Prospects for the measurement of the bottom quark mass at the ILC

S. Tairafune[a], J. Fuster[b], A. Irles[b],
German Rodrigo[b], H. Yamamoto[b], R. Yonamine[a]
Tohoku University[a]
IFIC, Universitat de Valéncia and CSIC[b]

Background of This Study

b quark mass and New Physics

- $\overline{\text{MS}}$ running mass : **Mass which has energy dependence**

  QCD Renormalized Group Equation : $\mu^2 \frac{\partial m_q(\mu)}{\partial \mu^2} = -\gamma\left(\alpha_s(\mu)\right) m_q(\mu)$

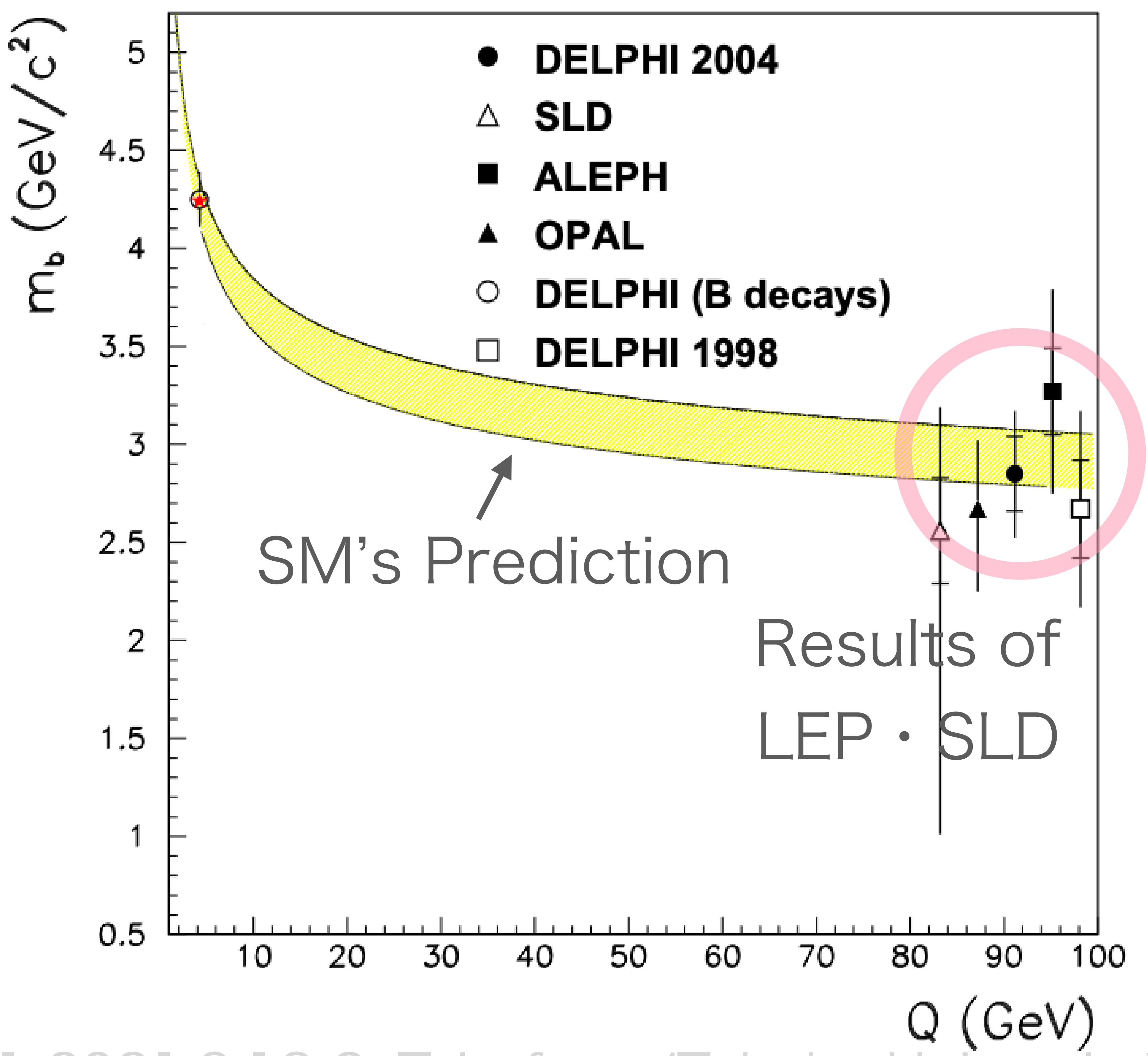
  $\gamma(\alpha_s(\mu))$: Perturbative function

  μ : energy scale of experiments

- Energy dependence deviates from SM's expectation by new particles's effect (SUSY etc).

- No indication of new physics in b quark mass at Z-pole (~91GeV).

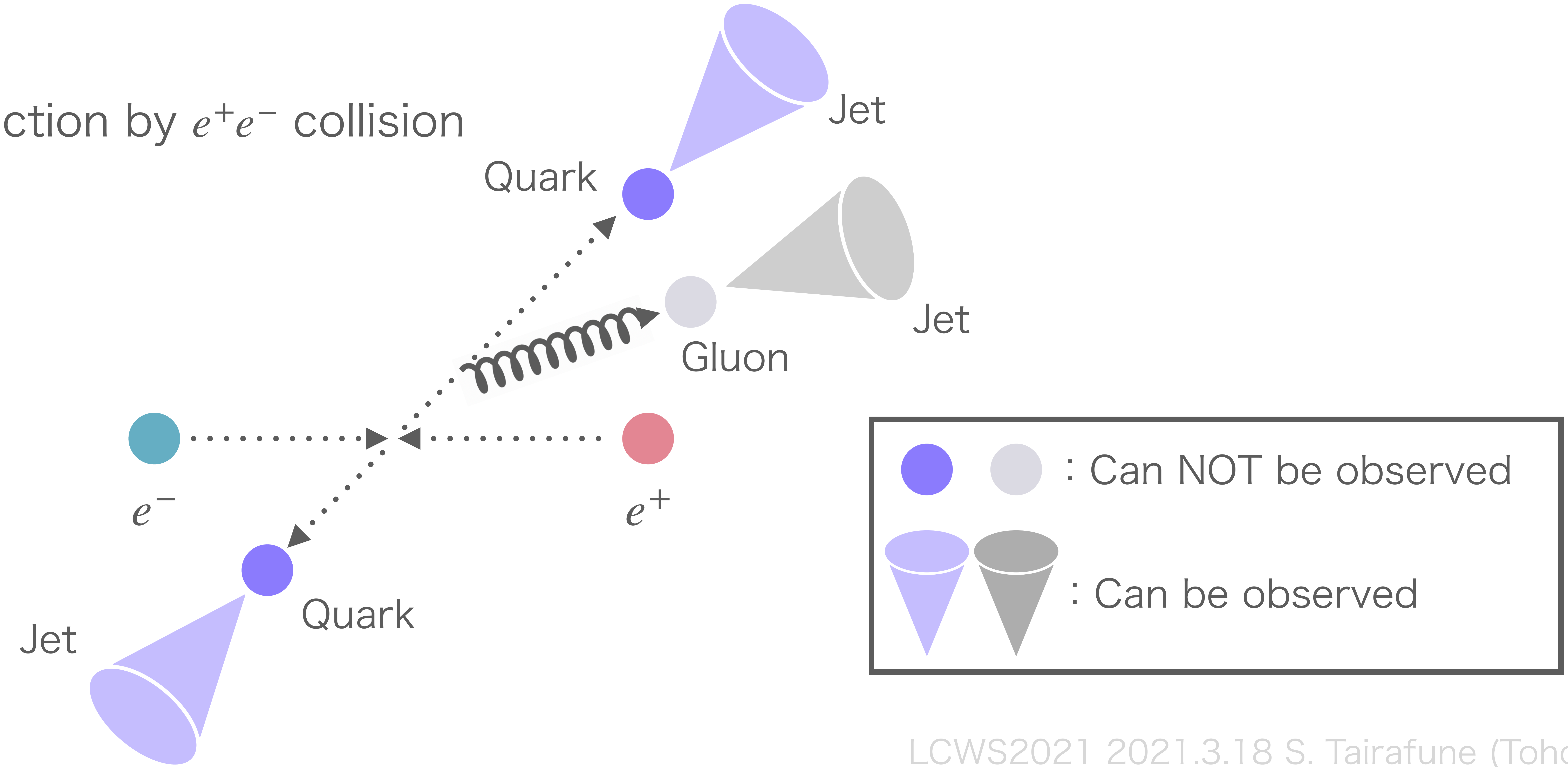
  → **b quark mass at higher energy scale above Z-pole can be SM's test and a probe of new physics.**



Purpose of This study

- **Estimate b quark mass at 250 GeV ILC through simulation.**

- $q\bar{q}$ production by e^+e^- collision



Definition of Observable

- **Heavier quark tends to be difficult to emit gluon $q \to q + g$.**

  $\to b$ quark mass sensitivity appears on 3-jet events after gluon radiation.

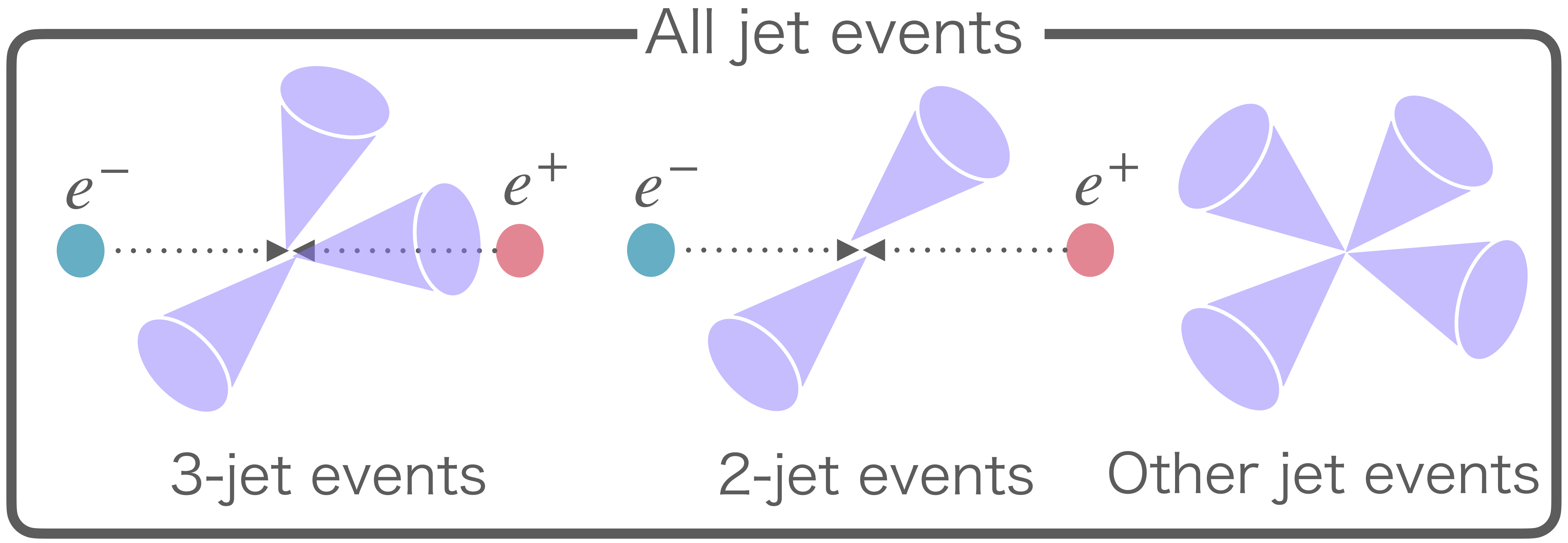
- Define **the fraction of 3-jet ratio** for $e^+e^- \to q\bar{q}$.

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l}$$

$$= 1 + \frac{\alpha_s}{\pi} a_{LO} + \frac{m_b^2}{s}\left(b_{LO}(m_b) + \frac{\alpha_s}{\pi} b_{NLO}(m_b)\right)$$

a_{LO} , b_{LO} : LO corrections b_{NLO} : NLO correction

$\sqrt{s}$: CM energy



N_b : $e^+e^- \to b\bar{b} \to$ all jet events number

N_{3b} : $e^+e^- \to b\bar{b} \to$ 3-jet events number

N_l : $e^+e^- \to l\bar{l} \to$ all jet events number $(l = u \text{ or } d \text{ or } s)$

N_{3l} : $e^+e^- \to l\bar{l} \to$ 3-jet events number $(l = u \text{ or } d \text{ or } s)$

Sensitivity of b quark mass

- For higher CM energy experiment, b quark mass becomes smaller relatively.
  → **b quark mass sensitivity be lower at higher energy scale.**

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l} = 1 + \frac{\alpha_s}{\pi} a_{LO} + \frac{m_b^2}{s}\left(b_{LO}(m_b) + \frac{\alpha_s}{\pi} b_{NLO}(m_b)\right)$$

b quark mass sensitivity on R_3^{bl} : $\Delta R_3^{bl} = 2\left(1 - R_3^{bl}\right)\frac{\Delta m_b}{m_b}$

CM energy	Z-pole	250GeV
Necessary precision of R_3^{bl} for $\Delta m_b = 0.4\mathrm{GeV}$	~1%	~0.1%

Flow of Simulation and Analysis

Flow of Simulation & Analysis

1. Generate Signal・Background events ··········▶ Measure R_3^{bl} @ Parton, R_3^{bl} @ Hadronization

   Signal : $e^+e^- \to q\bar{q} \to Jets$, Backgrounds : explain after

2. Detector Simulation・Event reconstruction

3. Cut of Background events ····················▶ Measure R_3^{bl} @ Reconstructed

4. Estimate corrections of hadronization and detector

5. Estimate b quark mass precision

Set up of Analysis

- Process : $e^+e^- \to q\bar{q} \quad (q = udscb)$
- CM energy : 250GeV
- 2 pure polarization configurations :

	e^-	e^+
Left components	100%	0%
Right components	0%	100%

Luminosity : $250\mathrm{fb}^{-1}$

	e^-	e^+
Left components	0%	100%
Right components	100%	0%

Luminosity : $250\mathrm{fb}^{-1}$

- Estimate b quark mass precision under the following polarizations by mixing above pure samples :

(-0.8,+0.3)

	e^-	e^+
Left components	90%	35%
Right components	10%	65%

Luminosity : $900\mathrm{fb}^{-1}$

(+0.8,-0.3)

	e^-	e^+
Left components	10%	65%
Right components	90%	35%

Luminosity : $900\mathrm{fb}^{-1}$

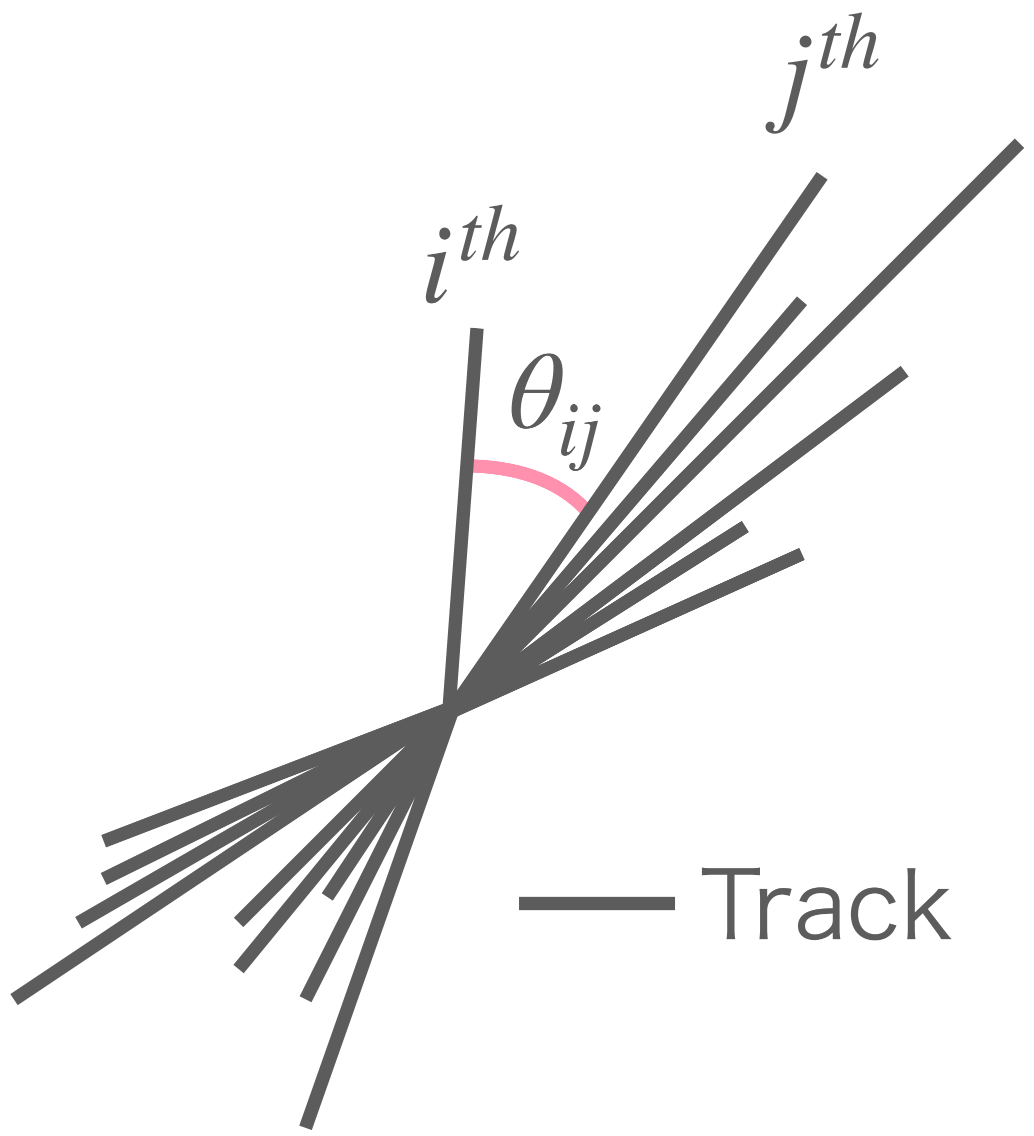
Reconstruction of Jets

- Define d_{ij} between i^{th} and j^{th} tracks (Cambridge algorithm) :

$$d_{ij} = \min(E_i^2, E_j^2)\frac{1-\cos\theta_{ij}}{1-\cos R} \quad (R = 1.25)$$

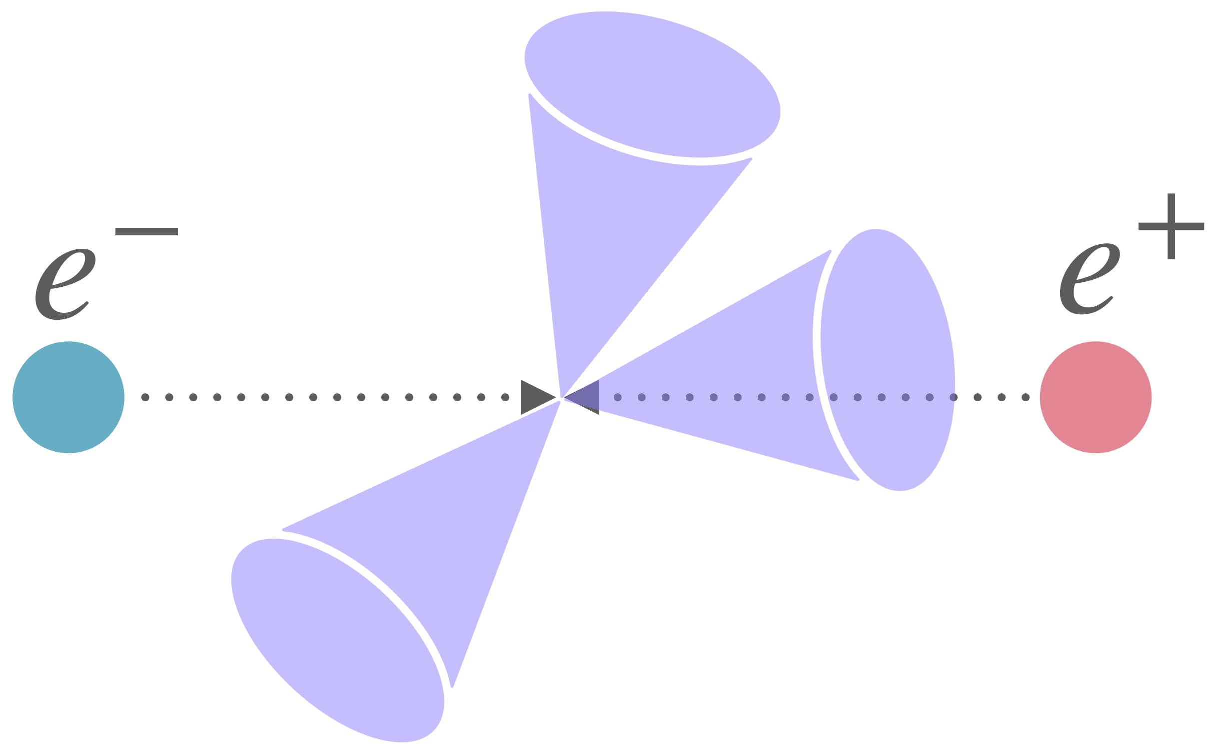
E_i : Energy of i^{th} track

If $d_{ij} < y_c$, these tracks are included in a same jet.



- R_3^{bl} has y_c dependence :

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l}$$
$$= 1 + \frac{\alpha_s}{\pi}a_{LO}(y_c) + \frac{m_q^2}{s}\left(b_{LO}(m_q, y_c) + \frac{\alpha_s}{\pi}b_{NLO}(m_q, y_c)\right)$$



3-jet event

- Focuses on $y_c = 0.01$.

- Bundle until 2-jet events by loosing y_c for later analysis of backgrounds.

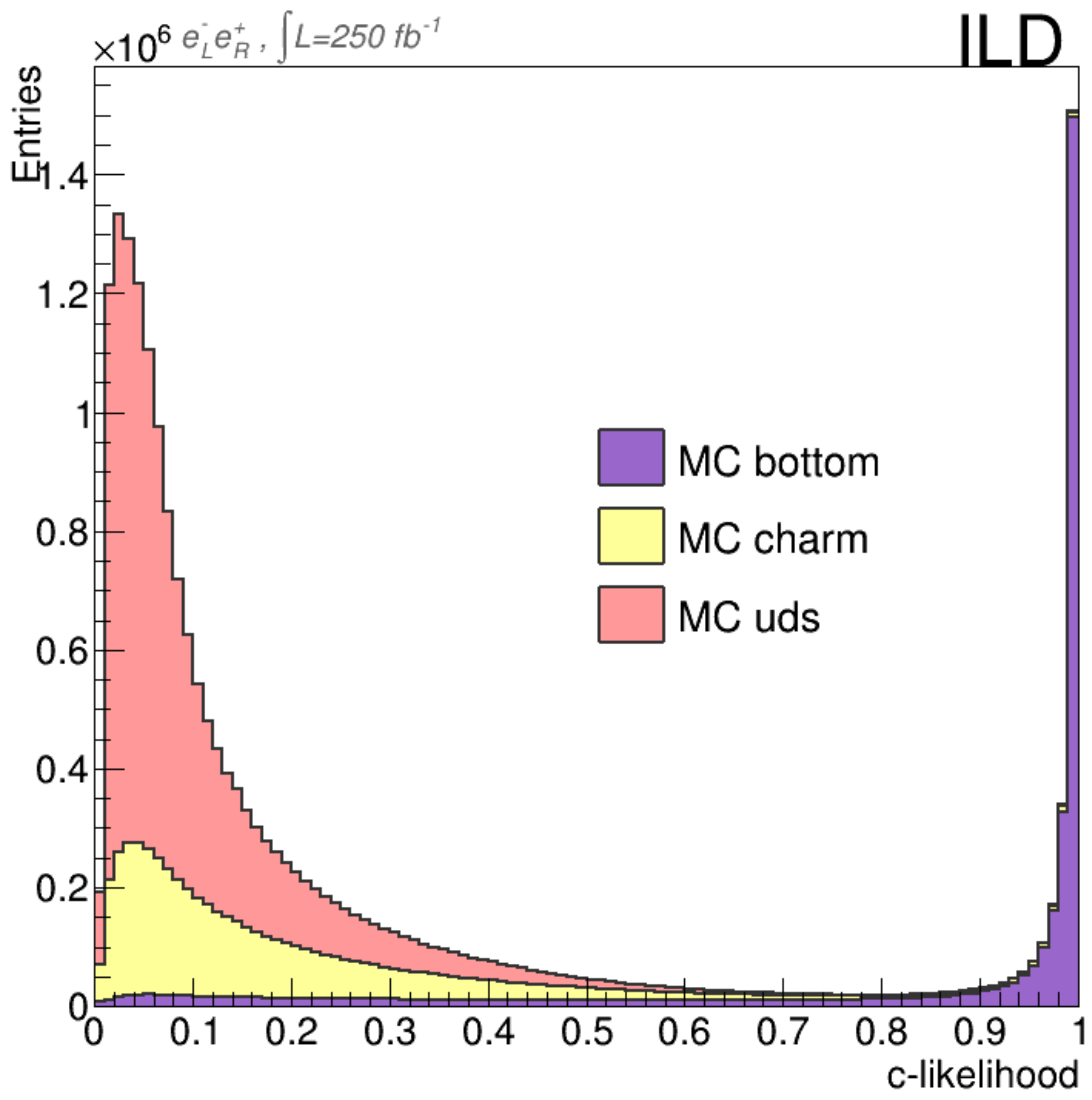
Flavor Identification

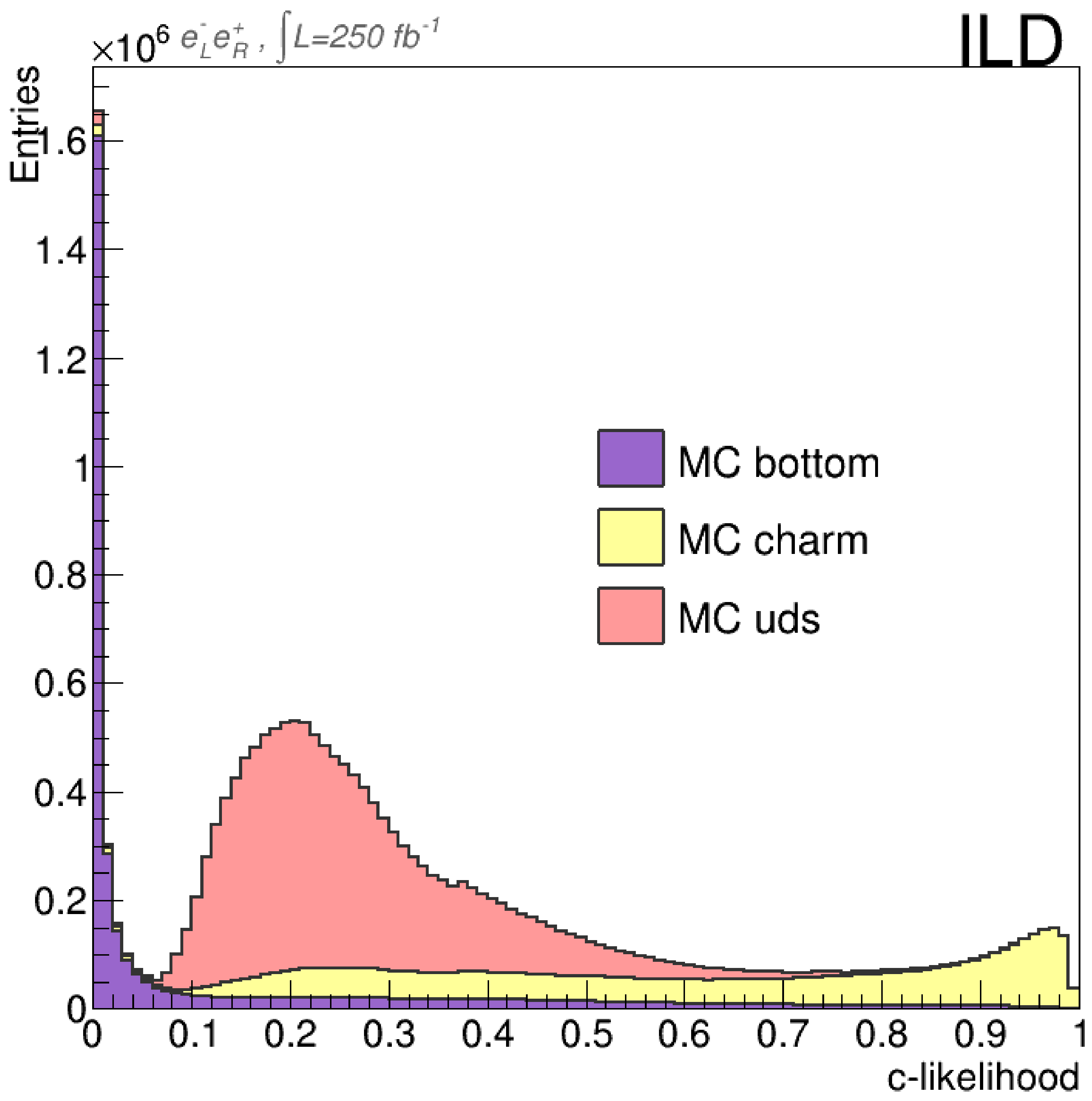
- Impose the following conditions to each jet

  b-likelihood > 0.8 → b events

  b-likelihood < 0.4 & c-likelihood < 0.25 → uds events

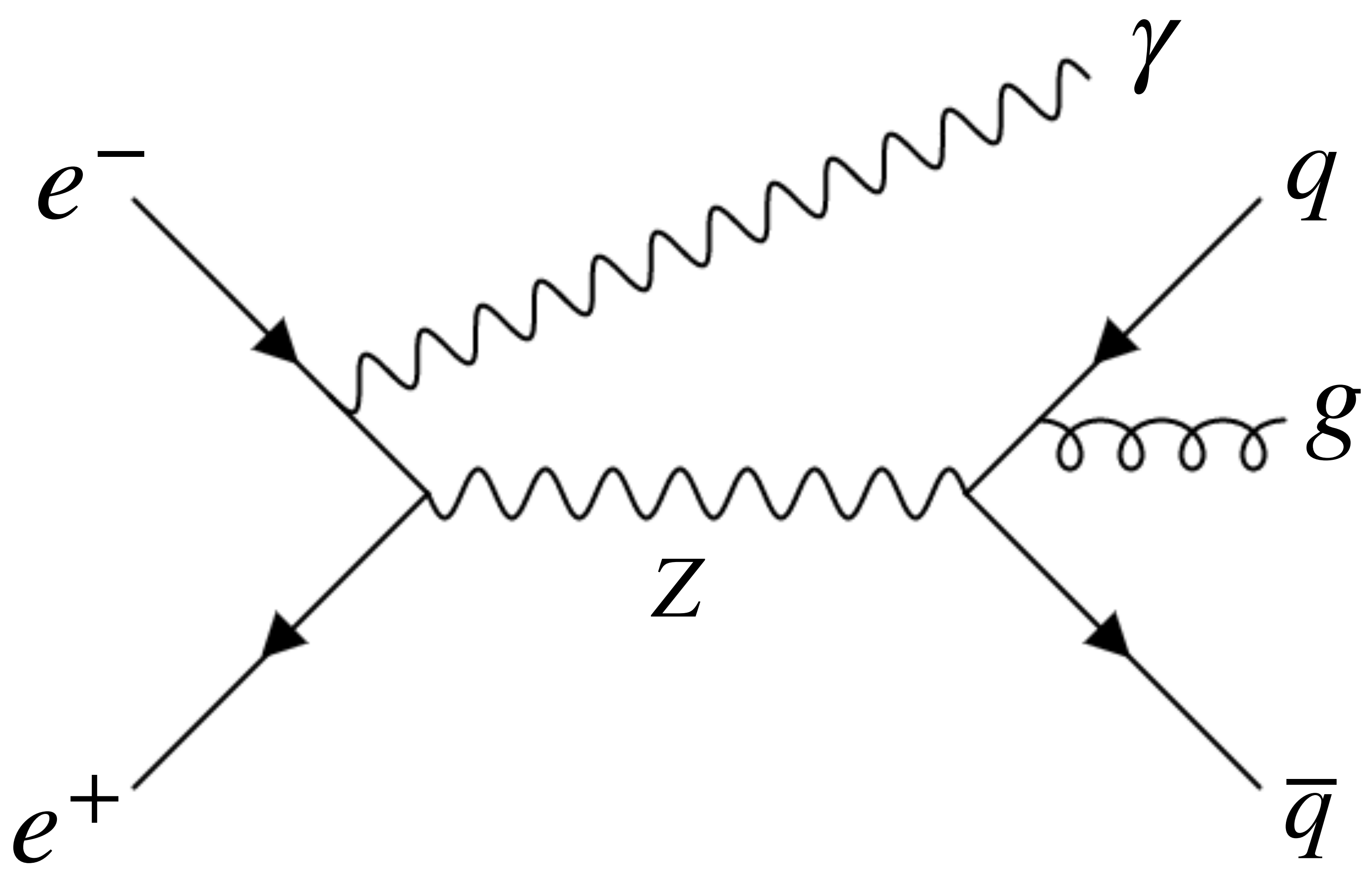
Efficiency of b ID	Purity of b ID	Efficiency of uds ID	Purity of uds ID
80%	98.7%	58%	96.1%





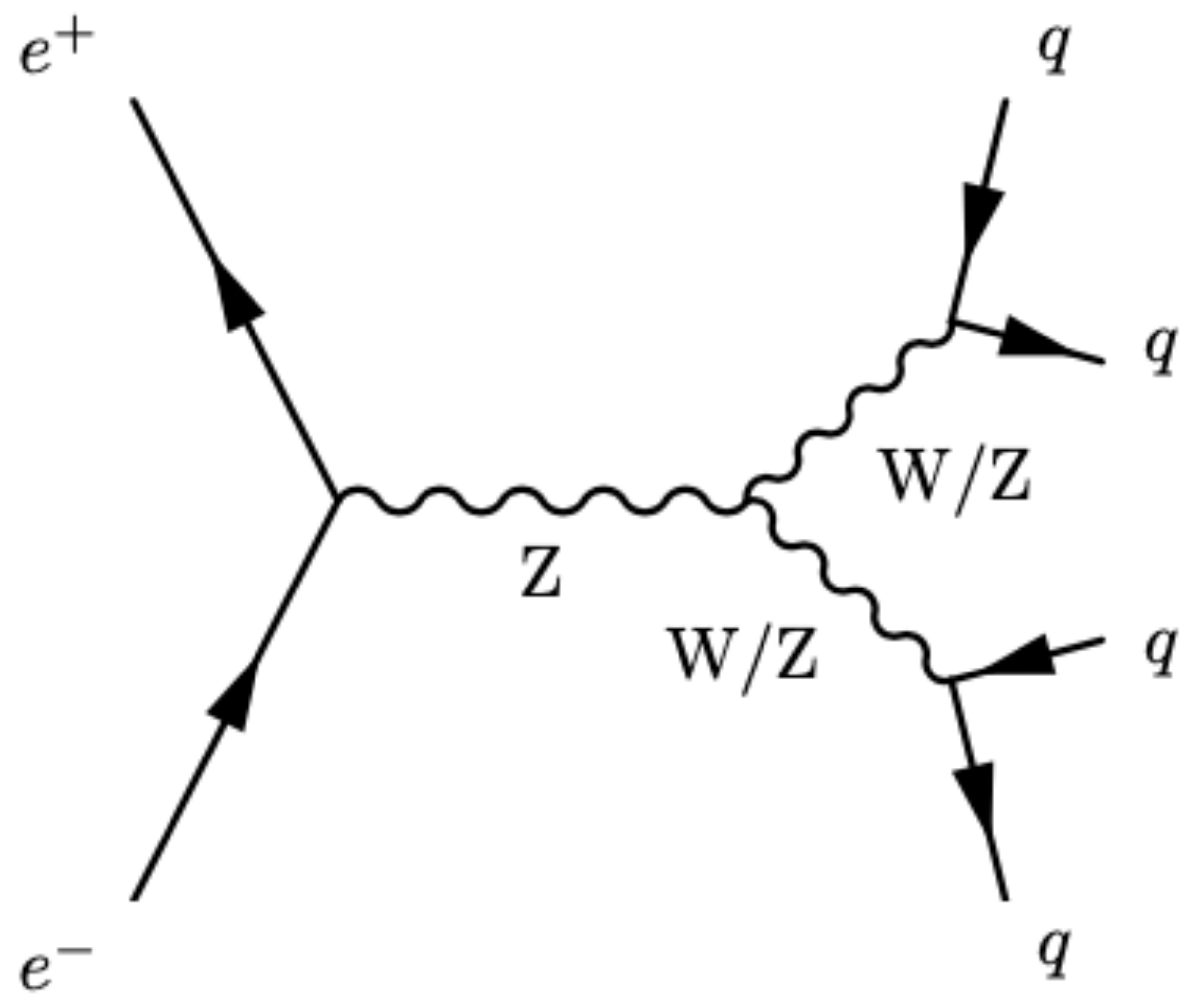
Cut of Background events

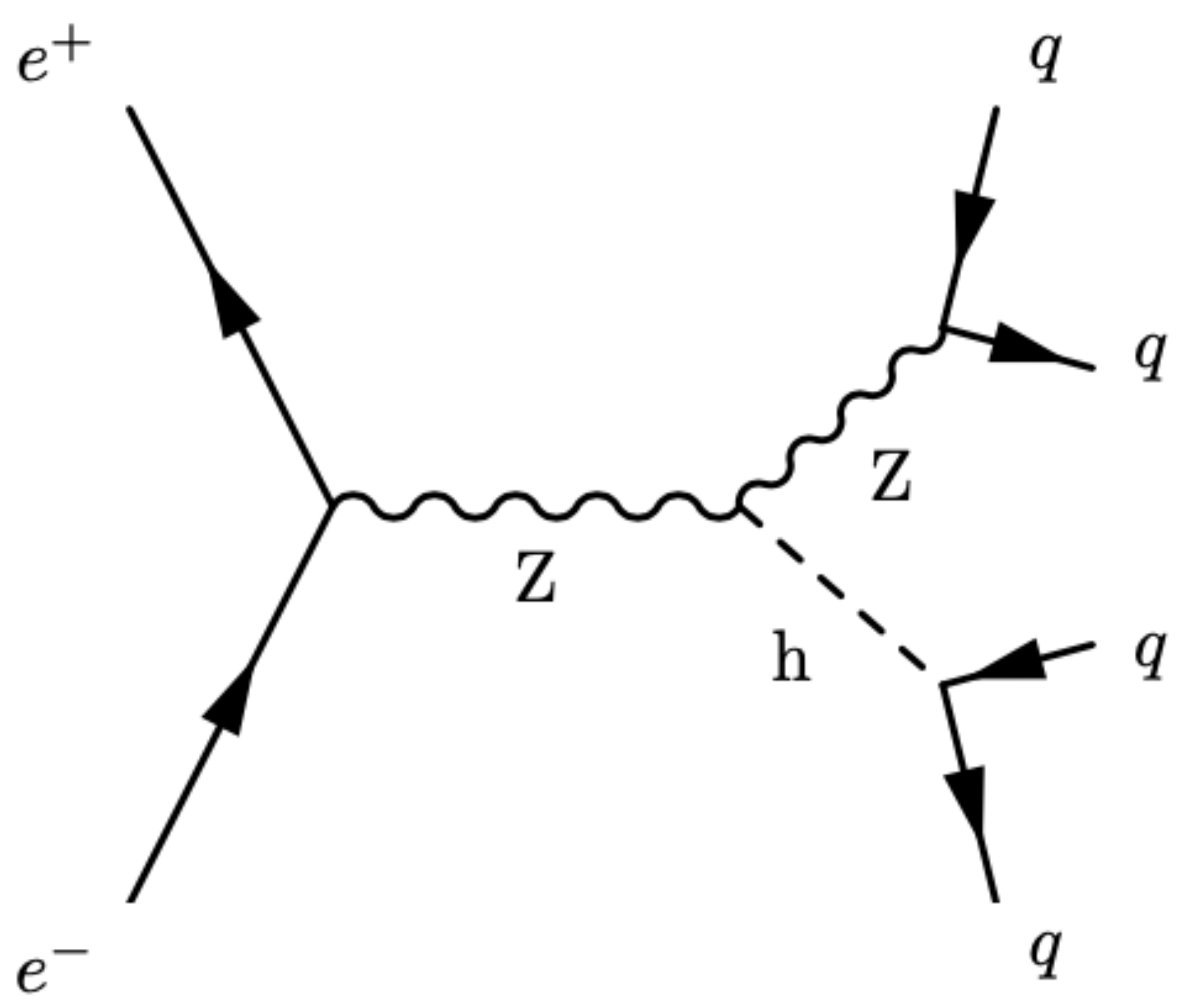
Types of Main Background events



Background event① : Radiative return

(Collision energy decreases by radiation)





Background event② : 4-jet events

(Can be mis-identified the number of jets)

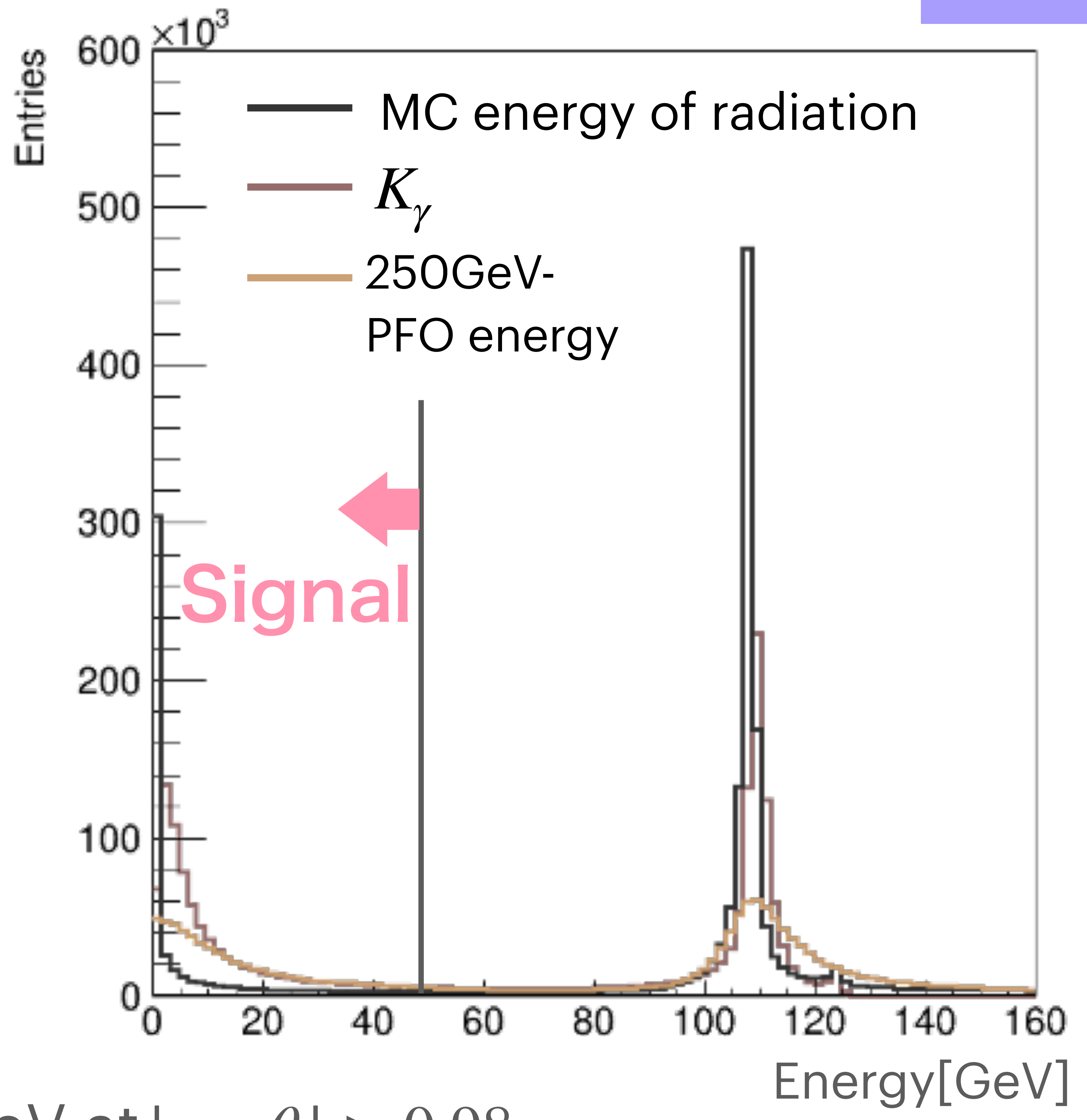
Cut of Radiative return

- If radiation can not detect…
  →Construct energy of radiation K_γ from angles of jets,
  and cut $K_\gamma > 50\text{GeV}$.

$$K_\gamma \equiv \frac{250GeV \cdot \sin\psi_{acol}}{\sin\psi_{acol} + \sin\theta_1 + \sin\theta_2}$$

ψ_{acol} : Angle between 2 jets

θ_1, θ_2 : Polar angles of each jet



- If radiation is detected, cut it by using neutral PFO.

1. Invariant mass of whole jets less than 130GeV
2. Each jet should include particles more than 5
3. Jets include neutral particles which have energy of more than 50 GeV at $|\cos\theta| > 0.98$

100% Left polarized	(Remained BKG)/(Selected Signal)
b quark	1.2%
uds quarks	1.3%

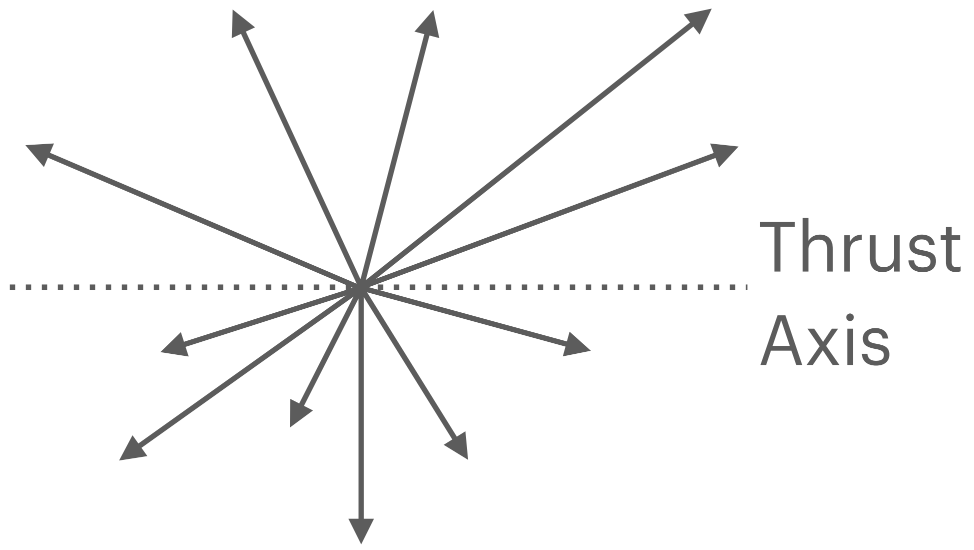
Cut of 4-jet events

- If 4-jet events are forced to reconstruct as 2-jet events, shapes tend to be wider.
  → Cut by using Thrust T ($T < 0.85$)

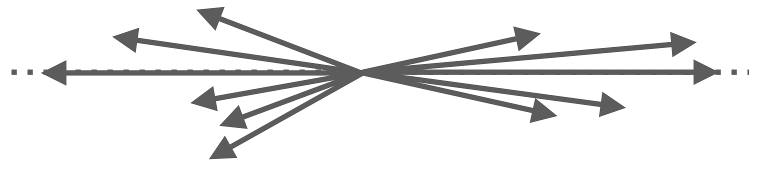
$$T \equiv \max_{\boldsymbol{n}} \frac{\sum_i^N |\boldsymbol{p}_i \cdot \boldsymbol{n}|}{\sum_i^N \boldsymbol{p}_i}$$

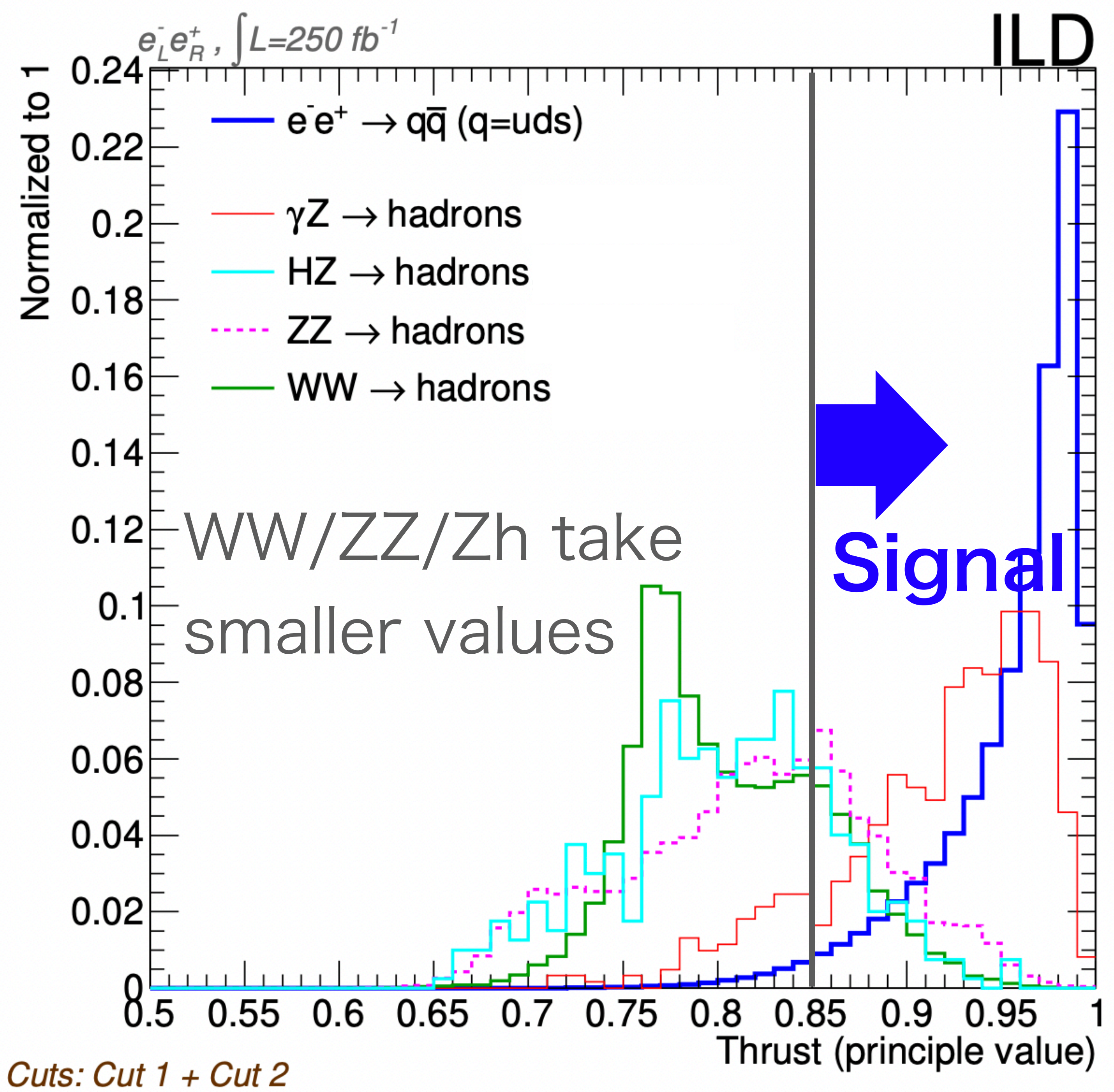
N : Total number of particles in jets
$\boldsymbol{p}_i$: momentum of each jet
$\boldsymbol{n}$: unit vector of the thrust axis

$T \sim 0.5$ (Isotopic)



$T \sim 1$ (Sharper)





Cuts: Cut 1 + Cut 2
+ double light-quark tag

	(Remained BKG)/(Selected Signal)		
	WW	ZZ	Zh
100% Left polarized			
b quark	0.0%	0.3%	0.3%
uds quarks	2.3%	0.2%	0.0%
100% Right polarized			
b quark	0.0%	0.9%	1.0%
uds quarks	0.0%	0.3%	0.0%

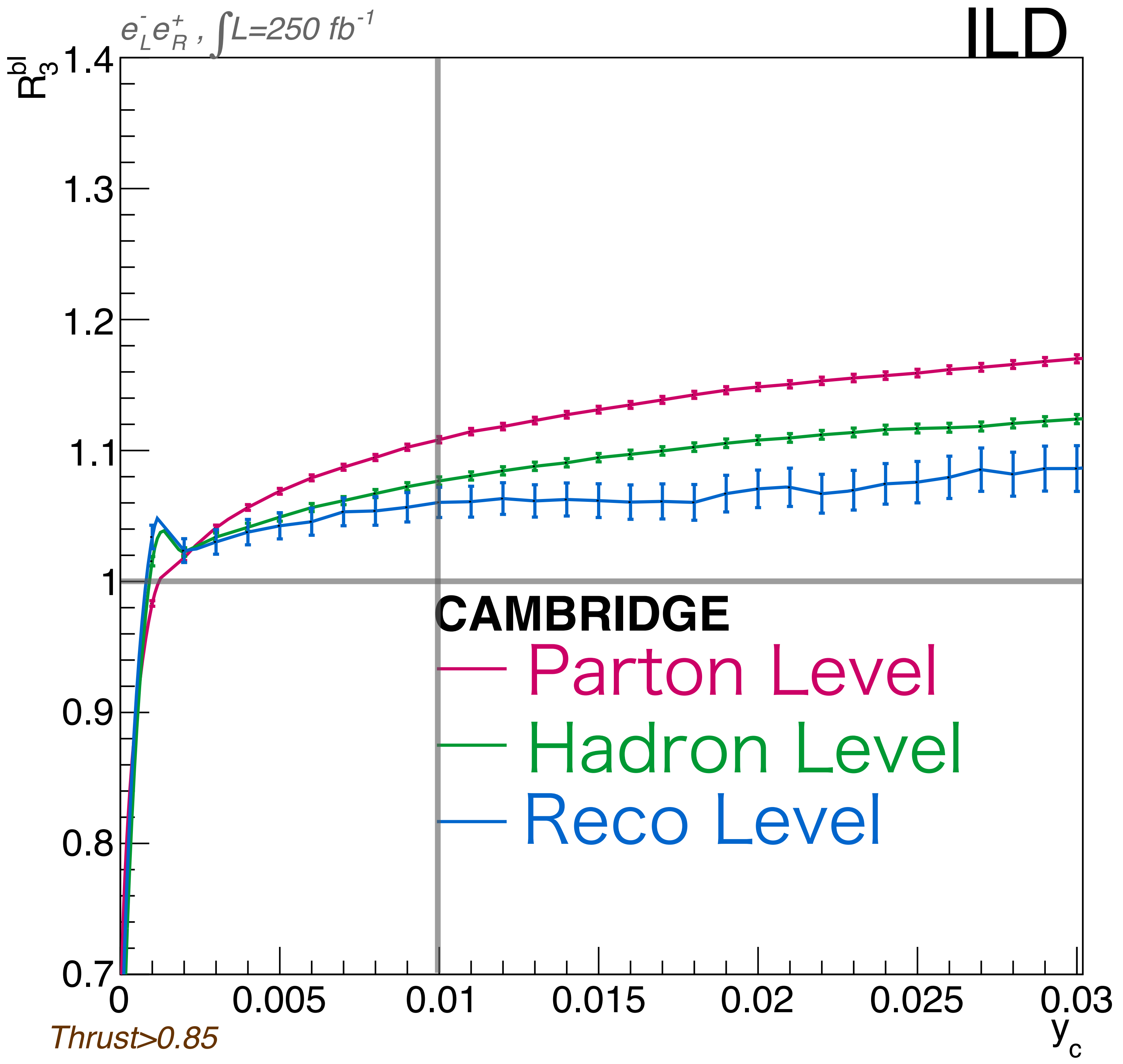
Measurement of the Observable

Measured Results of Observable

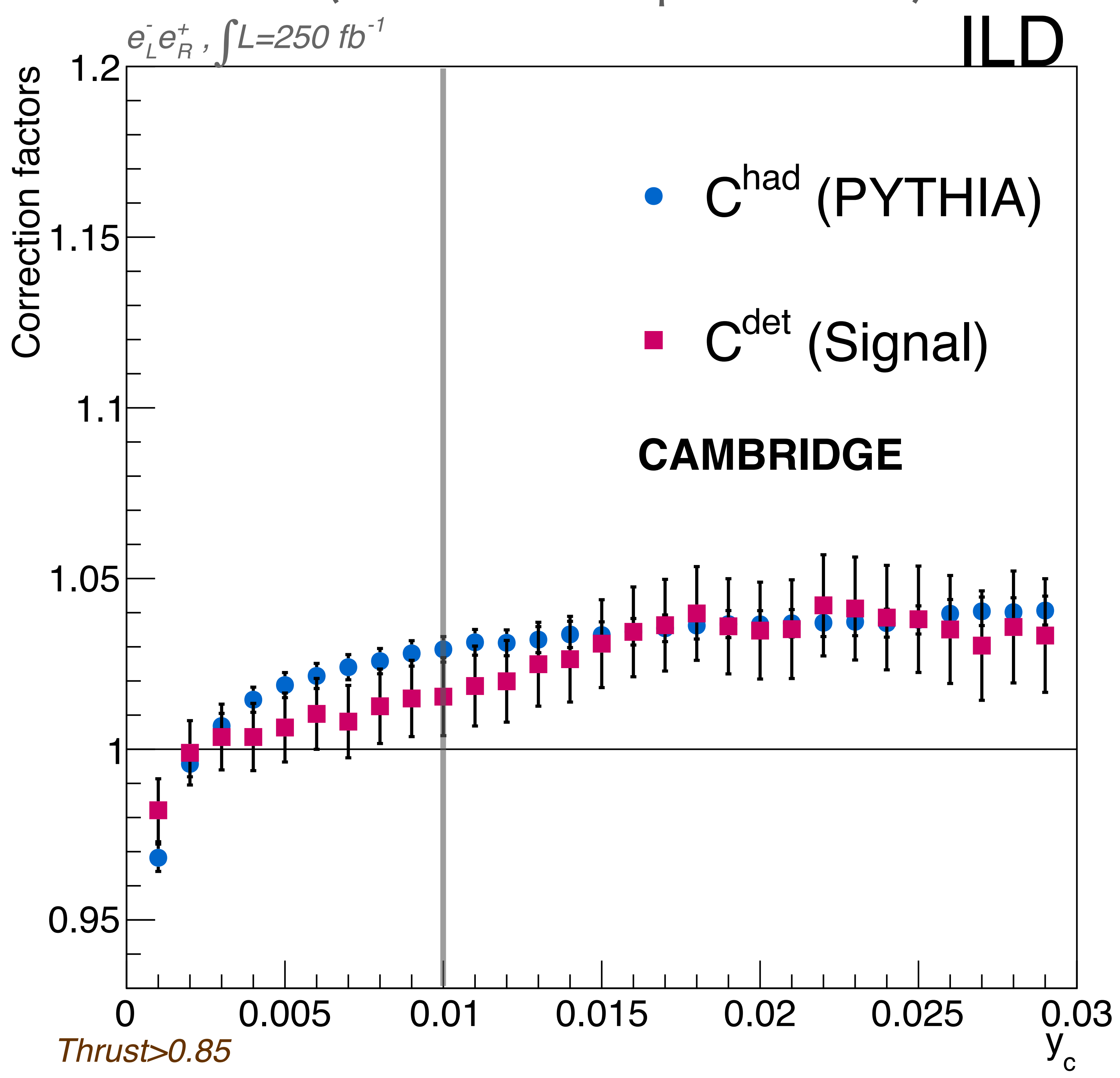
- Reco level is corrected to Parton level by corrections of hadronization and detector.
  Parton level can be compared to theory.

$$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec} \qquad C^{had} \equiv \frac{R_3^{bl\ Par}}{R_3^{bl\ Had}} \quad C^{det} \equiv \frac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$$

Each level R_3^{bl} by MC
(100% Left polarized)



Corrections C^{had}, C^{det}
(100% Left polarized)



Consideration of obtained results

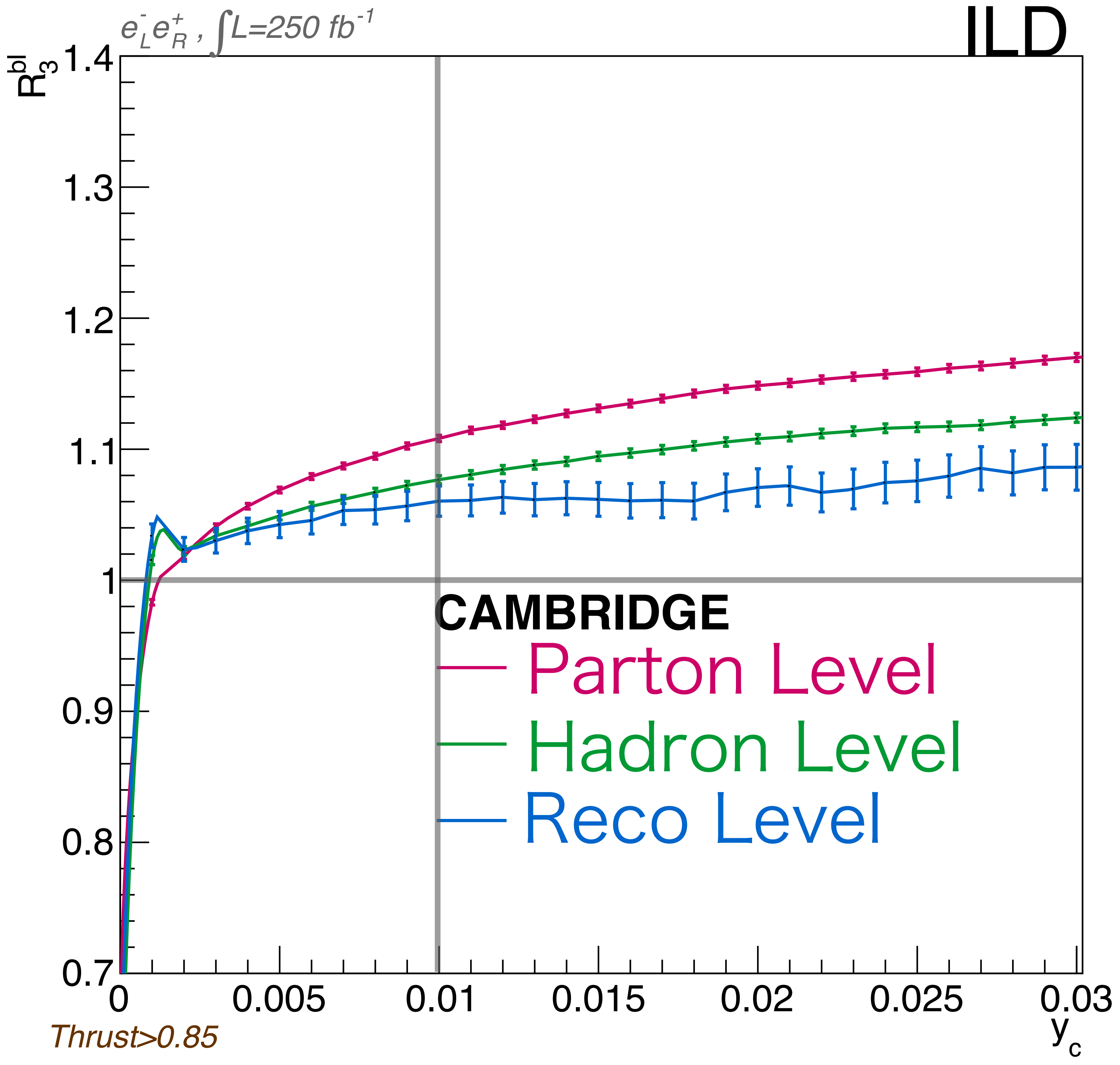
- b is heavier than uds → $R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l} < 1$

  But obtained result is $R_3^{bl} > 1$

- Numbers of 3-jet events are ~10 times less than the SM's expectation.
  → MC has some problems?

- When examine MC …
  b quark is massless, gluon radiation is LO in WHIZARD.
  → MC is updating to NLO+massive now.

Each level R_3^{bl} by MC
(100% Left polarized)



Estimation of Errors

Estimation of Statistical error

■ Statistical error on R_3^{bl} :

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l} \rightarrow \frac{\Delta R_3^{bl}}{R_3^{bl}}(stat.) = \frac{1}{\sqrt{N_{3b}}} + \frac{1}{\sqrt{N_{3l}}} + \frac{1}{\sqrt{N_b}} + \frac{1}{\sqrt{N_l}}$$

N_{3q} : 3-jet events number
N_q : all jet events number

■ Estimate under 2ab-1 of 250 GeV ILC

Polarization (P_{e^-}, P_{e^+})	(-0.8,+0.3)	(+0.8,-0.3)	(-0.8,-0.3)	(+0.8,+0.3)
Integrated luminosity	900fb-1	900fb-1	100fb-1	100fb-1

■ Assumed the proportion of 3-jet events is 30% of the all jet events number.

	All jet events (MC)		3-jet events (仮定)	
	N_b	N_l	$N_{3b} = 0.3N_b$	$N_{3l} = 0.3N_l$
(-0.8,+0.3%)	1,210,601	1,659,628	363,180	497,888
(+0.8%,-0.3%)	341,956	596,210	102,587	178,863

30%

$$\frac{\Delta R_3^{bl}}{R_3^{bl}}(stat.) = 0.25\,\% \text{ for } (-0.8, +0.3),\ 0.45\,\% \text{ for } (+0.8, -0.3)$$

Types of Systematic errors

- Appear systematic errors on corrections C^{had}, C^{det}.

Parton Level

Reco Level

$$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec}$$

Corrections of hadronization and detector

1. Hadronization error : Estimate the uncertainty on $C^{had} \equiv \dfrac{R_3^{bl\ Par}}{R_3^{bl\ Had}}$.

2. Detector error : Estimate the uncertainty on $C^{det} \equiv \dfrac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$.

Estimations of Systematic errors

■ Hadronization model error

- Mis-identification of jets will be decreased than LEP
- Understanding of hadronization will be developed than LEP

→ Assume the half of LEP's result : $\Delta C^{had}/C^{had} = 0.1\,\%$

■ Detector error

Estimate appeared uncertainties on C^{det} from 3 element's uncertainties by ToyMC.

Common uncertainties for jet's number and flavor are cancelled.

	Each Uncertainty	Uncertainties on C^{det}	
		100% Left polarized	100% Right polarized
Flavor tag	0.5%	0.07%	0.06%
Signal selection	1%	0.06%	0.06%
Contamination of BKG	1%	0.20%	0.10%
Total	—	0.22%	0.13%

Result and Summary

Result of This study

- Precision of R_3^{bl} :

$$\frac{\Delta R_3^{bl}}{R_3^{bl}} = 0.25(stat.) \pm 0.22(exp.) \pm 0.1(had.)[\,\%\,] \text{ for } (-0.8, +0.3)$$

$$\frac{\Delta R_3^{bl}}{R_3^{bl}} = 0.45(stat.) \pm 0.13(exp.) \pm 0.1(had.)[\,\%\,] \text{ for } (+0.8, -0.3)$$

(3-jet events number is assumed to be 30% of all jet events number.)

- Estimation of b quark mass precision :

b quark mass sensitivity on R_3^{bl} : $\dfrac{\Delta m_b}{m_b} = \dfrac{\Delta R_3^{bl}}{2(1 - R_3^{bl})}$

Precision for $R_3^{bl} = 0.996,\ m_b = 2.75\text{GeV}$:

$$\Delta m_b\left(250\right) = 0.85(stat.) \pm 0.75(exp.) \pm 0.34(had.) \pm 0.07(theo.)\text{GeV} \text{ for } \left(-80\,\%, +30\,\%\right)$$

$$\Delta m_b\left(250\right) = 1.53(stat.) \pm 0.44(exp.) \pm 0.34(had.) \pm 0.07(theo.)\text{GeV} \text{ for } \left(+80\,\%, -30\,\%\right)$$

Theoretical uncertainty : come from renormalized scale and quark mass definition

ILC Prospects

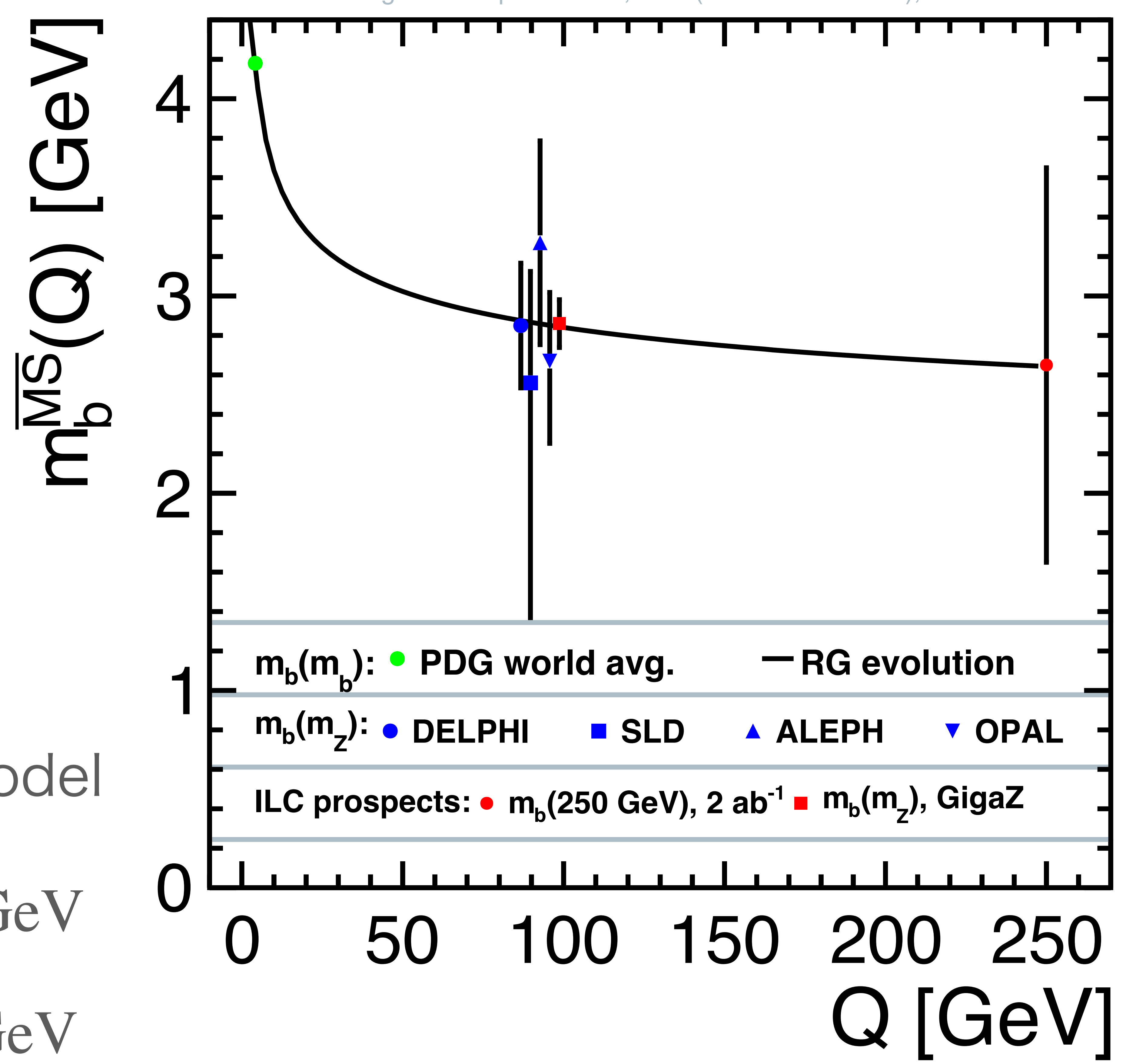
- 250GeV measurement is challenging, but it will provide an extra point at never probed energies.

- **Giga-Z ILC (ILC@Z-pole)**
  - ・Statistics is 100 times larger
  - ・ILD superior the performance of flavor tagging
  - ・Hadronization error will be half thanks for development of model

LEP : $\Delta m_b\left(M_Z\right) = 0.18(stat.) \pm 0.13(exp.) \pm 0.19(had.) \pm 0.12(theo.)$ GeV

ILC : $\Delta m_b\left(M_Z\right) = 0.02(stat.) \pm 0.02(exp.) \pm 0.09(had.) \pm 0.06(theo.)$ GeV

The running bottom quark mass, IFIC (U. Valencia/CSIC), U. Tohoku



Summary

- b quark mass precision at 250 GeV :

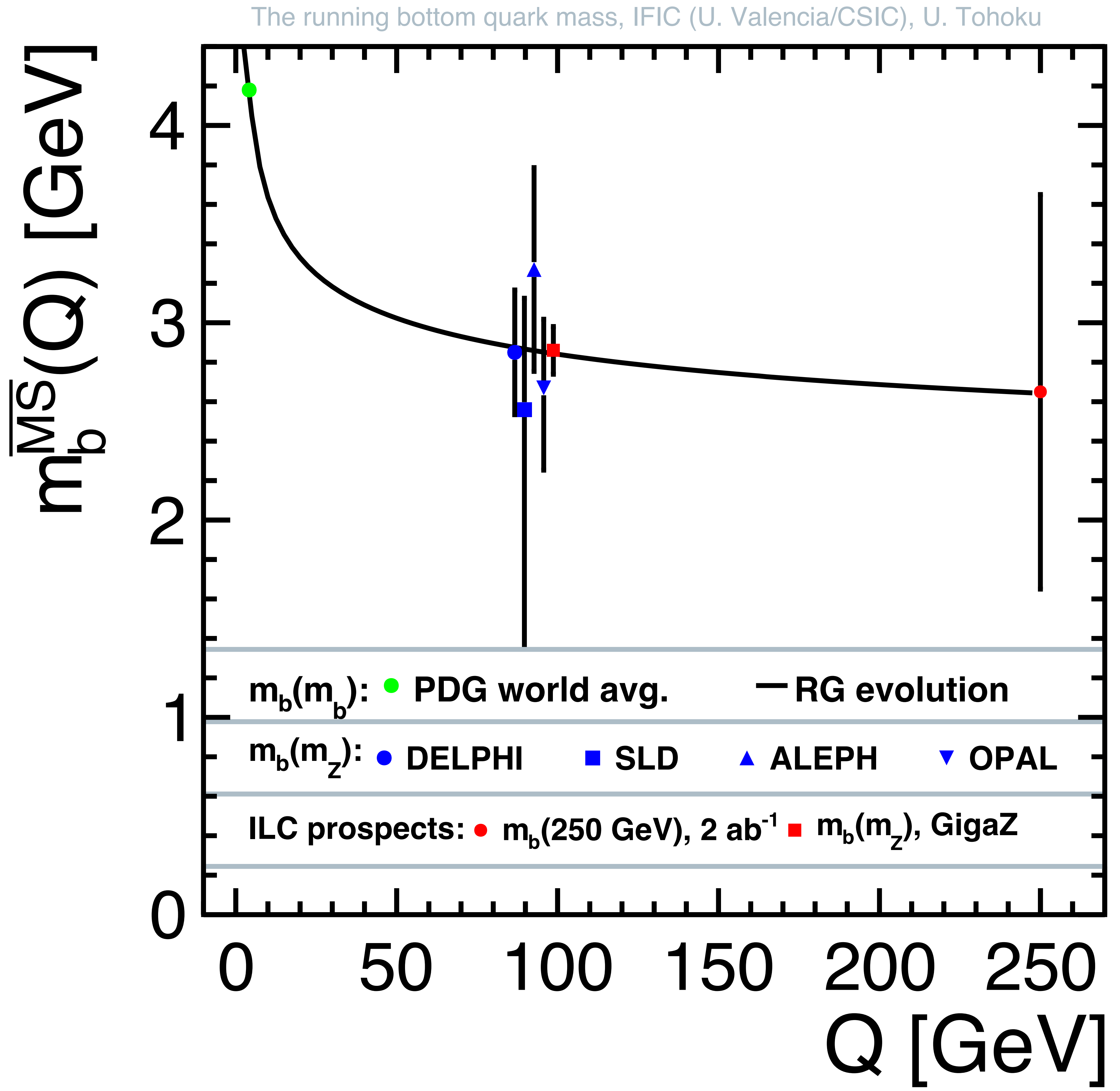
$$\Delta m_b(250) = 0.85(stat.) \pm 0.75(exp.) \pm 0.34(had.) \pm 0.07(theo.)\ \text{GeV}$$
for $(-0.8, +0.3)$

$$\Delta m_b(250) = 1.53(stat.) \pm 0.44(exp.) \pm 0.34(had.) \pm 0.07(theo.)\ \text{GeV}$$
for $(+0.8, -0.3)$

- WHIZARD is updating to NLO+massive quarks.

- Giga-Z ILC provides better measurement of b quark mass at Z-pole :

$$\Delta m_b(M_Z) = 0.02(stat.) \pm 0.02(exp.) \pm 0.09(had.) \pm 0.06(theo.)\ \text{GeV}$$



Backup

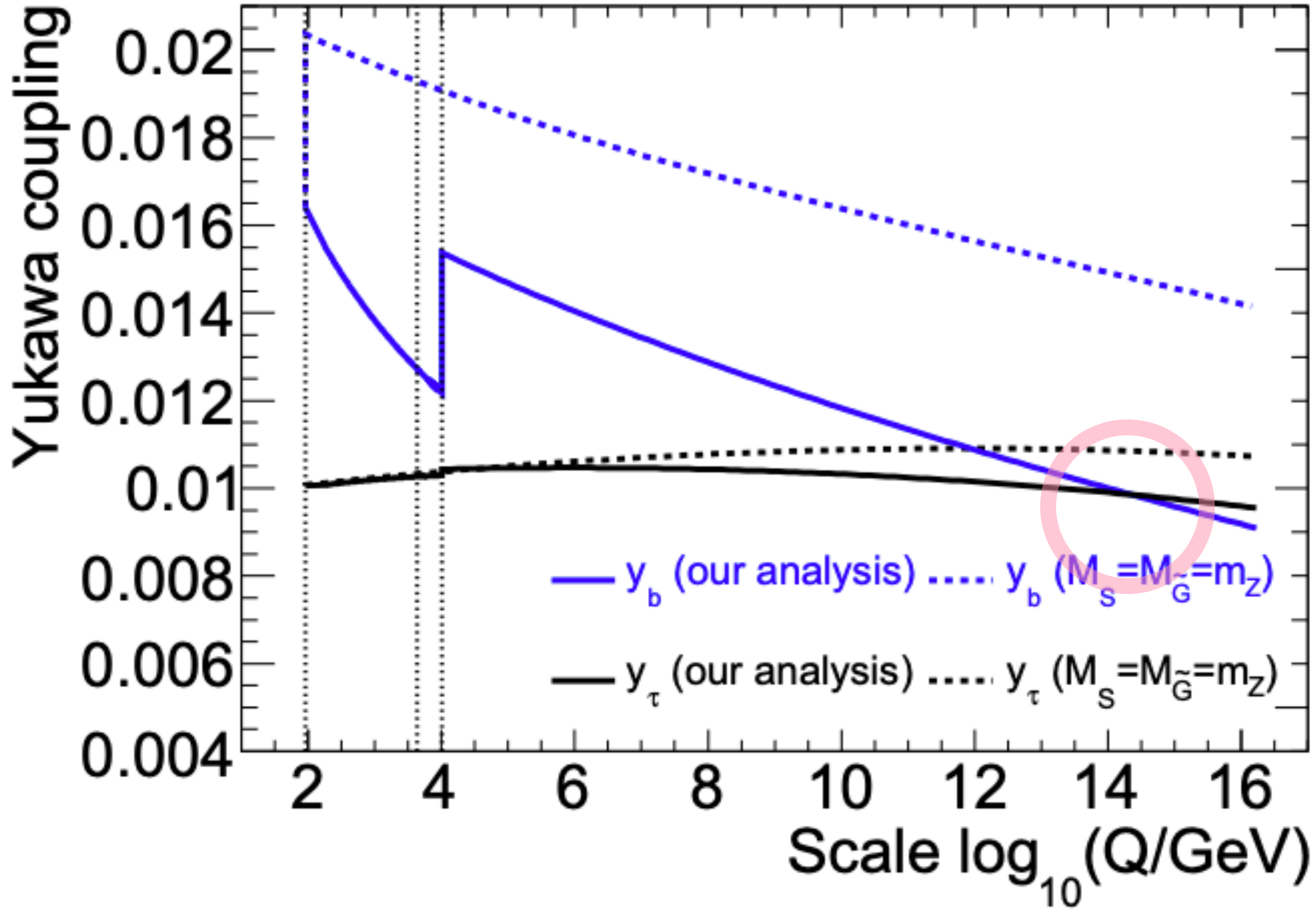
Unification of 3rd generation particles

- Scenario changes according to the ratio of vacuum expectation value of SUSY Higgs $\tan\beta$.

arXiv:1201.4412 [hep-ph]
arXiv:1604.02156 [hep-ph]

$\tan\beta \sim 3-11$: b-τ Yukawa coupling is unified

$\tan\beta \sim 35-60$: b-τ-t Yukawa coupling is unified



Polarization

■ Polarization rate : $P_{e^{-(+)}} \equiv \frac{f_R - f_L}{f_R + f_L}$

f_L : proportion of left handed components

f_R : proportion of right handed components

(P_{e^-}, P_{e^+}) = (+80%,+30%)

	e^-	e^+
Left	10%	35%
Right	90%	65%

Luminosity : 100fb^{-1}

$e_L^- e_R^+$: 17fb-1

(P_{e^-}, P_{e^+}) = (-80%,-30%)

	e^-	e^+
Left	90%	65%
Right	10%	35%

Luminosity : 100fb^{-1}

$e_L^- e_R^+$: 83fb-1

(P_{e^-}, P_{e^+}) = (-80%,+30%)

	e^-	e^+
Left	90%	35%
Right	10%	65%

Luminosity : 900fb^{-1}

$e_L^- e_R^+$: 846fb-1

(P_{e^-}, P_{e^+}) = (+80%,-30%)

	e^-	e^+
Left	10%	65%
Right	90%	35%

Luminosity : 900fb^{-1}

$e_L^- e_R^+$: 50fb-1

■ How to mix :

100% Left polarized

100% Right polarized

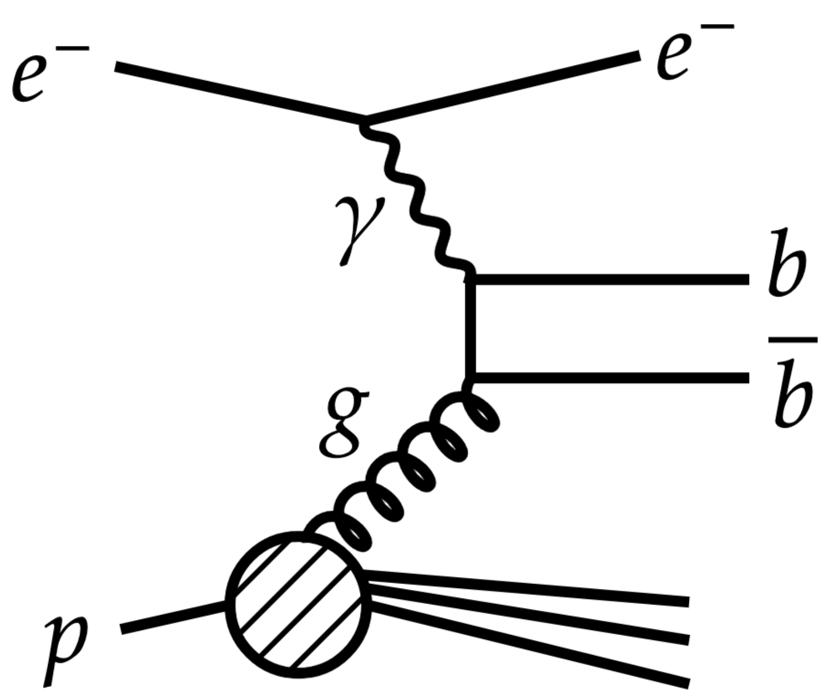
$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4}\left(\left(1 - P_{e^-}\right)\left(1 + P_{e^+}\right)\sigma_L + \left(1 + P_{e^-}\right)\left(1 - P_{e^+}\right)\sigma_R\right)$$

$e_L^- e_L^+$ and $e_R^- e_R^+$ for quark pair production from e^+e^- are vanished by angular momentum conservation.

About the number of $m_b(m_b)$

■ Green marker is the word average of PDG

$$m_b(m_b) = 4.18^{+0.03}_{-0.02}\ \mathrm{GeV}$$



Main experiments：

ZEUS @ HERA (2018) arXiv:1804.01019 **[hep-ex]**

The e-p deep inelastic scattering depends on b mass.

BABAR @ PEP-II (2009) arXiv:0908.0415 **[hep-ex]**

Belle @ KEKB (2008) arXiv:0803.2158 **[hep-ex]**

Semi-leptonic decay of B meson(e.g. $B \to X_c l \nu_l$) depends on b mass.

B中間子の崩壊例)