The ILC (?), SUSY, & the Tiny (g-2) Muon Wobble

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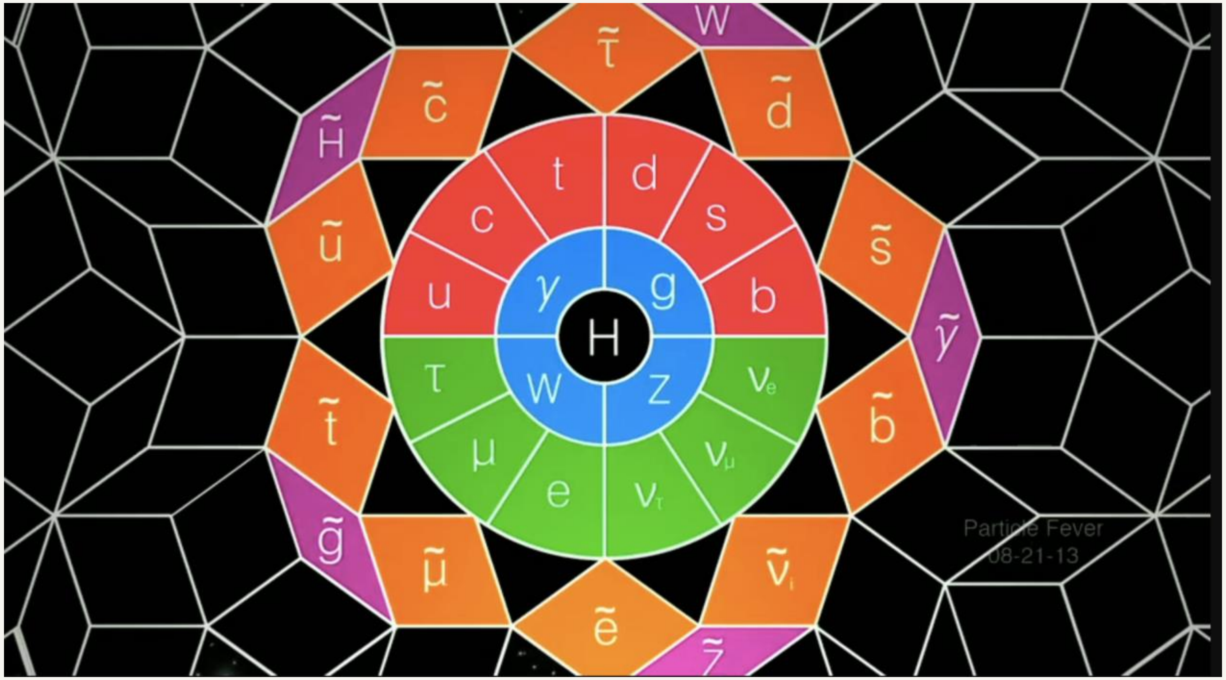
Baum, Carena, NRS, Wagner, arXiv: 2104.03302

ILC IDT WG3 – Mini Symposium on Muon (g-2)

Thursday May 27, 2021

“Particle Fever” – The movie



THE ANSWER TO *EVERYTHING*:

NOTORIOUS

SUPERSYMMETRY

Let's start at the VERY beginning.
A VERY good place to start!

MSSM Charginos & Neutralinos

SM Gauge Bosons and Higgs (2HDM in SUSY) Superpartners →	Bunch of weakly coupled neutral Majorana fermions, lightest stable due to R-parity: **WIMP!**
B: SU(2) singlet	Bino M_1 g_1
$\{W^0, W^1, W^2\}$: SU(2) triplet	Winos M_2 g_2 1 neutral, 2 charged
$\{H_u\ H_d\}$: SU(2) doublets	Higgsinos μ Yukawas etc 2 neutral, 2 charged

MSSM: 4 neutral “Neutralinos”, mixtures of interaction states (Also 2 charged “Charginos” mixtures of wino and Higgsinos).

$$\chi = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_d + N_{14}\tilde{H}_u$$

MSSM Charginos & Neutralinos:
Mass Matrices

charginos

in $(\tilde{W}^-, \tilde{H}^-)$ basis

$$\begin{pmatrix} M_2 & \sqrt{2} m_W c_\beta \\ \sqrt{2} m_W s_\beta & \mu \end{pmatrix}$$

neutralinos

in $(\tilde{B}^0, \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0)$ basis

$$\begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\ 0 & M_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0 \end{pmatrix}$$

At Tree-level:

Charginos: M_2, μ, $\tan\beta$
Neutralinos: $+ M_1$

A good starting point for SUSY parameter/mass spectrum determination

Slepton Sector

Masses are determined by mixing between the left and right-handed sleptons.

$$M^2_{\tilde{\mu}} = \begin{pmatrix} m^2_{\tilde{\mu}L} & m^2_{LR} \\ m^2_{LR} & m^2_{\tilde{\mu}R} \end{pmatrix},$$

$$v_i = < H_i >$$

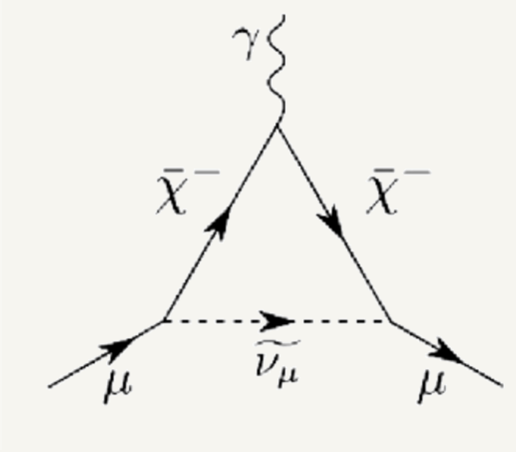
$$m_\mu = h_\mu v_1$$

$$\begin{aligned} m^2_{\tilde{\mu}L} &= m^2_L + m^2_Z \cos 2\beta \left(\sin^2\theta_W - \frac{1}{2} \right) \\ m^2_{\tilde{\mu}R} &= m^2_R - m^2_Z \cos 2\beta \sin^2\theta_W, \\ m^2_{LR} &= y_\mu \mu_H \langle H_2 \rangle + A_\mu \langle H_1 \rangle. \end{aligned}$$

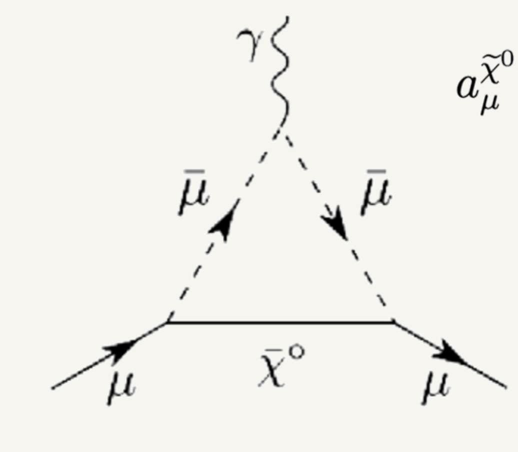
Mixing most relevant for the case of tau sleptons (staus).

Dominant Contributions:

Barbieri, Maiani,'82; Ellis et al,'82; Grifols and Mendez,'82;
Moroi,'95; Carena, Giudice, Wagner, '95; Martin and Wells, '00 …



$$a_\mu^{\tilde{\chi}^\pm-\tilde{\upsilon}_\mu} \simeq \frac{\alpha m_\mu^2 \mu M_2 \tan\beta}{4\pi \sin^2\theta_W m_{\tilde{\upsilon}_\mu}^2}\left[\frac{f_{\chi^\pm}\left(M_2^2/m_{\tilde{\upsilon}_\mu}^2\right)-f_{\chi^\pm}\left(\mu^2/m_{\tilde{\upsilon}_\mu}^2\right)}{M_2^2-\mu^2}\right]$$



$$a_\mu^{\tilde{\chi}^0-\tilde{\mu}} \simeq \frac{\alpha m_\mu^2 M_1\left(\mu\tan\beta - A_\mu\right)}{4\pi\cos^2\theta_W\left(m_{\tilde{\mu}_R}^2-m_{\tilde{\mu}_L}^2\right)}\left[\frac{f_{\chi^0}\left(M_1^2/m_{\tilde{\mu}_R}^2\right)}{m_{\tilde{\mu}_R}^2}-\frac{f_{\chi^0}\left(M_1^2/m_{\tilde{\mu}_L}^2\right)}{m_{\tilde{\mu}_L}^2}\right]$$

$$f_{\chi^\pm}(x) = \frac{x^2-4x+3+2\ln(x)}{(1-x)^3}\,,$$

$$f_{\chi^0}(x) = \frac{x^2-1-2x\ln(x)}{(1-x)^3}\,;$$

Rough Approximation

If ALL weakly interacting SUSY particle masses same, AND the gaugino masses had the same sign:

$$(\Delta a_\mu)^{\rm SUSY} \simeq 150 \times 10^{-11} \left(\frac{100\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta$$

$$\Delta a_\mu \equiv (a_\mu^{\rm exp} - a_\mu^{\rm SM}) = (251 \pm 59) \times 10^{-11}$$

Anomaly could be explained:

For $\tan\beta = 10$, $m_{\rm SUSY} \sim 250$ GeV

For $\tan\beta = 60$, $m_{\rm SUSY} \sim 700$ GeV

(consistent with the unification of the top and bottom Yukawas).

What about electron (g – 2)??

The same contributions apply to the electron case (with appropriate substitutions).

In particular, both contributions are suppressed by the square of the **electron** mass rather than the muon mass!

$$(\Delta a_e)^{\mathrm{SUSY}} \simeq 150 \times 10^{-11} \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta \left(\frac{m_e}{m_\mu}\right)^2$$

$$\simeq 4 \times 10^{-14} \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta$$

To explain the muon anomaly, **$(100\ \mathrm{GeV}/m_{\mathrm{SUSY}})^2 \tan\beta \sim 1.5$**

Δa_e contribution **order of magnitude smaller** than current uncertainties!

BUT ISN'T SUSY ALREADY RULED OUT BELOW THE TEV SCALE ??

LHC: All about the Strong Stuff

Guinos:

If decay directly to 3rd generation squarks, gluinos must be heavier than about 1.5 to 2.2 TeV

Cascade decays into intermediate chargino/neutralino states and compressed spectrum present the weakest limits, and the bound falls short of 2 TeV for non-compressed spectrum. Bound of 2.2 TeV in the most extreme case.

Hard to evade the TeV bound.

Stops:

Higgs mass implies stops masses heavier than ~ 1 TeV

Combining all searches, in the simplest decay scenarios, it is hard to avoid the constraints 600 GeV - 1.2 TeV for stops.

We are just starting to explore the mass region suggested by the Higgs mass!

All that **Weak** stuff?

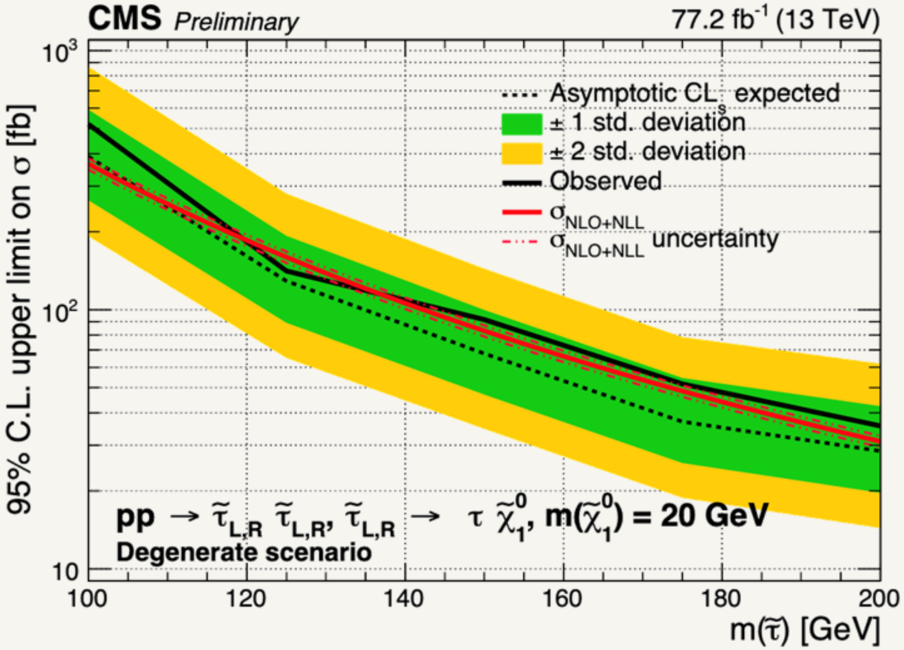
Situation here is far less well defined than in the strongly interacting sector.

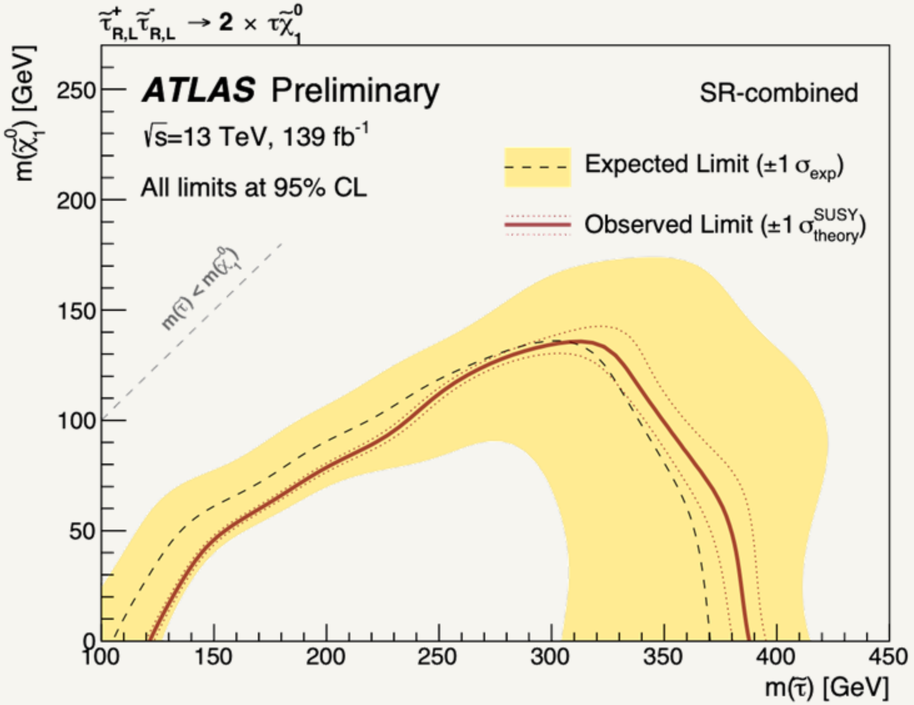
Sleptons, in particular staus, are only weakly constraint beyond the LEP limits.

Winos as NLSP’s are the strongest constrained particles.

In general, a SUSY scenario with large cascade decays with light electroweakinos is the **most natural one** and the **least constrained** so far.

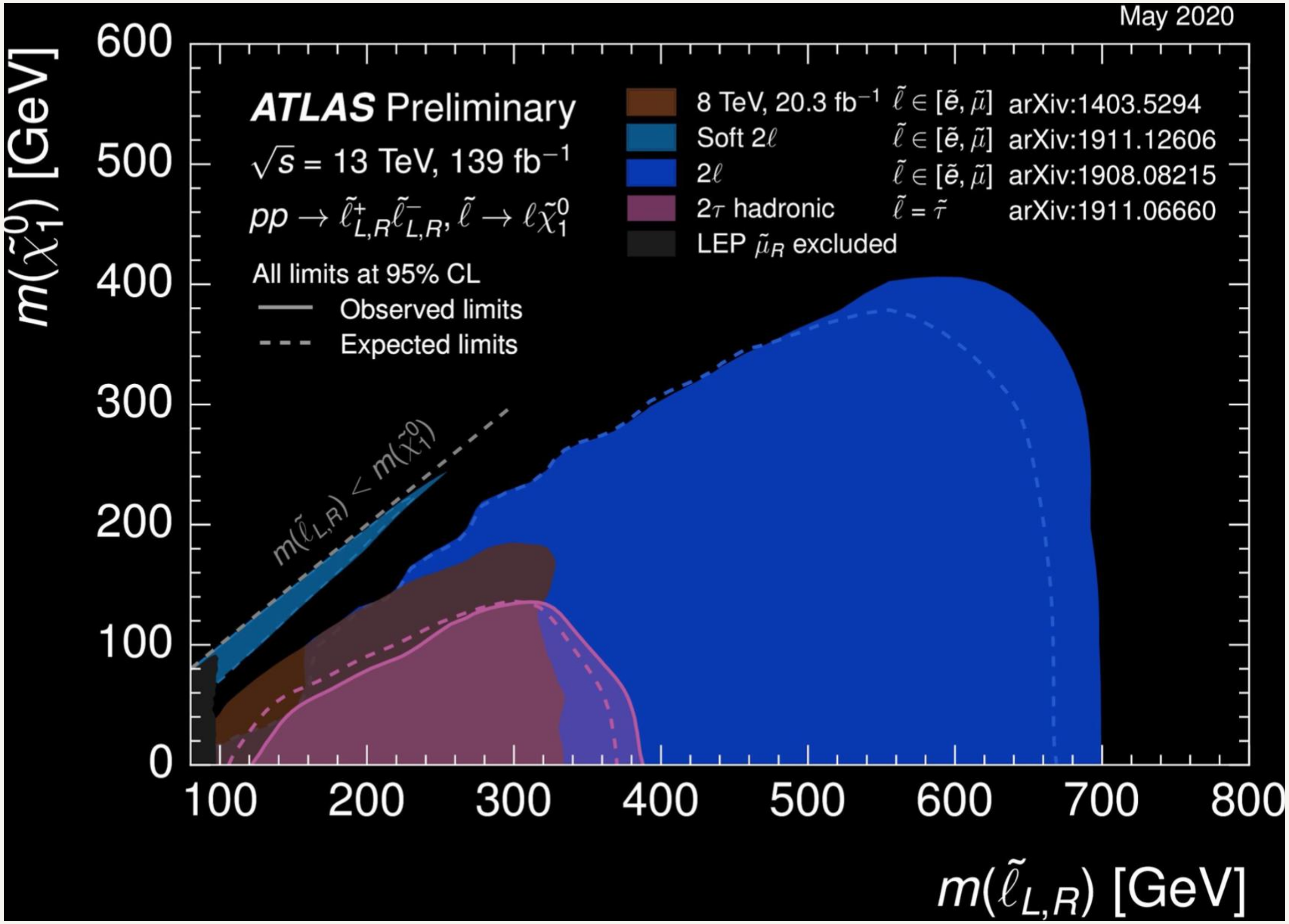
Stau searches: Bounds depend on mixing!





Weak limit at this point, we are now exploring limits beyond LEP.
Observe that this assumes degenerate stau masses.

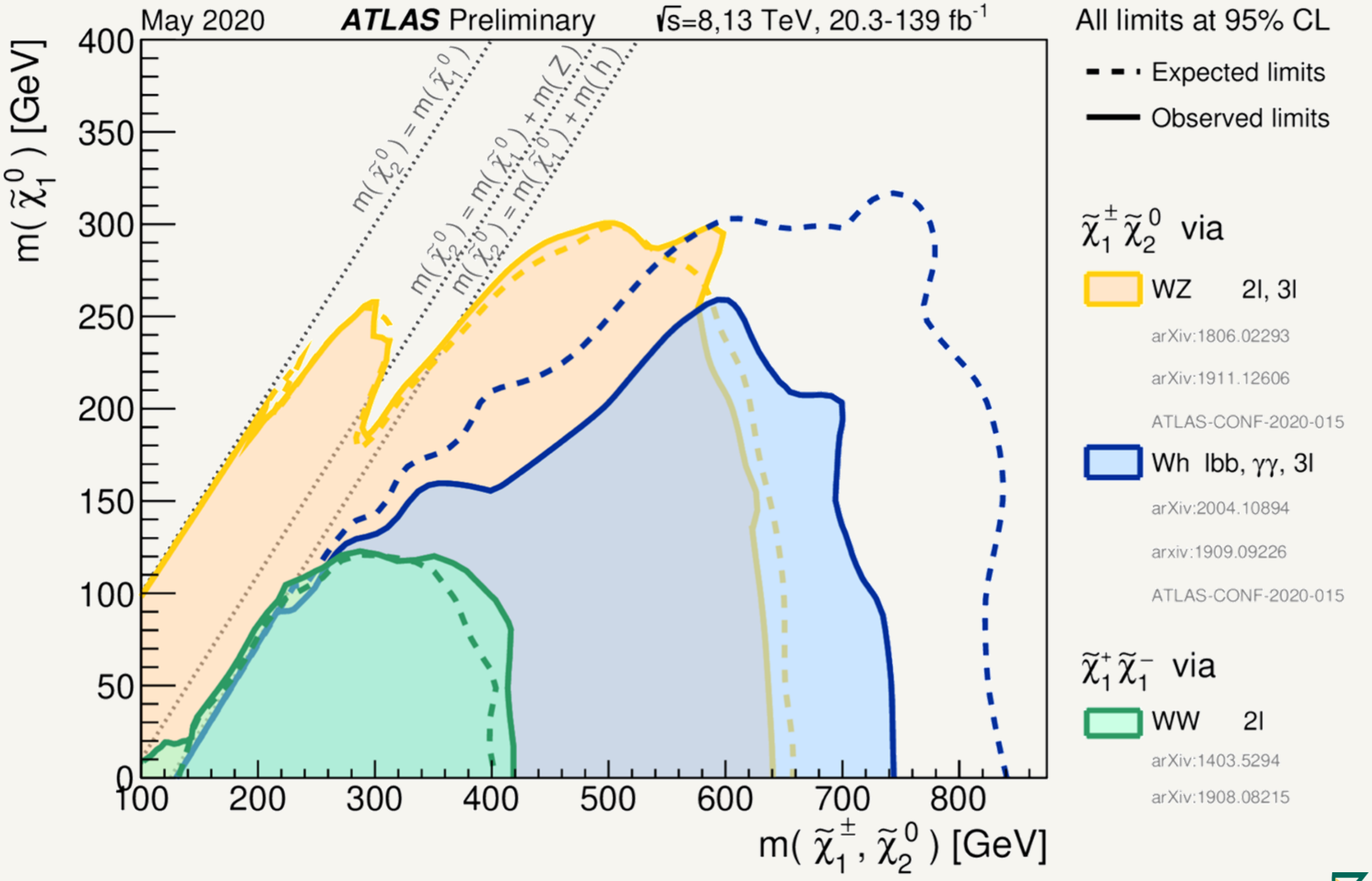
Slepton Searches



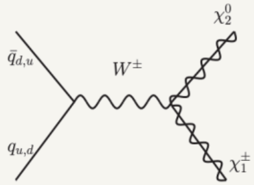
Assuming all leptons are degenerate, bound can be as large as 700 GeV. Bounds are significantly relaxed if mass difference between sleptons and neutralinos is smaller than ~100 GeV.

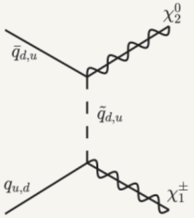
Current Electroweakino Mass Bounds

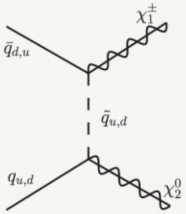
Wino NLSP, BR = 1



But also… The Squarks!



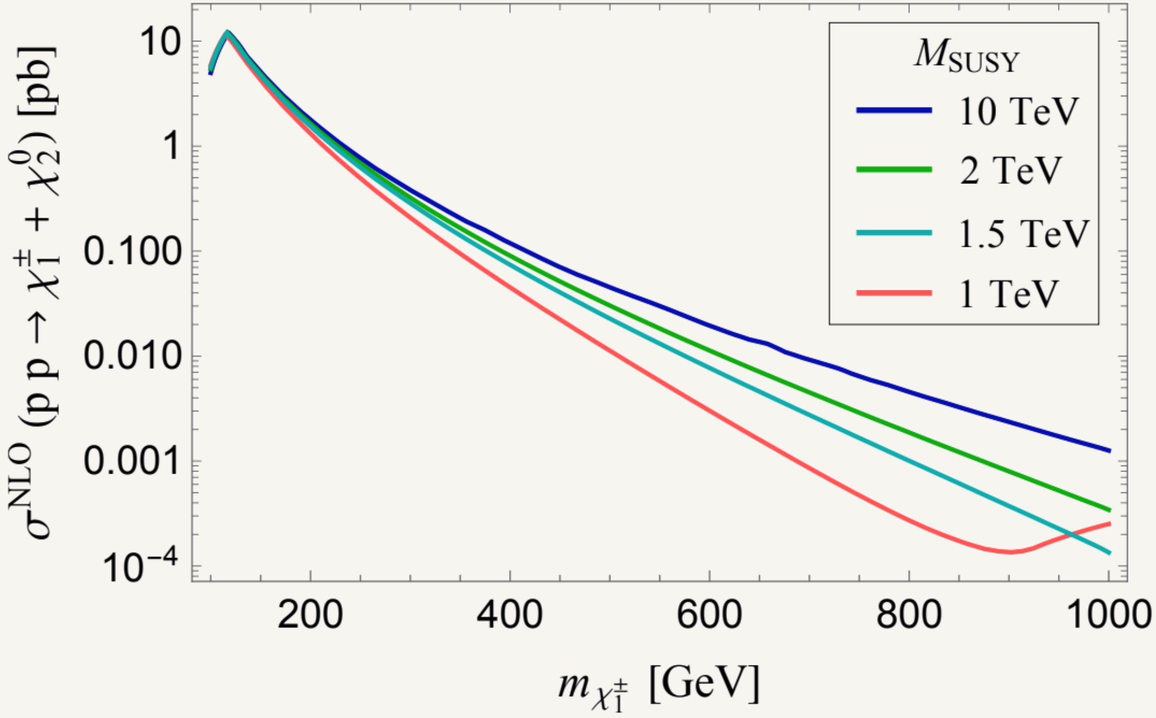




In evaluating these bounds the squarks have been decoupled.

But cross section depends on the squark masses due to *t* and *u* channel contribution to them

Liu, McGinnis, Wagner, Wang, arXiv:2008.11847



The resulting cross sections may differ by factors of a few!

So … ($g_\mu - 2$) wants

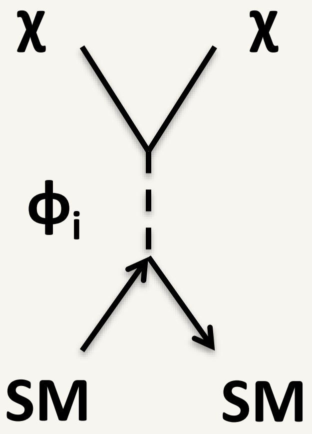
light neutralinos, charginos, and sleptons,

& so far LHC seems OK with it…

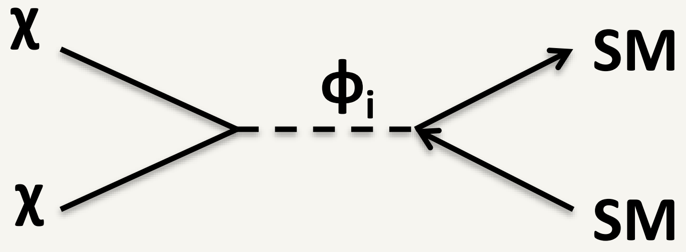
HOW ABOUT NEUTRALINO WIMPS FOR DARK MATTER?

SI DD + Ωh^2 ??

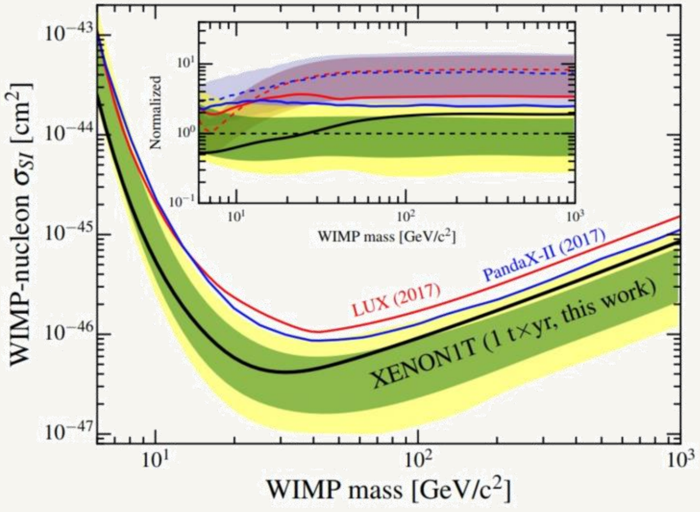
SI DD



Ωh^2



XENON1T arXiv:1805.12562



m_χ ~ few 100 GeV

Break the Connection!

Co-annihilation/resonance

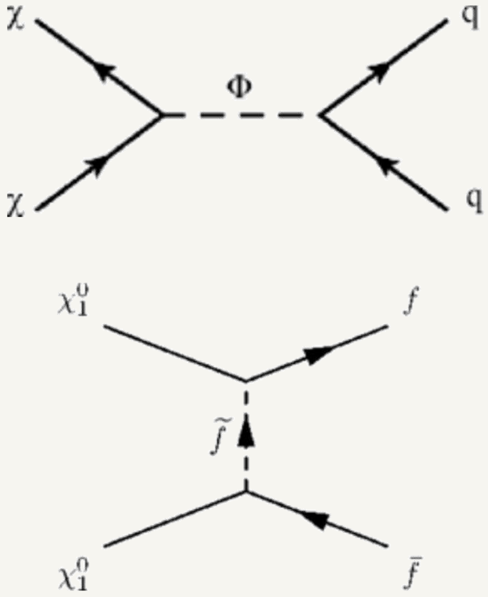
Multiple mediators for destructive interference

ALL consistent with (g_μ – 2)!!

Relic Density: Annihilations

Light Binos with mass close to M_1 excellent candidates for Dark Matter.

s-channel Resonance:
When the neutralino mass is close to a half of the mediating particle mass (eg: Higgs particle).
Highly constrained for the light Higgs.



NRS, Pierce, Freese'13

Direct search bounds on Heavy Higgs seriously limit resonant annihilation of light neutralinos consistent with ($g_\mu - 2$).

t-channel annihilation with light staus.
Happens also in a natural way at large tanβ.

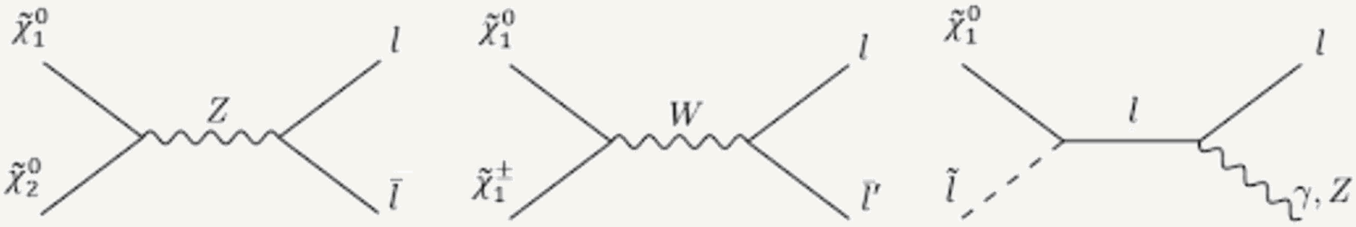
Relic Density: Co - Annihilations

Happen when the DM can annihilate against other rapidly annihilating particles.

Mass difference of DM with the other rapidly annihilating particles must be ~*O*(few 10s GeV).

Naturally leads to **COMPRESSED SPECTRUM** for searches in the missing energy channel.

Some relevant channels in the case of sleptons or Winos
(too light Higginos/small μ leads to large SD cross sections).



Direct Detection

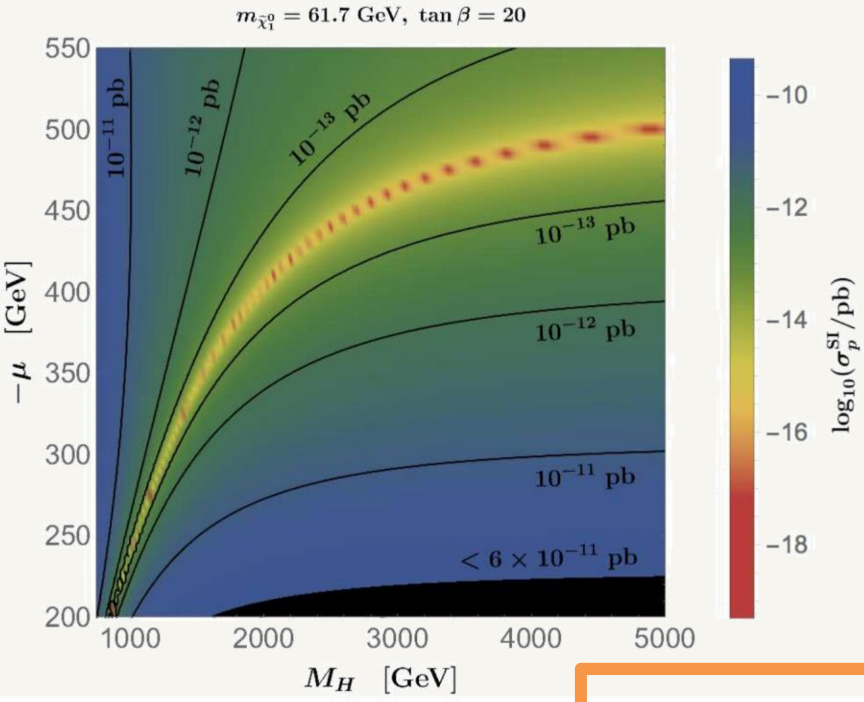
$$\sigma_p^{\rm SI} \propto \frac{m_Z^4}{\mu^4}\left[2(m_{\tilde{\chi}_1^0}+2\mu/\tan\beta)\frac{1}{m_h^2}+\mu\tan\beta\frac{1}{m_H^2}+(m_{\tilde{\chi}_1^0}+\mu\tan\beta/2)\frac{1}{m_{\tilde{Q}}^2}\right]^2$$

$$2\left(m_{\tilde{\chi}_1^0}+2\frac{\mu}{\tan\beta}\right)\frac{1}{m_h^2} \simeq -\mu\tan\beta\left(\frac{1}{m_H^2}+\frac{1}{2m_{\tilde{Q}}^2}\right)$$

$$\mu \times m_{\tilde{\chi}^0} < 0$$

$$m_{\tilde{\chi}^0} \simeq M_1$$

Cheung, Hall, Ruderman ‘12,
Huang, Wagner ‘14,
Cheung, Papucci, NRS, Stanford, Zurek ‘14,
Han, Liu, Makhapadhyay, Wang ‘18.



Carena, Osborne, NRS, Wagner, ‘18

Small SI DD can easily be obtained via **blind spots**.

Negative values of μ x M1 :
Much weaker spin-independent direct detection bounds

SD DD:
No cancellations!
Lower limit on **μ**

$$\sigma^{\rm SD} \propto \frac{m_Z^4}{\mu^4}\cos^2(2\beta)$$

$(g_\mu - 2)$ &

DARK MATTER DIRECT DETECTION

Baum, Carena, NRS, Wagner, arXiv:2104.03302

So, What Do We Have?

Small SI DD: ***negative* values of μ x M_1**.

SI DD also suppressed for ***large* values of μ**.

(g_μ -2) has two contributions:

Bino: proportional to **μ x M_1**

Chargino: proportional to **μ x M_2**

Bino contribution is ***negative*** in proximity of blind spot

but becomes ***subdominant*** at ***smaller*** values of μ.

Chargino contribution ***dominant*** if all masses of the same order

and ***suppressed*** at ***large*** μ .

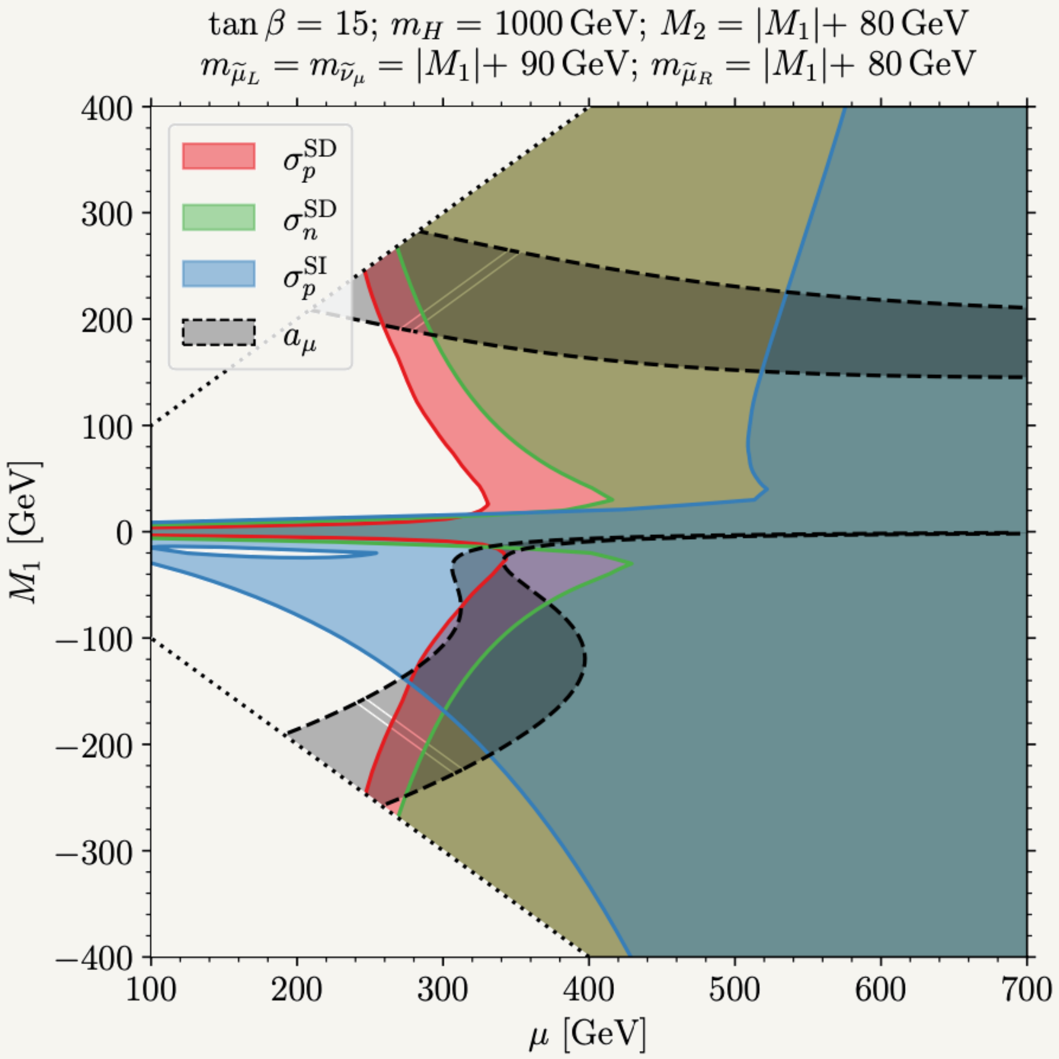
(g_μ - 2) needs to be **positive** => (g_μ - 2) + SI DD:

Large values of μ, or

Smaller values of μ and ***opposite sign*** for the gaugino masses.

Baum, Carena, NRS, Wagner, arXiv:2104.03302

A Qualitative Picture



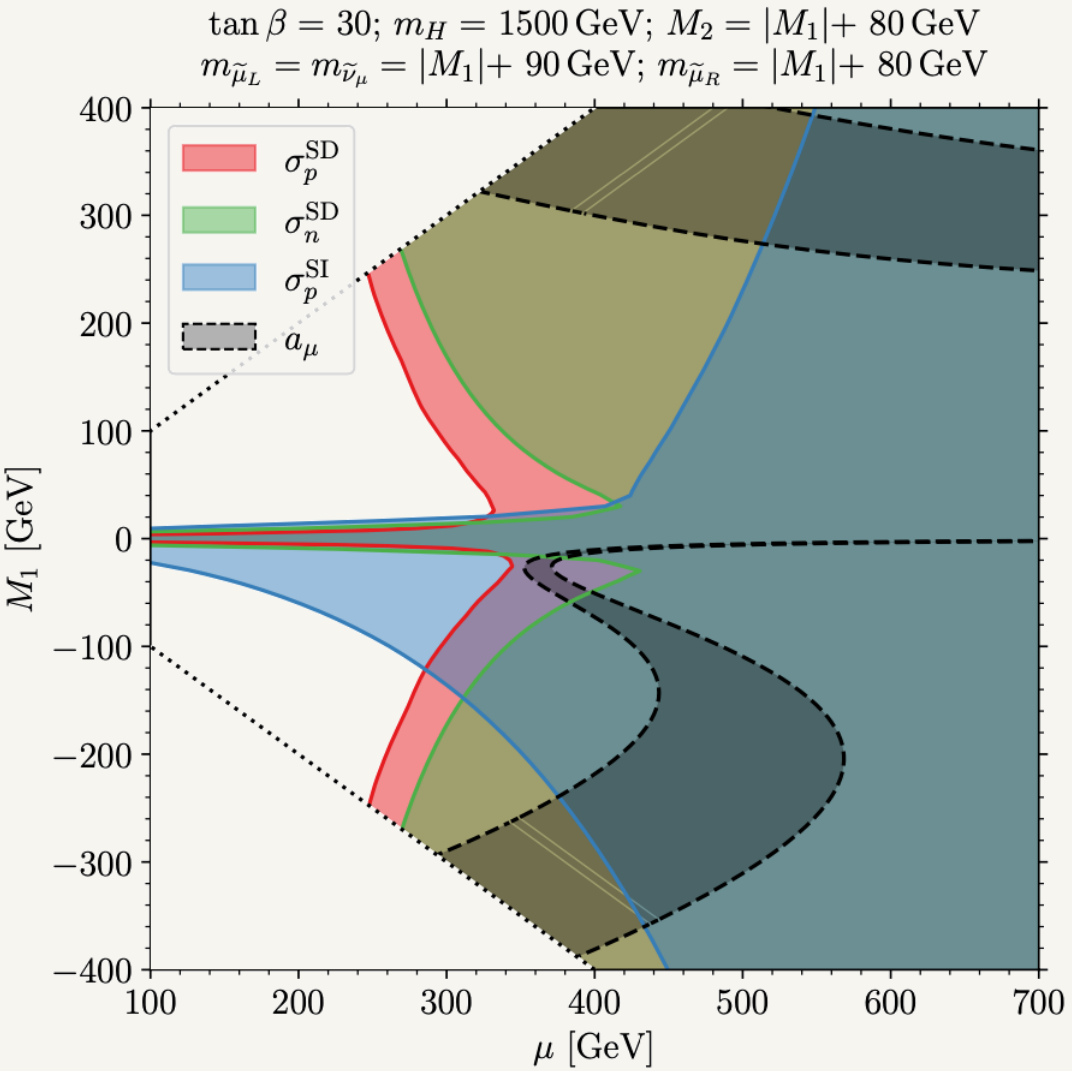
Shaded regions are allowed.

Large hierarchy of values of μ between + and - values of the Bino mass parameter is observed.

Compatibility of Direct Detection and (g_μ -2) Constraints for a representative example of a compressed spectrum. Stau co-annihilation is assumed

Baum, Carena, NRS, Wagner, arXiv:2104.03302

A Qualitative Picture



Small values of μ < 500 GeV may only be obtained for **negative** values of the Bino mass parameter.

Compatibility of Direct Detection and g_μ -2 Constraints for a representative example of a compressed spectrum. Stau co-annihilation is assumed

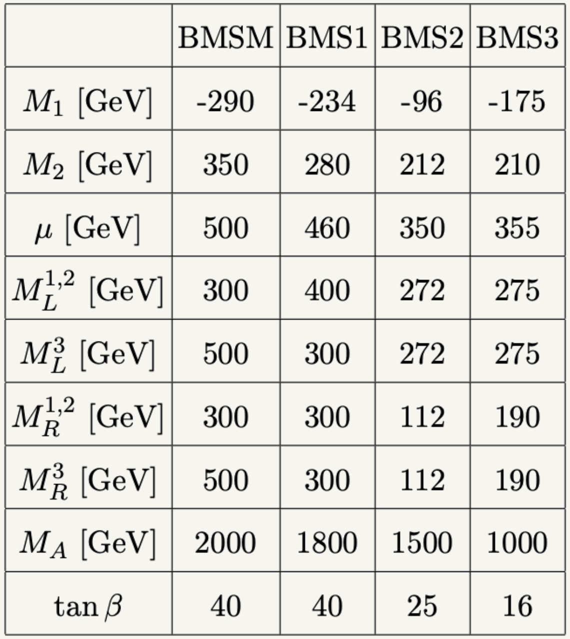
Benchmarks for **Negative** ($\mu \times M_1$)

BMSM :
Co-annihilation with a muon sneutrino,

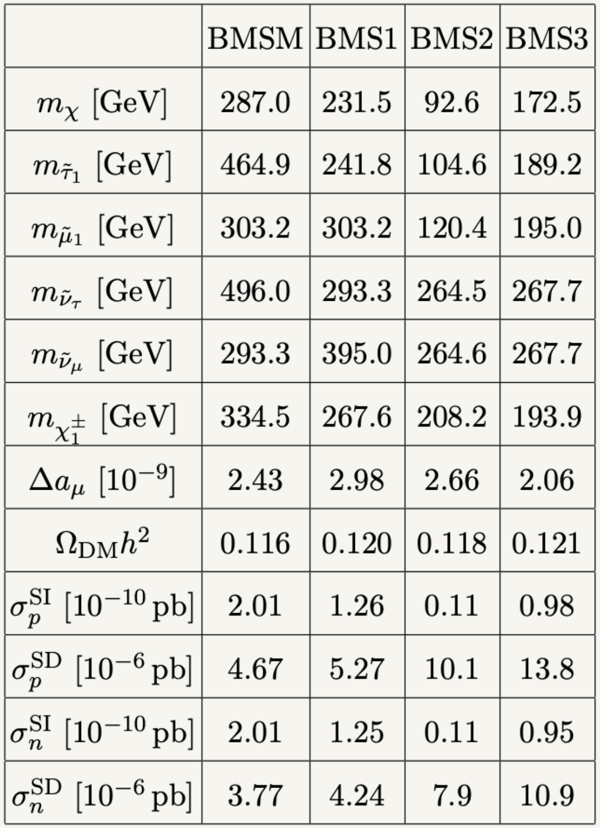
BMS1 :
Co- annihilation with a light stau,

BMS2 :
Annihilating via stau mediated *t*-channel

BMS3 :
Co-annihilation with staus and charginos.



	BMSM	BMS1	BMS2	BMS3
M_1 [GeV]	-290	-234	-96	-175
M_2 [GeV]	350	280	212	210
μ [GeV]	500	460	350	355
$M_L^{1,2}$ [GeV]	300	400	272	275
M_L^3 [GeV]	500	300	272	275
$M_R^{1,2}$ [GeV]	300	300	112	190
M_R^3 [GeV]	500	300	112	190
M_A [GeV]	2000	1800	1500	1000
$\tan\beta$	40	40	25	16



	BMSM	BMS1	BMS2	BMS3
m_χ [GeV]	287.0	231.5	92.6	172.5
$m_{\tilde{\tau}_1}$ [GeV]	464.9	241.8	104.6	189.2
$m_{\tilde{\mu}_1}$ [GeV]	303.2	303.2	120.4	195.0
$m_{\tilde{\nu}_\tau}$ [GeV]	496.0	293.3	264.5	267.7
$m_{\tilde{\nu}_\mu}$ [GeV]	293.3	395.0	264.6	267.7
$m_{\chi_1^\pm}$ [GeV]	334.5	267.6	208.2	193.9
Δa_μ $[10^{-9}]$	2.43	2.98	2.66	2.06
$\Omega_{\rm DM}h^2$	0.116	0.120	0.118	0.121
$\sigma_p^{\rm SI}$ $[10^{-10}\,{\rm pb}]$	2.01	1.26	0.11	0.98
$\sigma_p^{\rm SD}$ $[10^{-6}\,{\rm pb}]$	4.67	5.27	10.1	13.8
$\sigma_n^{\rm SI}$ $[10^{-10}\,{\rm pb}]$	2.01	1.25	0.11	0.95
$\sigma_n^{\rm SD}$ $[10^{-6}\,{\rm pb}]$	3.77	4.24	7.9	10.9

Benchmarks for Negative ($\mu \times M_1$) & Resonance

BMW :

Co-annihilation with a Wino.

BMH :

s-channel resonant annihilation via the SM-like Higgs boson.

	BMW	BMH
M_1 [GeV]	-227	63
M_2 [GeV]	260	700
μ [GeV]	450	470
$M_L^{1,2}$ [GeV]	332	720
M_L^3 [GeV]	332	720
$M_R^{1,2}$ [GeV]	330	720
M_R^3 [GeV]	330	725
M_A [GeV]	1500	3000
$\tan\beta$	25	65

	BMW	BMH
m_χ (m_h) [GeV]	224.6	61.0 (124.9)
$m_{\tilde{\tau}_1}$ [GeV]	303.3	680.1
$m_{\tilde{\mu}_1}$ [GeV]	332.9	721.4
$m_{\tilde{\nu}_\tau}$ [GeV]	325.9	717.2
$m_{\tilde{\nu}_\mu}$ [GeV]	325.9	717.2
$m_{\chi_1^\pm}$ [GeV]	247.9	469.6
Δa_μ $[10^{-9}]$	2.13	1.99
$\Omega_{\rm DM} h^2$	0.117	0.121
$\sigma_p^{\rm SI}$ $[10^{-10}\,{\rm pb}]$	1.20	0.31
$\sigma_p^{\rm SD}$ $[10^{-6}\,{\rm pb}]$	5.7	3.0
$\sigma_n^{\rm SI}$ $[10^{-10}\,{\rm pb}]$	1.19	0.33
$\sigma_n^{\rm SD}$ $[10^{-6}\,{\rm pb}]$	4.6	2.3

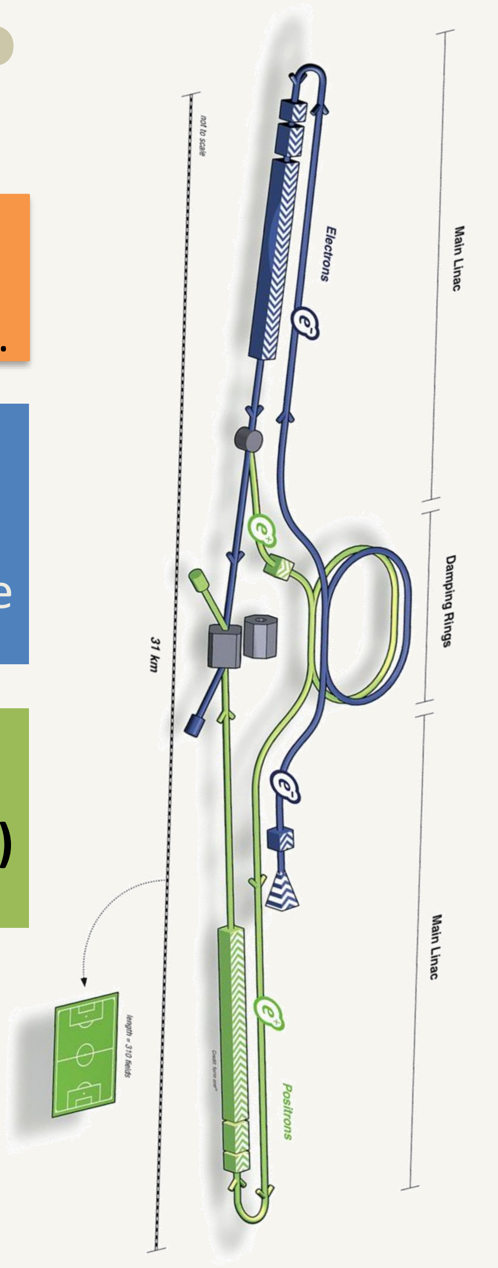
ILC?

High-Lumi LHC: Most gain in such spectra
-- statistics rather than energy limited.
But bounds on Higgsinos will remain weak.

ILC:

e+e- machine: can DIRECTLY probe weak physics!
IF kinematically accessible

Compressed Spectra NOT a bottle neck!
(See S. Heinemeyer talk **)**



Thank You!

SM WORKS TOO WELL!

Fermilab (g_μ - 2) confirms previous Brookhaven result!
Other Flavor Anomalies may also be present… Need confirmation

May be hint of New Physics

What are the right questions?

***Natural* SUSY:**

Light Higgsinos, Neutralinos, Charginos & Sleptons
With relative signs, can accommodate (g_μ - 2) & Dark Matter!

Data + Theory: Where to look next!

ILC:

Could probe relevant mass spectra – IF enough energy!

~~“May we live in interesting times.”~~

BACK UP SLIDES

s-channel resonant Annihilation with heavy Higgs

- Bounds on the heavy Higgs boson masses, seriously restrict this possibility at low values of μ.

- Indeed, assuming the Bino mass to be half of the heavy Higgs boson masses, and all the sparticle masses to be close to it to maximize the contribution to g-2, from

$$m_H \geq 250 \text{ GeV} \times \sqrt{\tan\beta} \sim 2\, m_h \sqrt{\tan\beta}$$

$$\Delta a_\mu \simeq 1.3 \times 10^{-9} \tan\beta \times \left(\frac{100 \text{ GeV}}{\widetilde{m}}\right)^2 ,$$

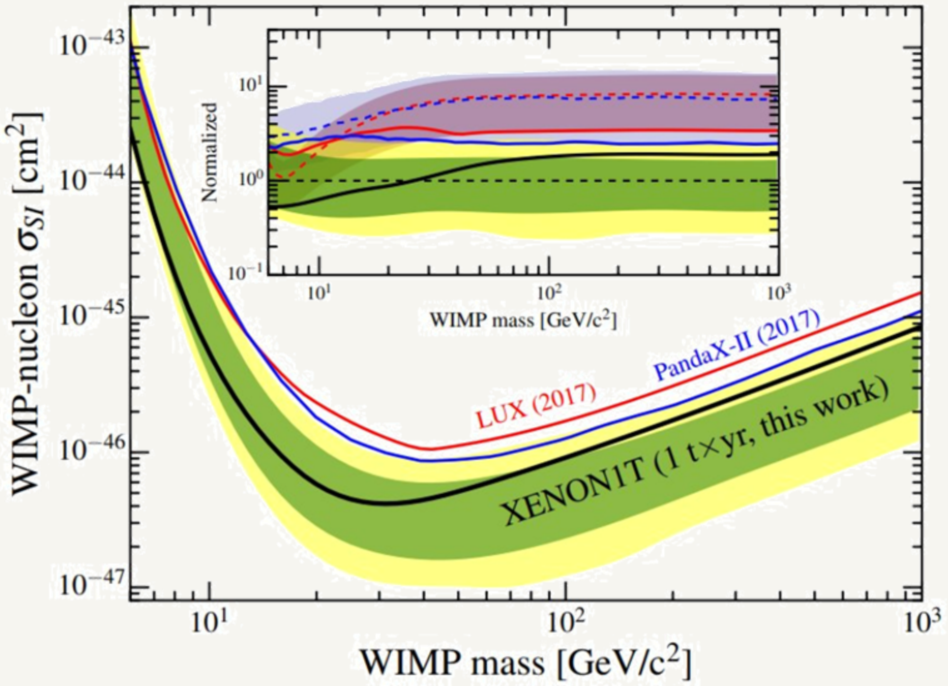
- We obtain

$$\Delta a_\mu \simeq 10^{-9} \tan\beta \frac{4}{m_H^2} (100 \text{ GeV})^2 \lesssim 7 \times 10^{-10},$$

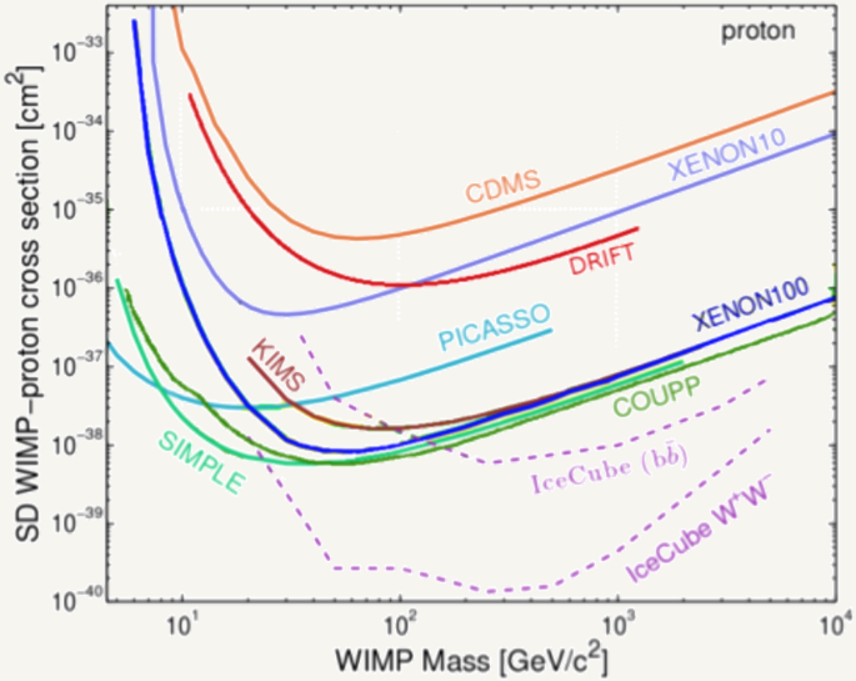
Current Bounds from Direct Dark Matter Detection

Current Limits

$$1\ \mathrm{pb} = 10^{-36}\ \mathrm{cm}^2, \qquad 1\ \mathrm{zb} = 10^{-45}\ \mathrm{cm}^2$$



Spin Independent Interactions



Spin Dependent Interactions

$$\Delta a_\mu \equiv (a_\mu^{\rm exp} - a_\mu^{\rm SM}) = (251 \pm 59) \times 10^{-11}$$

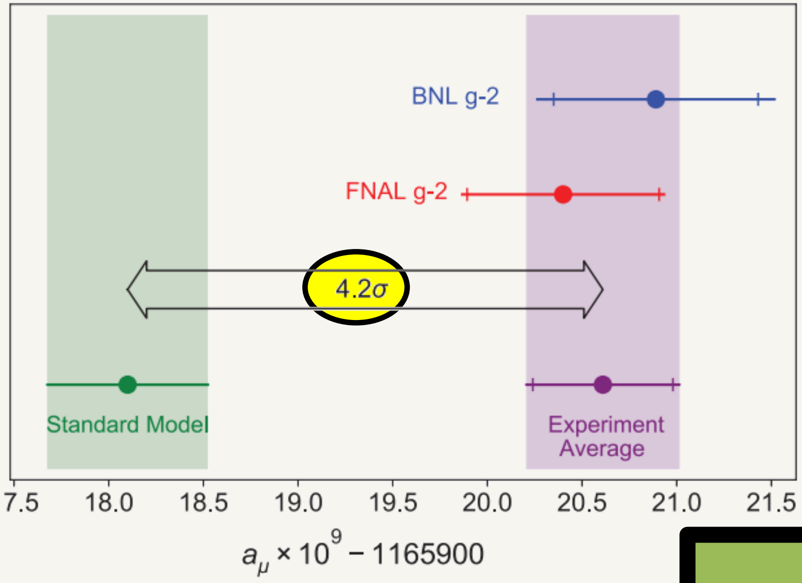
Flavor Anomalies!!

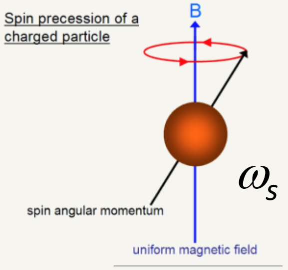
Anomalous magnetic moment of the muon

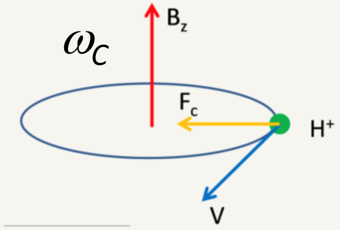
Fermilab Muon g-2

$(g_\mu - 2)/2 = 0.00116592061(41)$

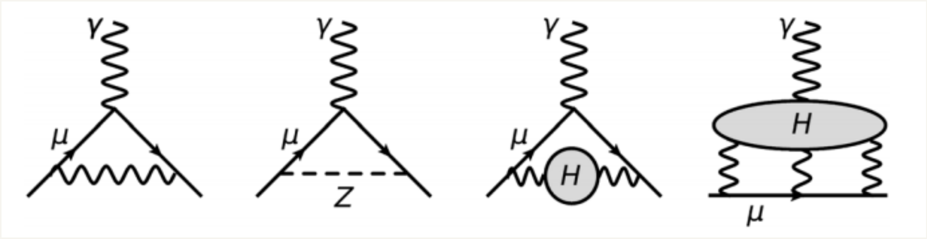
FNAL Muon g-2, PRL Apr 2021.







$$\omega_a = \omega_C - \omega_s = \left(\frac{g-2}{2}\right)\frac{qB}{m}$$



MUONS??

Hadronic Vacuum Polarization Contribution – BMW Lattice calculation: Is there in fact tension with the SM??

The Lattice results should be taking seriously. However, we should clearly wait for other lattice groups to corroborate the BMW result.

- HVP effects would impact the variation of the fine structure constant, affecting precision measurements at M_Z. Any correction from the current values should be limited to energies below 0.9 GeV, as is also confirmed by the BMW study.

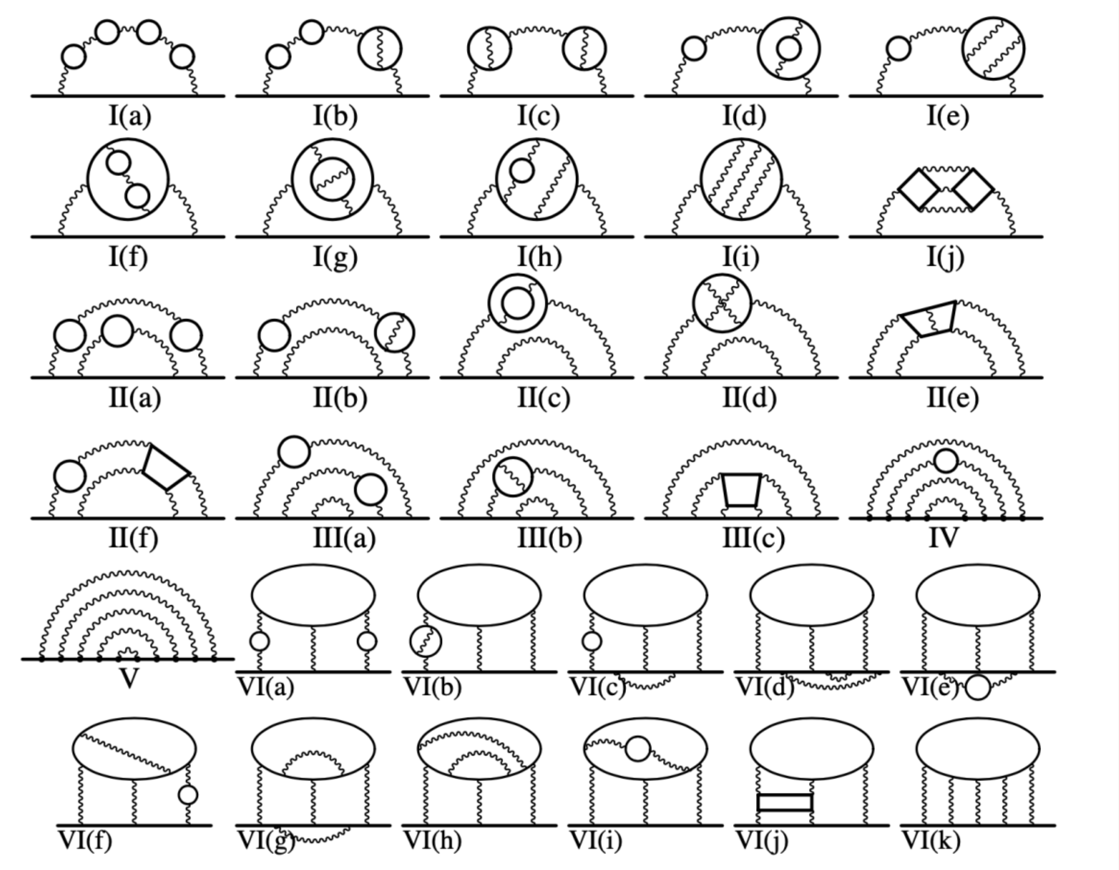
Crivellin et al, 2003.04886; Kezhavarzi, Marciano, Pasera, Sirlin, arXiv: 2006.12666

- Tension with low energy data could be resolved by large systematic errors in the cross section evaluations or by new physics contributing to them. Both possibilities look unlikely, but certainly not impossible.

- May also be resolved by some unaccounted systematic error in the lattice evaluations. BMW provides a detailed account of their error estimates and will be double checked by other lattice groups (expected in the next year or so).

5-Loop QED Contributions!

Electrons??



The QED corrections to (g-2) of the electron are known up to 5-loops.

The agreement between theory and experiments is one of the greatest triumphs of science and of the SM!

$$\Delta a_e \equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = (-88 \pm 36) \times 10^{-14}$$

Parker et al, 1812.04130