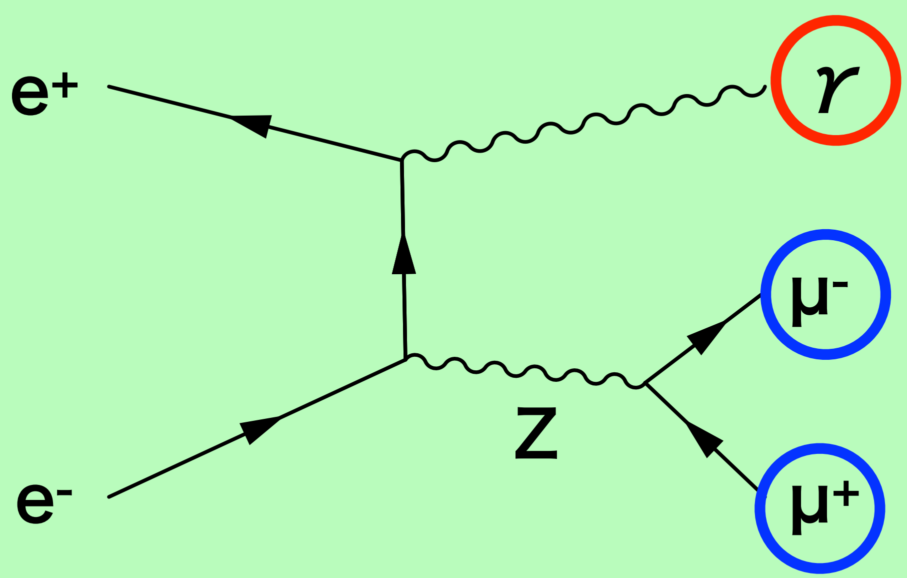
# Introduction

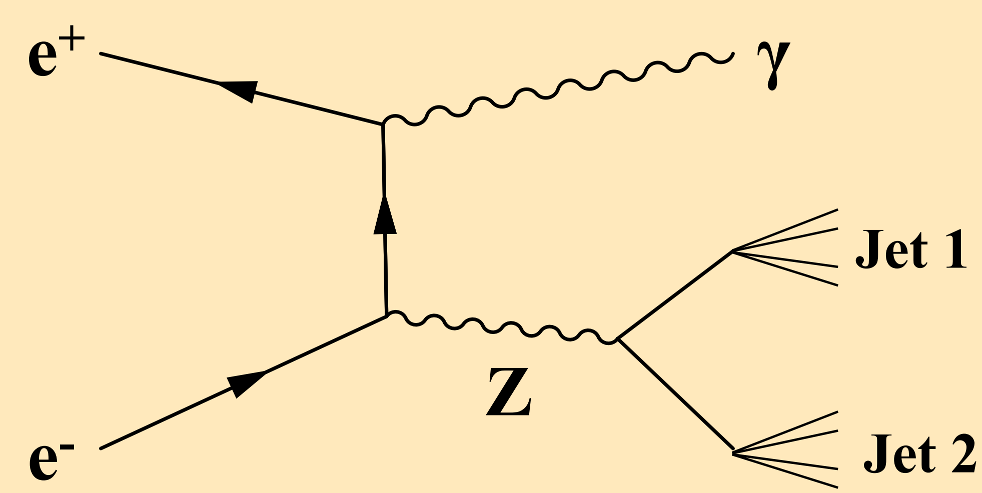
## Detector Benchmark Motivation

- In the previous study, it has been shown that photon energy can be calibrated using the $e^+e^- \rightarrow \gamma Z$ process.
- Using similar energy reconstruction methods as the photon energy reconstruction, the jet energies in the $e^+e^- \rightarrow \gamma Z$, $Z \rightarrow 2\text{Jets}$ can be reconstructed.
- If the jet energies can be correctly reconstructed, the $e^+e^- \rightarrow \gamma Z$ process is useful for the jet energy calibration.

**Photon Energy Scale Calibration**

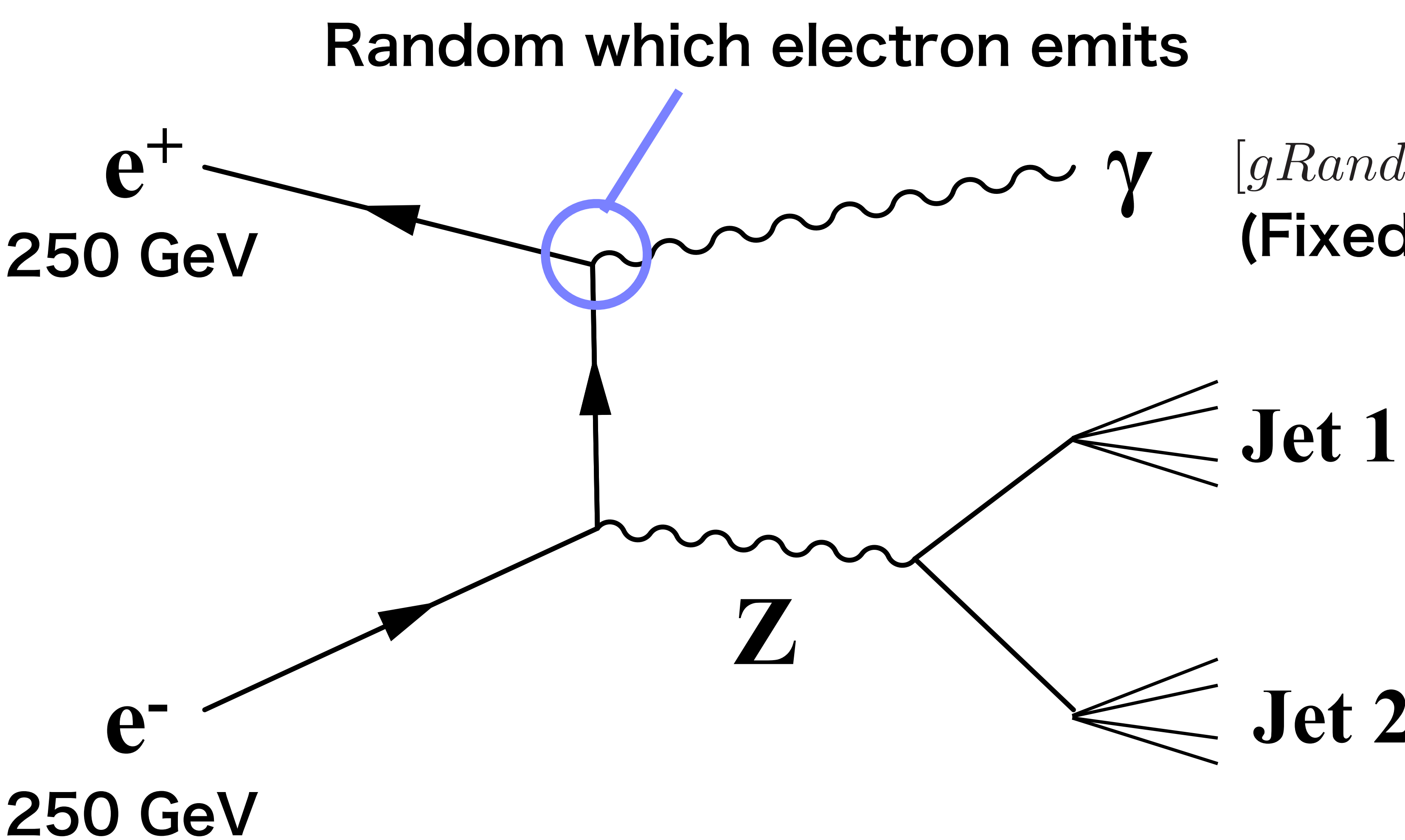



**Jet Energy Scale Calibration**



# Jet Energy Scale Calibration

- Process: $e^+e^- \rightarrow \gamma Z,\ Z \rightarrow 2\text{Jets}$
- As a first step of this calibration, Toy MC simulation to reconstruct the jet energy is performed.
  $E_{CM}$= 500 GeV, $M_{jet}$ is quoted from the MCTrue jet mass distribution in $e^+e^- \rightarrow \gamma Z,\ Z \rightarrow 2\text{Jets}$ process.



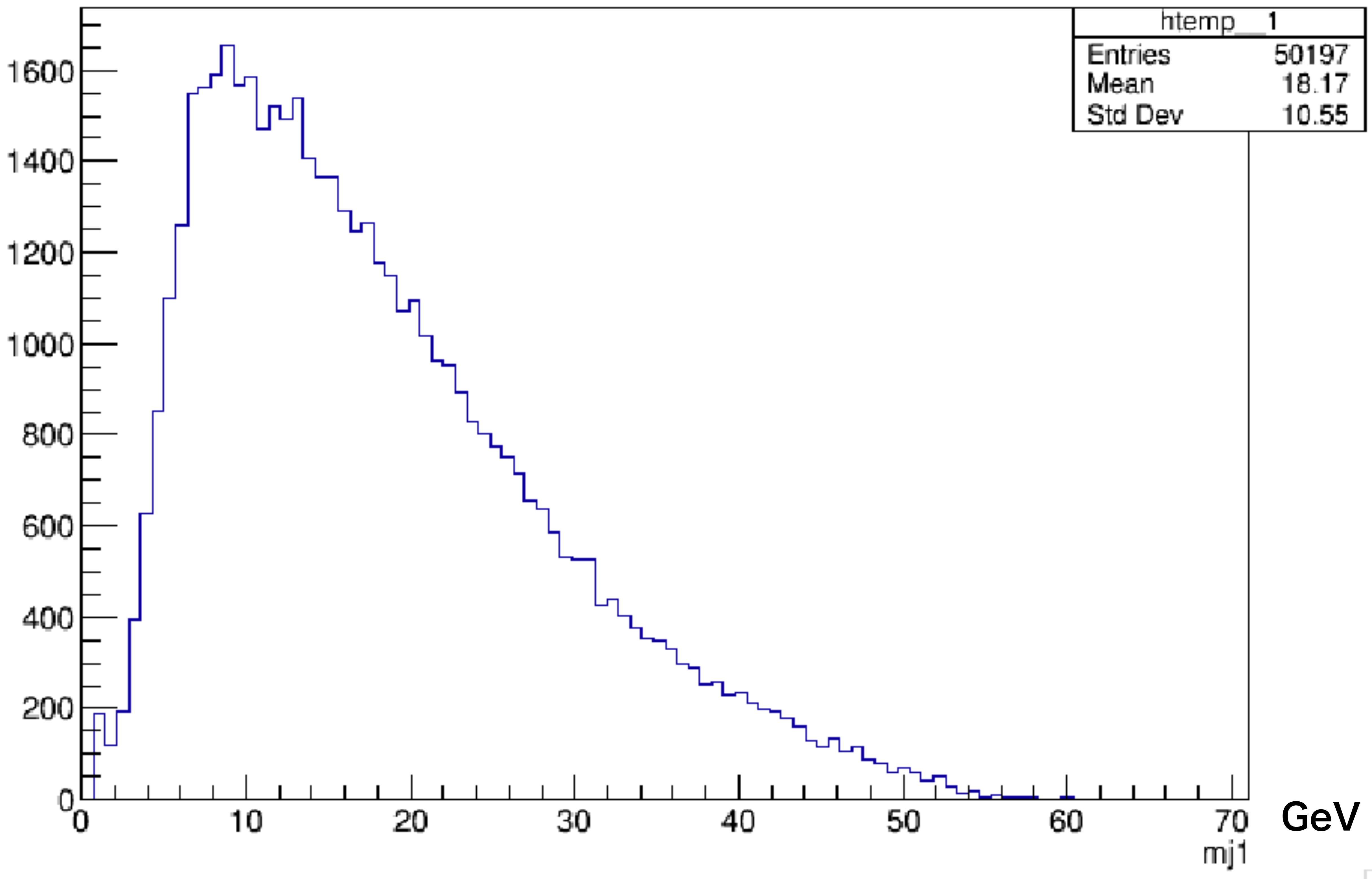

$[gRandom->Uniform()]^{\frac{1}{\beta}}$
(Fixed to be <250 GeV)

**$M_{jet}$ :From the $M_{jet}$ distribution in the $e^+e^- \rightarrow \gamma Z$ process**

# Jet Mass Input (Before smearing)



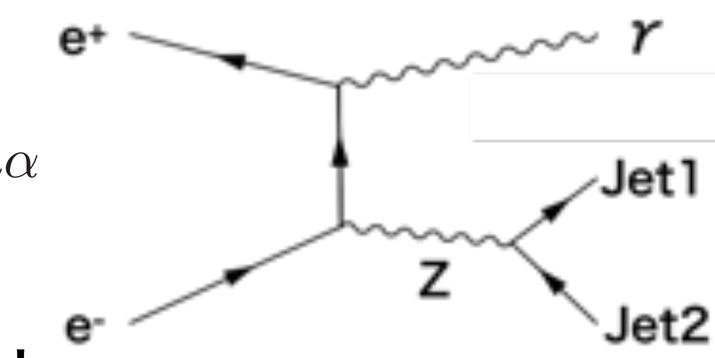
## MC particles for e+e- -> gamma Z events at Ecm=500 GeV, by the physsim



# Reconstruction Method

Based on 4-momentum conservation

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \\ P_{J1} sin\theta_{J1} cos\phi_{J1} + P_{J2} sin\theta_{J2} cos\phi_{J2} + P_\gamma sin\theta_\gamma cos\phi_\gamma + |P_{ISR}| sin\alpha = 500 sin\alpha \\ P_{J1} sin\theta_{J1} sin\phi_{J1} + P_{J2} sin\theta_{J2} sin\phi_{J2} + P_\gamma sin\theta_\gamma sin\phi_\gamma = 0 \\ P_{J1} cos\theta_{J1} + P_{J2} cos\theta_{J2} + P_\gamma cos\theta_\gamma \pm |P_{ISR}| cos\alpha = 0 \end{cases}$$




**Beam Crossing Angle ≡2α : α = 7.0 mrad**

Direction Angle
θ: polar angle
ϕ: azimuthal angle

- ISR photon = **additional** unseen photon
- Several reconstruction methods (Method 1, 2', 2, and 3) are considered.

**Method 1: Ignore ISR**

Using ($\theta_{J1}, \theta_{J2}, \theta_\gamma, \varphi_{J1}, \varphi_{J2}, \varphi_\gamma, m_{J1}, m_{J2}$) -> Determine ($P_{J1}, P_{J2}, P_\gamma$)

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| = 500 \\ \begin{pmatrix} sin\theta_{J1} cos\phi_{J1} & sin\theta_{J2} cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1} sin\phi_{J1} & sin\theta_{J2} sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} 500 sin\alpha \\ 0 \\ 0 \end{pmatrix} \end{cases}$$

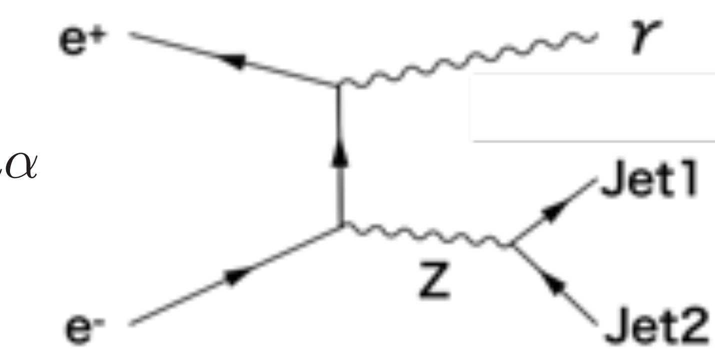
Matrix A → Inverse

# Reconstruction Method



Based on 4-momentum conservation

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \\ P_{J1} sin\theta_{J1} cos\phi_{J1} + P_{J2} sin\theta_{J2} cos\phi_{J2} + P_\gamma sin\theta_\gamma cos\phi_\gamma + |P_{ISR}| sin\alpha = 500 sin\alpha \\ P_{J1} sin\theta_{J1} sin\phi_{J1} + P_{J2} sin\theta_{J2} sin\phi_{J2} + P_\gamma sin\theta_\gamma sin\phi_\gamma = 0 \\ P_{J1} cos\theta_{J1} + P_{J2} cos\theta_{J2} + P_\gamma cos\theta_\gamma \pm |P_{ISR}| cos\alpha = 0 \end{cases}$$




**Beam Crossing Angle ≡2$\alpha$ : $\alpha$ = 7.0 mrad**

Direction Angle
$\theta$: polar angle
$\phi$: azimuthal angle

In Method 2’ and 2, measured $P_\gamma$ is used as input.

**Measured $P_\gamma$ is smeared as**

$$P_\gamma = P_{\gamma MC} \times 0.15\sqrt{P_{\gamma MC}} \times gRandom- > Gaus(0., 1.)$$

**corresponding to the photon energy resolution**

$$\frac{\sigma_{E\gamma}}{E_\gamma} = \frac{0.15}{\sqrt{E_\gamma}}$$

# Reconstruction Method

**Method 2’: Ignore ISR and use smeared $P_\gamma$**

Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2},P_\gamma)$ -> Determine $(P_{J1},P_{J2})$

$$\left\{ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \end{pmatrix} = \begin{pmatrix} 500sin\alpha - sin\theta_\gamma cos\phi_\gamma P_\gamma \\ -sin\theta_\gamma sin\phi_\gamma P_\gamma \end{pmatrix} \right.$$

**Method 2: Consider ISR and use smeared $P_\gamma$**

Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2},P_\gamma)$ -> Determine $(P_{J1},P_{J2},P_{ISR})$

$$\left\{ \begin{array}{l} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \quad ① \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\alpha \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & 0 \\ cos\theta_{J1} & cos\theta_{J2} & \pm cos\alpha \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ |P_{ISR}| \end{pmatrix} = \begin{pmatrix} 500sin\alpha - sin\theta_\gamma cos\phi_\gamma P_\gamma \\ -sin\theta_\gamma sin\phi_\gamma P_\gamma \\ -cos\theta_\gamma P_\gamma \end{pmatrix} \end{array} \right.$$

Matrix A

Inverse

**2 solutions for each sign of $P_{ISR}$**

**-> choose the best answer which satisfies ① better**

# Reconstruction Method

**Method 3: Consider ISR and solve the full equation**

Using ($\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2}$) -> Determine ($P_{J1},P_{J2},P_\gamma,P_{ISR}$)

$$
\begin{cases}
\sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}|=500 \\
\begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix}\begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix}=\begin{pmatrix} (500-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix}
\end{cases}
$$

Matrix A

Inverse

**Inserting $P_{J1},P_{J2},P_\gamma$ into the first equation**

**-> 8 Possible Solutions!**

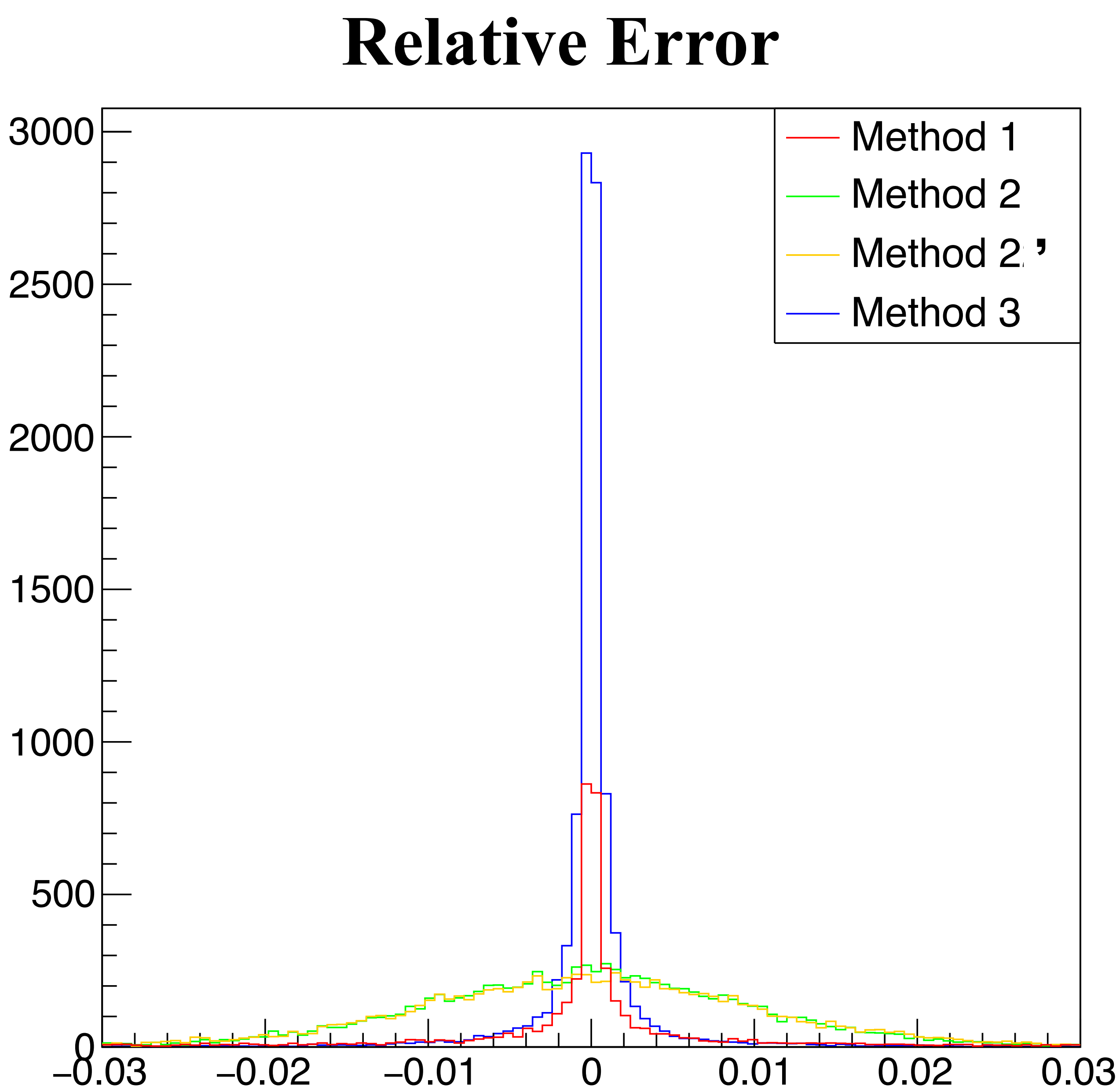
**4: Quartic Equation of $|P_{ISR}|$ X 2: sign of ISR**

- **Choose real and positive solutions**
- **Solved $P_\gamma$ close to the measured (smeared) $P_\gamma$**

# Method Comparison Result



**In every method,**
**Jet Mass Inputs: Smeared 3% in Sigma**
**Jet Angles: No Smearing**




**|Relative Error| >0.1 rate in 10000 events**

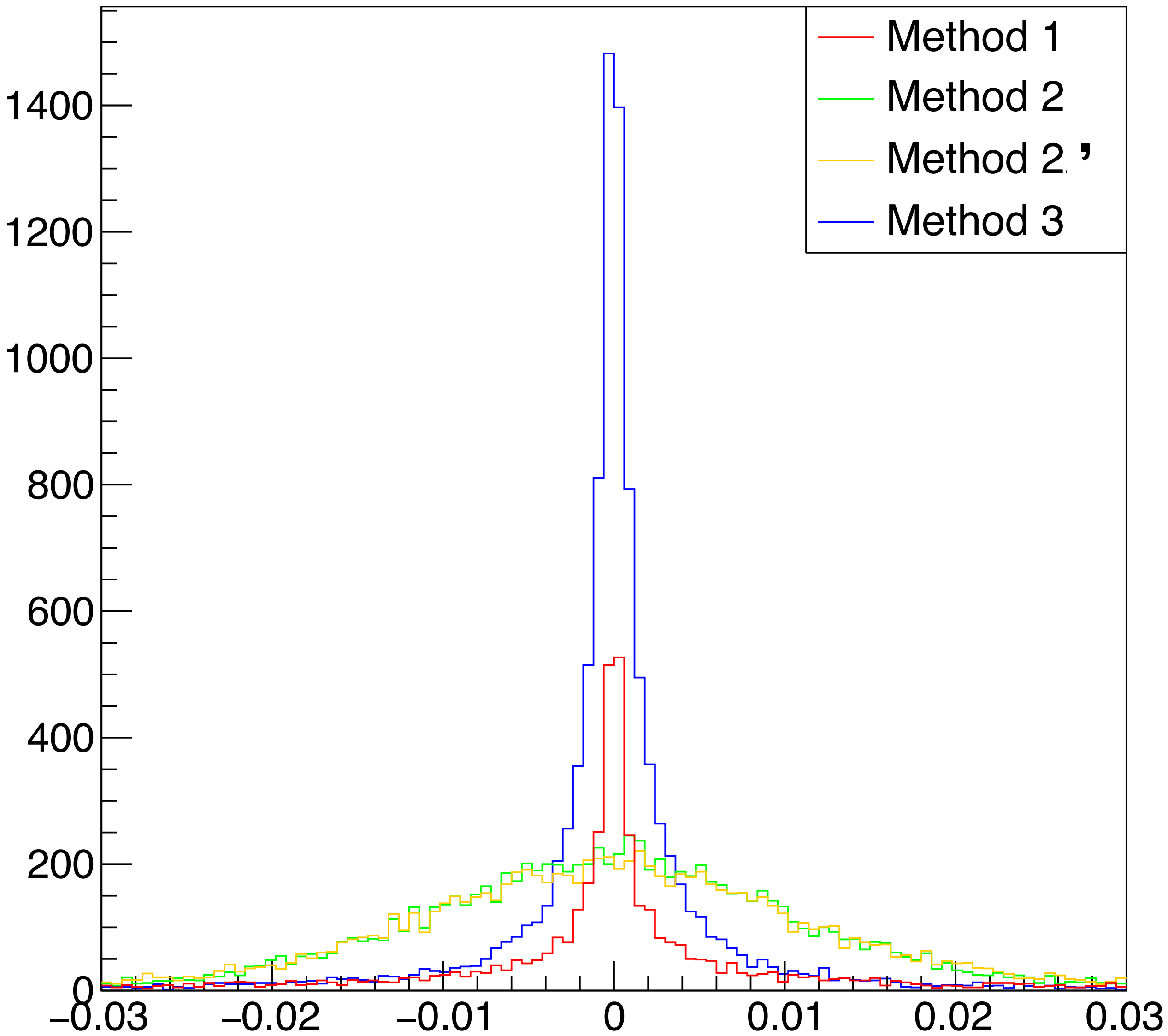
**Method1 : 0.5177**
**Method2 : 0.0041**
**Method2': 0.0096**
**Method3: 0.0006**

# Method Comparison Result



**In every method,**
**Jet Mass Inputs: Smeared 10% in Sigma**
**Jet Angles: No Smearing**

**Relative Error**




**|Relative Error| >0.1 rate in 10000 events**

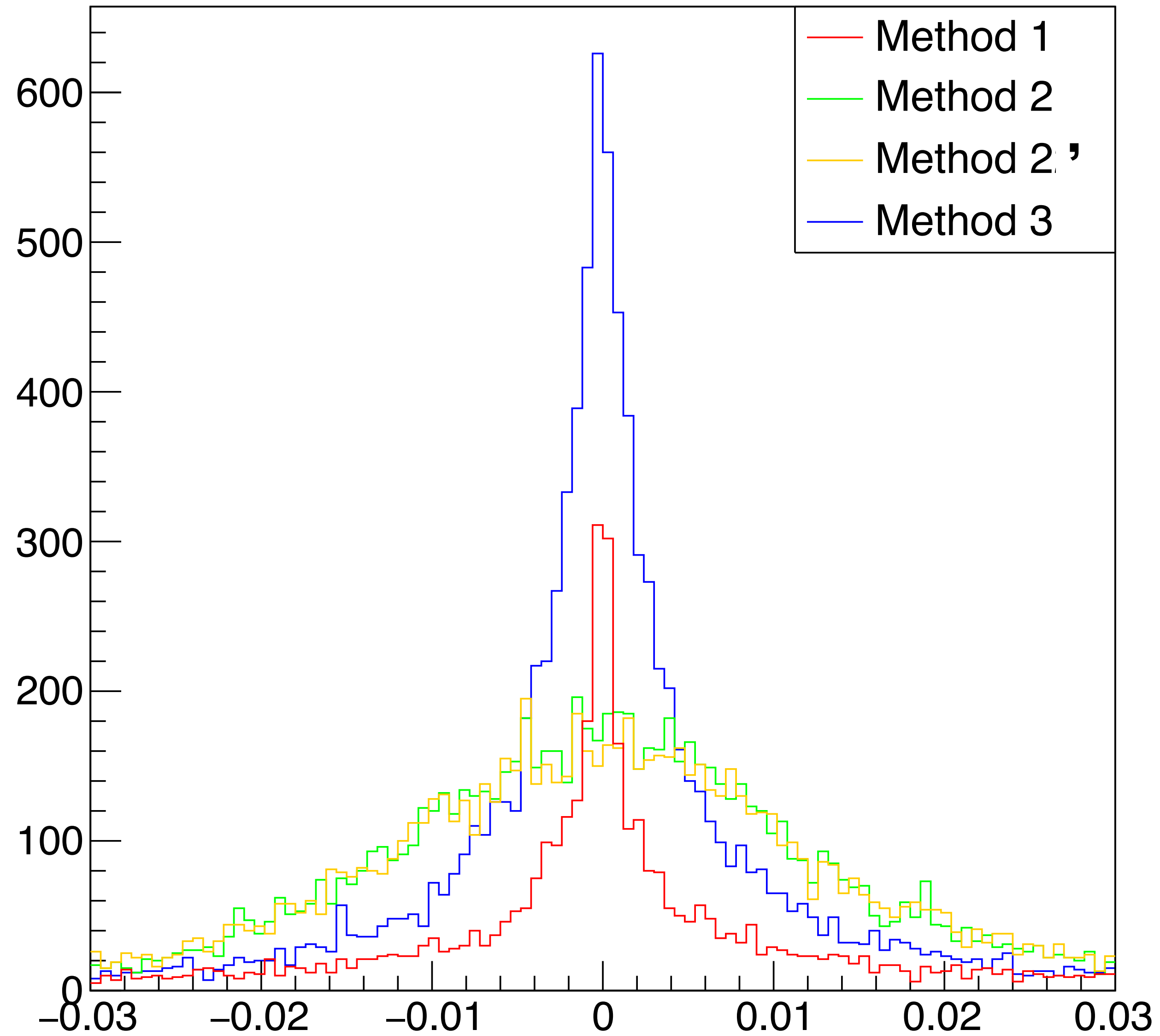
**Method1 : 0.5189**
**Method2 : 0.0081**
**Method2': 0.0130**
**Method3: 0.0043**

# Method Comparison Result



**In every method,**
**Jet Mass Inputs: Smeared 30% in Sigma**
**Jet Angles: No Smearing**

**Relative Error**



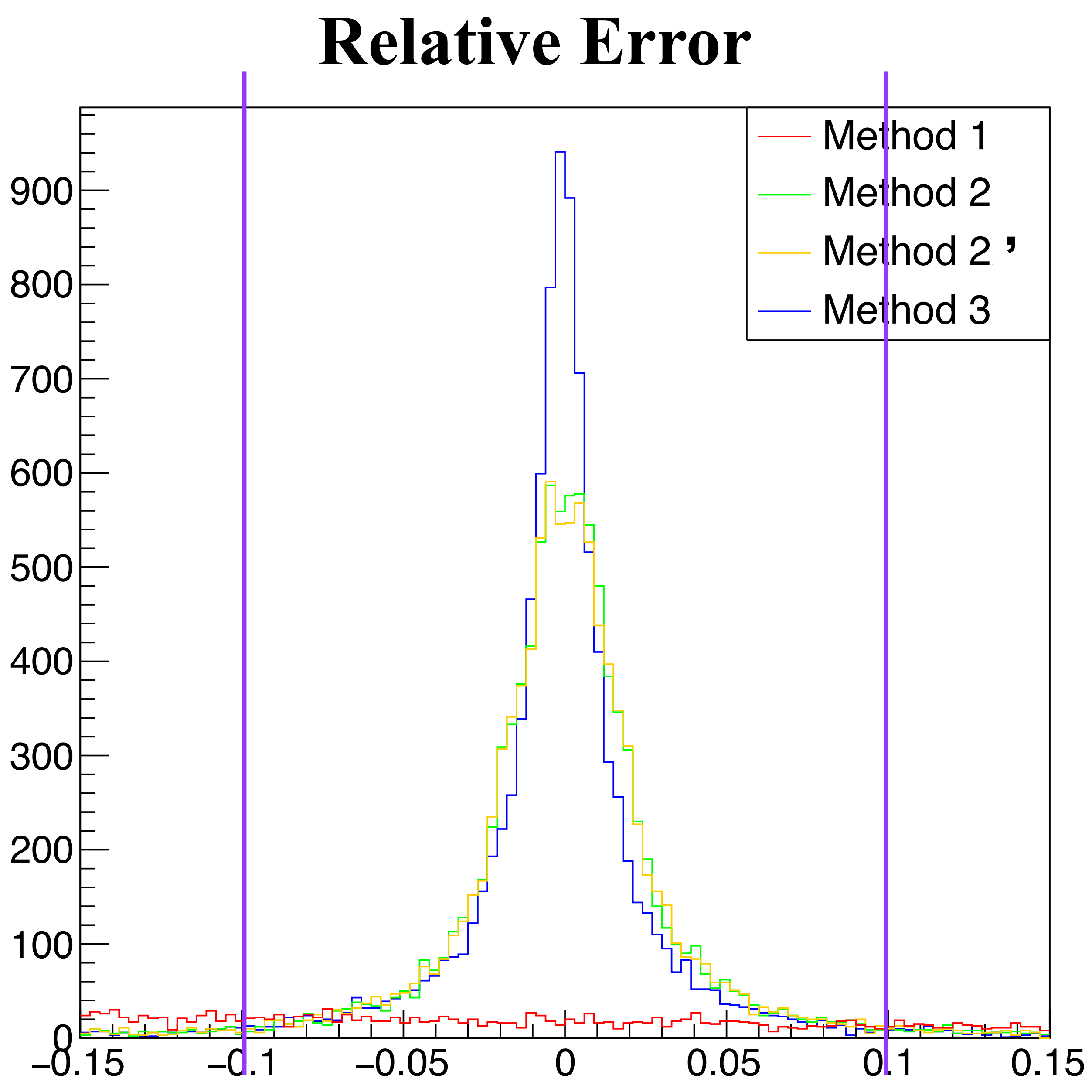

**|Relative Error| >0.1 rate in 10000 events**

**Method1 : 0.5333**
**Method2 : 0.0397**
**Method2’: 0.0449**
**Method3: 0.0339**

# Method Comparison Result



**In every method,**
**Jet Mass Inputs: Smeared 3% in Sigma**
**Jet Angles: Smeared 0.3 degree**




**|Relative Error| >0.1 rate in 10000 events**

**Method1 : 0.8810**
**Method2 : 0.0636**
**Method2': 0.0679**
**Method3: 0.0632**

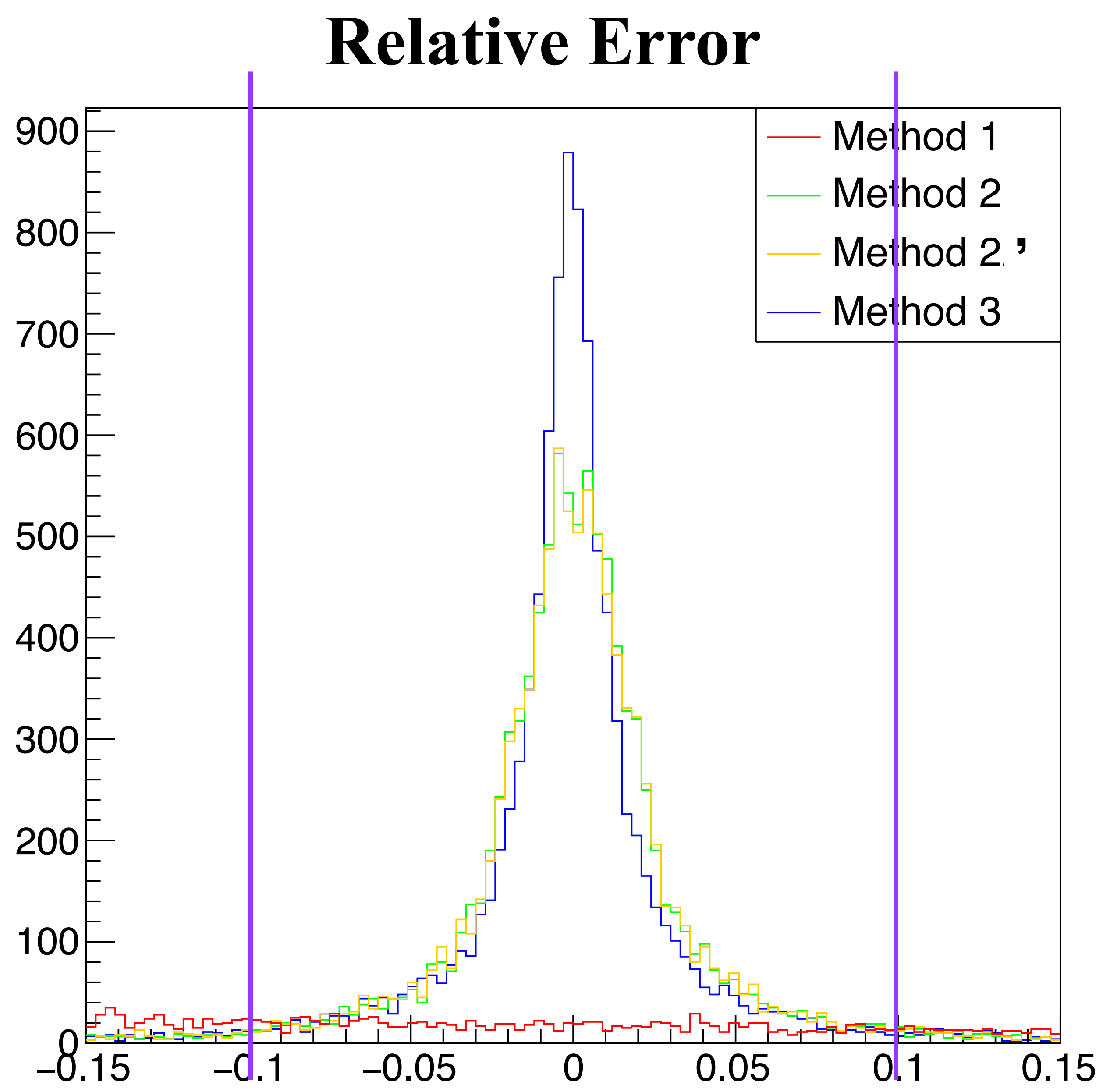
# Method Comparison Result



**In every method,**

**Jet Mass Inputs: Smeared 10% in Sigma**

**Jet Angles: Smeared 0.3 degree**



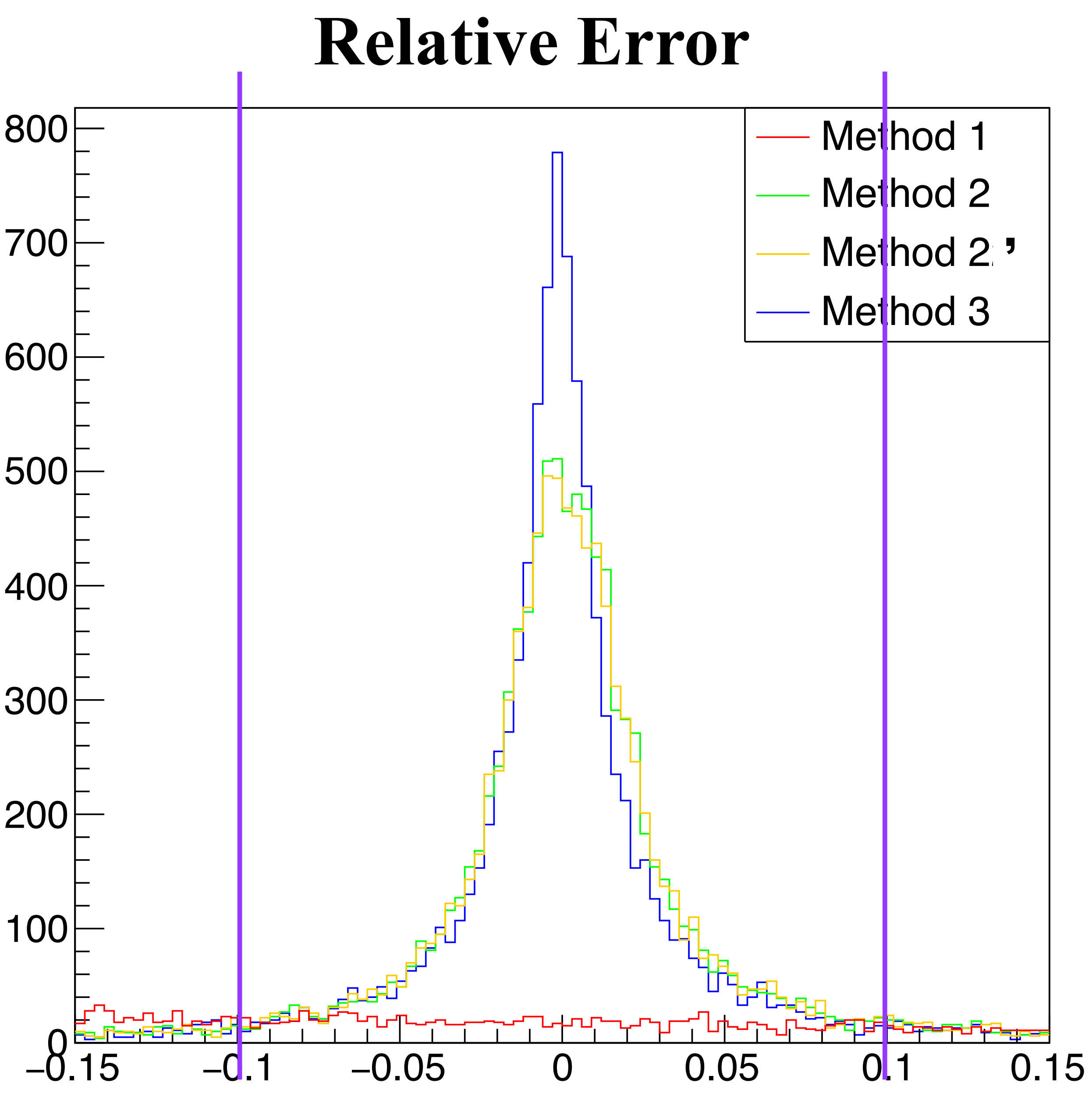

**|Relative Error| >0.1 rate in 10000 events**

**Method1 : 0.8807**
**Method2 : 0.0668**
**Method2’: 0.0722**
**Method3: 0.0668**

# Method Comparison Result



**In every method,**
**Jet Mass Inputs: Smeared 30% in Sigma**
**Jet Angles: Smeared 0.3 degree**




**|Relative Error| >0.1 rate in 10000 events**

**Method1 : 0.8804**
**Method2 : 0.0956**
**Method2’: 0.1003**
**Method3: 0.0929**

# Result

**Method3 is the best for the jet energy reconstruction.**

**As Method3 has 8 solutions,**
**I investigated why we can choose the best answer almost every time.**

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**In Method3, the criteria to choose the best answer are following:**

- **Choose real and positive solutions**
- **Solved $P_\gamma$ close to the measured (smeared) $P_\gamma$**

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**I found that the second criteria has great influence.**

**The difference (solved $P_\gamma$) - (measured $P_\gamma$) is shown as DIFF in next page.**

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# Result



```
************
I31=70.6428
I33=406.849
solv= 0.0118054 31.0622
solv= 0.0653349 6.88158
Best PISR= 0.0118054
 ROOT[0]= 0.0118054 ROOT[1]= 0.0653349 ROOT[2]= 6.88158 ROOT[3]= 31.0622
 DIFF[0]= 1.28653 DIFF[1]= 30.0707 DIFF[2]= 2799.81 DIFF[3]= 12649.2
************
I31=69.0833
I33=-234.084
solv= 0.491862 2.60819
solv= 0.000413702 31.4852
Best PISR= 0.000413702
 ROOT[0]= 0.000413702 ROOT[1]= 0.491862 ROOT[2]= 2.60819 ROOT[3]= 31.4852
 DIFF[0]= 2.05969 DIFF[1]= 112.934 DIFF[2]= 607.296 DIFF[3]= 7387.17
************
I31=87.4963
I33=-63.6165
solv= 19.3529
solv= 0.0387745 1.02363 106.951
Best PISR= 1.02363
 ROOT[0]= 1.02363 ROOT[1]= 0.0387745 ROOT[2]= 19.3529 ROOT[3]= 106.951
 DIFF[0]= 0.125377 DIFF[1]= 63.1295 DIFF[2]= 1284.9 DIFF[3]= 6803.59
************
```

# Reconstruction Method

**After we get solutions of equation $|P_{ISR}|$,**
**$P_\gamma$ is calculated by using the inversed matrix $A^{-1}$,**
**which has huge components (shown as $I_{31}$ and $I_{33}$ in the previous page).**

**Method 3: Consider ISR and solve the full equation**
Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2})$ -> Determine $(P_{J1},P_{J2},P_\gamma,P_{ISR})$

$$\begin{cases} \sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}|=500 \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} (500-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{cases}$$

Matrix A
Inverse

**If $|P_{ISR}|$ is shifted even slightly, the "DIFF" has large value.**
**That's why we can choose the best answer almost every time.**

# Conclusion and future work

**Method3 is the best for the jet energy reconstruction in the Toy MC study.**

**I will move on to the realistic simulation.simulation using 2f_z_h samples.**