Precision calculations for electroweak processes

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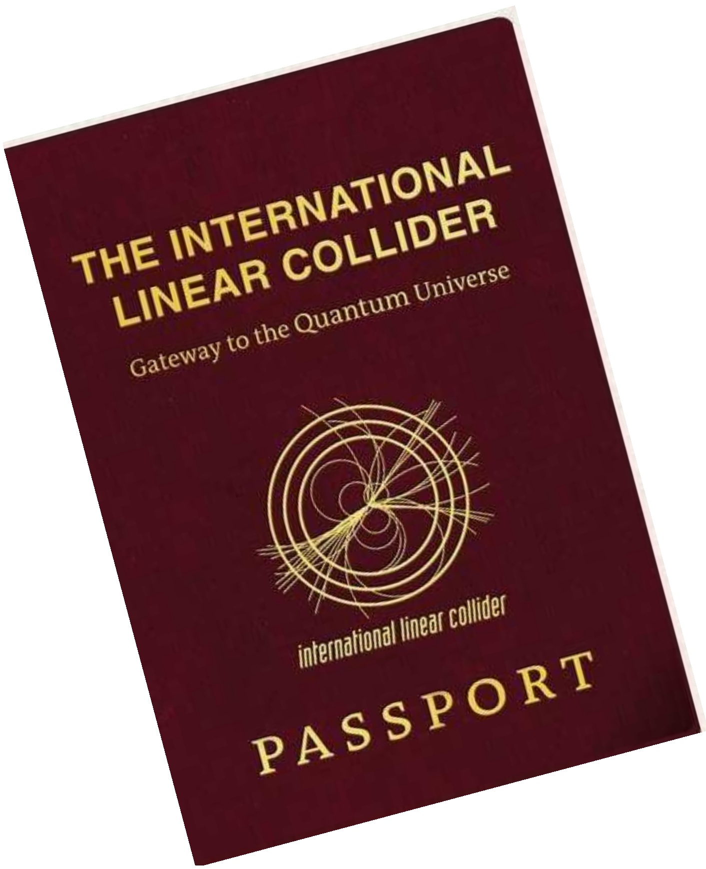


- Z pole
- WW production
- Precision tests of (B)SM physics
- Higgs physics $\longrightarrow$ talk by Mück

based on arXiv:1906.05379

Need for theory input

- Comparison of EWPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ QED/QCD, some “boring” SM effects

- “Other” eletroweak parameters (**“input” parameters**)
  $\longrightarrow$ $m_{\rm t}$, $\alpha_{\rm s}$, etc. extracted from other processes $\longrightarrow$ talk by Moult

1906.05379:

“Theoretical uncertainties for electroweak and Higgs-boson measurements at FCC-ee”

Future e^+e^- colliders

	Current exp.	CEPC	FCC-ee	ILC_{250}	CLIC_{380}
M_W [MeV]	12	1	1	2.5	?
Γ_Z [MeV]	2.3	0.5	0.1	–	–
$R_\ell = \Gamma_Z^\mathrm{had}/\Gamma_Z^\ell$ $[10^{-3}]$	25	2	1	14	38
$R_b = \Gamma_Z^b/\Gamma_Z^\mathrm{had}$ $[10^{-5}]$	66	4.3	6	23	38
$\sin^2\theta_\mathrm{eff}^\ell$ $[10^{-5}]$	13^*	2.3	0.5	2	7.8

* naive combination of LEP/SLC/TeV/LHC

$\longrightarrow$ Improved measurements of several EWPOs necessary to improve global fit

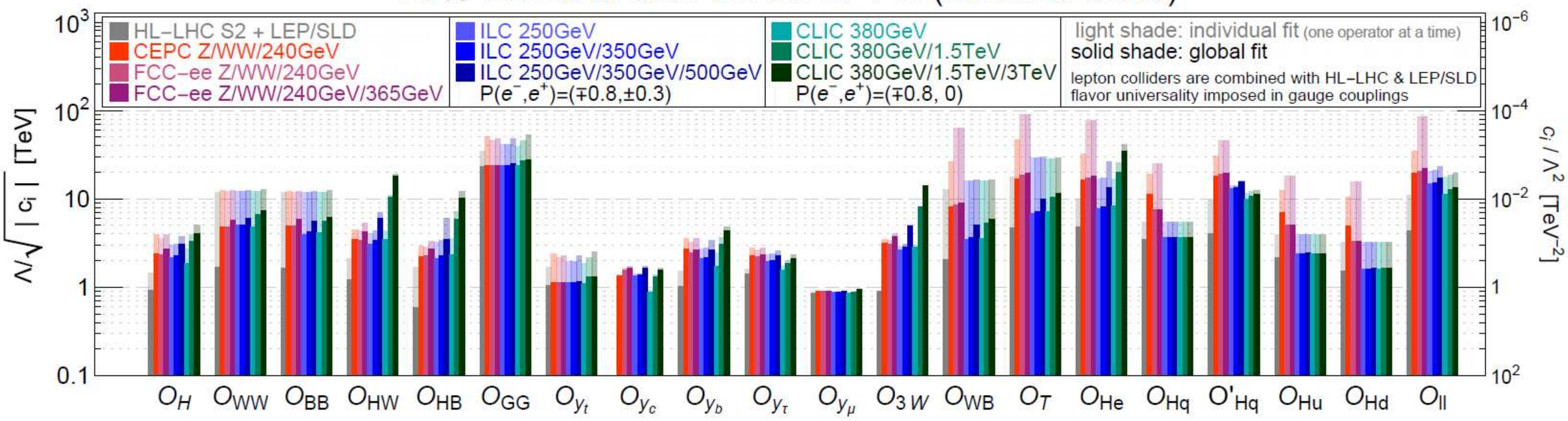
$\longrightarrow$ Challenge for theorists (multi-loop calc. and MC tools)

New physics reach

- More operators than observables
  $\longrightarrow$ Need to make assumptions, e.g. U(3) or U(2) flavor universality

- Strong correlations between some operators
  $\longrightarrow$ All exp. inputs are important

95% CL reach from the full EFT fit (modified SILH')



de Blas, Durieux, Grojean, Gu, Paul ’19

Theory uncertainties

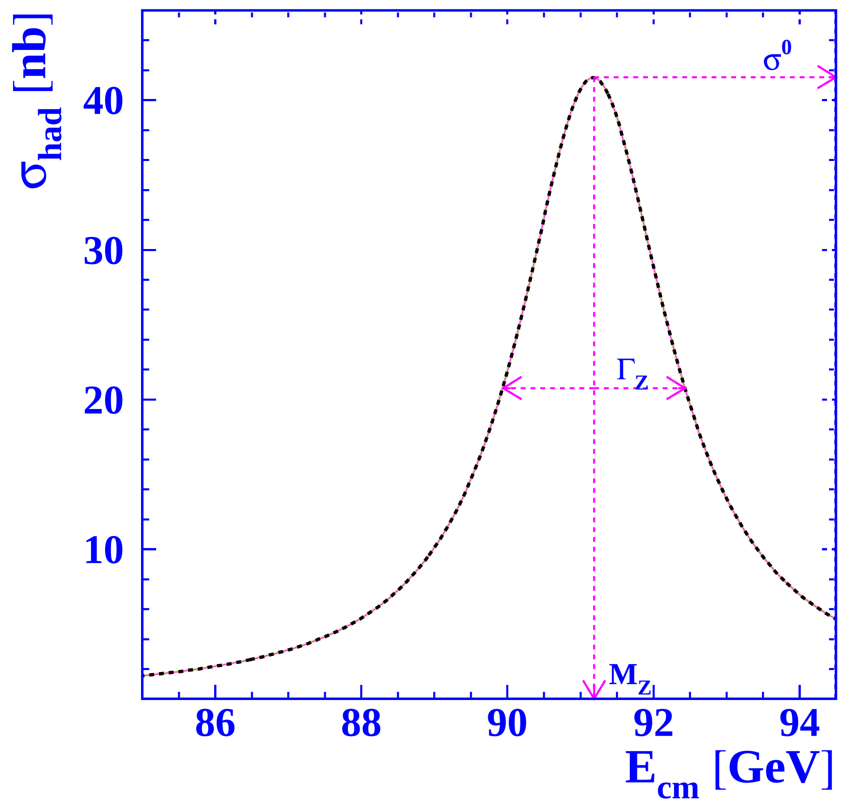
- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

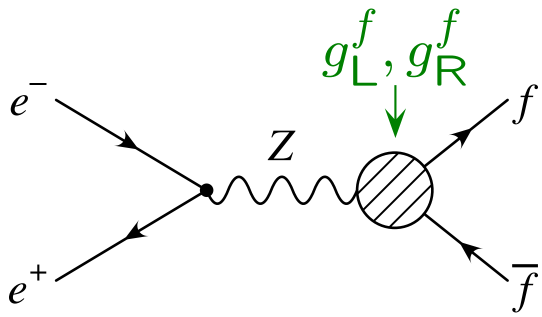
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2} R_e R_f$

$$\Gamma_{ff} = C\left[(g_{\text{L}}^f)^2 + (g_{\text{R}}^f)^2\right]$$



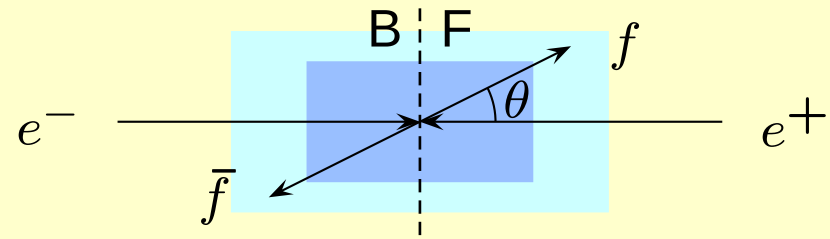


Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\mathsf{FB}} \equiv \frac{\sigma_{\mathsf{F}} - \sigma_{\mathsf{B}}}{\sigma_{\mathsf{F}} + \sigma_{\mathsf{B}}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$



Other asymmetries:

- Average τ pol. in $e^+e^- \to \tau^+\tau^-$, $\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$
- Left-right asymmetry (for polarized e^-): $A_{\mathsf{LR}} = \frac{1}{P_{e^-}} \frac{\sigma_{\mathsf{L}} - \sigma_{\mathsf{R}}}{\sigma_{\mathsf{L}} + \sigma_{\mathsf{R}}} = -\mathcal{A}_e$

Z lineshape

Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

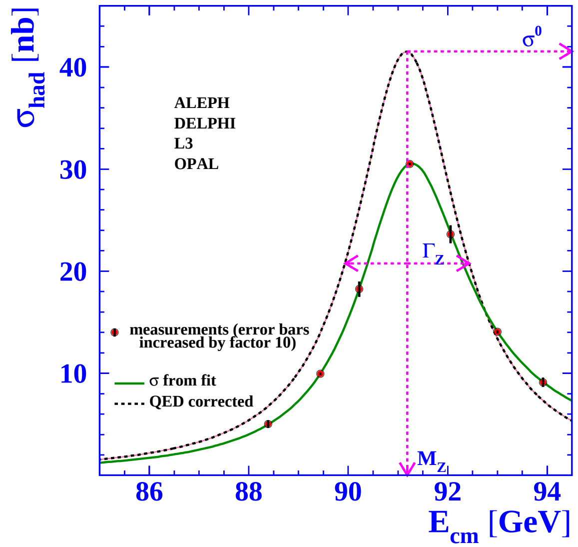
$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

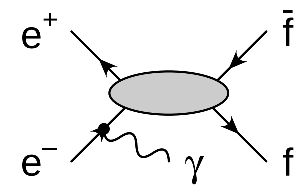
Universal ($m{=}n$) logs known to $n=6$,
also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools → talk by Jadach

LEP EWWG '05

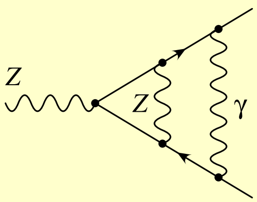




Z decay

Factorization of massive and QED/QCD FSR:

$$\Gamma_f \approx \frac{N_c M_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma'_Z}\right]_{s=M_Z^2}$$



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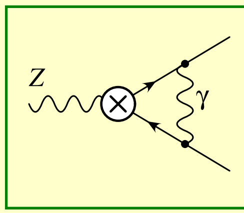
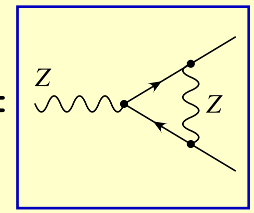


$+$ finite, with $\otimes =$



$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;

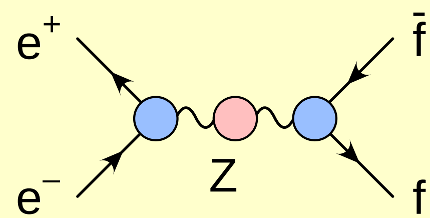
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

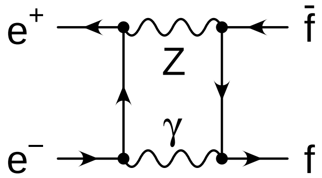
Baikov, Chetyrkin, Kühn, Rittinger '12

g_V^f, g_A^f, Σ'_Z: Electroweak corrections

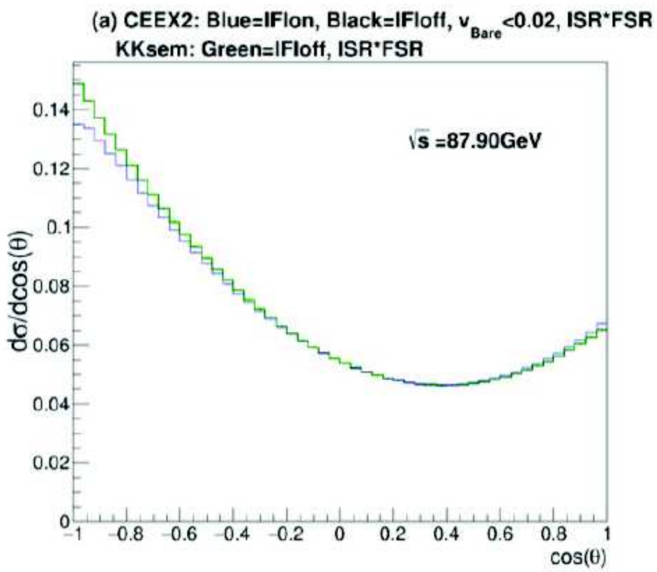


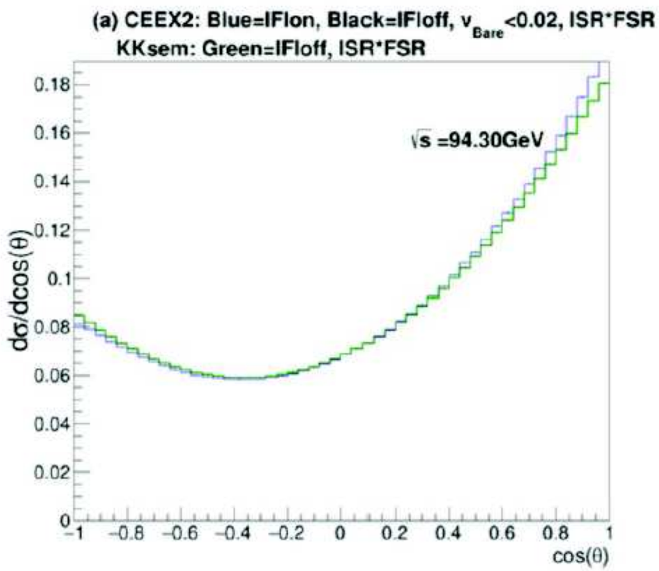
Z-boson initial-final inference

- Interference between ISR and FSR suppressed by Γ_Z/M_Z on Z resonance



- Still relevant for high precision an off-resonance





Jadach, Yost ’18

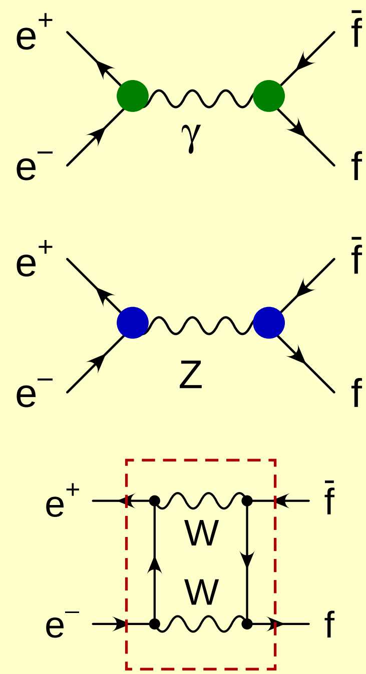
- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost ’18
Greco, Pancheri-Srivastava, Srivastava ’75

"Hard" matrix element

Consistent (gauge-invariant) theory setup:

Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:



$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings

At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

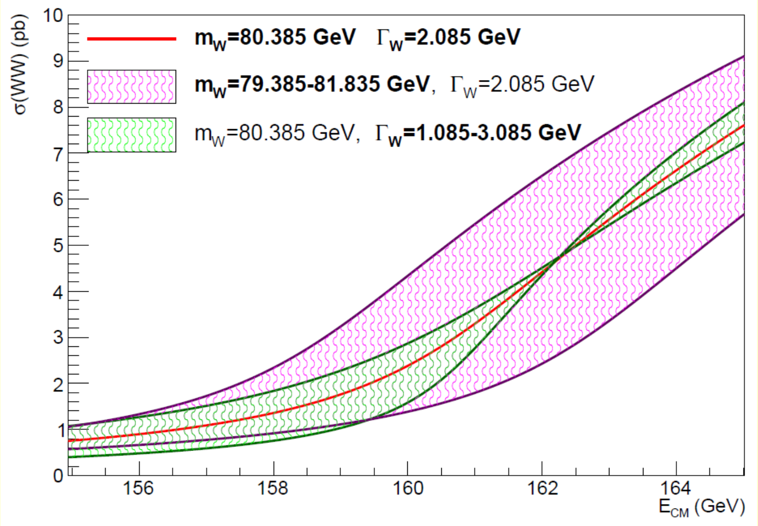
Current state of art: full one-loop for S, T

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed) — see, e.g., Bardin, Grünewald, Passarino '99

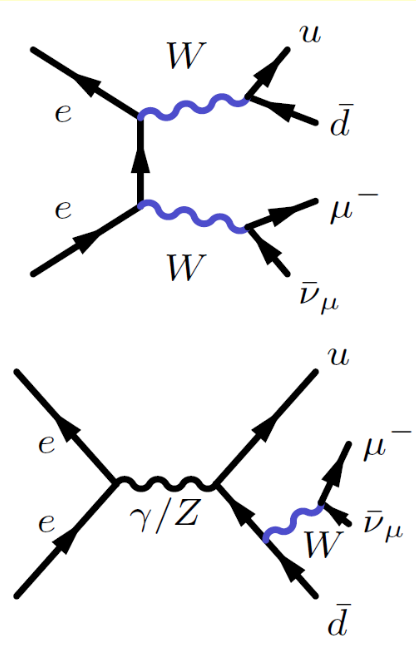
$\longrightarrow$ Sensitivity to some NP beyond EWPO

WW threshold

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by
  $1/\beta$ and $\ln\beta$
  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$
  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$
  Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\text{th}} M_W \sim 3$ MeV
    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\text{th}} M_W \lesssim 0.6$ MeV



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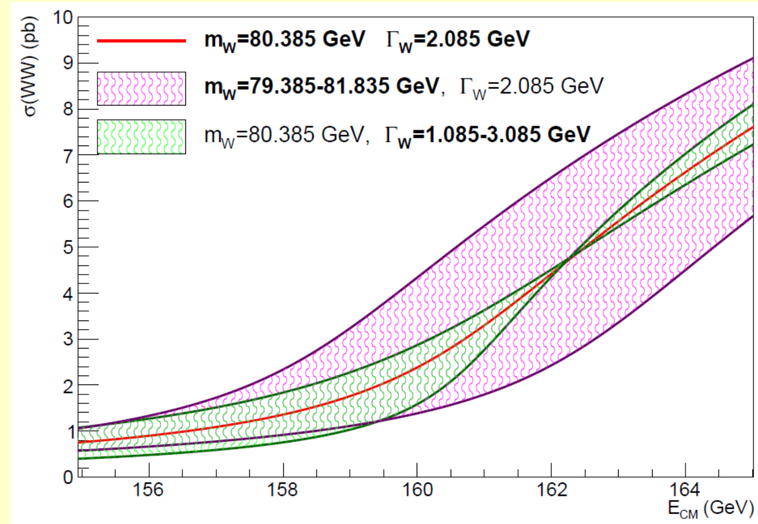
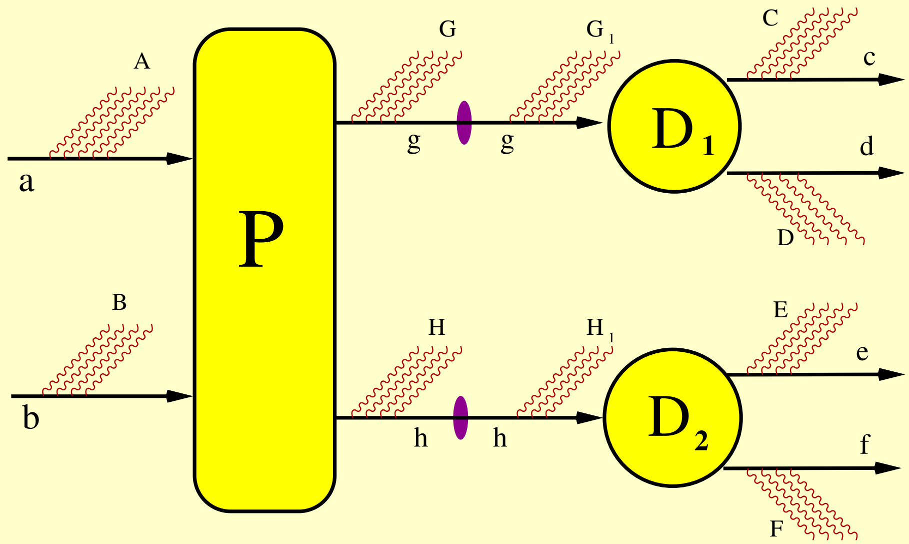


- Resummation of soft photon radiation

Jadach, Płaczek, Skrzypek ’19



Precision test of (B)SM physics

	Current exp.	Current th.	ILC exp.
M_{W} [MeV]	12	4	2.5
$R_\ell = \Gamma_{\mathrm{Z}}^{\mathrm{had}}/\Gamma_{\mathrm{Z}}^{\ell}$ $[10^{-3}]$	25	5	14
$R_b = \Gamma_{\mathrm{Z}}^{b}/\Gamma_{\mathrm{Z}}^{\mathrm{had}}$ $[10^{-5}]$	66	10	23
$\sin^2\theta_{\mathrm{eff}}^{\ell}$ $[10^{-5}]$	13	4.5	2

$\longrightarrow$ To probe NP, need to compare with SM theory predictions ($\longrightarrow$ theory error)

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC!

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_{W}, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '13,14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhanced by y_{t} and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

Projected future theory uncertainties

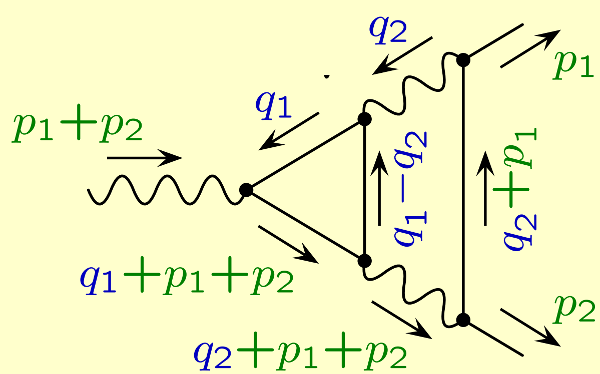
	Current exp.	Current th.	ILC exp.	perturb. error with 3/4-loop†
M_W [MeV]	12	4	2.5	1
$R_\ell = \Gamma_\text{Z}^\text{had}/\Gamma_\text{Z}^\ell$ $[10^{-3}]$	25	5	14	1.5
$R_b = \Gamma_\text{Z}^b/\Gamma_\text{Z}^\text{had}$ $[10^{-5}]$	66	10	23	5
$\sin^2\theta^\ell_\text{eff}$ $[10^{-5}]$	13	4.5	2	1.5

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\text{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\text{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\text{s})$,
leading 4-loop $\mathcal{O}(\alpha_\text{t}^{4-n}\alpha_\text{s}^n)$,
$[N_f^n$ = at least n closed fermion loops, $\alpha_\text{t} = y_\text{t}^2/(4\pi)]$

Also “parametric” uncertainties from inputs for m_t, α_s, $\Delta\alpha$

Theory calculations

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(1000) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods are more general, but computing intensive
- Special numerical techniques can balance precision and evaluation time

Analytic calculations

- Mostly used for diagrams with few mass scales
- Reduce to **master integrals** with integration-by-parts and other identities
  Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...

  Public programs:

Public programs:	`Reduze`	von Manteuffel, Studerus '12
	`FIRE`	Smirnov '13,14
	`LiteRed`	Lee '13
	`KIRA`	Maierhoefer, Usovitsch, Uwer '17

  $\longrightarrow$ Large need for computing time and memory

- Evaluate master integrals with differential equations or Mellin-Barnes rep.
  Kotikov '91; Remiddi '97; Smirnov '00,01; Henn '13; ...

  $\longrightarrow$ Result in terms of Goncharov polylogs / multiple polylogs

  $\longrightarrow$ Some problems need iterated elliptic integrals / elliptic multiple polylogs
  Broedel, Duhr, Dulat, Trancredi '17,18
  Ablinger er al. '17

  $\longrightarrow$ Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios,
  $e.\,g.\ M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

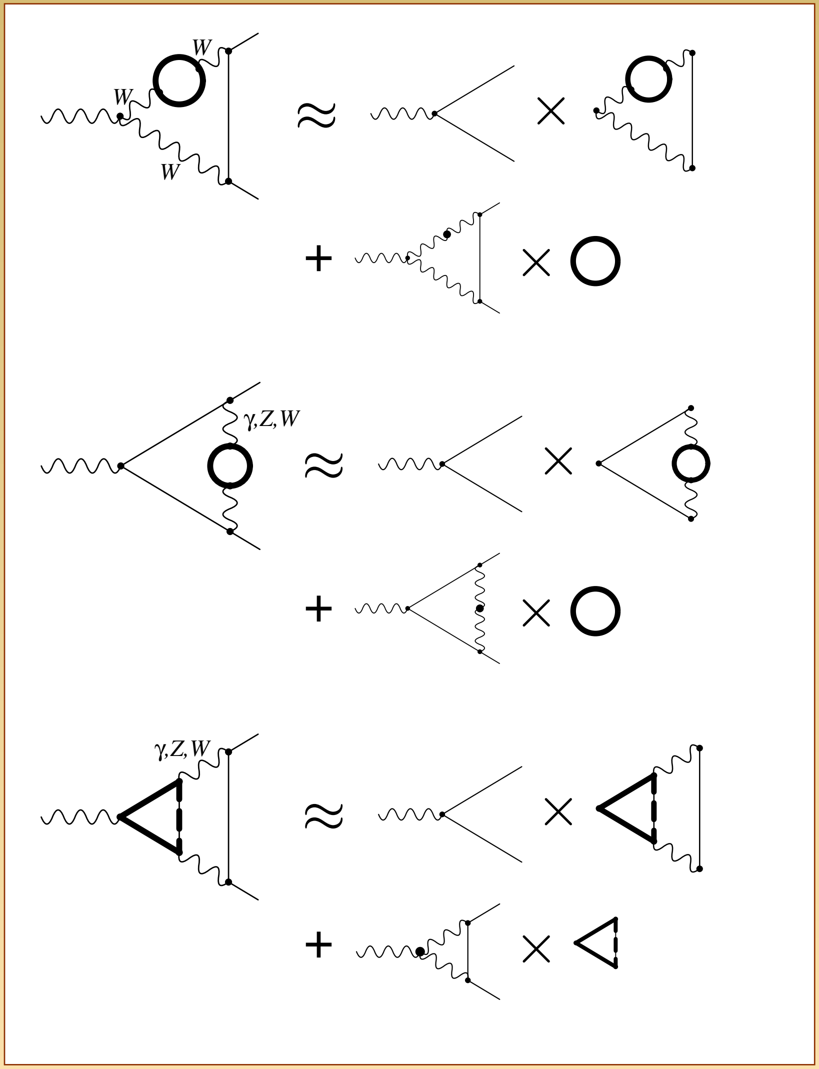
⟶ Used in some 2/3-scale problems

⟶ Public programs:

exp Harlander, Seidensticker, Steinhauser '97
asy Pak, Smirnov '10

⟶ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Direct numerical integration

Two general approches:

⟶ Automated treatment of UV/IR divergencies

⟶ No restriction on number of loops or legs

- **Sector decomposition:**

  Public programs: `SecDec` Carter, Heinrich '10; Borowka et al. '12,15,17
  `FIESTA` Smirnov, Tentyukov '08; Smirnov '13,15

- **Mellin-Barnes representations:**

  Public programs: `MB/MBresolve` Czakon '06; Smirnov, Smirnov '09
  `AMBRE/MBnumerics` Gluza, Kajda, Riemann '07
  Dubovyk, Gluza, Riemann '15
  Usovitsch, Dubovyk, Riemann '18

- Diagrams with internal thresholds can cause numerical instabilities
- Specialized techniques (for some type of diagrams) often improve computing time, robustness, precision (but not automated)

Summary

- **Electroweak precision tests** at future e^+e^- colliders require 1–2 orders improvement in SM theory calculations and tools
  - Z-pole: 3-loop & leading 4-loop EW + multi-loop/leg merging for QED MC
  - off Z-pole / backgrounds: ($\geq$2)-loop EW
  - WW 2-loop EW for 2$\rightarrow$2 processes (+ 4-loop QCD)
    ($\geq$1)-loop for backgr. and non-resonant terms

- Improvements needed both for **fixed-order loop corrections** as well as **MC tools**

- Development of **new calculational techniques** (numerical/semi-numerical) may be crucial

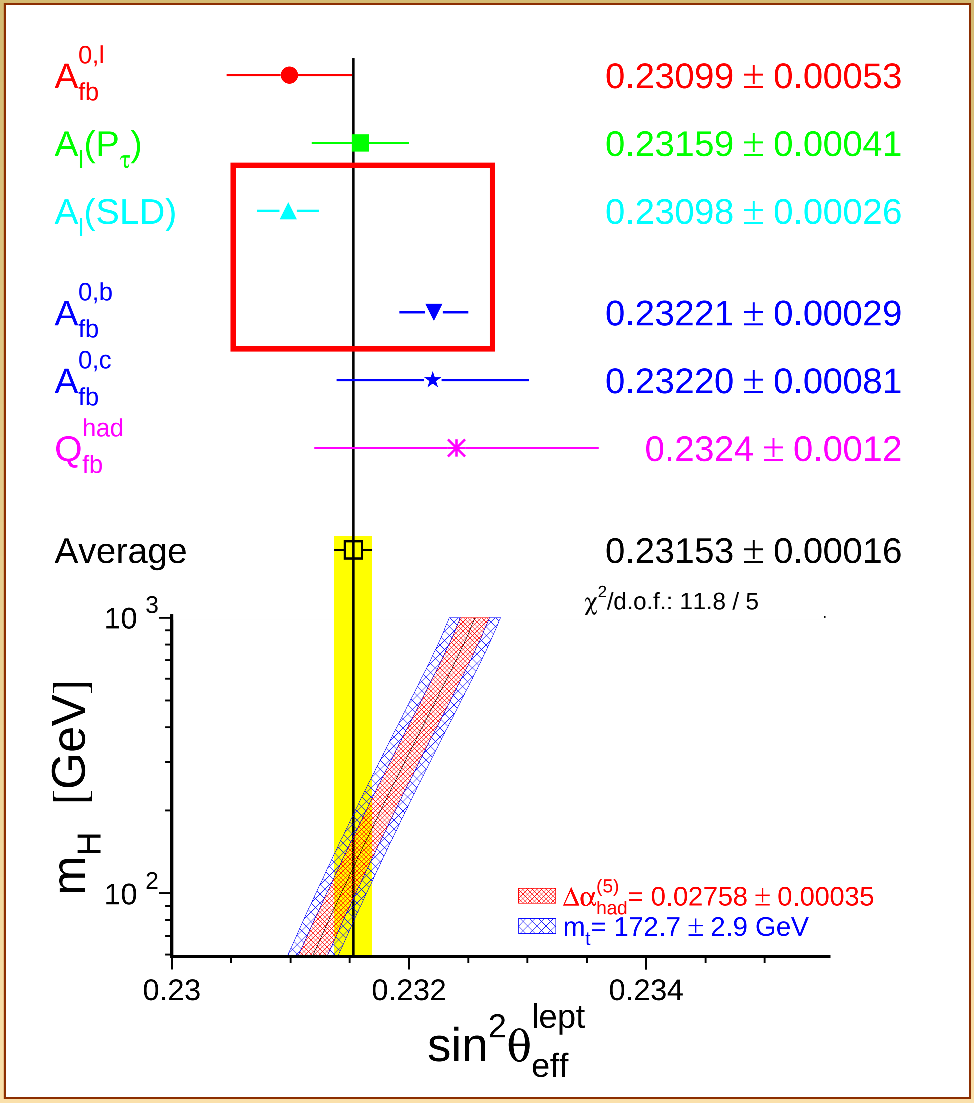
  $\longrightarrow$ Results can only be shared as numerical grid or fit formula

- Besides massive community effort of theorists, no obvious show-stopper in achieving the goals

Backup slides

Present status of $\sin^2\theta^\ell_{\rm eff}$

- Most precise determination from $A_{\rm LR}$ at SLD and $A^b_{\rm FB}$ at LEP
- Disagreement by $\sim 4\sigma$
  $\longrightarrow$ Underestimated systematics?
- Default at CEPC, FCC-ee:
  $\sin^2\theta^\ell_{\rm eff}$ from $A^{\mu\mu}_{\rm FB}$



LEP EWWG '05

Z-pole asymmetries

Left-right asymmetry: (using polarization e^- beams)

$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e + \Delta A_{\gamma Z} + \Delta A_\gamma$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta_{\mathrm{eff}}^f)}{1 + (1 - 4\sin^2\theta_{\mathrm{eff}}^f)^2} \qquad \sin^2\theta_{\mathrm{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Limited by systematic uncertainty of P_{e^-}

0.5% at SLD, 0.1% possible in future

Karl, List ’17

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4} \quad \Rightarrow \quad \delta \sin^2 \theta^\ell_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

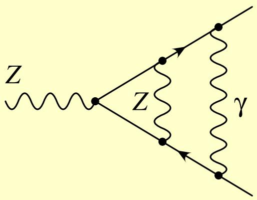
$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV

Z decay

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma'_Z}\right]_{s=\overline{M}_Z^2}$$



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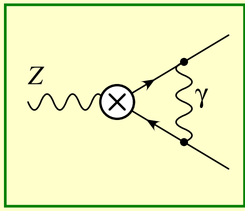
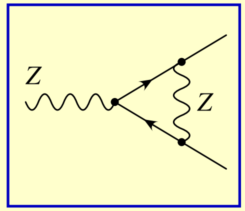
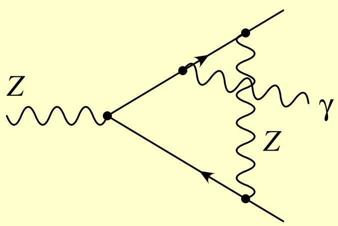


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

$\longrightarrow$ How to account for in MC simulations?

Other electroweak precision parameters

- $M_Z,\ \Gamma_Z$: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ $\delta M_Z,\ \delta\Gamma_Z \sim 0.1$ MeV could be achievable

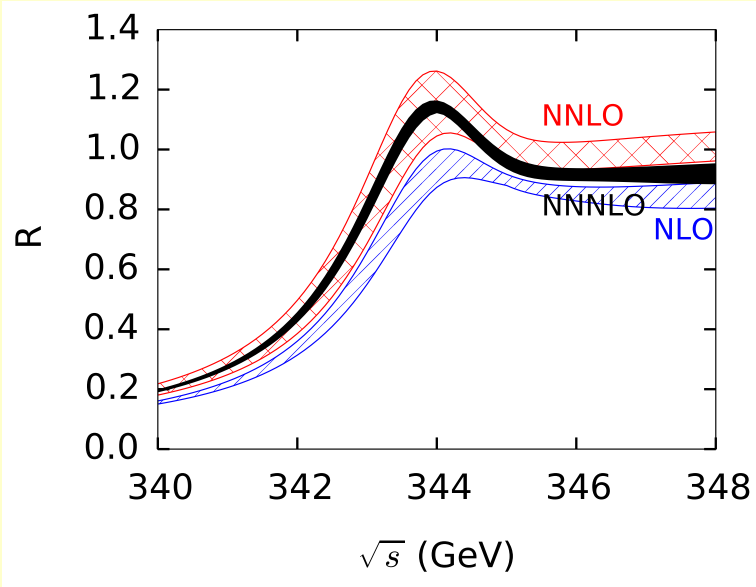
- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:

$$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp} \oplus [50\text{ MeV}]_{\rm QCD} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [70\text{ MeV}]_{\alpha_s}$$

$$> 100\text{ MeV}$$



Beneke et al. '15

Other electroweak precision parameters

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From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}}$	$[20\text{ MeV}]_{\text{exp}}$	
$\oplus\ [50\text{ MeV}]_{\text{QCD}}$	$\oplus\ [30\text{ MeV}]_{\text{QCD}}$	(h.o. resummation)
$\oplus\ [10\text{ MeV}]_{\text{mass def.}}$	$\oplus\ [10\text{ MeV}]_{\text{mass def.}}$	
$\oplus\ [70\text{ MeV}]_{\alpha_s}$	$\oplus\ [15\text{ MeV}]_{\alpha_s}$	$(\delta\alpha_s \lesssim 0.0002)$
> 100 MeV	$\lesssim 50$ MeV	

Other electroweak precision parameters

- $m_b,\ m_c$: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\text{MS}}} \sim 30$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 25$ MeV LHC HXSWG '16

  $\longrightarrow$ estimated improvements $\delta m_b^{\overline{\text{MS}}} \sim 13$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 7$ MeV
  Lepage, Mackenzie, Peskin '14

- M_H: from kinematic constraint fits $HZ(\ell\ell)$, $H(b\bar{b})Z$
  $\longrightarrow$ $\delta M_\text{H} \sim 10...20$ MeV
  $\longrightarrow$ theory errors subdominant

Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

    $\longrightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$

    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

    $\longrightarrow$ Subject to sizeable non-pertubative power corrections

    $\longrightarrow$ Systematic uncertainties in power corrections?

  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^{\ell}$):

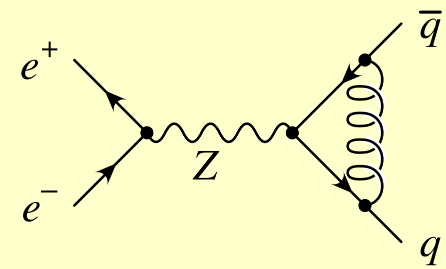
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    $\longrightarrow$ No (negligible) non-perturbative QCD effects

    FCC: $\delta R_\ell \sim 0.001$

    $\Rightarrow \quad \delta\alpha_s < 0.0002$ (subj. to theory error)

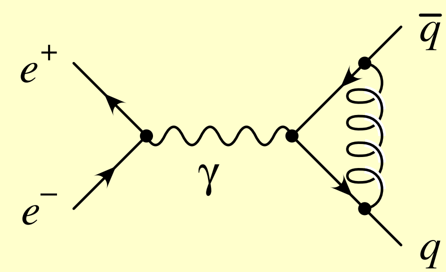
    Caviat: R_ℓ could be affected by new physics



  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$

    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$

    Kühn, Steinhauser, Teubner '07



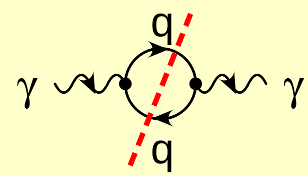
    $\longrightarrow$ dominated by s-channel photon, less room for new physics

    $\longrightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Shift of finestructure constant

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- Hadronic effects from $e^+e^- \to$ had. data

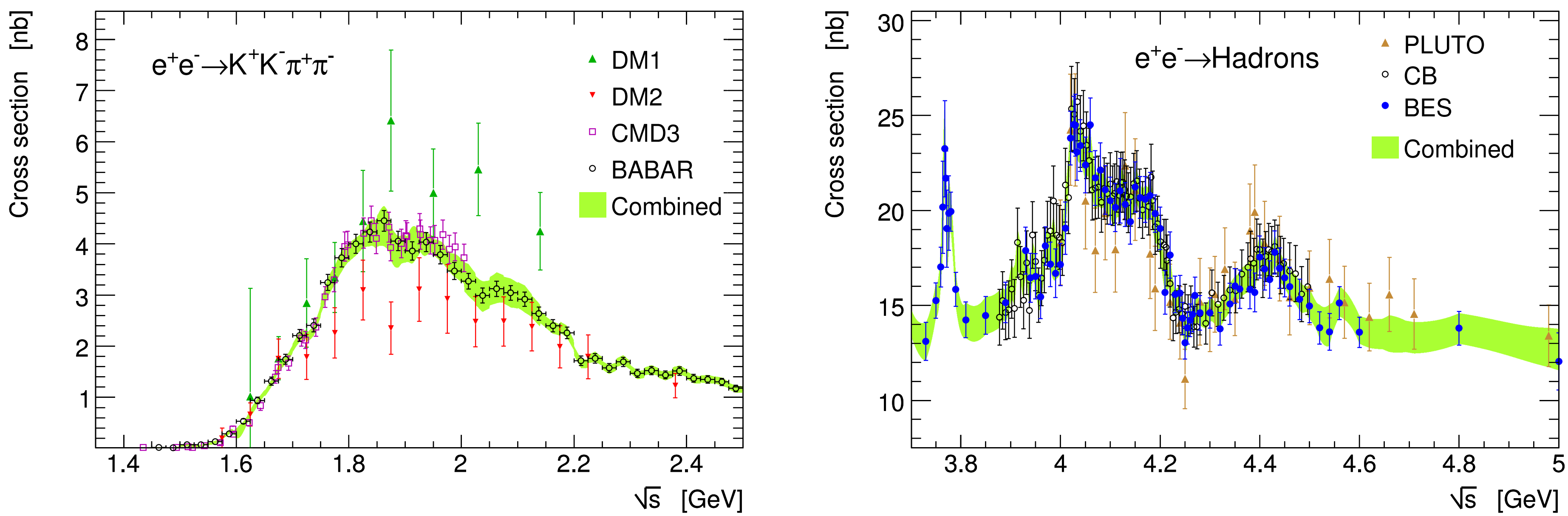


- Last 5 years: new data from BaBar, VEPP, BES
  $\longrightarrow$ Robust precision $\sim 10^{-4}$

Davier et al. '17,19; Jegerlehner '17
Keshavarzi, Nomura, Teubner '18

- With future data from BES, VEPP, Belle and improvements in QCD:
  $\delta(\Delta\alpha) \sim 5 \times 10^{-5}$

Jegerlehner '19



Davier et al. '17

Shift of finestructure constant

- $\Delta\alpha_{\text{had}}$: Could be limiting factor

a) From $e^+e^- \to$ had. using dispersion relation
Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely

b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
(i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
$\to \delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$

Janot '15

Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions