Status on e^+e^- -> γZ process Benchmark

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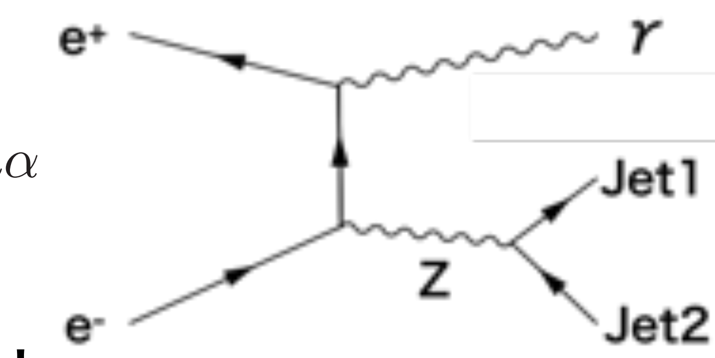
- **ILD note: circulated**
  -> I got 2 comments and implementing them now.

- **Jet Energy Scale calibration**
  1. Try new method (2') to reconstruct the jet energy
  2. Investigate why we can choose the best answer among 8 solutions in method 3.
  -> Explain today

Reconstruction Method

Based on 4-momentum conservation

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \\ P_{J1} sin\theta_{J1} cos\phi_{J1} + P_{J2} sin\theta_{J2} cos\phi_{J2} + P_\gamma sin\theta_\gamma cos\phi_\gamma + |P_{ISR}| sin\alpha = 500 sin\alpha \\ P_{J1} sin\theta_{J1} sin\phi_{J1} + P_{J2} sin\theta_{J2} sin\phi_{J2} + P_\gamma sin\theta_\gamma sin\phi_\gamma = 0 \\ P_{J1} cos\theta_{J1} + P_{J2} cos\theta_{J2} + P_\gamma cos\theta_\gamma \pm |P_{ISR}| cos\alpha = 0 \end{cases}$$



Beam Crossing Angle ≡2α : α = 7.0 mrad

Direction Angle
θ : polar angle
φ : azimuthal angle

- ISR photon = **additional** unseen photon
- Several reconstruction methods (Method 1, 2', 2,and 3) are considered.

Method 1: Ignore ISR

Using ($\theta_{J1}, \theta_{J2}, \theta_\gamma, \varphi_{J1}, \varphi_{J2}, \varphi_\gamma, m_{J1}, m_{J2}$) -> Determine ($P_{J1}, P_{J2}, P_\gamma$)

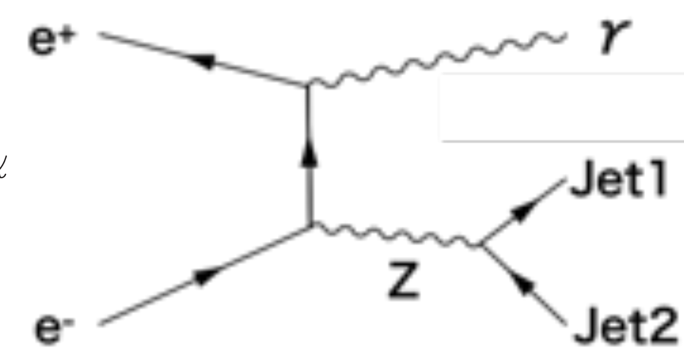
$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| = 500 \\ \begin{pmatrix} sin\theta_{J1} cos\phi_{J1} & sin\theta_{J2} cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1} sin\phi_{J1} & sin\theta_{J2} sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} 500 sin\alpha \\ 0 \\ 0 \end{pmatrix} \end{cases}$$

Matrix A —— Inverse

Reconstruction Method

Based on 4-momentum conservation

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \\ P_{J1} sin\theta_{J1} cos\phi_{J1} + P_{J2} sin\theta_{J2} cos\phi_{J2} + P_\gamma sin\theta_\gamma cos\phi_\gamma + |P_{ISR}| sin\alpha = 500 sin\alpha \\ P_{J1} sin\theta_{J1} sin\phi_{J1} + P_{J2} sin\theta_{J2} sin\phi_{J2} + P_\gamma sin\theta_\gamma sin\phi_\gamma = 0 \\ P_{J1} cos\theta_{J1} + P_{J2} cos\theta_{J2} + P_\gamma cos\theta_\gamma \pm |P_{ISR}| cos\alpha = 0 \end{cases}$$



Beam Crossing Angle ≡2α : α = 7.0 mrad

Direction Angle
θ: polar angle
ϕ: azimuthal angle

In Method 2’ and 2, measured P_γ is used as input.

Measured P_γ is smeared as

$$\mathbf{P_{\gamma MC}} + P_{\gamma MC} \times 0.15\sqrt{P_{\gamma MC}} \times gRandom- > Gaus(0., 1.)$$

corresponding to the photon energy resolution

$$\frac{\sigma_{E\gamma}}{E_\gamma} = \frac{0.15}{\sqrt{E_\gamma}}$$

Reconstruction Method

Method 3: Consider ISR and solve the full equation

Using $(\theta_{J1},\theta_{J2},\theta_{\gamma},\varphi_{J1},\varphi_{J2},\varphi_{\gamma},m_{J1},m_{J2})$ -> Determine $(P_{J1},P_{J2},P_{\gamma},P_{ISR})$

$$\begin{cases} \sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_{\gamma}|+|P_{ISR}|=500 \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_{\gamma}cos\phi_{\gamma} \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_{\gamma}sin\phi_{\gamma} \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_{\gamma} \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_{\gamma} \end{pmatrix} = \begin{pmatrix} (500-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{cases}$$

Matrix A

Inverse

Inserting P_{J1},P_{J2},P_{γ} into the first equation

-> 8 Possible Solutions!

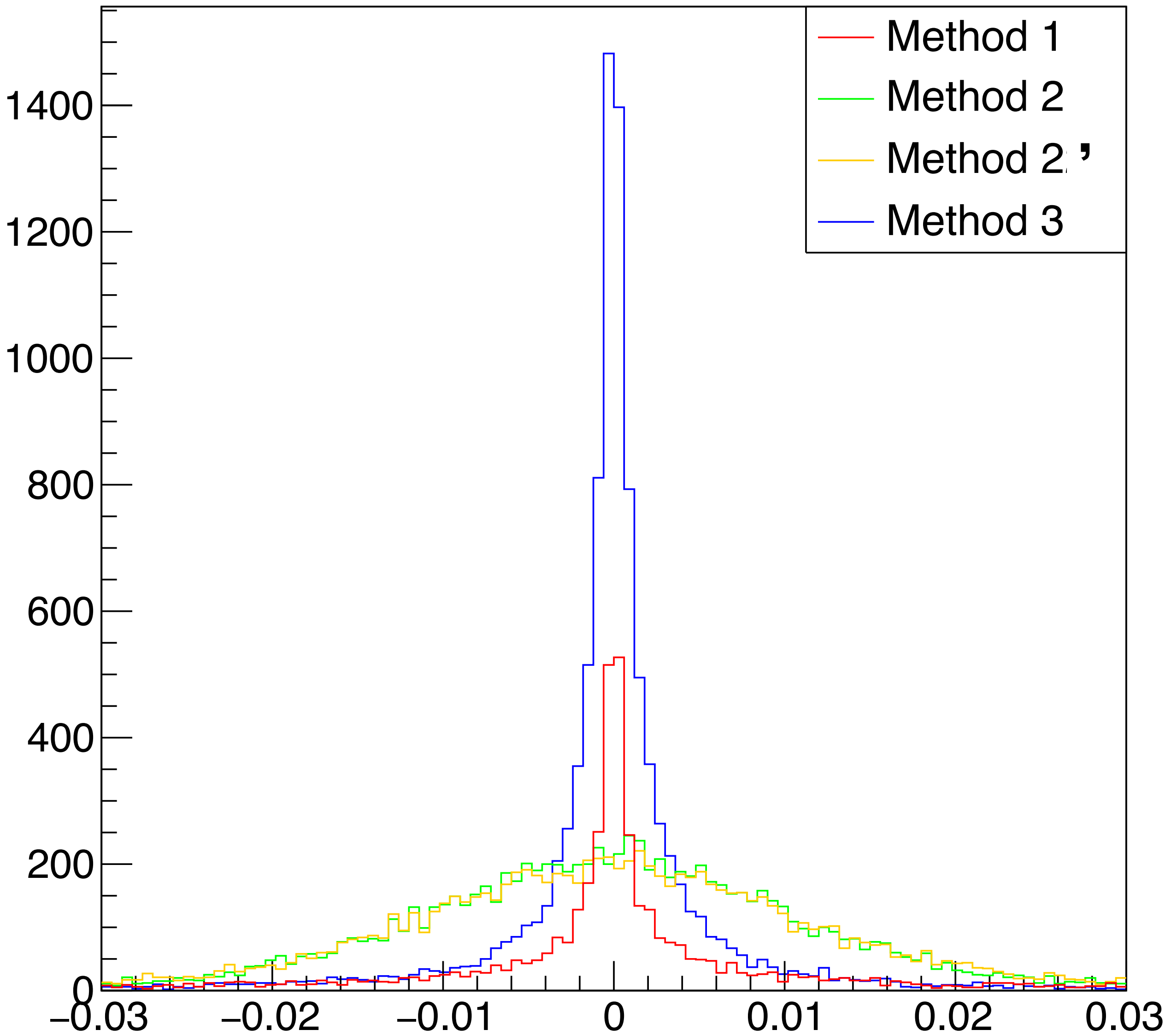
4: Quartic Equation of $|P_{ISR}|$ X 2: sign of ISR

- **Choose real and positive solutions**
- **Solved P_{γ} close to the measured (smeared) P_{γ}**

Method Comparison Result

In every method, Jet Mass Inputs: Smeared 10% in Sigma Jet Angles: No Smearing

Relative Error



|Relative Error| >0.1 rate in 10000 events

Method1 : 0.5189
Method2 : 0.0081
Method2’: 0.0130
Method3: 0.0043

Result

Method3 is the best for the jet energy reconstruction.

As Method3 has 8 solutions,
I investigated why we can choose the best answer almost every time.

In Method3, the criteria to choose the best answer are following:

- **Choose real and positive solutions**
- **Solved P_γ close to the measured (smeared) P_γ**

Result

Method3 is the best for the jet energy reconstruction.

As Method3 has 8 solutions,
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In Method3, the criteria to choose the best answer are following:

- **Choose real and positive solutions**
- **Solved P_γ close to the measured (smeared) P_γ**

I found that the second criteria has great influence.

The difference (solved P_γ) - (measured P_γ) is shown as DIFF in next page.

Result


```
************
I31=70.6428
I33=406.849
solv= 0.0118054 31.0622
solv= 0.0653349 6.88158
Best PISR= 0.0118054
 ROOT[0]= 0.0118054 ROOT[1]= 0.0653349 ROOT[2]= 6.88158 ROOT[3]= 31.0622
 DIFF[0]= 1.28653 DIFF[1]= 30.0707 DIFF[2]= 2799.81 DIFF[3]= 12649.2
************
I31=69.0833
I33=-234.084
solv= 0.491862 2.60819
solv= 0.000413702 31.4852
Best PISR= 0.000413702
 ROOT[0]= 0.000413702 ROOT[1]= 0.491862 ROOT[2]= 2.60819 ROOT[3]= 31.4852
 DIFF[0]= 2.05969 DIFF[1]= 112.934 DIFF[2]= 607.296 DIFF[3]= 7387.17
************
I31=87.4963
I33=-63.6165
solv= 19.3529
solv= 0.0387745 1.02363 106.951
Best PISR= 1.02363
 ROOT[0]= 1.02363 ROOT[1]= 0.0387745 ROOT[2]= 19.3529 ROOT[3]= 106.951
 DIFF[0]= 0.125377 DIFF[1]= 63.1295 DIFF[2]= 1284.9 DIFF[3]= 6803.59
************
```

Reconstruction Method

After we get solutions of equation $|P_{ISR}|$,
P_γ is calculated by using the inversed matrix A^{-1},
which has huge components (shown as I_{31} and I_{33} in the previous page).

Method 3: Consider ISR and solve the full equation
Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2})$ -> Determine $(P_{J1},P_{J2},P_\gamma,P_{ISR})$

$$\begin{cases} \sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}|=500 \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} (500-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{cases}$$

Matrix A

Inverse

If $|P_{ISR}|$ is shifted even slightly, the "DIFF" has large value.
That's why we can choose the best answer almost every time.

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Future work

- **Prepare a presentation at ICCEP Symposium**
- **Jet Energy Scale calibration using 2f_z_h samples Realistic simulation**