Investigating High Precision Tracker p-Scale Calibration

with K^0_S, Λ, D^0, J/ψ

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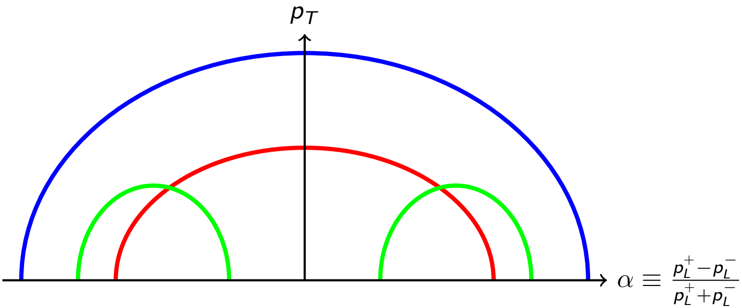


Introduction

- ILC can and should make precision measurements of the **masses** of known fundamental particles (M_{H}, M_{t}, M_{W}, M_{Z}), and Γ_{Z}. Measure new ones M_{X}.
- A primary issue for most determinations is the measurement of the **absolute center-of-mass energy scale**. A method, the so-called $\sqrt{s}_p$ method, has been proposed using only the momenta of muons in di-muon events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Up to now, foresaw $J/\psi \to \mu^+\mu^-$ with mass known to 1.9 ppm as the gold standard method. Statistically limited though in $\mathrm{e}^+\mathrm{e}^-$ collisions.

More details in my older studies of $\sqrt{s}_p$ from DESY, ECFA LC2013, and of momentum-scale from Fermilab, AWLC 2014.

Today,

1. Explore method based on the Armenteros-Podolanski construction (inspired by Rodriguez et al.) using mainly $\mathrm{K}^0_{\mathrm{S}}$, Λ.
2. Bonus: potential to also improve masses of parent and daughter particles.

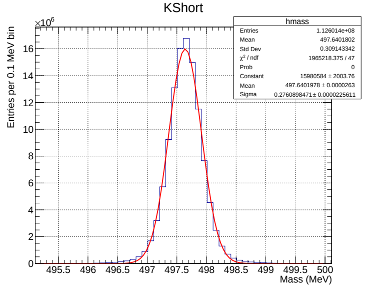
Particles of Interest

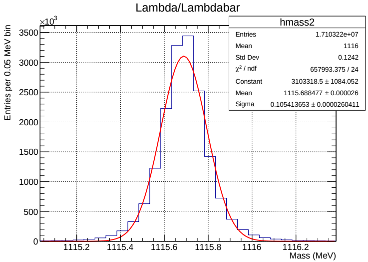
- Indeed J/ψ is the best measured parent particle, but other particles much more prevalent
- Daughter masses are known very well (except $K^{\pm}$)
- ppm = parts per million, ppb = parts per billion (10^9)

Particle	Mode	$\Delta M/M$ (PDG-2020)	$n_Z^{\mathrm{had}} B$
$\mathrm{K_S^0}$	$\pi^+\pi^-$	26 ppm	0.71
Λ	p π^-	5.4 ppm	0.25
$\mathrm{D^0}$	$\mathrm{K}^-\pi^+$	27 ppm	0.018
J/ψ	$\mu^+\mu^-$	1.9 ppm	0.00031
Z	$\mu^+\mu^-$	23 ppm	(0.047)
μ	–	23 ppb	
$\pi^{\pm}$	–	1.3 ppm	
$K^{\pm}$	–	32 ppm	
p	–	6.4 ppb	

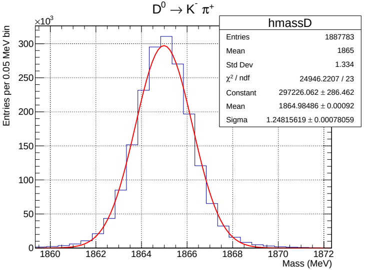
Following slide has the expected mass peaks from the main decay modes for a sample of 250 M hadronic Z's (91 GeV). Uses ILD momentum resolution parametrization. Hierarchy: $\mathrm{K_S^0}, \Lambda, \mathrm{D^0}, J/\psi$.

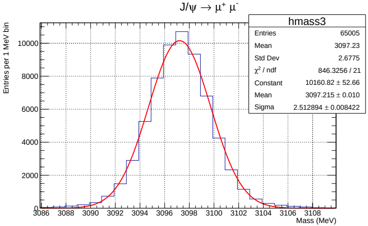
Mass Peaks in 250 M hadronic Z's (91 GeV)





(this is just for Λ - multiply by two).





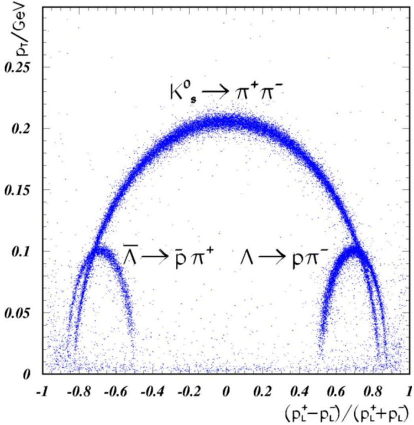
113M, 34M, 3.8M, 65k

Caveats

- Study uses lots of events. Sufficient that many of the known particle masses could be improved by factors of up to 75 for 250M hadronic Zs.
- Likely a real reach in terms of eventual systematics. Discussed more later.
- Study so far is like an estimate of the average *B*-field absolute scale statistical uncertainty.
- For now I decided to be somewhat cavalier for several reasons.
  1. High event counts are needed to uncover residual systematics in the fit procedure and figure out how to correct them.
  2. Some channels need very high statistics to be viable especially in the context of the analysis method chosen.
  3. A high statistics run at the Z for EW precision measurements with potentially 4000 M Z's would have 16 times this statistics
  4. This study should give an idea of how well point-to-point systematics might be controllable for scan observables.

Armenteros-Podolanski Method for 2-body V Decays

J. Podolanski and R. Armenteros, "Analysis of V-events", Phil. Mag. 1954.
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the daughter momenta in the lab into components tranverse and parallel to the parent momentum.
The resulting distributions of (daughter p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) are related by the CM decay angle, θ^*, the underlying masses, (M, m_1, m_2), that determine, p^*, and β.

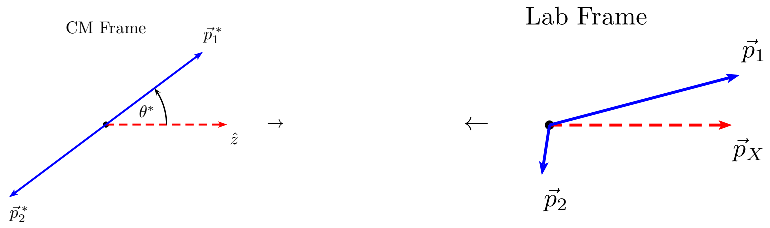


- K^0-decay
  - $M = 0.498$ GeV
  - $m_{1,2} = 0.140$ GeV
  - $p_{cm} = 0.206$ GeV
  - $\alpha_0 = 0$
  - $r_\alpha = 0.827$
- Λ-decay
  - $M = 1.116$ GeV
  - $m_1 = 0.938$ GeV
  - $m_2 = 0.140$ GeV
  - $p_{cm} = 0.101$ GeV
  - $\alpha_0 = \pm 0.691$
  - $r_\alpha = 0.181$

plot from talk by M. Schmelling

Armenteros-Podolanski II

CM frame, so $p_1^* = p_2^* = p^* = \frac{\sqrt{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}}{2M}$.
Transverse momentum, $p_T^* = p^* \sin\theta^* = p_T$ in CM and lab.



One can derive, that $\alpha = \frac{2p^*}{\beta M}\cos\theta^* + \frac{m_+^2 - m_-^2}{M^2}$ and can rewrite as

$$\alpha = \alpha_0 + \frac{r_\alpha}{\beta}\cos\theta^* \quad \text{where} \;\; \alpha_0 \equiv \frac{m_+^2 - m_-^2}{M^2}, \quad r_\alpha \equiv \frac{2p^*}{M}$$

$$\text{AP Ellipse}: \quad \left(\frac{\alpha - \alpha_0}{(r_\alpha/\beta)}\right)^2 + \left(\frac{p_T}{p^*}\right)^2 = 1 \tag{1}$$

Elliptical Transformation a la Rodriguez

Building on the recent preprint, arXiv:2012.03620 (Rodriguez et al), one can "flatten" the measured AP ellipse into (r, ϕ) variables defined using

$$\begin{cases} \alpha(r,\phi) = \widetilde{\alpha}_0 \quad + \quad \frac{\widetilde{r}_\alpha}{\beta} r \cos\phi \\ p'_T(r,\phi) = S_p \; p_T(r,\phi) = \widetilde{p}^* r \sin\phi \end{cases} , \tag{2}$$

where the tilde based values assume some reference masses, p'_T is the measured p_T biased by a momentum scale factor, S_p. Straightforward solution is:

$$\begin{cases} r^2(\alpha,p'_T,\beta) = \left(\frac{\alpha - \widetilde{\alpha}_0}{(\widetilde{r}_\alpha/\beta)}\right)^2 + \left(\frac{p'_T}{\widetilde{p}^*}\right)^2 \\ \phi(\alpha,p'_T,\beta) = \text{atan2}(\frac{p'_T}{\widetilde{p}^*}, \frac{\alpha - \widetilde{\alpha}_0}{(\widetilde{r}_\alpha/\beta)}) \end{cases} \tag{3}$$

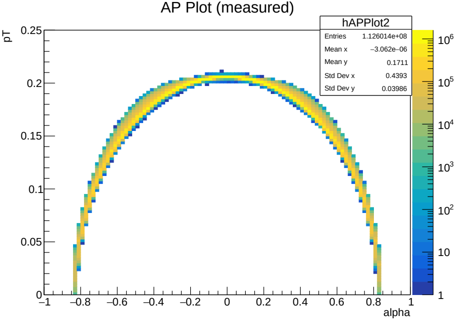
Equation 1 results in a quadratic in r that depends on ϕ with coefficients defined by combinations of true masses, reference masses, S_p, and β.

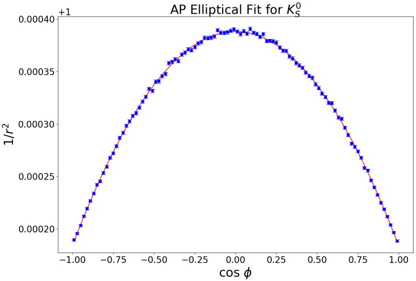
$$\left(\frac{(\widetilde{\alpha}_0 - \alpha_0) + (\widetilde{r}_\alpha/\beta) r \cos\phi}{(r_\alpha/\beta)}\right)^2 + \left(\frac{\widetilde{p}^* r \sin\phi}{S_p \; p^*}\right)^2 = 1 \;, \tag{4}$$

If all masses are as assumed, and $S_p = 1$, expect $r(\phi) = 1$ for $m_1 = m_2$.

Illustrate with $\mathrm{K_S^0}$ (113M accepted decays)

Simulate measurement of (p_T, α). For each decay, measure corresponding (r, ϕ) using Eqn. 3 with reference masses. For each bin in $\cos\phi$ (approximately the same as $\cos\theta^*$), find the mean value of $1/r^2$. For $S_p = 1.0001$.





Fit $1/r^2$ vs $\cos\phi$ for underlying parameters ($m_{\mathrm{K_S^0}}$ and S_p) with m_π assumed known perfectly. Uses reference (tilde) value of the PDG $\mathrm{K_S^0}$ mass + 0.1 MeV.
Results: $\chi^2/\mathrm{dof} = 108/98$ and relative uncertainties on $(m_{\mathrm{K_S^0}}, S_p)$ of (0.3, 0.5) ppm with correlation of -97.9%.
Deviations are $(+2.6, -3.9)\sigma$ (residual systematics? fix...)

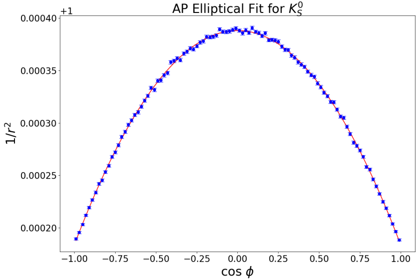
Parametric Form (for $m_1 = m_2 = m$)

For this simpler case, the solutions of the quadratic equation (with $B = 0$) lead to

$$\frac{1}{r^2} = -\frac{A}{C} = \left(\frac{\widetilde{r}_\alpha}{r_\alpha}\right)^2 \cos^2\phi + \left(\frac{\widetilde{p}^*}{S_p\, p^*}\right)^2 \sin^2\phi \; . \tag{5}$$

Note: no dependence on β. Define, $c_r = \frac{\widetilde{r}_\alpha}{r_\alpha}$, $c_p = \frac{\widetilde{p}^*}{S_p\, p^*}$. Eqn. 5 becomes

$$\frac{1}{r^2} = c_p^2 + (c_r^2 - c_p^2)\cos^2\phi \tag{6}$$



- Can measure both r_α and the product, $S_p\, p^*$.
- These depend on M, m, and S_p.
- When m is well known. Can measure M and S_p.
- $r^{-2}(\cos\phi = 0, \pm 1) = (c_p^2, c_r^2)$
- p'_T depends on S_p. α does not.

Experimental Methods Used in Current Study*

Caveat: short-term goal to establish whether this method can work and get first estimates of precisions especially at the Z. Not meant to be an end-to-end study.

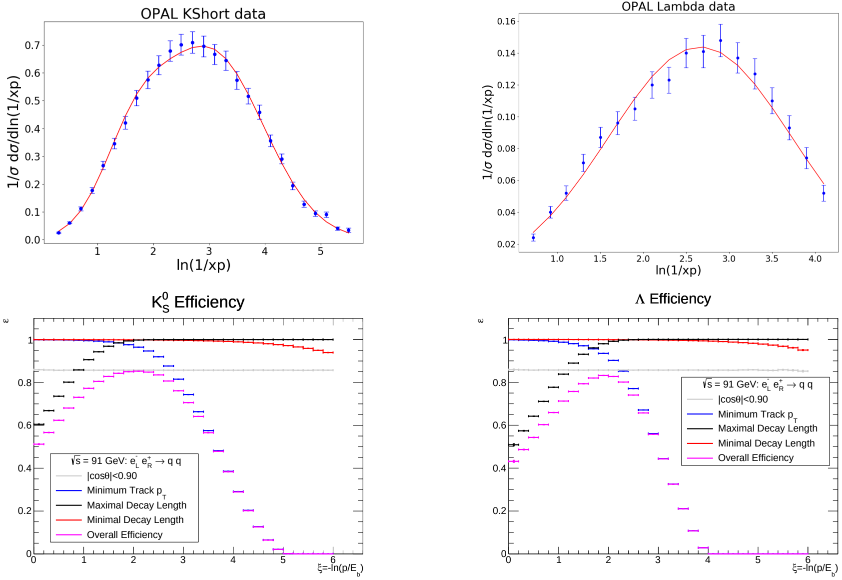
Methods

- Use toy MC generator with momentum spectrum from PYTHIA 6.
- Use parametrization of "normal" ILD momentum resolution (from fits to DBD curves) - checked with SGV. (trackresmodel.py code attached)
- Scale a and b parameters for reduced lever arm (decays). Scale b by $1/\beta$.
- $\frac{d\sigma}{d\cos\theta} = 1 + \cos^2\theta$. Rates from PDG.
- Neglect angular resolution and backgrounds.

Cuts

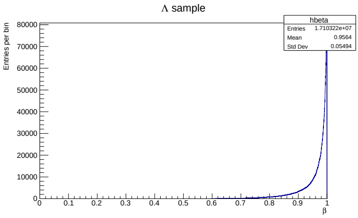
- Angular acceptance: $|\cos\theta| < 0.90$
- Minimum track detector p_T of 0.25 GeV
- Fiducial cut (decay vertex within 20 cm of outer edge of TPC in r and z)
- Require decay radius exceeds 250 μm (only for $\mathrm{K^0_S}$, Λ)
- Require momentum resolution of each track $< 1\%$ (for late decays ...)

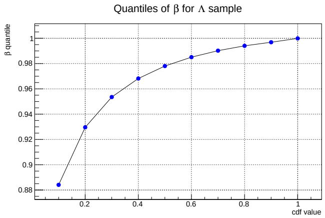
ξ Distributions and Efficiency



Note: efficiency plots neglect the 1% resolution cut (recent addition)

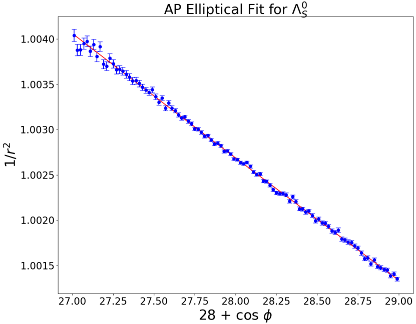
Lambda Fits (Eqn 4 with $\widetilde{\alpha}_0 \neq \alpha_0$)





Lowest decile ($\beta < 0.885$) is excluded from all fits for now. (suspect β-bite is currently too big.)

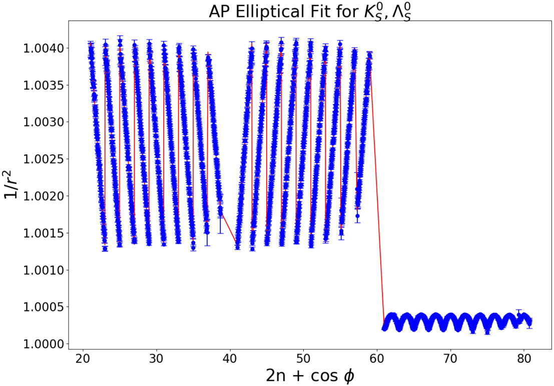
r vs ϕ dependence depends on β. So divide and conquer. Divide β range in tenths. Use $< \beta >$ in fit for now. Example with $\beta \in [0.9853, 0.9904]$.



Again for $S_p = 1.0$ and $M + 0.1$ MeV. Fit $\chi^2/\text{dof} = 109/98$. Find (M, S_p) to (0.3, 5) ppm, $\rho = -0.962$. Uses 1.7M Λ's. Overall fit has $\times 18$ (for 250M Z).

Mega Fit for K_S^0, Λ, $\bar{\Lambda}$ with 250M Z statistics

Fit deciles for each particle type (necessary for $\Lambda/\bar{\Lambda}$).
9 deciles of Λ, 9 deciles of $\bar{\Lambda}$, 10 deciles of K_S^0.



$\chi^2/\nu = 3117/2714$ (NYGE)

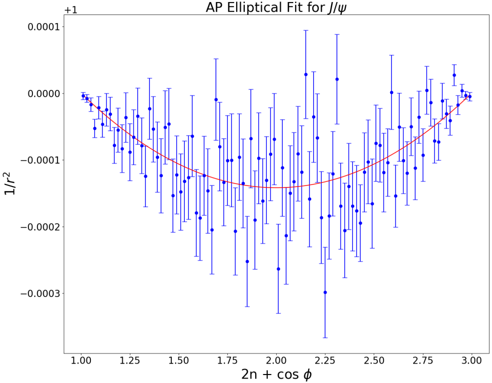
1. $m_{K_S^0}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

	$M(K_S^0)$	$M(\Lambda)$	m_π	S_p
$M(K_S^0)$	1.0	0.934	0.814	−0.942
$M(\Lambda)$	–	1.0	0.914	−0.818
m_π	–	–	1.0	−0.601
S_p	–	–	–	1.0

Here m_p is fixed. Technically, fits are set up with χ^2 penalty terms that constrain the particles to the PDG masses within known uncertainties. So in essence the fitted masses are new world averages.

J/ψ stand-alone fit (250M Z's, 65k accepted J/ψ)

Fix m_μ.



1. $m_{J/\psi}$: 1.9 ppm (no improvement wrt PDG)
2. S_p: 4.4 ppm
3. Correlation: -0.44

- Consistent with prior estimate of 1.0 ppm statistical uncertainty on S_p from $J/\psi \to \mu^+\mu^-$ in 4 GZ hadronic (find 1.03 ppm here).
- Fit with $m(J/\psi)$ fixed gives 4.0 ppm uncertainty on S_p.

Intrinsic S_p sensitivity (mostly for reference)

For each particle decay separately, carry out fit with all particles set to correct masses, and fit only for S_p (again using 250M hadronic Zs).

Results:

Particle	Mode	S_p uncertainty	$n_Z^{\mathrm{had}} B$	S
$\mathrm{K_S^0}$	$\pi^+\pi^-$	0.104 ppm	0.71	0.00139
Λ	p π^-	0.297 ppm	0.25	0.00235
$\mathrm{D^0}$	$\mathrm{K}^-\pi^+$	0.538 ppm	0.018	0.00114
J/ψ	$\mu^+\mu^-$	3.98 ppm	0.00031	0.00111

Sensitivity, S, defined as S_p relative uncertainty per event, ie,

$$S = [\Delta(S_p)/S_p]\sqrt{250 \times 10^6 (n_Z^{\mathrm{had}} B)}$$

Note that the sensitivity to S_p differs quite a lot (due to different Q values).

I am also looking into adding other decay modes into the mix. For now, adding $\mathrm{D^0}$ with only the $\mathrm{K}^-\pi^+$ decay mode would add uncertainty from both the $\mathrm{D^0}$ mass and the charged kaon mass. $\phi(1020)$ is not so interesting given the width.

Other modes

For example,

1. $D^+ \to K_S^0\ \pi^+$ (1.56%)
2. $D^+ \to K^-\ \pi^+\ \pi^+$ (9.38%)
3. $D^0 \to K_S^0\ \pi^+\ \pi^-$ (2.80%)

So far fits to D^0 with only $K^-\pi^+$ (3.95%) with two free parameters, S_p and $m(K)$, (ie. $m(D^0)$ fixed) give S_p to 0.69 ppm and $m(K)$ to 2.0 ppm for 250M Zs. ($\rho = +0.63$).

More realistic and improved methods / systematics?

Full Reconstruction

- Need to develop performant V0 finder and fitter for large IP.
- With nano-beams - much potential.
- Looper reconstruction
- Angular resolution
- Backgrounds
- Expect some degradation in resolution for Si-poor tracks.

Systematics

- Field map precision
- Tracker alignment/survey
- Material distribution
- Energy loss corrections
- Radiative tails
- Variations with p, $\cos\theta$, decay point
- Smaller bins / better β treatment
- Understand $r, \cos\phi$ resolution

- Current measurements are based on the sample average from TProfile plots. Scope for improved measurement uncertainties simply from fits to the r distributions in each $\cos\phi$ bin and better use of errors.
- Note β estimate uses the measured P and measured mass of the two tracks.

$$M^2 = m_1^2 + m_2^2 + 2p_1p_2(\frac{1}{\beta_1\beta_2} - \cos\psi_{12}) \; , \;\; \beta = P/\sqrt{P^2 + M^2}$$

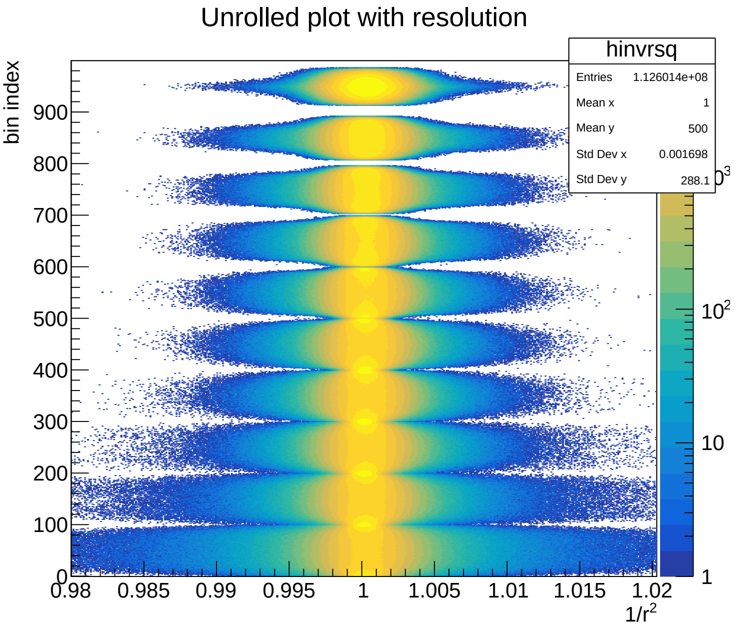
Conclusions

- Tremendous opportunity to target ppm type uncertainties on the momentum scale factor, S_p, at the Z.
- Would open up precision measurements of the masses of lots of known particles at the ppm level. In particular: K^0_S, Λ, $\pi^{\pm}$, $\mathrm{K}^{\pm}$.
- Would guarantee similar precision in the center-of-mass energy scale which would open a high precision Z program.
- When I started working on this a few years ago, 10 ppm seemed a sensible but very challenging goal. Maybe the bar should be set a bit higher still.
- Convincing people that this is realistic when typical experiments are at best at the 100 ppm level needs a lot of more realistic work, and work on designing this kind of functionality in from the start.
- (Now need to evaluate better limiting non-S_p systematics for $\sqrt{s}_p$ method for $\sqrt{s}$).

Backup: Current simulation approach

Some simplifications:

- Each of the decayed tracks is assumed to be in the same direction as the parent particle in terms of detector, θ (should fix). This affects the simulated detector acceptance and the momentum resolution formula.
- Each track's momentum magnitude along its momentum direction is smeared as detailed on p11. So no angular smearing.
- The AP variables are calculated using the components of the smeared decay particle momentum perpendicular and parallel to the parent particle's true flight direction.
- In particular the AP p_T variable is calculated as the average of the $|p_T|$ of the two tracks.
- Note that once the tracks are fitted to a common vertex and this neutral vertex is constrained to the nano beam spot, it is expected that the above assumptions are not unreasonable, especially in $r - \phi$.


Unrolled plot with resolution
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Entries 1.126014e+08
Mean x 1
Mean y 500
Std Dev x 0.001698
Std Dev y 288.1
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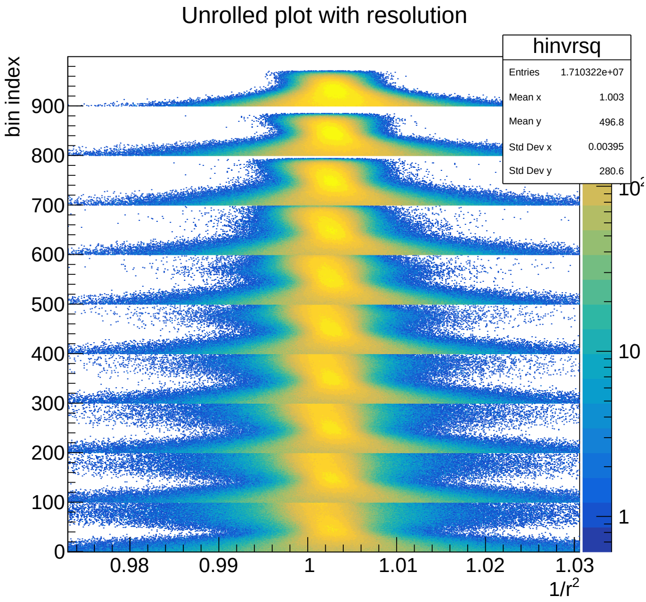
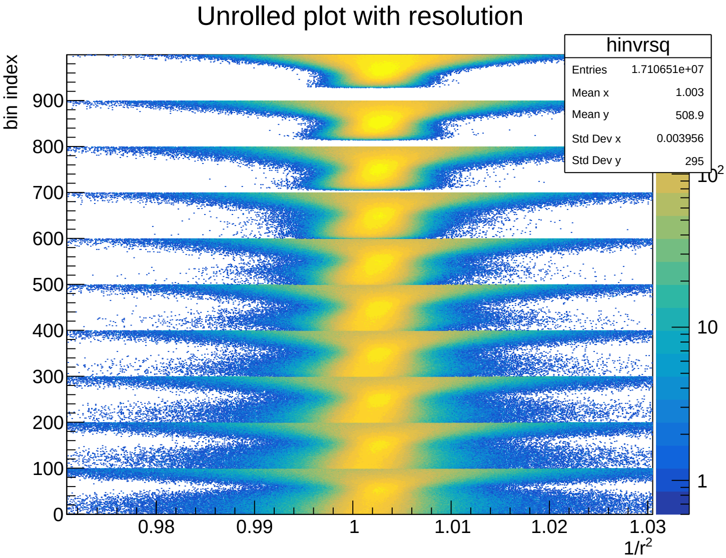
1/r²
10²
10
1

Backup: Λ plot (β deciles and $\cos\phi$ bins)

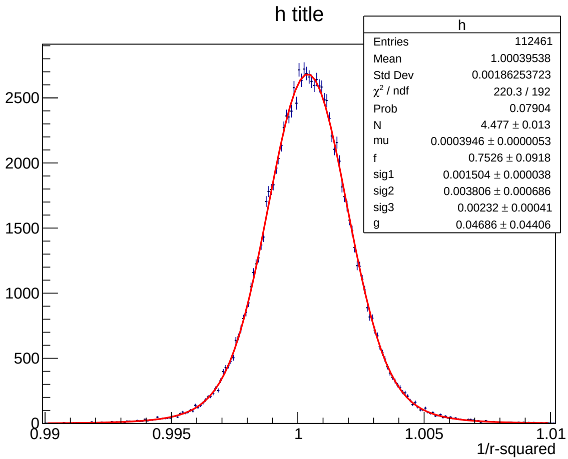


Backup: $\overline{\Lambda}$ plot (β deciles and $\cos\phi$ bins)



Backup: Example $1/r^2$ Distribution

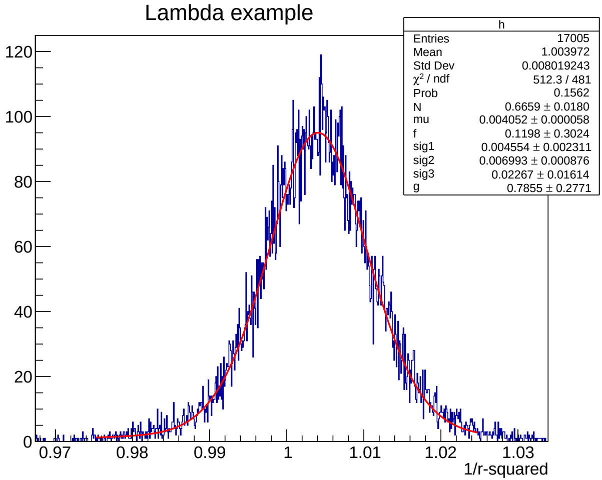
K_S^0, $\beta \in [0.99037, 0.99418]$, $\cos\phi \in [-0.02, 0.0]$



Triple Gaussian fit with common mean of $1 + \mu$. (Note that histogram mean is currently used - overly sensitive to far tails ...)

Backup: Example $1/r^2$ Distribution

Λ, same β as slide 13, $\cos\phi \in [-1.0, -0.98]$



Triple Gaussian fit with common mean of $1 + \mu$. (Note that histogram mean is currently used)

Backup: Plots from Rodriguez Preprint

Note here ϕ definition differs by $\pi/2$.

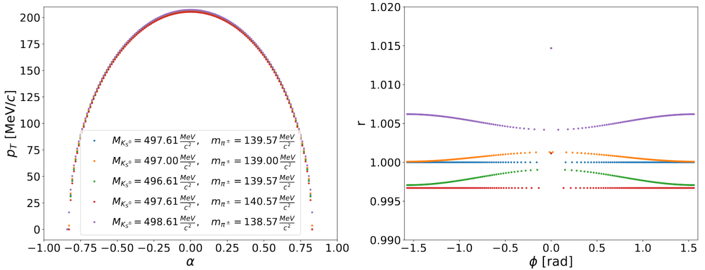


Figure 2: Functional form of a set of symmetric decays ($m_1 = m_2 = m$) in the Armenteros plot (left) and in the plot with elliptical coordinates (right).

In this case ($S_p = 1$). I included this mainly for illustration. It does not comport well with the plotting convention I used, nor the idea that there is a true AP ellipse, and a series of flattened elliptical coordinate plots for different reference values.

I think what is done is 5 different (M, m) assumptions are made, and the right plot is the analysis based on the PDG as reference (leading to the blue set at $r = 1$).