Jet Error Parametrization and Application in Kinematic Fit

ILD software and analysis meeting

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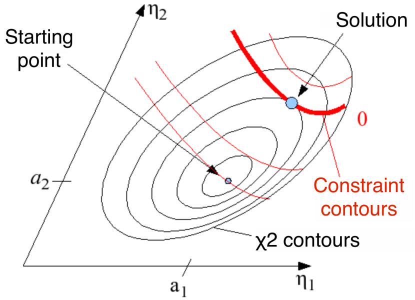
Motivation

- Kinematic fit: a powerful mathematical tool to improve the measurements beyond the detector capability/resolution
  - Vary quantities within their uncertainties to satisfy the kinematic requirements
  - Provides better understanding on source of uncertainties
  - Evaluation of kinematic fit performance based on fit probability
- New production samples:
  - new software version: ILCSoft v02-01-02
  - $e^+e^- \to \mu\bar{\mu}H$ with $H \to b\bar{b}$ at $\sqrt{s} = 250$ GeV
- ErrorFlow: use full covariance matrix of PFOs to estimate uncertainties on jet parameters

Kinematic fit

- Kinematic fit: adjustment of measured quantities under certain kinematic constraints:
  - Energy and momentum conservation
  - Invariant masses of particles



- Minimize χ^2:

$$\chi^2(\boldsymbol{a},\boldsymbol{\xi},\boldsymbol{f}) = (\boldsymbol{\eta}-\boldsymbol{a})^T \boldsymbol{V}^{-1}(\boldsymbol{\eta}-\boldsymbol{a}) - 2\boldsymbol{\lambda}^T \boldsymbol{f}(\boldsymbol{a},\boldsymbol{\xi})$$

$\boldsymbol{\eta}$: vector of measured kinematic variables (x)

$\boldsymbol{a}$: vector of fitted quantities

$\boldsymbol{\xi}$: vector of unmeasured kinematic variables

$\boldsymbol{V}$: covariance matrix

$\boldsymbol{\lambda}$: Lagrange multipliers

$\boldsymbol{f}(\boldsymbol{a},\boldsymbol{\xi})$: vector of constraints

- Measures of performance:

$F(\chi^2;\text{ndf})$: cumulative χ^2 distribution for a certain ndf

$P(\chi^2)$: fit probability

$$pull(x) = \frac{x_{fitted} - x_{measured}}{\sqrt{\sigma^2_{fitted} - \sigma^2_{measured}}}$$

$$P(\chi^2) = 1 - F(\chi^2;\text{ndf})$$

Exploit well-known initial state in e^+e^- colliders

Significantly improved estimation of the underlying kinematics of event

Error flow and application in kinematic fit

by Aliakbar Ebrahimi (DESY-THESIS-2017-045)

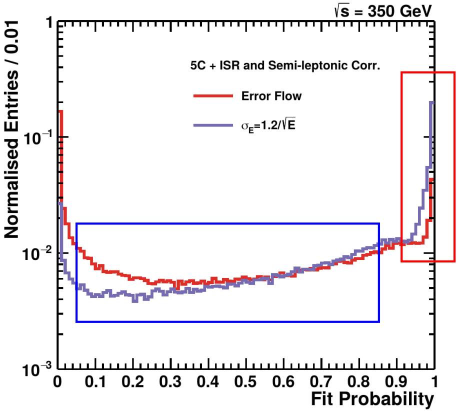
Jet specific energy resolution for $e^+e^- \rightarrow ZH \rightarrow q\bar{q}b\bar{b}$ process at $\sqrt{s} = 350$ GeV

- Full 4×4 CovMatrix on 4-momentum of jets $\sigma(\vec{p}, E)$:
  - σ_{Det}: computed using subdetector momentum/energy resolution
  - σ_{Conf}: computed using jet energy and particle content (charged, neutral and photon)
  - $\sigma_{\nu} = 0.73.E_l$
  - σ_{Had}, σ_{Clus} are not accounted for error flow procedure yet.
- Fixed (and wide) angular resolution: $\sigma_\theta = \sigma_\phi = 100$ mrad

Kinematic fit: vary jet quantities (E,θ,ϕ) within uncertainties (σ_E,σ_θ,σ_ϕ)

Improved fit probability by applying Error Flow on jet energy

$\Rightarrow$ Further improvements by jet error parametrization



Error parametrisations for jets

- Jets parametrized by $(E_{jet},\ \theta_{jet},\ \phi_{jet},\ \sigma_{E_{jet}},\ \sigma_{\theta_{jet}},\ \sigma_{\phi_{jet}},\ m_{jet})$:

$$(p_x , p_y , p_z , E , \text{CovMat}(\vec{p} , E)) \rightarrow (E , \theta , \phi , \sigma_E , \sigma_\theta , \sigma_\phi , m)$$

$$\tan\theta = \frac{\sqrt{p_x^2+p_y^2}}{p_z} ,\ \tan\phi = \frac{p_y}{p_x} ,\ m = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)}$$

σ_θ^2: Error propagation of $(\sigma_{p_x}^2, \sigma_{p_z}^2, \sigma_{p_z}^2, \ldots)$, σ_ϕ^2: Error propagation of $(\sigma_{p_x}^2, \sigma_{p_y}^2, \ldots)$

$$\sigma_{\theta_j}^2 = |\frac{\partial\theta}{\partial p_x}|^2\sigma_{p_x}^2 + |\frac{\partial\theta}{\partial p_y}|^2\sigma_{p_y}^2 + |\frac{\partial\theta}{\partial p_z}|^2\sigma_{p_z}^2 + 2\ \frac{\partial\theta}{\partial p_x}\frac{\partial\theta}{\partial p_y}\sigma_{p_x}\sigma_{p_y} + 2\ \frac{\partial\theta}{\partial p_x}\frac{\partial\theta}{\partial p_z}\sigma_{p_x}\sigma_{p_z} + 2\ \frac{\partial\theta}{\partial p_y}\frac{\partial\theta}{\partial p_z}\sigma_{p_y}\sigma_{p_z}$$

$$\sigma_{\phi_j}^2 = |\frac{\partial\theta}{\partial p_x}|^2\sigma_{p_x}^2 + |\frac{\partial\phi}{\partial p_y}|^2\sigma_{p_y}^2 + 2\ \frac{\partial\theta}{\partial p_x}\frac{\partial\theta}{\partial p_y}\sigma_{p_x}\sigma_{p_y}$$

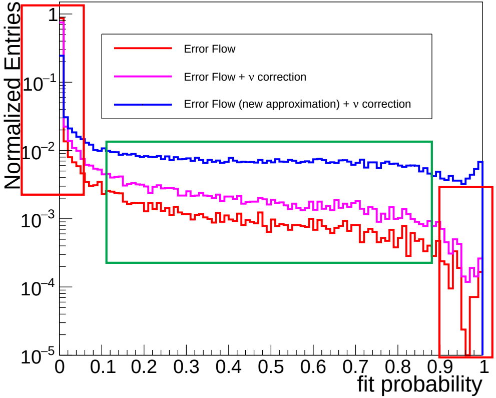
$$\frac{\partial\theta}{\partial p_x} = \frac{p_x p_z}{p^2 p_T} \checkmark ,\ \frac{\partial\theta}{\partial p_y} = \frac{p_y p_z}{p^2 p_T} \checkmark ,\ \frac{\partial\theta}{\partial p_z} = -\frac{p_T}{p^2} \checkmark$$

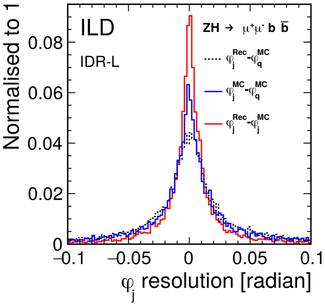
$$\frac{\partial\phi}{\partial p_x} = -\frac{p_y}{p_x^2+p_y^2} \checkmark ,\ \frac{\partial\phi}{\partial p_y} = \frac{p_x}{p_x^2+p_y^2} \checkmark$$

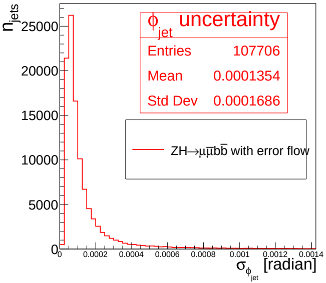
σ_θ and σ_ϕ are not fixed $\Rightarrow$ Angular resolutions are calculated for each individual jet (fixed bugs)

Planned TO DO: input full CovMatrix to fit objects

New approximation on jet angle uncertainties







Improved fit probability using individual jet angle resolution

new approximation: $100 \times (\sigma_{\theta_{jet}}, \sigma_{\phi_{jet}})$

Error Flow under-estimates the uncertainties on jet angles.

jet uncertainties (CovMat) in ErrorFlow

- charged PFOs (identified by charge of PFO $\neq 0$):
  PFO CovMat elements are added to the corresponding jet CovMat elemets
- neutral PFOs (identified by charge of PFO $= 0$):
  only $\sigma^2_{E_{PFO}}$ is added to $\sigma^2_{E_{jet}}$
  $\sigma^2_{E_{PFO}}$ is calculated using E_{PFO} and numerical parameters:
  $\sigma^2_{E_{PFO}} = a^2.E_{PFO} + c^2.E^2_{PFO}$
  - photons: $a_{ECAL} = 0.16$ and $c_{ECAL} = 0.01$ (doi:10.1016/j.nima.2009.07.026)
  - hadrons: $a_{HCAL} = 0.50$ and $c_{HCAL} = 0.01$ (2012 JINST 7 P09017)

ErrorFlow issues:

- full CovMat of neutral PFOs are not included
- neutral PFOs decaying to TWO charged particles can be treated as charged PFOs:
  $\gamma \rightarrow e^+e^-$ and $K^0_S \rightarrow \pi^+\pi^-$

Physics questions in PandoraPFOs collection

Neutral PFOs are assumed massless but identified as massive particle: $E = |\vec{p}|$ and $M \neq 0$

```
 event #1 in: /pnfs/desy.de/ilc/prod/ilc/mc-opt/ild/dst-merged/250-SetA/test/ILD_l5_o1_v02_nobg/v02-01/rv02-01.sv02-01.mILD_l5_o1_v02_nobg.E250-SetA.I401006.Pe2e2h.eL.pR.n000.d_dstm_14717_1.slcio


 collection name : PandoraPFOs
 parameters:

-------------- print out of ReconstructedParticle collection --------------
 [   id   ] |com|type|     momentum( px,py,pz)       | energy | mass   | charge |        position ( x,y,z)      |pidUsed|GoodnessOfPID|
------------|---|----|-------------------------------|--------|--------|--------|-------------------------------|-------|-------------|
 [00001407] |  0|2112|+6.98e-02, -4.04e-01, +3.07e-01|5.13e-01|9.40e-01|0.00e+00|+3.17e+02, -1.84e+03, +1.40e+03|0000000|     0.00e+00|

    covariance( px,py,pz,E) : (3.43e-03, -1.98e-02, 1.15e-01, 1.51e-02, -8.74e-02, 6.65e-02, 2.52e-02, -1.46e-01, 1.11e-01, 1.85e-01)
    particles ( [   id   ] ):
    tracks ( [   id   ] ):
    clusters ( [   id   ] ): [00001124]
    particle ids ( [id], PDG, (type)):
         |   Type   |    PDG   | Likelihood |Algorithm type|
         |         0|         0|    0.00e+00|             5
         |         0|         0|    0.00e+00|             6
         |         0|         0|    0.00e+00|             7
         |         0|         0|    0.00e+00|             8

    vertices: startVertex( id:[ 00000000id_aRP: 00000000]  endVertex( id:[00000000], id_aRP:[00000000]


 collection name : PandoraClusters
 parameters:

-------------- print out of Cluster collection --------------
 [   id   ] |type|  energy  |energyerr |      position ( x,y,z)           |  itheta  |   iphi
------------|----|----------|----------|----------------------------------|----------|----------
 [00001124] |   0|+5.127e-01|+4.299e-01|+3.170e+02, -1.836e+03, +1.396e+03|+8.556e-01|-1.552e+00 errors (6 pos)/( 3 dir): (+2.169e+01, -2.567e-01, +1.287e+02, +3.989e+00, -1.103e+02, +9.746e+01, )/
(+3.574e-02+4.888e-03+7.600e-04)
 clusters(e):
 subdetector energies : +4.369e-01, +0.000e+00, +0.000e+00, +0.000e+00, +0.000e+00, +0.000e+00,
```

needs recalibration of Pandora $\Rightarrow$ run standard JER and check the impact of PID (mass) assigned to each neutral hadron, not feasible at this stage!

Bug in CovMat of neutral PFOs:

- wrong CovMat calculation in AddClusterProperties for neutral PFOs
  Jacobian as $\frac{\partial(\vec{p},E)}{\partial(\vec{r},E)}$ (for cluster errors):

$$J_C = \begin{pmatrix} \frac{\partial p_x}{\partial x} & \frac{\partial p_y}{\partial x} & \frac{\partial p_z}{\partial x} & \frac{\partial E}{\partial x} \\ \frac{\partial p_x}{\partial y} & \frac{\partial p_y}{\partial y} & \frac{\partial p_z}{\partial y} & \frac{\partial E}{\partial y} \\ \frac{\partial p_x}{\partial z} & \frac{\partial p_y}{\partial z} & \frac{\partial p_z}{\partial z} & \frac{\partial E}{\partial z} \\ \frac{\partial p_x}{\partial E} & \frac{\partial p_y}{\partial E} & \frac{\partial p_z}{\partial E} & \frac{\partial E}{\partial E} \end{pmatrix} = \begin{pmatrix} E\frac{r^2+x^2}{r^3} & E\frac{x.y}{r^3} & E\frac{x.z}{r^3} & 0 \\ E\frac{x.y}{r^3} & E\frac{r^2+y^2}{r^3} & E\frac{y.z}{r^3} & 0 \\ E\frac{x.z}{r^3} & E\frac{y.z}{r^3} & E\frac{r^2+z^2}{r^3} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} & 1 \end{pmatrix}$$

$$|\vec{p}| = E \; , \; p_x = E\frac{x}{\sqrt{x^2+y^2+z^2}} \; , \; r = \sqrt{x^2+y^2+z^2} \Rightarrow \frac{\partial p_x}{\partial x} = E\frac{r^2-x^2}{r^3} \; , \; \frac{\partial p_x}{\partial y} = -E\frac{xy}{r^3} \; , \; \frac{\partial p_x}{\partial z} = -E\frac{xz}{r^3}$$

$$\Rightarrow J_C = \begin{pmatrix} E\frac{r^2-x^2}{r^3} & -E\frac{x.y}{r^3} & -E\frac{x.z}{r^3} & 0 \\ -E\frac{x.y}{r^3} & E\frac{r^2-y^2}{r^3} & -E\frac{y.z}{r^3} & 0 \\ -E\frac{x.z}{r^3} & -E\frac{y.z}{r^3} & E\frac{r^2-z^2}{r^3} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} & 1 \end{pmatrix}$$

- CovMat for neutral PFOs (without track):

$C_p = J_C^T C_{RC} J_C$

C_p: CovMatrix of ($\vec{p}$,E), C_{RC}: cluster errors (cluster energy error and cluster position (x, y and z) errors),

J_C: Jacobian matrix (cluster parameters)

$$C_{RC} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xE} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{yE} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{zE} \\ \sigma_{xE} & \sigma_{yE} & \sigma_{zE} & \sigma_E^2 \end{pmatrix} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_E^2 \end{pmatrix}$$

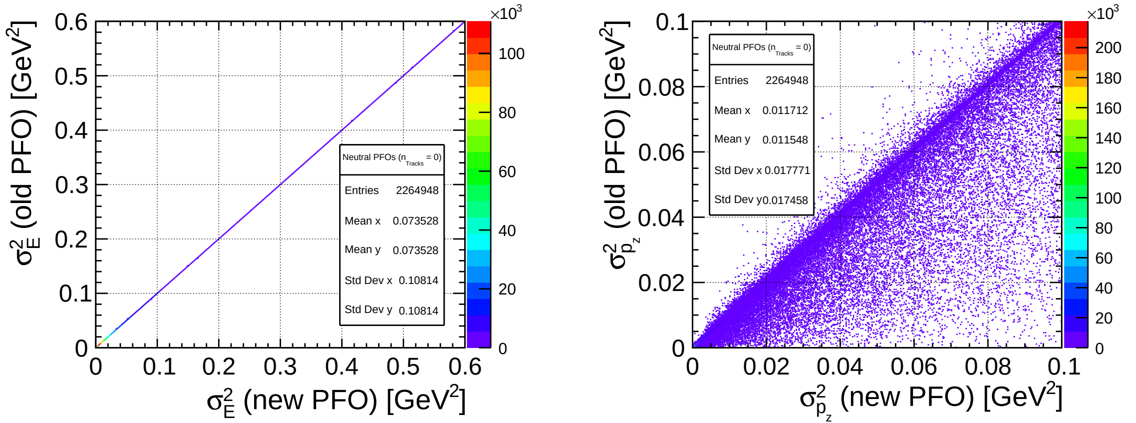
σ_x^2, σ_{xy}, ... from cluster position errors

σ_E^2 from cluster energy error (σ_E)

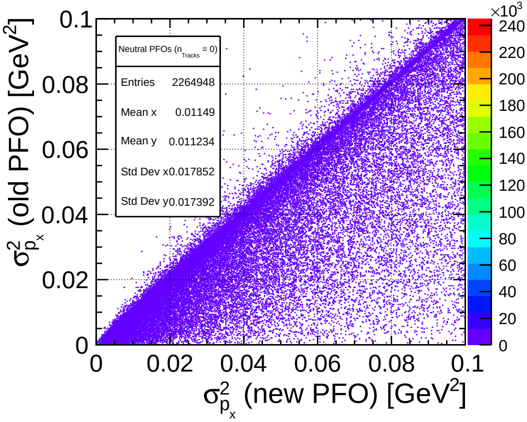
implemented in AddFourMomentumCovMatAllPFOs

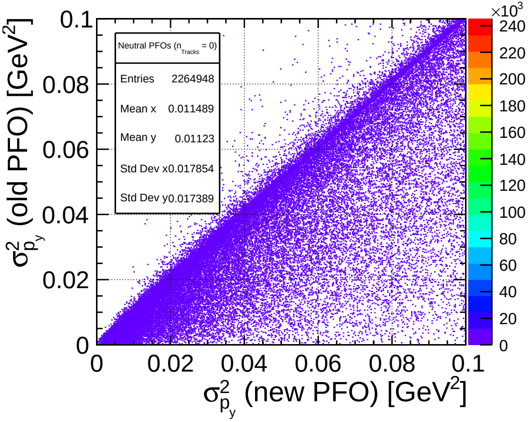
(`https://github.com/yradkhorrami/AddFourMomentumCovMatAllPFOs`)

CovMat elements with new calculations:

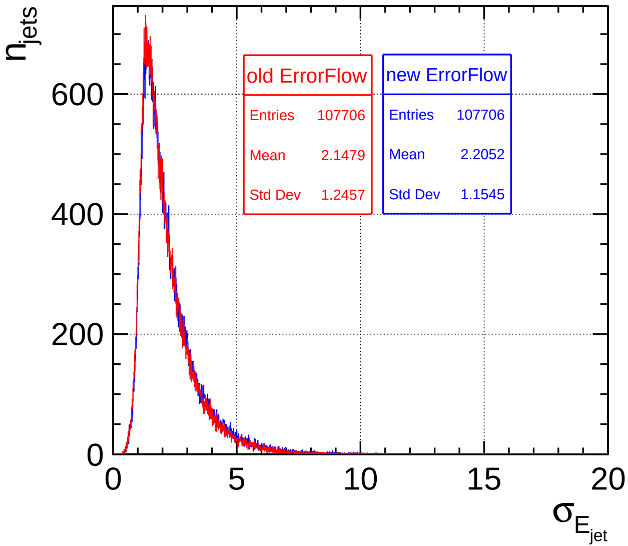


CovMat elements with new calculations:

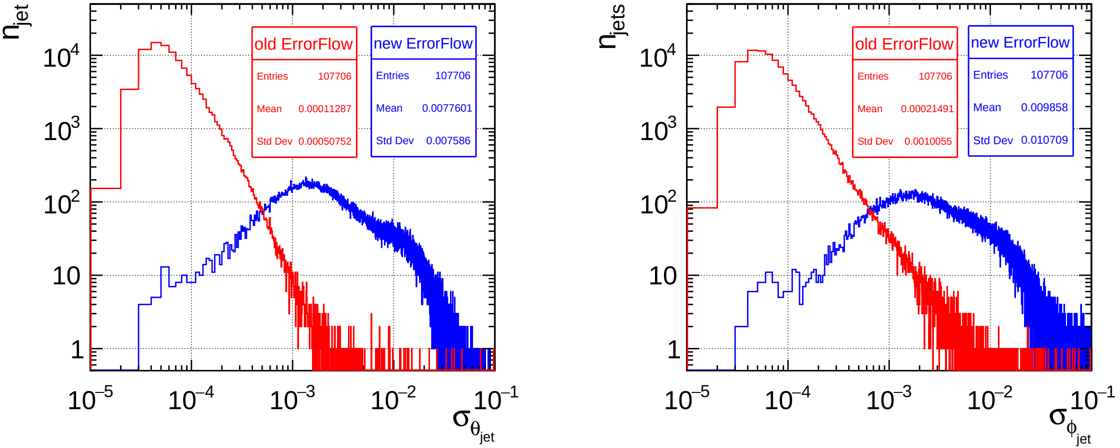




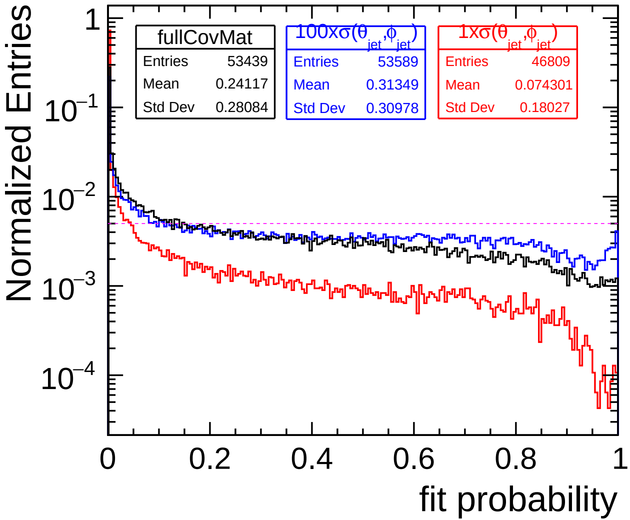
uncertainty on jet energy is unchanged (as expected) with new ErrorFlow calculation



uncertainties on jet angles are improved (worth factor of ∼100) with new ErrorFlow calculation



improved fit probability with new ErrorFlow calculation



Summary

- uncertainties on jet angles are parametrized for individual jets (new in MarlinKinFit)
  - neutral PFOs with TWO tracks: CovMat is included in Error Flow
  - neutral PFOs without track: fixed bug in AddClusterProperties
- improved fit probability with updated resolution on jet angles

open issues:

- Jacobian for neutral PFOs: with E or p? (preliminary with p)
- Further improvement on error estimation:
  - Full $(E, \vec{p})$ covariance matrix for JetFitObject (include $\sigma_{p_x E}$,$\sigma_{p_y E}$,$\sigma_{p_z E}$)
  - Estimate parton shower & hadronisation effects
  - Use proper masses, momenta and CovMatrices of PFOs from tracks refitted with correct mass hypothesis