A new way of understanding the role of each measurement in global SMEFT fit @ e+e-

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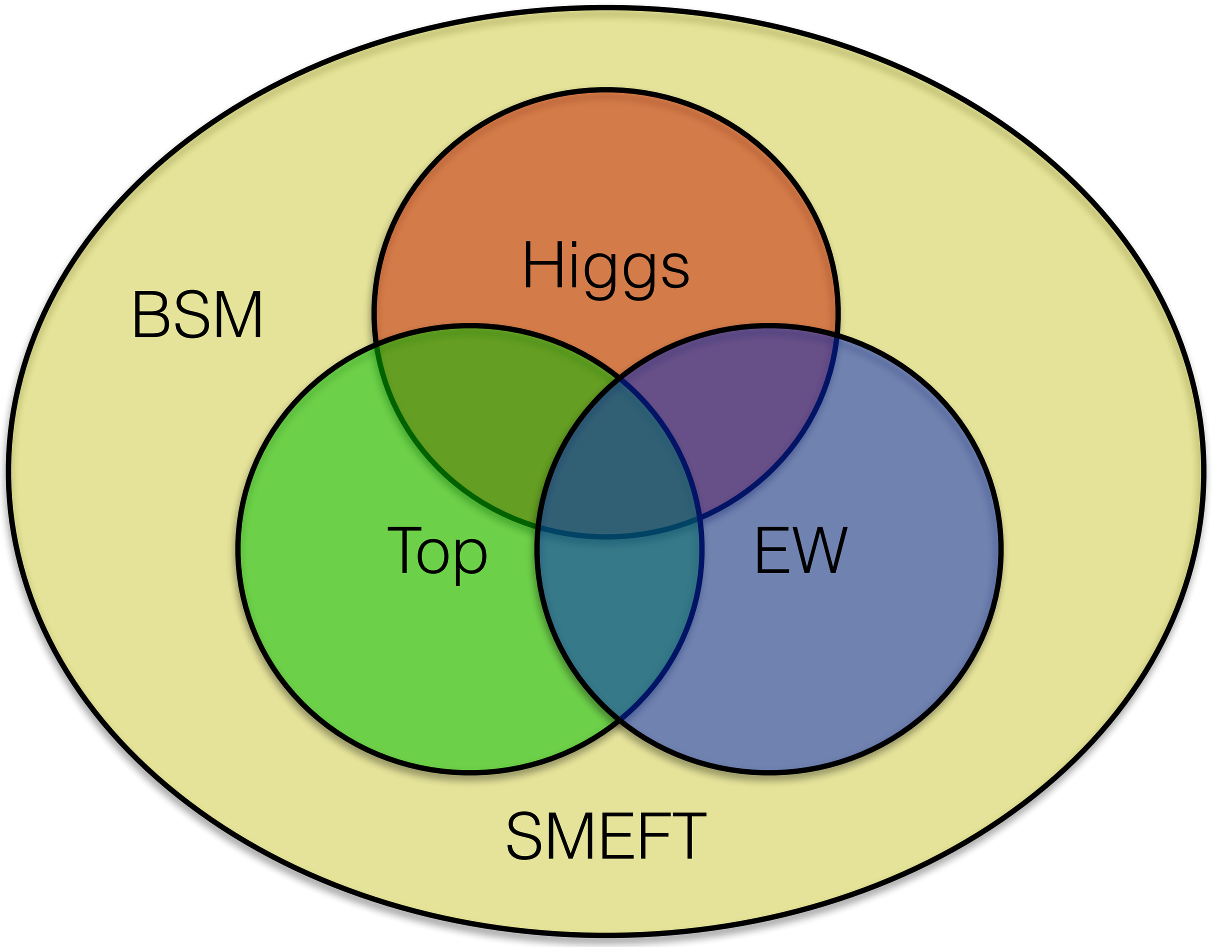
based on earlier work: Barklow et al, arXiv:1708.09079; 1708.08912

global SMEFT fit & roles of measurements



- When $m_{BSM} >> m_{EW}$, all the SM measurements can fit into a SMEFT framework, providing coherent tests of BSM physics
- A global EFT fit involves many fitting parameters and many input measurements, making the accurate understanding of roles of each measurement increasingly difficult
- Roles of **EWPO / TGC / Beam polarizations / Top EW couplings** for Higgs coupling determination @ e+e- get highly recognized

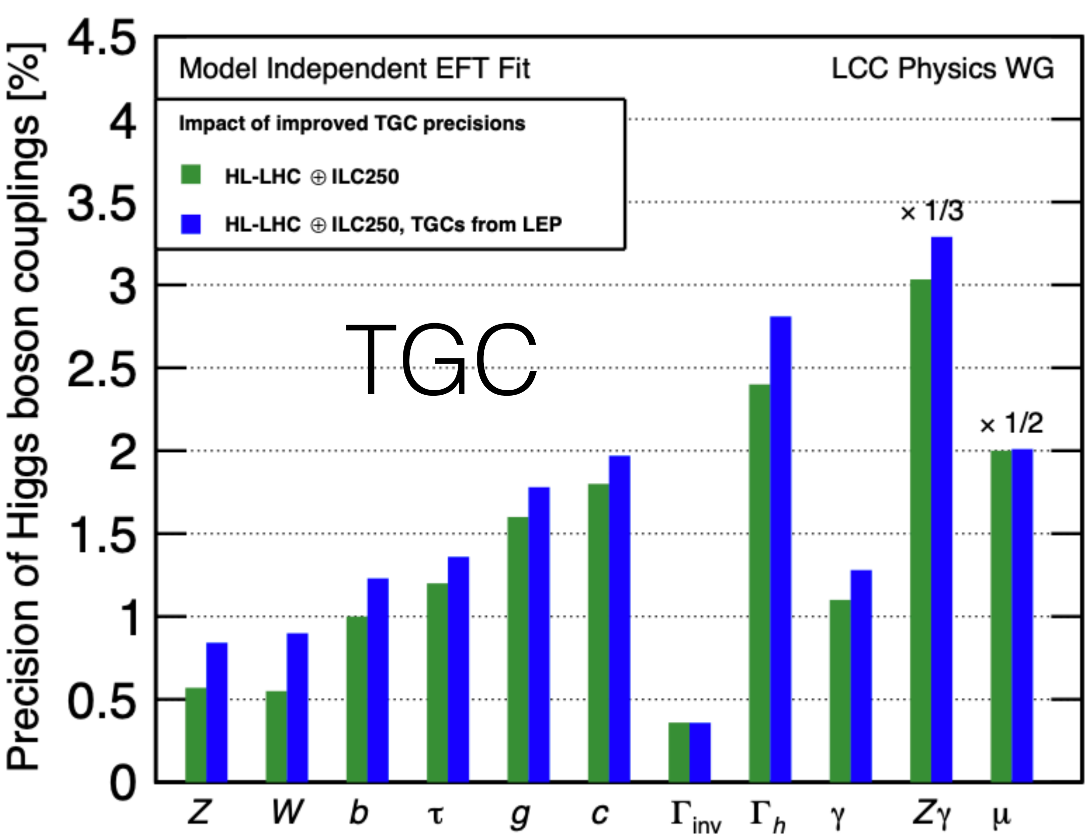
this talk:

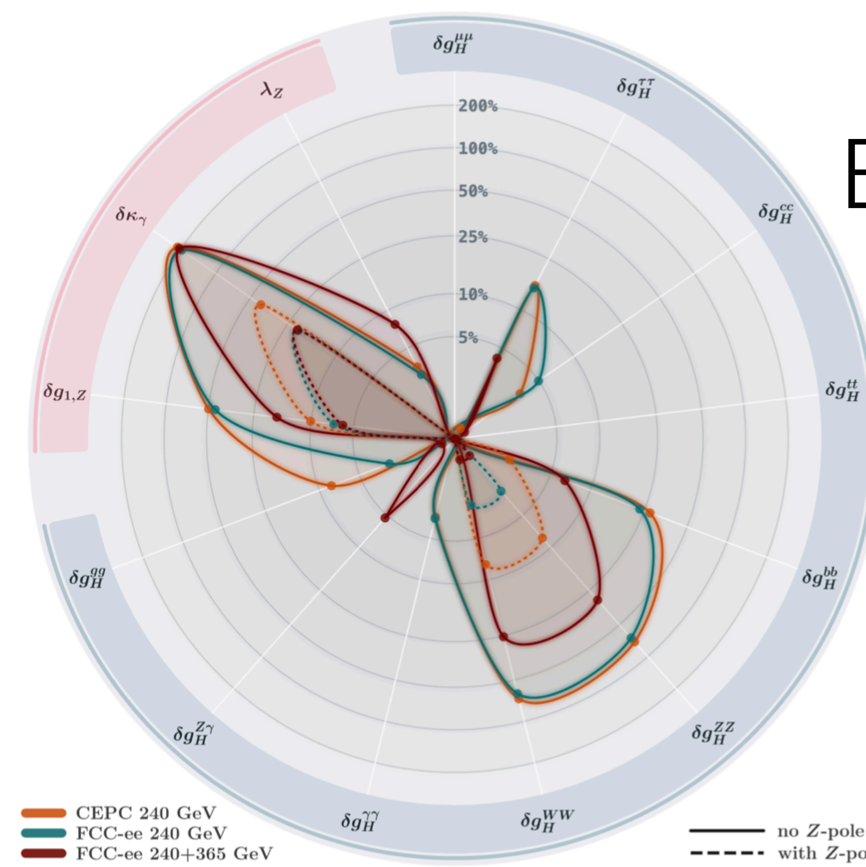
- ☐ a brief recap on qualitative & quantitative understanding (backup)
- ☑ a new way of more transparent understanding

classical way of evaluating roles of measurements

- do global fit numerically: vary certain input measurements, and see how the precision of Higgs couplings vary
- hard to figure out synergies among multiple (>>2) measurements

—> Is there a more transparent way?





EWPO

[Bambade, et al, arXiv:1903.01629]

[de Blas, et al, arXiv:1907.04311]

a new way for more transparent understanding

can we solve the global fit analytically?

we would like to express the uncertainty in a Higgs coupling (Δg)

analytically   in terms of the uncertainties in observables (ΔO)

e.g. $$\Delta g_{hXX} = x_1 \Delta O_1 \;\oplus\; x_2 \Delta O_2 \;\oplus\; x_3 \Delta O_3 \;\oplus\; \cdots$$

(all in physical quantities; should be EFT basis-independent)

as an intermediate step, we must get first the expression for

the uncertainties in Wilson coefficients (Δc)

basic notations (I)

$c_i \quad i = 1,2,\ldots,n$

Fitting parameters (Wilson coefficients)
n = number of fitting parameters

$y_i \quad i = 1,2,\ldots,m$

Observables (deviation w.r.t. SM values)
m = number of observables

$$y_i = v_{ij} c_j$$

$j = 1,2,\ldots,n$

$i = 1,2,\ldots,m$

v_{ij} known from theory computation

in matrix form

$$y = \mathrm{V} c$$

y: column vector of y_i, m x 1
c: column vector of c_j, n x 1
V: matrix of v_{ij}, m x n

basic notations (II)

in matrix form

$$y = \mathrm{V}c$$

y: column vector of y_i, m x 1
c: column vector of c_j, n x 1
V: matrix of v_{ij}, m x n

measurements

$$\mathrm{E}_y$$

covariance matrix for all observables y_i

which can be diagonalized

$$\mathrm{E}_y = \begin{pmatrix} \Delta_1^2 & 0 & . & 0 \\ 0 & \Delta_2^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \Delta_m^2 \end{pmatrix}$$

where $\boldsymbol{\Delta_i}$ is measurement uncertainty for $\boldsymbol{y_i}$

global fit

minimizing

$$\chi^2 = y^T \mathrm{E}_y^{-1} y$$

$$= c^T \mathrm{V}^T \mathrm{E}_y^{-1} \mathrm{V} c \;\; = c^T \mathrm{D} c$$

$$\mathrm{D} \equiv \mathrm{V}^T \mathrm{E}_y^{-1} \mathrm{V}$$

(i,j) element $d_{ij} = \sum_{k=1}^{m} \frac{v_{ki} v_{kj}}{\Delta_k^2}$

$\mathbf{D}^{-1}$ is exactly the covariance matrix of fitting parameters

global fit = how to obtain $\mathbf{D}^{-1}$ = how to invert $\mathrm{V}^T \mathrm{E}_y^{-1} \mathrm{V}$

numerical solution is easy, e.g. Barklow et al, 1708.09079; 1708.08912

global fit: analytic solution

$$D \equiv V^T E_y^{-1} V$$

global fit = how to obtain D^{-1} = how to invert $V^T E_y^{-1} V$

dimensions:

V: m x n

E_y: m x m

V^T: n x m

D: n x n

- **if m = n**, namely there is no redundant measurement

solution is easy:

$$D^{-1} = V^{-1} E_y (V^T)^{-1}$$

V is invertible, otherwise global fit doesn't converge

analytic solution: application of non-redundant case

for unpolarized e+e- at 250 GeV, there is almost no redundant measurement

(except W-fusion vvh, angular Zh; either small contribution)

$$\delta g_{hZZ} = \frac{1}{2}\delta\sigma_{Zh} + 6.4\delta\Gamma_l + 5.3\delta g_{Z,eff} - 0.015\delta R_{\gamma Z} - 2.4\delta\kappa_{A,eff} + 8.9\delta m_h + 0.098\delta A_l + \cdots$$

plug in measurement uncertainties for current EWPO + 2 ab^{-1}

$$= 41 \oplus 66 \oplus 30 \oplus 23 \oplus 14 \oplus 11 \oplus 8.7 \oplus \cdots \times 10^{-4}$$

σ_{Zh}

EWPO

TGC

BR(h->γZ)

Higgs mass

importance hierarchy

analytic solution: application of non-redundant case

- **if m = n**, namely there is no redundant measurement

$$\mathrm{D}^{-1} = \mathrm{V}^{-1}\mathrm{E}_y(\mathrm{V}^T)^{-1}$$

the expression in previous slide can be abstracted as

$$\Delta^2 c_i = \sum_{k=1}^{n} \frac{|\bar{\mathrm{V}}_{ki}|^2}{|\mathrm{V}|^2}\Delta_k^2 = \frac{\sum_S^{C_{n-1}^n} \frac{|\mathrm{V}_S^i|^2}{\Delta_S^2}}{\sum_L^{C_n^n} \frac{|\mathrm{V}_L|^2}{\Delta_L^2}}$$

global fit: analytic solution for general case (m>n)

$$\Delta^2 c_i = \frac{\sum_S^{C_{n-1}^m} \frac{|\mathrm{V}_S^i|^2}{\Delta_S^2}}{\sum_L^{C_n^m} \frac{|\mathrm{V}_L|^2}{\Delta_L^2}}$$

$$L = \{l_1, l_2, \cdots, l_n\}$$

n-combination of {1,2,…,**m**}

$$\Delta_L = \prod_{i=1}^{n} \Delta_{l_i}$$

V_L n x n matrix formed by Rows ***L*** of V

$$S = \{s_1, s_2, \cdots, s_{n-1}\}$$

(n-1)-combination of {1,2,…,**m**}

$$\Delta_S = \prod_{i=1}^{n-1} \Delta_{s_i}$$

V_S^i n-1 x n-1 matrix formed by Rows ***S*** of V & **eliminating Column *i***

summary

- developed a new way for more transparent understanding which is based on analytic expressions in terms of meas. uncertainties
- applied to non-redundant case: clear synergies among many meas.
- stay tuned for applications to redundant cases (of high interests: polarizations; multiple ECM; multiple Higgs prod. channels)

backup

global SMEFT fit @ future e+e-

[Barklow, Fujii, Jung, Peskin, JT, arXiv:1708.09079]

$$
\begin{aligned}
\Delta\mathcal{L} = & \frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) + \frac{c_T}{2v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6\lambda}{v^2}(\Phi^\dagger\Phi)^3 \\
& + \frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2}\Phi^\dagger t^a\Phi W^a_{\mu\nu}B^{\mu\nu} \\
& + \frac{g'^2 c_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho W^{c\rho\mu} \\
& + i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\overline{L}\gamma_\mu L) + 4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu\Phi)(\overline{L}\gamma_\mu t^a L) \\
& + i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\overline{e}\gamma_\mu e)\ .
\end{aligned}
$$

"Warsaw" basis, Grzadkowski et al, arXiv:1008.4884

22 parameters in total

Φ: higgs field
W, B: SU(2), U(1) gauge
L, e: left/right electron

- most of the non-trivial relations come from 7 operators

c_H

c_{HL} c'_{HL} c_{HE}

c_{WW} c_{WB} c_{BB}

recap 1: absolute Higgs couplings (unique role of inclusive σ_{Zh})

$$\frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi)$$

$$\frac{c_H}{2}\partial^\mu h\partial_\mu h$$ → renormalize SM Higgs field

h → $(1-c_H/2)h$

→ **shift all SM Higgs couplings by $-c_H/2$**

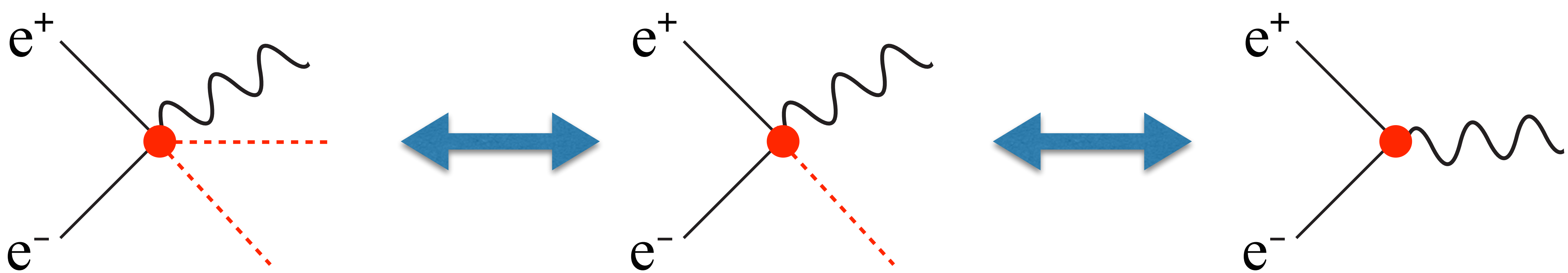
- c_H can not be determined by any BR or ratio of couplings

- c_H has to rely on inclusive cross section of $e^+e^- \to Zh$, enabled by recoil mass technique at e+e-

(precision of hZZ, hWW ~1/2 Δc_H)

recap 2: role of Electroweak Precision Observable (EWPO)

$$i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^{\mu}\Phi)(\overline{L}\gamma_\mu L) \qquad +(c'_{HL},\, c_{HE})$$



$e^+e^- \rightarrow Zhh$ $\qquad$ $e^+e^- \rightarrow Zh$ $\qquad$ Z-pole

- very useful EWPO at Z-pole: $\boldsymbol{A_{LR}}$, $\boldsymbol{\Gamma_{Z\rightarrow ee}}$
- Z-e-e couplings can also get helped by $\boldsymbol{\sigma_{Zh}}$: $\boldsymbol{\delta\sigma_{Zh} \sim s/m^2_Z}$

recap 3: roles of Higgs measurements at (HL-)LHC

$$\boxed{\frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger \Phi W^a_{\mu\nu} W^{a\mu\nu}} \qquad +(c_{BB},\ c_{WB})$$

$$\delta\Gamma(h \to \gamma\gamma) = -c_H + 122(8c_{WW} - 16c_{WB} + 8c_{BB}) + \cdots$$

$$\delta\Gamma(h \to \gamma Z) = -c_H + 122(8c_{WW} - 5.6c_{WB} - 2.4c_{BB}) + \cdots$$

$$\delta\Gamma(h \to ZZ^*) = -c_H - 0.4(8c_{WW} + 3.7c_{WB} + 0.6c_{BB}) + \cdots$$

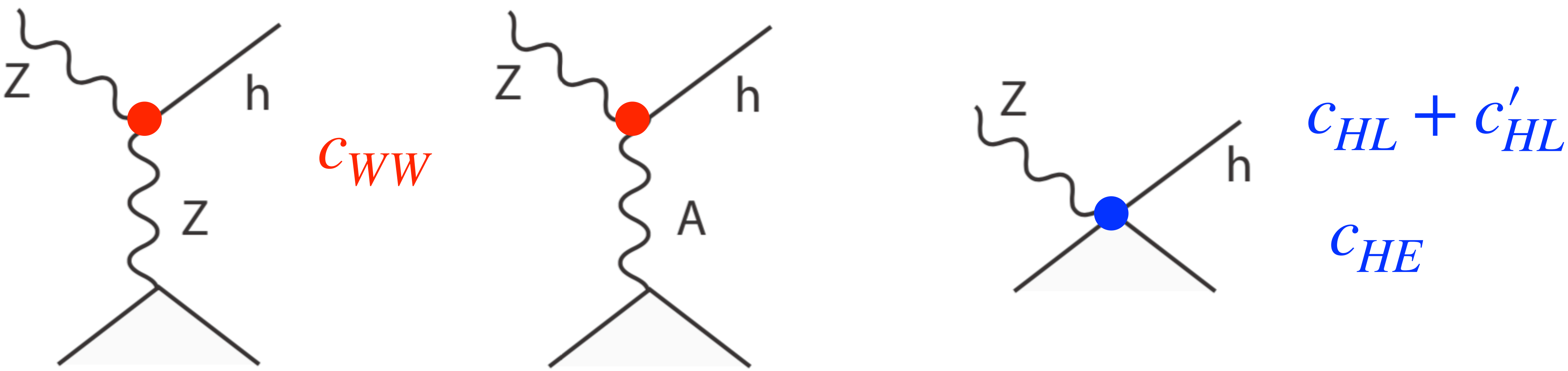
- loop induced h → γγ/γZ depend strongly on $c_{WW}/c_{BB}/c_{WB}$

- very useful measurements: **BR(h → γγ/γZ)** at (HL-)LHC

$$R_{\gamma\gamma} = \frac{BR(h \to \gamma\gamma)}{BR(h \to ZZ^*)} \qquad R_{\gamma Z} = \frac{BR(h \to \gamma Z)}{BR(h \to ZZ^*)}$$

use ratio of BR to keep model-independence

recap 4: role of beam polarizations



P(e⁻, e⁺) $\sqrt{s}$=250 GeV

(-0.8, +0.3) $$\delta\sigma_L = -c_H + 10(8c_{WW}) + 27(c_{HL} + c'_{HL}) + \cdots$$

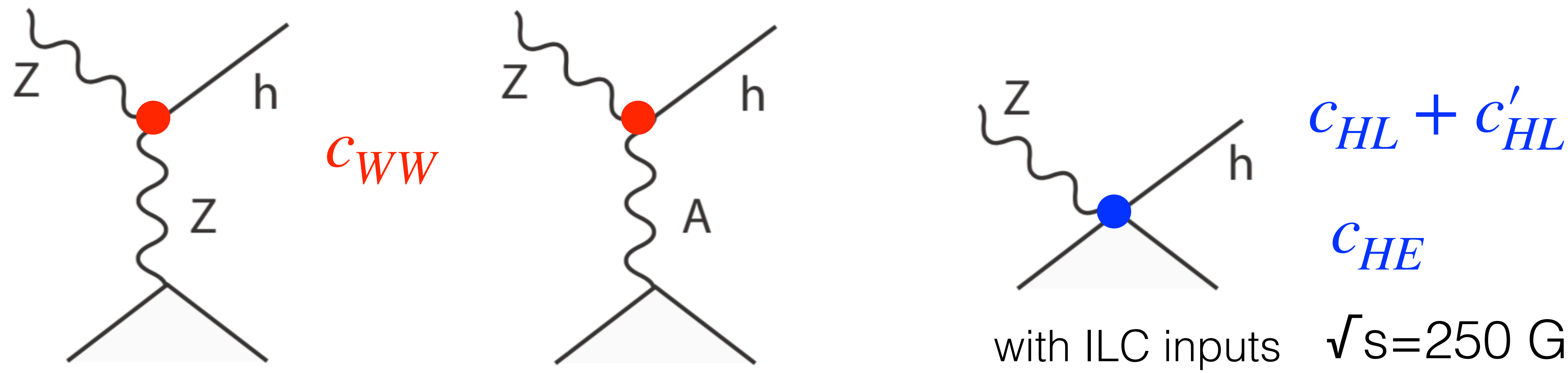
(+0.8, -0.3) $$\delta\sigma_R = -c_H + 1.6(8c_{WW}) + 2.1(c_{HL} + c'_{HL}) + \cdots$$

(0, 0) $$\delta\sigma_0 = -c_H + 6.6(8c_{WW}) + 16(c_{HL} + c'_{HL}) + \cdots$$

- **σ_R** has much weaker dependences on c_{WW} & c_{HL}+c_{HL}' (suppression of chiral new physics effects) -> results in better determination of c_H

- redundant **σ_L** in turn improves c_{WW}, c_{HL}+c_{HL}'

recap 4: role of beam polarizations



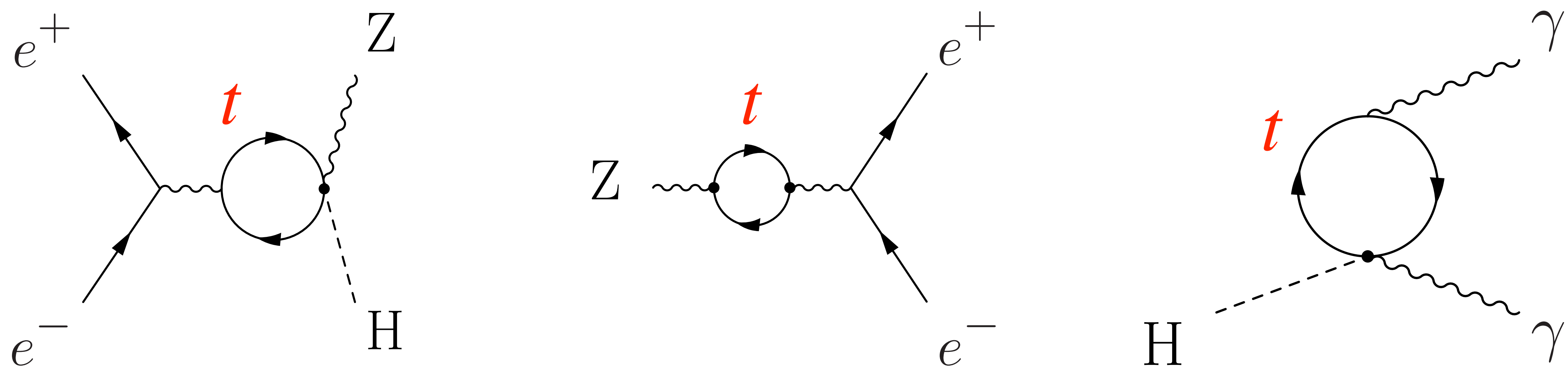
with ILC inputs $\sqrt{s}$=250 GeV

∫Ldt @ P(e-, e+)	Δc_H	$\Delta(c_{HL}+c_{HL}')$	$\Delta(8c_{WW})$	Δc_{HE}
2 ab^{-1} @ (0,0)	148	9.2	14	4.0
2 ab^{-1} @ (+0.8,-0.3)	106	9.2	14	4.0
1 ab^{-1} @ (+0.8,-0.3) 1 ab^{-1} @ (-0.8,+0.3)	104	5.95	12	3.2
2 ab^{-1} @ (-0.8,+0.3)	245	9.23	14	4.0
5 ab^{-1} @ (0,0)	134	8.84	13	3.1

Δ in unit of 10^{-4}

role of top-quark EW couplings

arXiv:2006.14631



$\mathcal{O}_{Ht} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{t}\gamma^\mu t)$ $\quad$ $\mathcal{O}_{Hq}^{(3)} = (\Phi^\dagger i \overleftrightarrow{D}^a_\mu \Phi)(\bar{Q}\gamma^\mu \tau^a Q)$ $\quad$ $\mathcal{O}_{tB} = (\bar{Q}\sigma^{\mu\nu} t)\tilde{\Phi} B_{\mu\nu}$

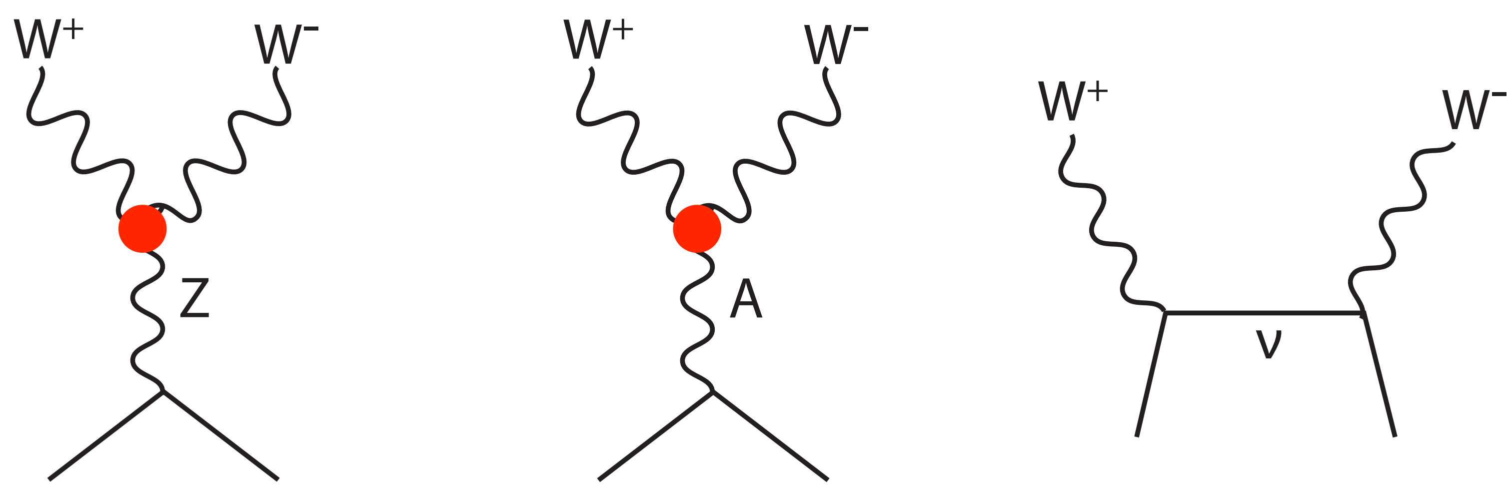
- NLO effects from top-quark operators are very important for Higgs couplings determination
- top-quark EW couplings measurements @ (HL-)LHC are invaluable for future e+e-

see talk by Martín Perelló @ top/EW session today

roles of Triple Gauge Couplings (TGC) in e+e- → WW

$$\boxed{\frac{4gg'c_{WB}}{m_W^2}\Phi^\dagger t^a \Phi W^a_{\mu\nu} B^{\mu\nu}} \qquad +(c_{WW},\ c_{BB})$$

$e^+e^- \rightarrow WW$



$h \rightarrow ZZ$ $\qquad \zeta_Z \frac{h}{2v} Z_{\mu\nu} Z^{\mu\nu}$ $\qquad \zeta_Z = c_w^2(8c_{WW}) + 2s_w^2(8c_{WB}) + s_w^4/c_w^2(8c_{BB})$

- longitudinal modes of W/Z are from Higgs fields

- c_{WB} / c_{HL}' / c_{3W} helped by meas. of TGCs in $e^+e^- \rightarrow WW$

Higgs couplings are related to themselves

$$\Delta\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - (1+\eta_h)\bar{\lambda} v h^3 + \frac{\theta_h}{v} h \partial_\mu h \partial^\mu h$$
$$+(1+\eta_W)\frac{2m_W^2}{v} W_\mu^+ W^{-\mu} h + (1+\eta_{WW})\frac{m_W^2}{v^2} W_\mu^+ W^{-\mu} h^2$$
$$+(1+\eta_Z)\frac{m_Z^2}{v} Z_\mu Z^\mu h + \frac{1}{2}(1+\eta_{ZZ})\frac{m_Z^2}{v^2} Z_\mu Z^\mu h^2$$
$$+\zeta_W \hat{W}_{\mu\nu}^+ \hat{W}^{-\mu\nu}\left(\frac{h}{v} + \frac{1}{2}\frac{h^2}{v^2}\right) + \frac{1}{2}\zeta_Z \hat{Z}_{\mu\nu}\hat{Z}^{\mu\nu}\left(\frac{h}{v} + \frac{1}{2}\frac{h^2}{v^2}\right)$$
$$+\frac{1}{2}\zeta_A \hat{A}_{\mu\nu}\hat{A}^{\mu\nu}\left(\frac{h}{v} + \frac{1}{2}\frac{h^2}{v^2}\right) + \zeta_{AZ}\hat{A}_{\mu\nu}\hat{Z}^{\mu\nu}\left(\frac{h}{v} + \frac{1}{2}\frac{h^2}{v^2}\right).$$

(SM structure: kappa like)

$$\eta_h = \delta\bar{\lambda} + \delta v - \frac{3}{2}c_H + c_6$$
$$\eta_W = 2\delta m_W - \delta v - \frac{1}{2}c_H$$
$$\eta_{WW} = 2\delta m_W - 2\delta v - c_H$$
$$\eta_Z = 2\delta m_Z - \delta v - \frac{1}{2}c_H - c_T$$
$$\eta_{ZZ} = 2\delta m_Z - 2\delta v - c_H - 5c_T$$

(Anomalous: new Lorentz structure)

$$\theta_h = c_H$$
$$\zeta_W = \delta Z_W = (8c_{WW})$$
$$\zeta_Z = \delta Z_Z = c_w^2(8c_{WW}) + 2s_w^2(8c_{WB}) + s_w^4/c_w^2(8c_{BB})$$
$$\zeta_A = \delta Z_A = s_w^2\Big((8c_{WW}) - 2(8c_{WB}) + (8c_{BB})\Big)$$
$$\zeta_{AZ} = \delta Z_{AZ} = s_w c_w\Big((8c_{WW}) - (1 - \frac{s_w^2}{c_w^2})(8c_{WB}) - \frac{s_w^2}{c_w^2}(8c_{BB})\Big)$$

- hZZ/hWW/hγZ/hγγ highly related: SU(2)xU(1) gauge symmetries

role of each measurement: more transparent understanding

for example: unpolarized e+e- at 250 GeV

$$\delta g_{hZZ} = \frac{1}{2}\delta\sigma_{Zh} + 6.4\delta\Gamma_l + 5.3\delta g_{Z,eff} - 0.015\delta R_{\gamma Z} - 2.4\delta\kappa_{A,eff} + 8.9\delta m_h + 0.098\delta A_l + \cdots$$

$$\delta X = \frac{\Delta X}{X}$$

σ_{Zh}	:	cross section of e+e- -> Zh
A_l, Γ_l	:	A_{LR} and Γ(Z->ll) at Z-pole
g_{Zeff}, κ_{Aeff}	:	Triple Gauge Couplings
$R_{\gamma Z}$	:	BR(h->γZ) / BR(h->ZZ*)
m_h	:	Higgs mass

role of each measurement: more transparent understanding

for example: unpolarized e+e- at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab-1

$$\delta g_{hbb} = \frac{1}{2}\delta B_{bb} - \frac{1}{2}\delta B_{WW} + \frac{1}{2}\delta\sigma_{Zh} - 5.79\delta\Gamma_l - 0.016\delta\Gamma_{\gamma Z} + \cdots$$

$$= 28 \oplus 91 \oplus 41 \oplus 59 \oplus 32 \oplus \cdots \times 10^{-4}$$

BR(h->WW)

σ_{Zh} **EWPOs**

BR(h->bb) **BR(h->γZ)**

role of each measurement: more transparent understanding

for example: unpolarized e+e- at 250 GeV

plug in measurement precisions for current EWPOs + 2 ab-1

$$\delta g_{hcc} = \frac{1}{2}\delta B_{cc} - \frac{1}{2}\delta B_{WW} + \frac{1}{2}\delta\sigma_{Zh} - 5.79\delta\Gamma_l - 0.016\delta\Gamma_{\gamma Z} + \cdots$$

$$= 160 \oplus 91 \oplus 41 \oplus 59 \oplus 32 \oplus \cdots \times 10^{-4}$$

BR(h->cc)

BR(h->WW)

σ_{Zh}

EWPOs

BR(h->γZ)

global fit: analytic answer

correlation between c_i & c_j

$$\rho_{ij} \equiv \frac{e_{ij}}{\sqrt{e_{ii}e_{jj}}} = \frac{(-1)^{i+j} \sum_S^{C_{n-1}^m} \frac{|V_S^i V_S^j|}{\Delta_S^2}}{\sqrt{\sum_S^{C_{n-1}^m} \frac{|V_S^i|^2}{\Delta_S^2} \sum_S^{C_{n-1}^m} \frac{|V_S^j|^2}{\Delta_S^2}}}$$

global fit: analytic solution

$$\mathrm{D} \equiv \mathrm{V}^T \mathrm{E}_y^{-1} \mathrm{V}$$

global fit = how to obtain $\mathbf{D^{-1}}$ = how to invert $\mathrm{V}^T \mathrm{E}_y^{-1} \mathrm{V}$

- what **if m > n**, namely there are redundant measurements?

some hints from n=1,2,3 cases,

e.g., if n=1 (only one parameter c)

$$\frac{1}{\Delta_c^2} = \sum_{k=1}^{m} \frac{v_k^2}{\Delta_k^2} = \frac{1}{\Delta_{c1}^2} + \frac{1}{\Delta_{c2}^2} + \cdots + \frac{1}{\Delta_{cm}^2}$$

global fit: analytic answer

uncertainty of each fitting parameter: reorganize

$$\frac{1}{\Delta c_i^2} = \frac{\sum_L^{C_n^m} \frac{|V_L|^2}{\Delta_L^2}}{\sum_S^{C_{n-1}^m} \frac{|V_S^i|^2}{\Delta_S^2}} = \sum_L^{C_n^m} \frac{1}{\sum_S^{C_{n-1}^m} \frac{|V_S^i|^2}{|V_L|^2} \frac{\Delta_L^2}{\Delta_S^2}}$$