Physics background for the A_{LR} measurement in $e^+e^- \rightarrow \gamma Z$ process

Takahiro Mizuno
SOKENDAI

Contents

Steady understanding of the background of the analysis

I read several papers this week.
My understanding is not yet perfect, but I will show what I have learned.

References

- Model-Independent Determination of the Triple Higgs Coupling at e+e- Colliders [arXiv:1708.09079v3]
- Dimension-Six Terms in the Standard Model Lagrangian [arXiv:1008.4884v3]
- Phenomenology of the Higgs Effective Lagrangian via FeynRules [arXiv:1310.5150v2]
- Precision Electroweak Measurements on the Z Resonance [arXiv:0509008v3]

Measurable Asymmetries

Z: V+A Coupling

-> Asymmetries

- Angular distributions of the final-state fermions
- Dependence of Z production on the helicities of the colliding electrons and positrons
- Polarisation of the produced particles

A_{FB} A_{LR} A_{LRFB}

Measurable Asymmetries

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$$

$$A_{\mathrm{LR}} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} \frac{1}{\langle|\mathcal{P}_{\mathrm{e}}|\rangle}$$

$$A_{\mathrm{LRFB}} = \frac{(\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}})_{\mathrm{L}} - (\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}})_{\mathrm{R}}}{(\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}})_{\mathrm{L}} + (\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}})_{\mathrm{R}}} \frac{1}{\langle|\mathcal{P}_{\mathrm{e}}|\rangle} .$$

- "F" means that the produced fermions in the hemisphere defined by the direction of the e^- beam
- "L" means the value for left-handed electron bunches
- P_e: electron polarisation (+ for right-handed)

The interaction of the Z boson with fermions

Radiative corrections (boson and fermion loops, vertex corrections) -> renormalize in "on-shell" scheme

-> Couplings at the Z-pole is absorbed into complex form factors

- R_f: overall scale
- K_f: on-shell electroweak mixing angle

$$\mathcal{G}_{\mathrm{Vf}} = \sqrt{\mathcal{R}_{\mathrm{f}}}\,(T_3^{\mathrm{f}} - 2Q_{\mathrm{f}}\mathcal{K}_{\mathrm{f}}\sin^2\theta_{\mathrm{W}})$$

$$\mathcal{G}_{\mathrm{Af}} = \sqrt{\mathcal{R}_{\mathrm{f}}}\,T_3^{\mathrm{f}}\,.$$

$\rho_0=1$ is assumed.

Asymmetry parameters

Considering the Z exchange diagrams and real couplings only, summed over final-state helicities, assuming an unpolarised positron beam but allowing polarisation of the electron beam,

Differential cross-section is:

$$\frac{\mathrm{d}\sigma_{\mathrm{f\bar{f}}}}{\mathrm{d}\cos\theta} = \frac{3}{8}\sigma_{\mathrm{f\bar{f}}}^{\mathrm{tot}}\left[(1-\mathcal{P}_{\mathrm{e}}\mathcal{A}_{\mathrm{e}})(1+\cos^2\theta)+2(\mathcal{A}_{\mathrm{e}}-\mathcal{P}_{\mathrm{e}})\mathcal{A}_{\mathrm{f}}\cos\theta\right]$$

$$\mathcal{A}_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2-g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2+g_{\mathrm{Rf}}^2} = \frac{2g_{\mathrm{Vf}}g_{\mathrm{Af}}}{g_{\mathrm{Vf}}^2+g_{\mathrm{Af}}^2} = 2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1+(g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$

$g_V=g_L+g_R$

$g_A=g_L-g_R$

Asymmetry parameters

Considering the Z exchange diagrams and real couplings only, summed over final-state helicities, assuming an unpolarised positron beam but allowing polarisation of the electron beam,

Differential cross-section is:

$$\frac{\mathrm{d}\sigma_{\mathrm{f\bar{f}}}}{\mathrm{d}\cos\theta} = \frac{3}{8}\sigma_{\mathrm{f\bar{f}}}^{\mathrm{tot}}\left[(1-\mathcal{P}_{\mathrm{e}}\mathcal{A}_{\mathrm{e}})(1+\cos^2\theta)+2(\mathcal{A}_{\mathrm{e}}-\mathcal{P}_{\mathrm{e}})\mathcal{A}_{\mathrm{f}}\cos\theta\right]$$

$$\mathcal{A}_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2-g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2+g_{\mathrm{Rf}}^2} = \frac{2g_{\mathrm{Vf}}g_{\mathrm{Af}}}{g_{\mathrm{Vf}}^2+g_{\mathrm{Af}}^2} = \boxed{2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1+(g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}}$$

$g_V=g_L+g_R$

$g_A=g_L-g_R$

Depend only on the ratio of the couplings

Asymmetry parameters

Considering the Z exchange diagrams and real couplings only, summed over final-state helicities, assuming an unpolarised positron beam but allowing polarisation of the electron beam,

Differential cross-section is:

$$\frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \frac{3}{8}\sigma_{f\bar{f}}^{tot}\left[(1-\mathcal{P}_e\mathcal{A}_e)(1+\cos^2\theta) + 2(\mathcal{A}_e - \mathcal{P}_e)\mathcal{A}_f\cos\theta\right]$$

A_{LR} A_{FB} A_{LRFB}

$$\mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2}$$

$g_V = g_L + g_R$

$g_A = g_L - g_R$

Depend only on the ratio of the couplings

Asymmetry parameters

$$\frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \frac{3}{8}\sigma_{f\bar{f}}^{tot}\left[(1-\mathcal{P}_e\mathcal{A}_e)(1+\cos^2\theta) + 2(\mathcal{A}_e - \mathcal{P}_e)\mathcal{A}_f\cos\theta\right]$$

A_{LR} A_{FB} A_{LRFB}

$$A_{FB}^{0,f} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

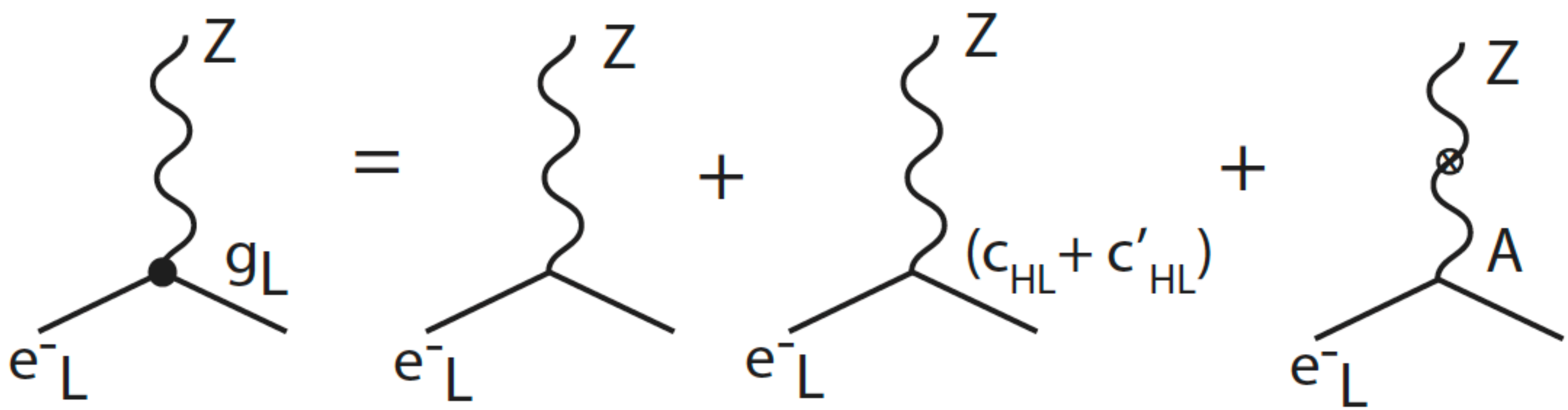
$$A_{LR}^{0} = \mathcal{A}_e$$

$$A_{LRFB}^{0} = \frac{3}{4}\mathcal{A}_f$$

Depend only on the ratio of the couplings

$$\mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2}$$

eeZ Coupling



Contributions to gL, the left-handed electron coupling to the Z, including the effects of contact interactions and AZ kinetic mixing

The contributions to gR have a similar structure.

eeZ Coupling

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \bar{c}_i \mathcal{O}_i = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm SILH} + \mathcal{L}_{CP} + \mathcal{L}_{F_1} + \mathcal{L}_{F_2} + \mathcal{L}_G$$

$$\begin{aligned}
\mathcal{L}_{F_1} =\ & \frac{i\bar{c}_{HQ}}{v^2}\left[\bar{Q}_L\gamma^\mu Q_L\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right] + \frac{4i\bar{c}'_{HQ}}{v^2}\left[\bar{Q}_L\gamma^\mu T_{2k} Q_L\right]\left[\Phi^\dagger T_2^k \overleftrightarrow{D}_\mu \Phi\right] \\
& + \frac{i\bar{c}_{Hu}}{v^2}\left[\bar{u}_R\gamma^\mu u_R\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right] + \frac{i\bar{c}_{Hd}}{v^2}\left[\bar{d}_R\gamma^\mu d_R\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right] \\
& - \left[\frac{i\bar{c}_{Hud}}{v^2}\left[\bar{u}_R\gamma^\mu d_R\right]\left[\Phi \cdot \overleftrightarrow{D}_\mu \Phi\right] + \text{h.c.}\right] \\
& + \frac{i\bar{c}_{HL}}{v^2}\left[\bar{L}_L\gamma^\mu L_L\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right] + \frac{4i\bar{c}'_{HL}}{v^2}\left[\bar{L}_L\gamma^\mu T_{2k} L_L\right]\left[\Phi^\dagger T_2^k \overleftrightarrow{D}_\mu \Phi\right] \\
& + \frac{i\bar{c}_{He}}{v^2}\left[\bar{e}_R\gamma^\mu e_R\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right],
\end{aligned}$$

Contact Interaction

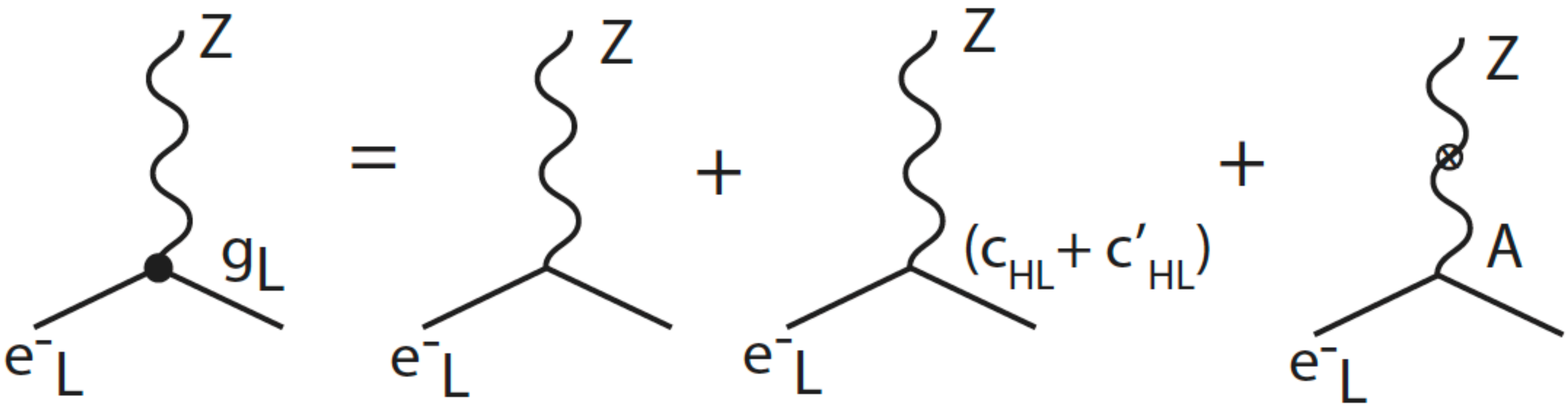
eeZ Coupling

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \bar{c}_i \mathcal{O}_i = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm SILH} + \mathcal{L}_{CP} + \mathcal{L}_{F_1} + \mathcal{L}_{F_2} + \mathcal{L}_G$$

$$\begin{aligned}
\mathcal{L}_{\rm SILH} = &\ \frac{\bar{c}_H}{2v^2}\partial^\mu\big[\Phi^\dagger\Phi\big]\partial_\mu\big[\Phi^\dagger\Phi\big] + \frac{\bar{c}_T}{2v^2}\big[\Phi^\dagger \overleftrightarrow{D}^\mu \Phi\big]\big[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\big] - \frac{\bar{c}_6\lambda}{v^2}\big[\Phi^\dagger\Phi\big]^3 \\
&- \Big[\frac{\bar{c}_u}{v^2} y_u \Phi^\dagger\Phi\ \Phi^\dagger\cdot\bar{Q}_L u_R + \frac{\bar{c}_d}{v^2} y_d \Phi^\dagger\Phi\ \Phi\bar{Q}_L d_R + \frac{\bar{c}_l}{v^2} y_\ell\ \Phi^\dagger\Phi\ \Phi\bar{L}_L e_R + \text{h.c.}\Big] \\
&+ \frac{ig\ \bar{c}_W}{m_W^2}\big[\Phi^\dagger T_{2k}\overleftrightarrow{D}^\mu\Phi\big] D^\nu W^k_{\mu\nu} + \frac{ig'\ \bar{c}_B}{2m_W^2}\big[\Phi^\dagger\overleftrightarrow{D}^\mu\Phi\big]\partial^\nu B_{\mu\nu} \\
&+ \frac{2ig\ \bar{c}_{HW}}{m_W^2}\big[D^\mu\Phi^\dagger T_{2k} D^\nu\Phi\big] W^k_{\mu\nu} + \frac{ig'\ \bar{c}_{HB}}{m_W^2}\big[D^\mu\Phi^\dagger D^\nu\Phi\big] B_{\mu\nu} \\
&+ \frac{g'^2\ \bar{c}_\gamma}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \frac{g_s^2\ \bar{c}_g}{m_W^2}\Phi^\dagger\Phi G^a_{\mu\nu}G_a^{\mu\nu}\ ,
\end{aligned}$$

AZ Mixing

eeZ Coupling



$$g_L = \frac{g}{c_w}\left[(-\frac{1}{2} + s_w^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}(c_{HL} + c'_{HL}) - s_w c_w \delta Z_{AZ}\right]$$

$$g_R = \frac{g}{c_w}\left[(+s_w^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}c_{HE} - s_w c_w \delta Z_{AZ}\right]$$

$$\delta Z_{AZ} = s_w c_w \Big((8c_{WW}) - (1 - \frac{s_w^2}{c_w^2})(8c_{WB}) - \frac{s_w^2}{c_w^2}(8c_{BB})\Big)$$

6 coefficients **c_{HL}, c'_{HL}, c_{HE}, c_{WW}, c_{WB}, c_{BB}** are relevant to the analysis.

Z production

LEP: **17 million** Z decays (ALEPH + DELPHI + L3+ OPAL, LEP-I, 1989-1995)
SLC: **600 thousand** Z decays (SLD, 1992-1998, polarization of e^-)

ILC250: **90 million** radiative return events

Current best measurement

A_{LR}= 0.1514±0.0019 (statistic error) ±0.0011(systematic error)

I would like to access how much we can improve these systematic errors.

Next step

- Concentrate on new sample checking next week.
- Continue to read papers for the steady understanding of the physics background.

Back up

e⁺e⁻ -> ff process

The differential cross-sections for fermion pair production around the Z resonance

After radiative correction, neglecting initial and final state photon radiation, final state gluon radiation and fermion masses, unpolarised beams

$$\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e^+e^-}\to\mathrm{f\bar{f}}) \; =$$

$$\underbrace{|\alpha(s)Q_{\mathrm{f}}|^2(1+\cos^2\theta)}_{\sigma^{\gamma}}$$

$$\underbrace{-8\Re\left\{\alpha^*(s)Q_{\mathrm{f}}\chi(s)\left[\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Vf}}(1+\cos^2\theta)+2\mathcal{G}_{\mathrm{Ae}}\mathcal{G}_{\mathrm{Af}}\cos\theta\right]\right\}}_{\gamma\text{–Z interference}}$$

$$\underbrace{\begin{array}{c}+16|\chi(s)|^2\left[(|\mathcal{G}_{\mathrm{Ve}}|^2+|\mathcal{G}_{\mathrm{Ae}}|^2)(|\mathcal{G}_{\mathrm{Vf}}|^2+|\mathcal{G}_{\mathrm{Af}}|^2)(1+\cos^2\theta)\right.\\ \left.+8\Re\left\{\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Ae}}{}^*\right\}\Re\left\{\mathcal{G}_{\mathrm{Vf}}\mathcal{G}_{\mathrm{Af}}{}^*\right\}\cos\theta\right]\end{array}}_{\sigma^{\mathrm{Z}}}$$

$$\chi(s) \;=\; \frac{G_{\mathrm{F}}m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\mathrm{Z}}^2+is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}}\,,$$