



Introduction of ILC-TPC

Jurina Nakajima(SOKENDAI)
belonging to LC-TPC Asian group



S O K E N D A I

Contents

1. Principle of TPC

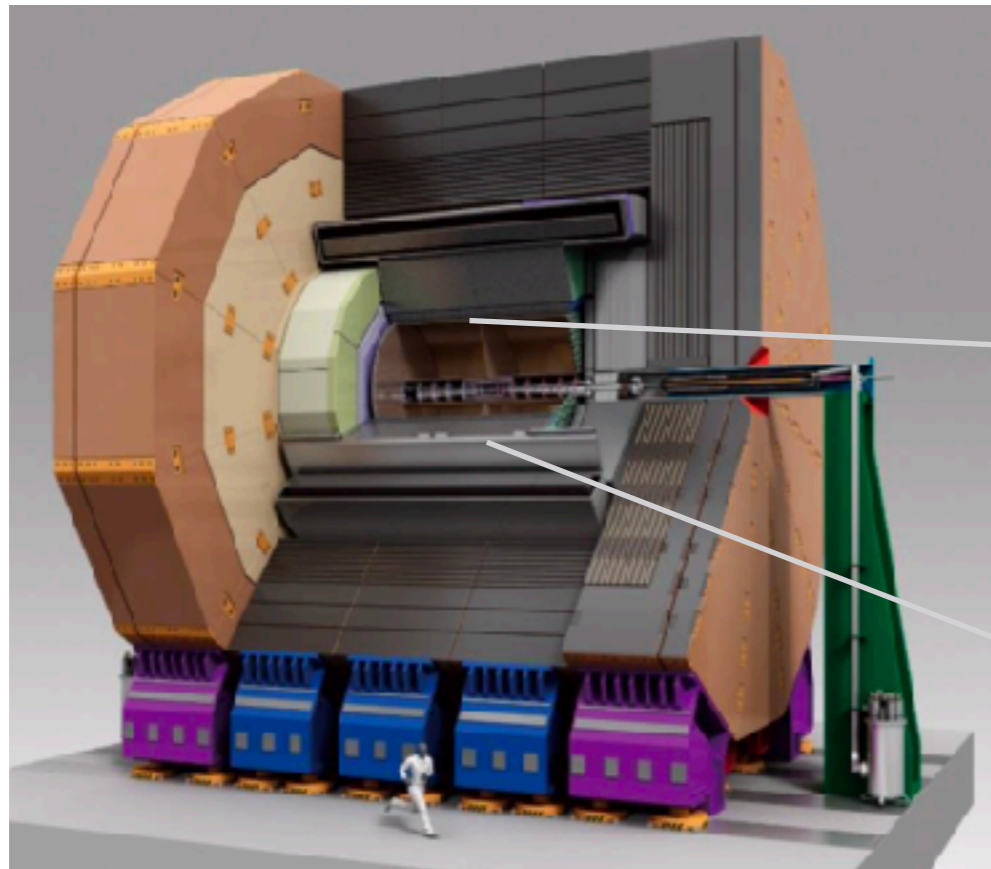
- What is TPC?
- What do we need to know at TPC?
- Momentum resolution
- Position resolution
- Gas

2. Readout module of TPC

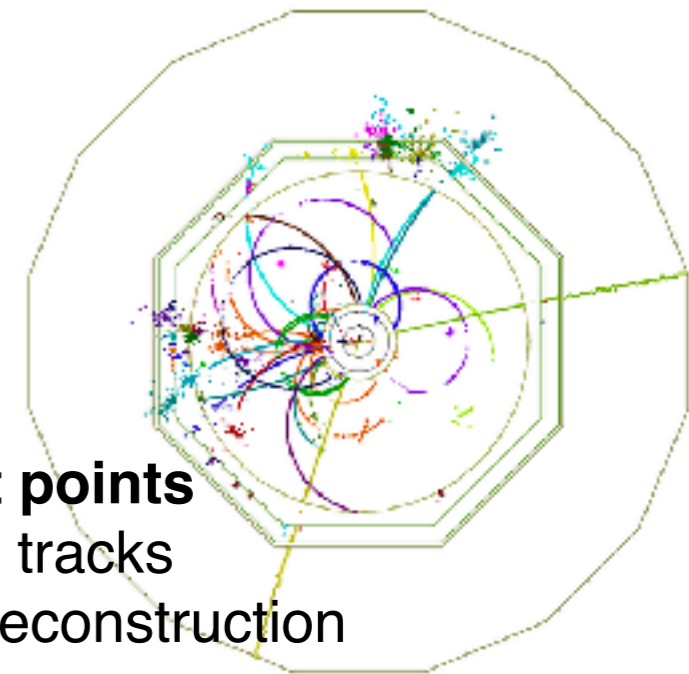
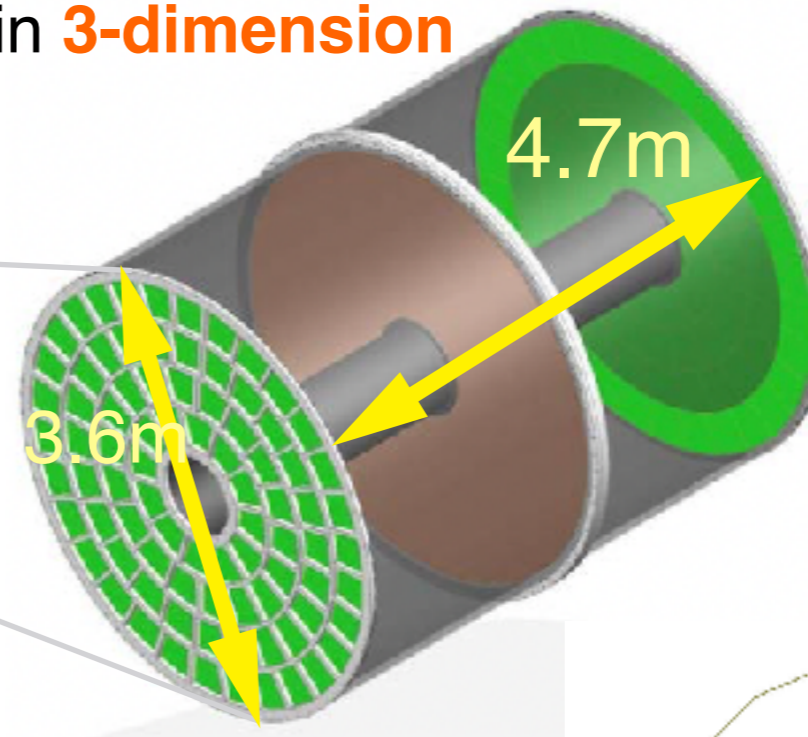
- Introduction of various modules
- New technology
- Ion backflow

3. Summary

Time Projection Chamber (TPC)



Reconstruct tracks of **charged** particles
in **3-dimension**



~200 hit points
Can see tracks
before reconstruction

What we want to know

“Two 4-vectors”

$$\mathbf{p}^\mu = (\mathbf{E}/c, \mathbf{p})$$

$$\mathbf{x}^\mu = (ct, \mathbf{x})$$

+ **Charge**

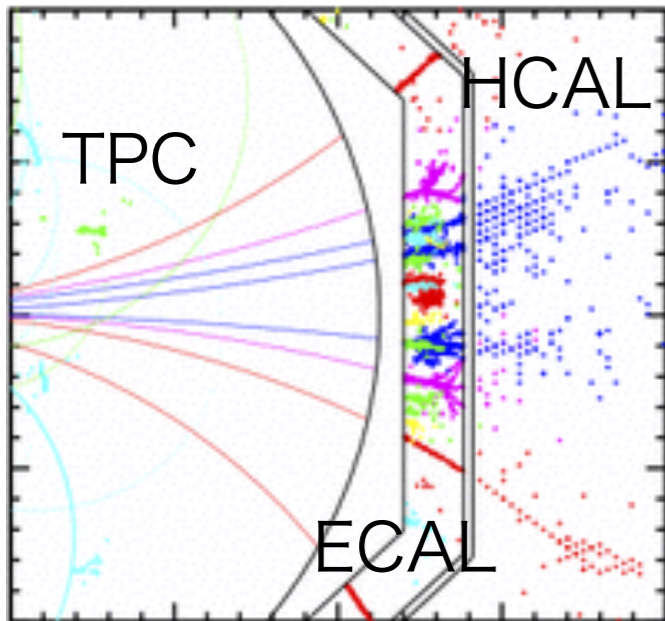
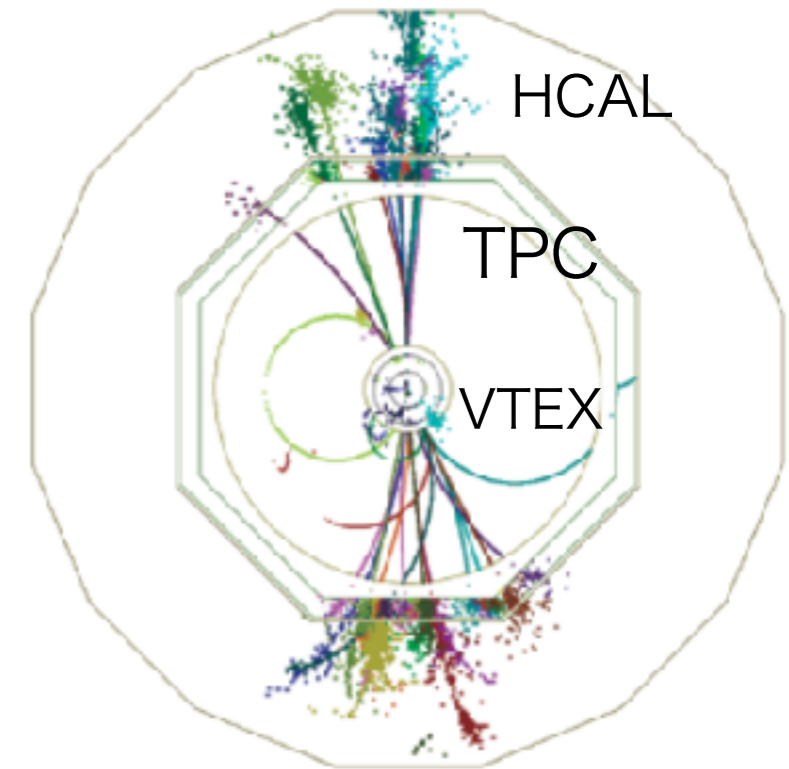
for every particle produced

What can we know at TPC?

1. Momentum measurement ← Today's topic

Measure the **curvature radius** of the tracks in **B=3.5T**

$$\frac{\sigma_{p_{\perp}}}{p_{\perp}} \simeq 1 \times 10^{-4} p_{\perp} [\text{GeV}/c] \quad (\text{TPC only})$$



<https://www.ilcild.org>

2.2-track separation

Make a 1:1 correspondence between track and Calorimeter

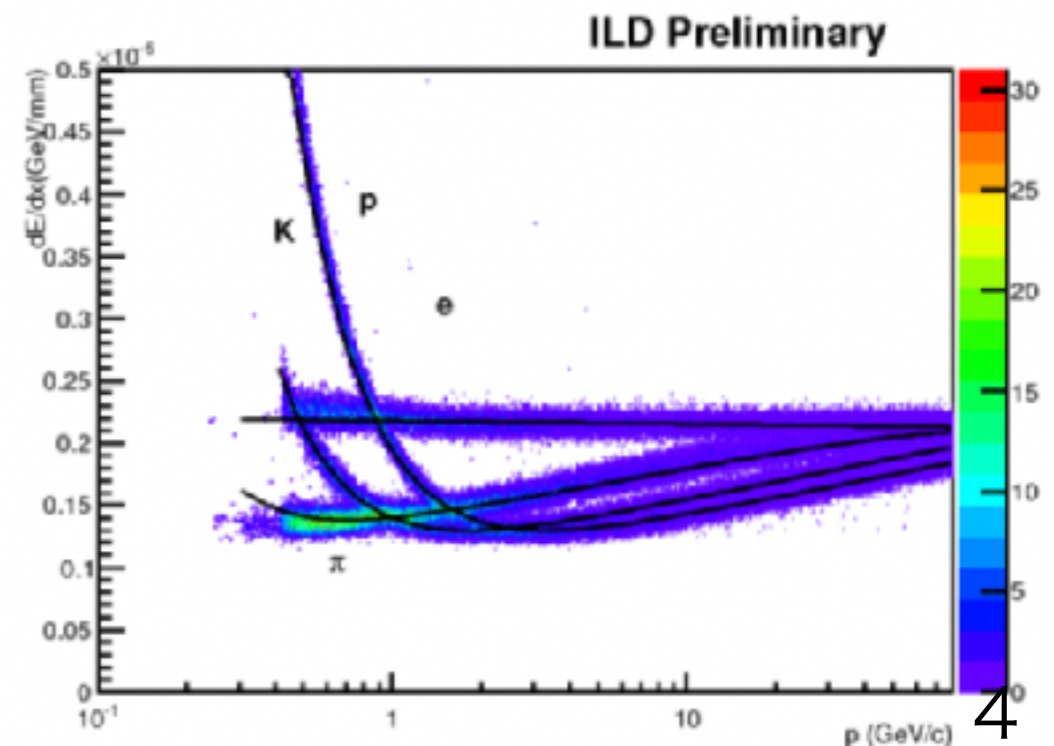
2hit resolution in $r\phi \simeq 2 \text{ mm}$

2hit resolution in $z \simeq 6 \text{ mm}$

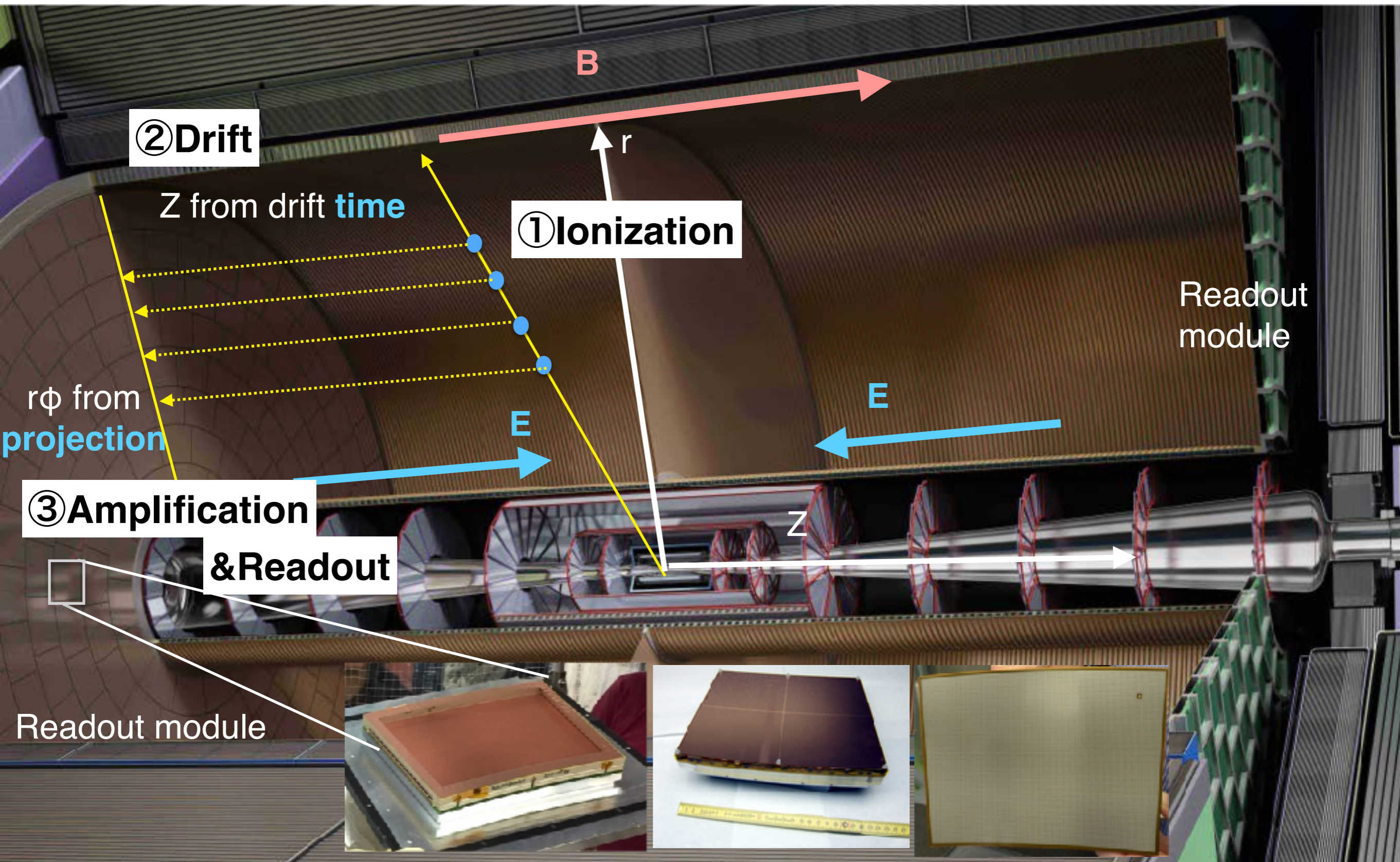
3. Particle ID

Measure the **dE/dx (loss energy of particles)**

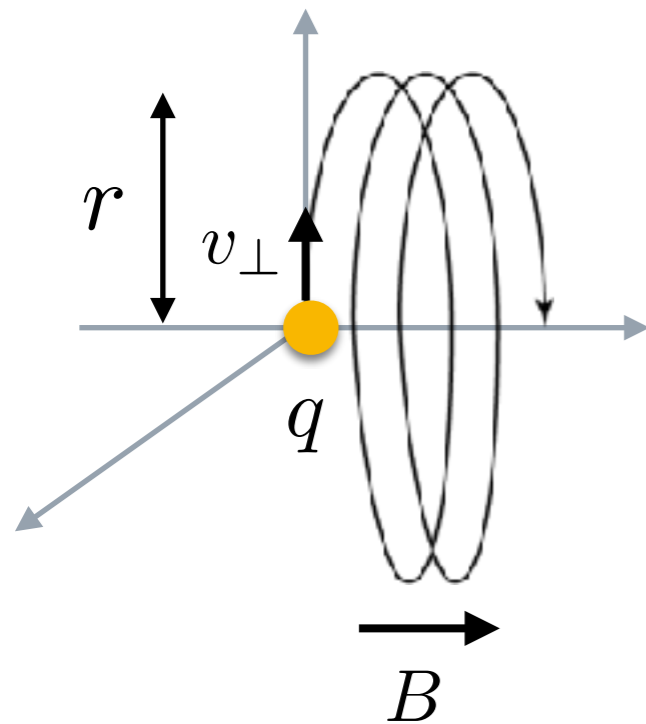
← dE/dx is different for each particle.



Fundamental principle of TPC



Momentum resolution



A charged particle follows a **helix** in uniform B-field

Radius: r

Transverse momentum: p_{\perp}

Velocity: v_{\perp}

Charge: q

$$F = qv_{\perp}B, \quad F = mv_{\perp}^2/r$$

$$\rightarrow p_{\perp} [\text{GeV}/c] = 0.3 \cdot B [\text{T}] \cdot \underline{r [\text{m}]}$$

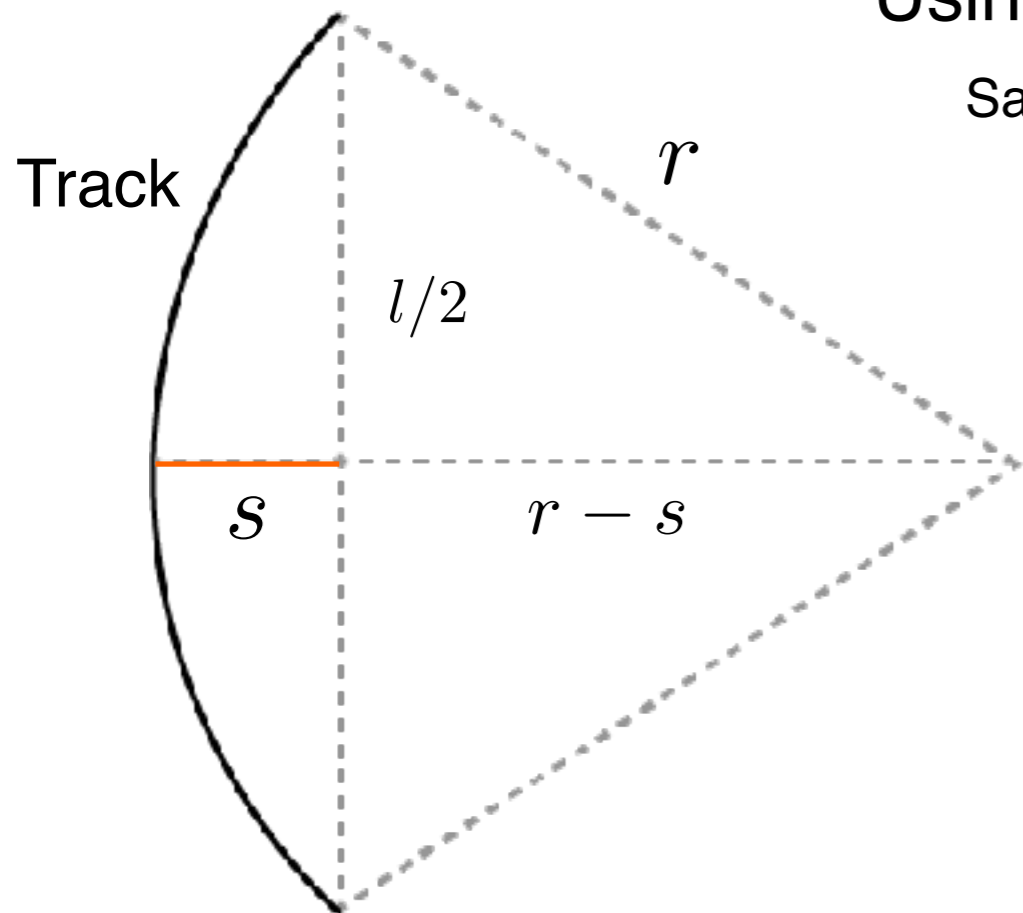
Using **sagitta** to calculate curvature radius of track

Sagitta: S curvature radius of track: r Arm length: l

$$r^2 = (l/2)^2 + (r - s)^2 \quad (s \ll r)$$

$$\rightarrow 2rs = (l/2)^2 + s^2 \simeq l^2/4$$

$$\rightarrow r \simeq l^2/(8s)$$



$$\kappa := \frac{1}{p_{\perp}} \simeq \left(\frac{8\alpha}{Bl^2} \right) s \rightarrow \sigma_{\kappa}^{meas} := \frac{\sigma_{p_{\perp}}}{p_{\perp}^2} \simeq \left(\frac{8\alpha}{Bl^2} \right) \sigma_s$$

Ex)Q. $p = 1\text{GeV}$, $B = 3\text{T}$? A. $r = \text{about } 1\text{m}$

Momentum resolution

Gluckstern Formula

$$\frac{\sigma_{p_{\perp}}}{p_{\perp}} = \sqrt{\underbrace{(\sigma_{\kappa}^{meas})^2}_{\text{Detector resolution}} + \underbrace{(\sigma_{\kappa}^{MS})^2}_{\text{Multiple scattering}}}$$

$$= \sqrt{\underbrace{\left(\frac{\alpha\sigma_x}{Bl^2}\right)^2 \left(\frac{720}{N+4}\right) p_{\perp}^2}_{\text{Detector resolution}} + \underbrace{\left(\frac{\alpha C}{Bl}\right)^2 \frac{10}{7} \left(\frac{X}{X_0}\right)^2}_{\text{Multiple scattering}}}$$

Transversmomentum: p_{\perp}

Spatial resolution in the r- ϕ plane per point: σ_x

The number of sampling points: N

Thickness measured in radiation length units: X/X_0

Leaver arm length: l Magnetic field: B

Const.: α, C

R. L. Glueckstern, NIM 24 (1963) 381

ILD detector requires overall

$$\frac{\sigma_{p_{\perp}}}{p_{\perp}} \simeq 2 \times 10^{-5} p_{\perp} [\text{GeV}/c]$$

TPC is required

$$\frac{\sigma_{p_{\perp}}}{p_{\perp}} \simeq 1 \times 10^{-4} p_{\perp} [\text{GeV}/c]$$



There are about **200** points of $\sigma_x \simeq 100[\mu\text{m}]$

in **B = 3.5 T** and drift length = **2.2 m**

Position resolution

$$\sigma_x = \sqrt{\sigma_0^2 + \frac{C_d^2 \cdot z}{N_{eff}}}$$

drift length: z

Transverse diffusion: C_d

The effective number of electrons: N_{eff}

Transverse diffusion constant C_d

$$C_d(B, E) = \frac{1}{\sqrt{1 + \omega^2 \tau^2}} C_d(0, E)$$

Cyclotron frequency: ω

Mean free time: τ

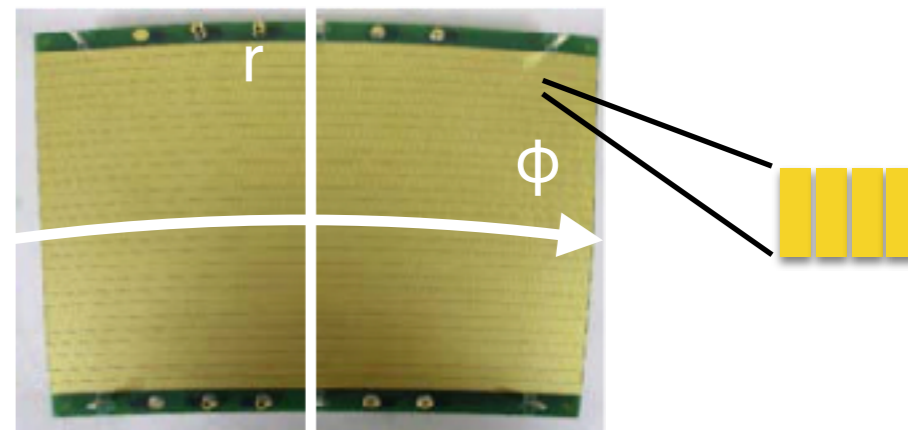
Electrons are affected by diffusion → position resolution is worse

 **A large $\omega\tau$ is needed to reduce transverse diffusion**

※ In TPC, Lorentz force suppresses transverse diffusion of drift electrons by curling them around the magnetic field

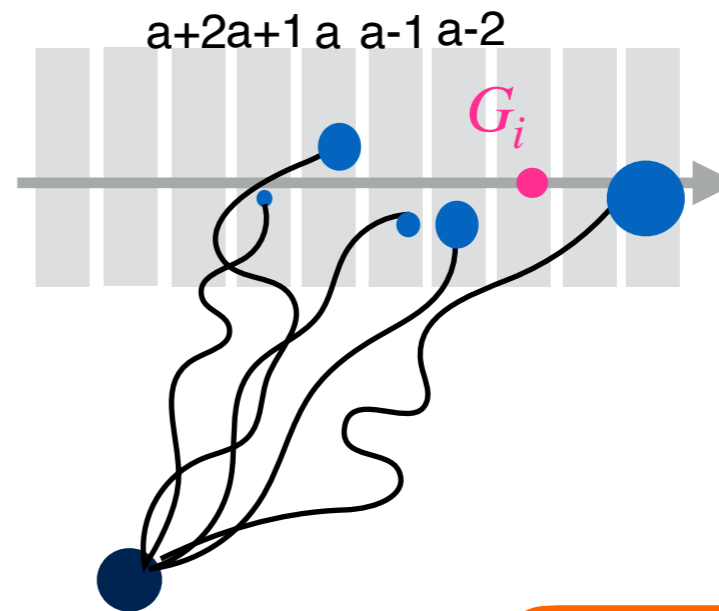
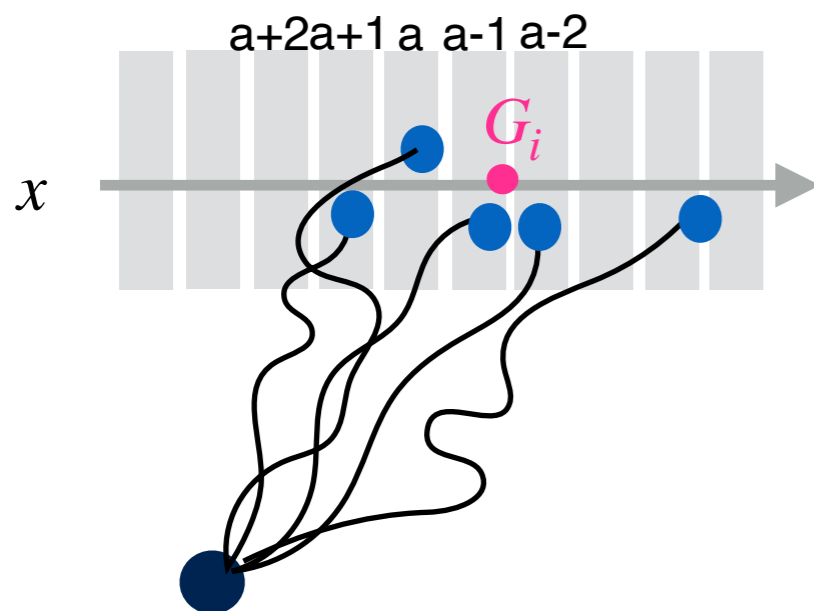
Effect of gas gain fluctuation

Effective number of electron N_{eff}



W/o gas gain fluctuation

W/ gas gain fluctuation



$$N_{eff} := \frac{1}{\langle 1/N \rangle} < \langle N \rangle$$

electrons

$$N_{eff} \simeq \left[\left\langle \frac{1}{\sum_i k_i} \right\rangle_{N,k} \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle_G \right]^{-1}$$

electrons gas gain fluctuation

Ex) 4 GeV pion, pad pitch 6mm, pure Ar

$$N_{eff} := \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle \right]^{-1} = 21 < \frac{1}{\langle 1/N \rangle} = 36 < \langle N \rangle = 71$$

We need to select “good” gas

What is “good” gas?

- Sufficient electrons
- Suppress diffusion in high B-field region

Gas mixture (T2K gas)

$$\text{Ar} : \underline{\text{CF}_4} : \text{Iso-C}_4\text{H}_{10} = 95 : 3 : 2 [\%]$$

“Quencher”

CF_4 “Quencher”

Ar - CF4 mixture has large τ

→ fast drift velocity as compared typical chamber gas

→ **Large $\omega\tau$** ex) 7.5 cm/ μs (230V/cm)

$$C_d(B, E) = \frac{1}{\sqrt{1 + \omega^2\tau^2}} C_d(0, E)$$

$\text{Iso-C}_4\text{H}_{10}$ “Quencher”

+ “Penning effect”

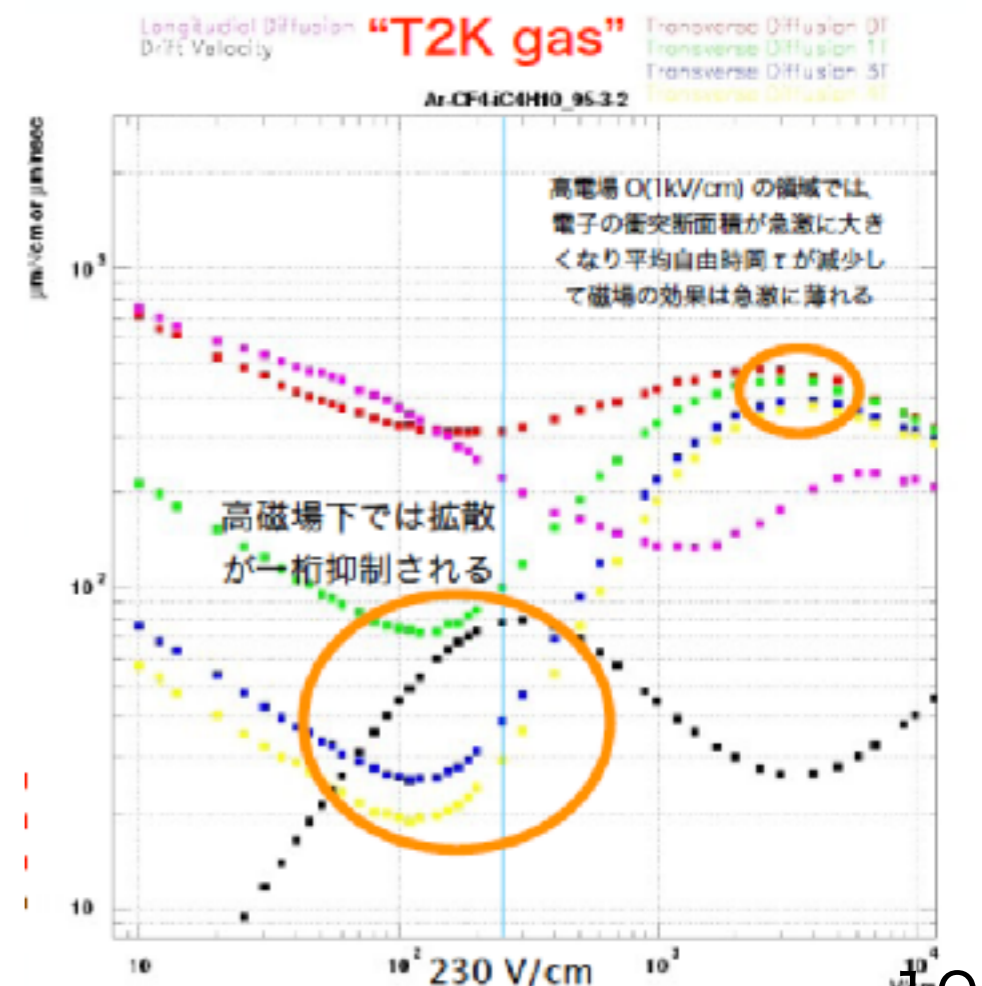
Ionization potential

$$\text{Iso-C}_4\text{H}_{10} < \text{Ar}$$



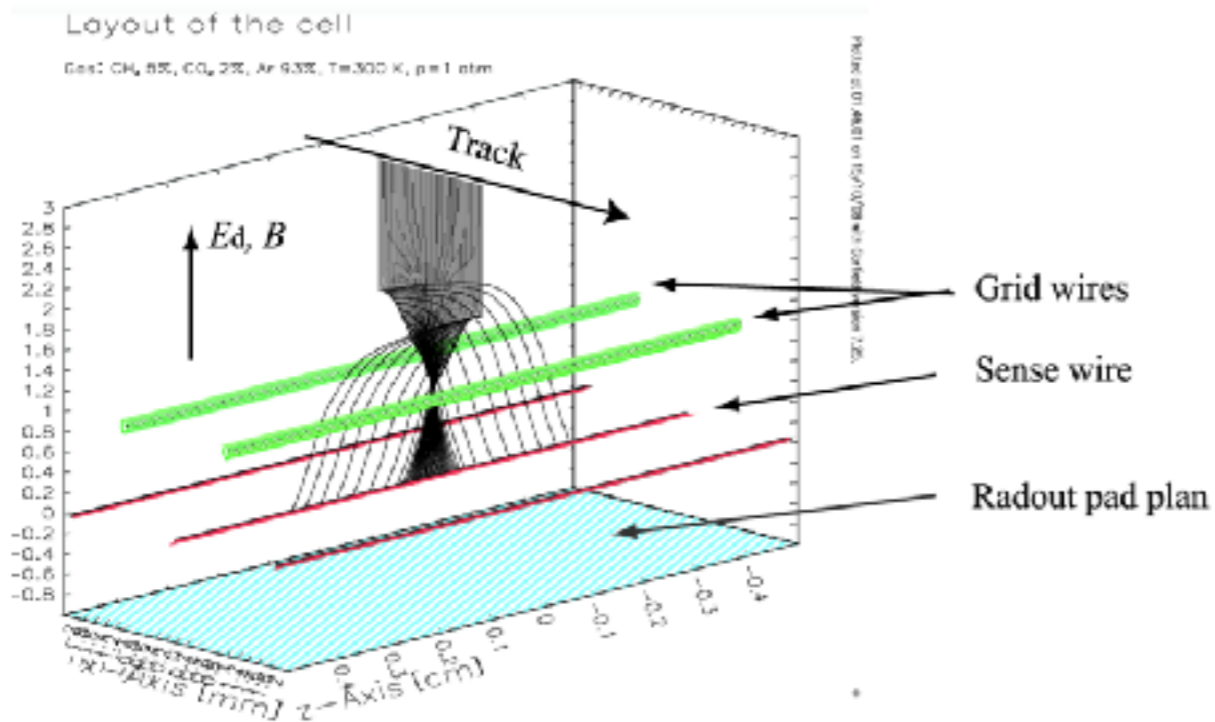
meta-stable Ar

Additional e



Readout Module

Multi-Wire Proportional Chamber (MWPC)

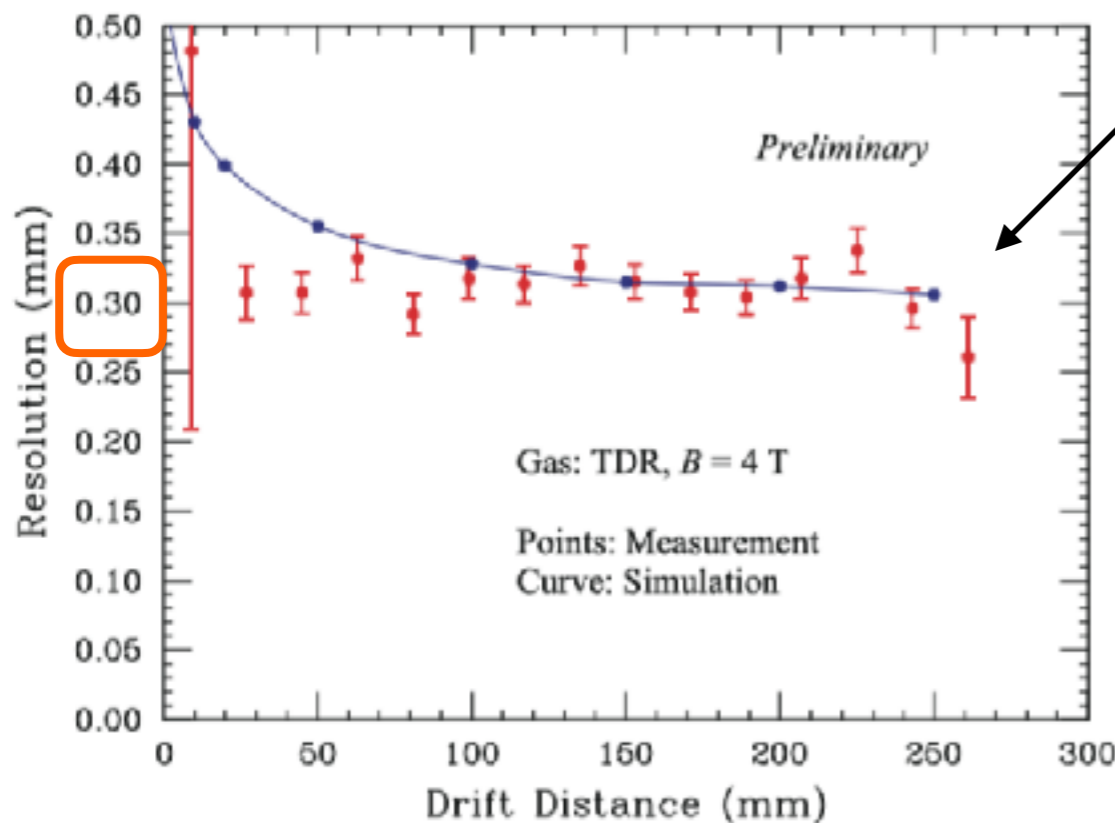


ILD: B-field 3.5T

- Limiting the spatial resolution
 - ← $E \times B$ spread seed electrons along sense wire

Ex) $\sigma_x = 300 \mu\text{m}$

Cannot achieve ILD requirement!

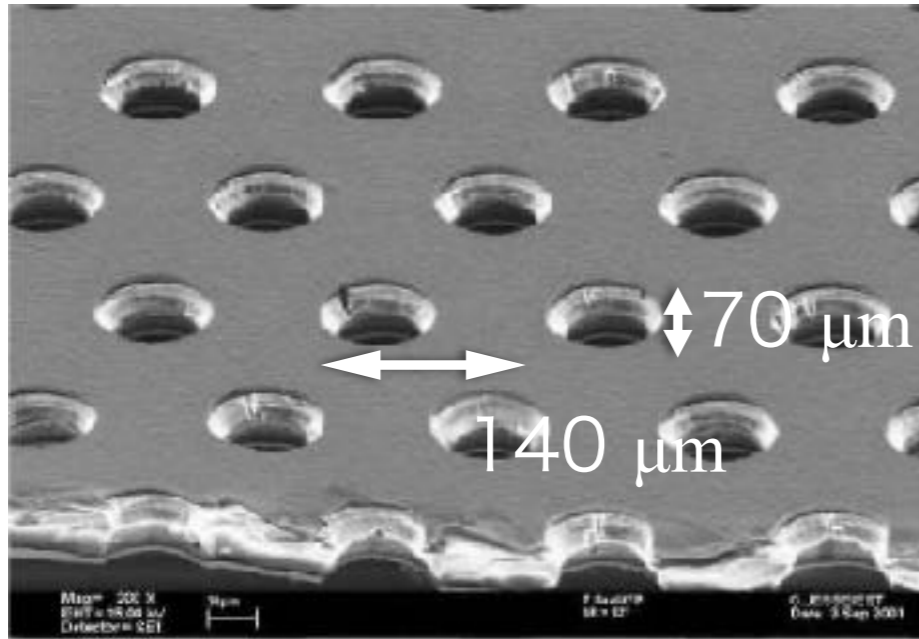


- 2mm 2-track separation is difficult
- Need support structure to tighten wire

→ MPGD

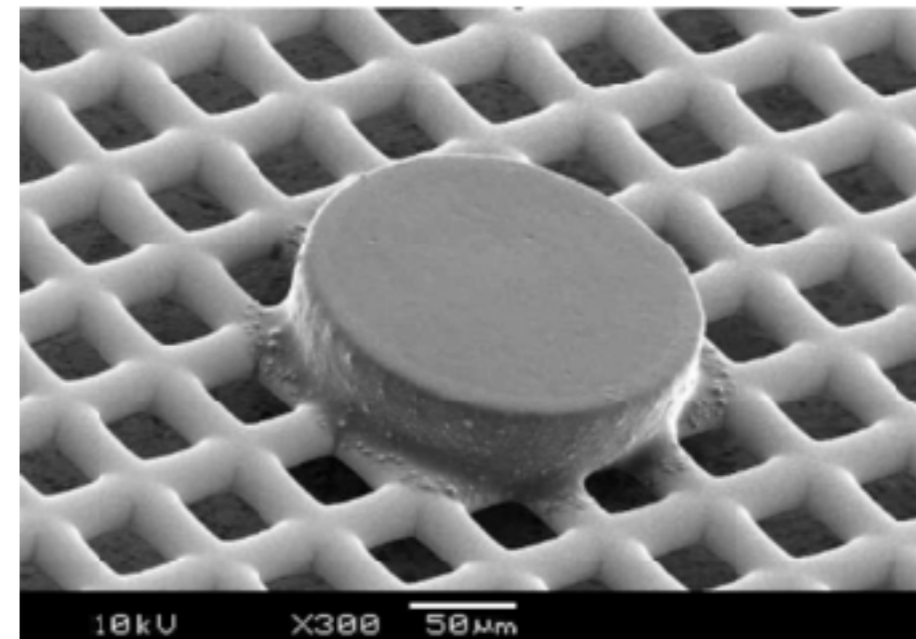
Micro-Pattern Gaseous Detector (MPGD)

GEM (Gas Electron Multiplier)



F.Sauli, NIM A 386(1997)531

MICROMEAS



SEM image of micro-mesh by scienergy company

Why MPGD?

- ☑ The distance between holes is small
 - $E \times B$ effect is small
 - **We can get more precise position information**

- ☑ Supporting structure is simplified (compared to multi-wire chamber)
 - **Dead regions are reduced**

MICROME GAS

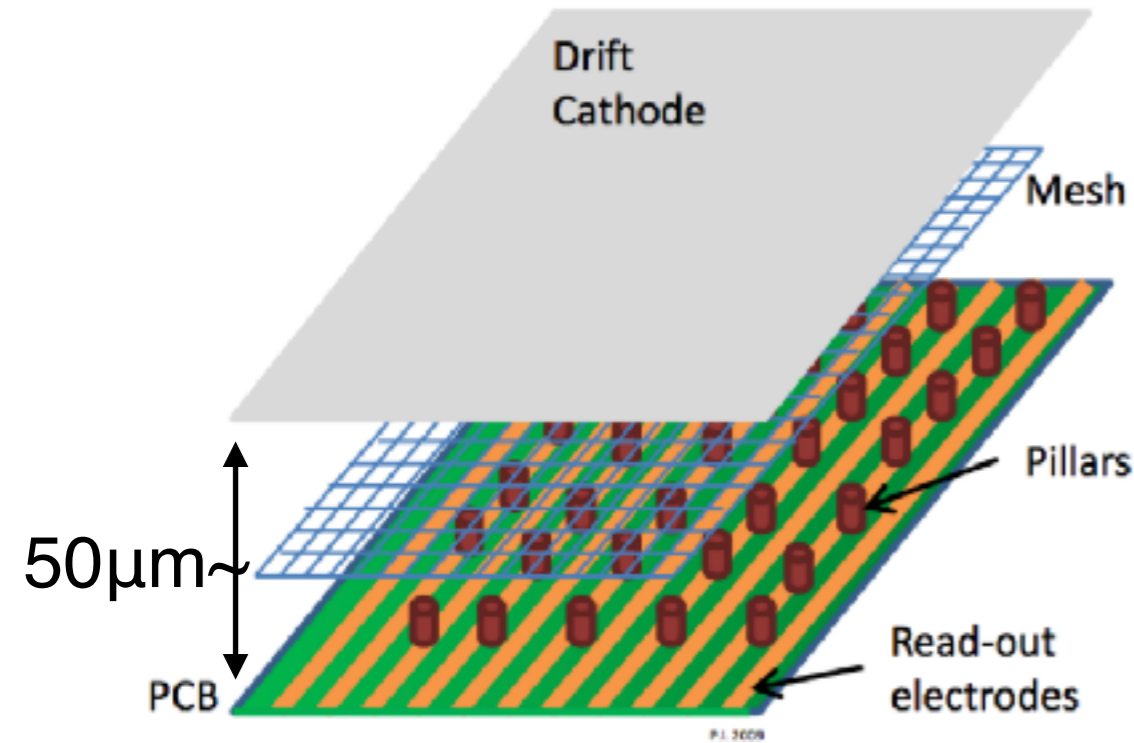
Micro-mesh Gaseous Detector Structure

☑ Y.Giomataris(Saclay) proposed in 1996

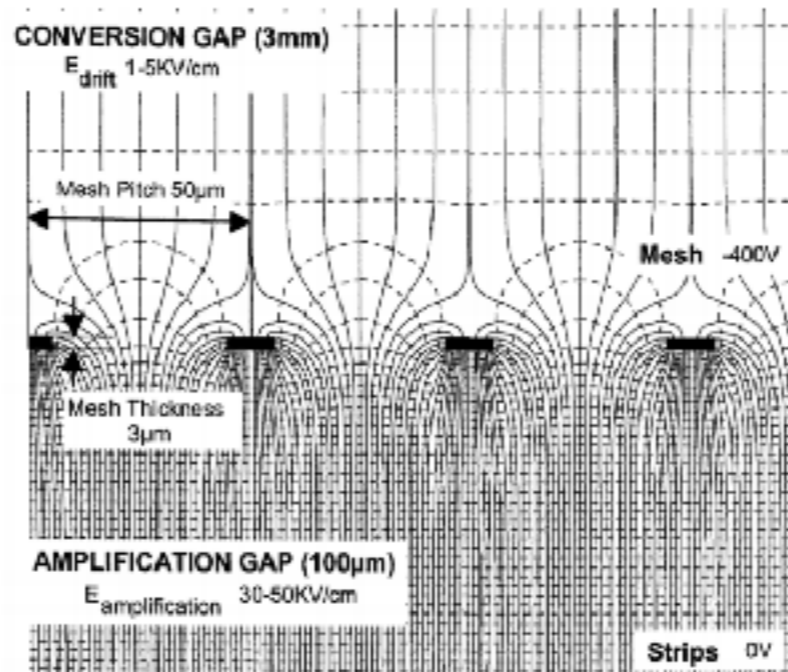
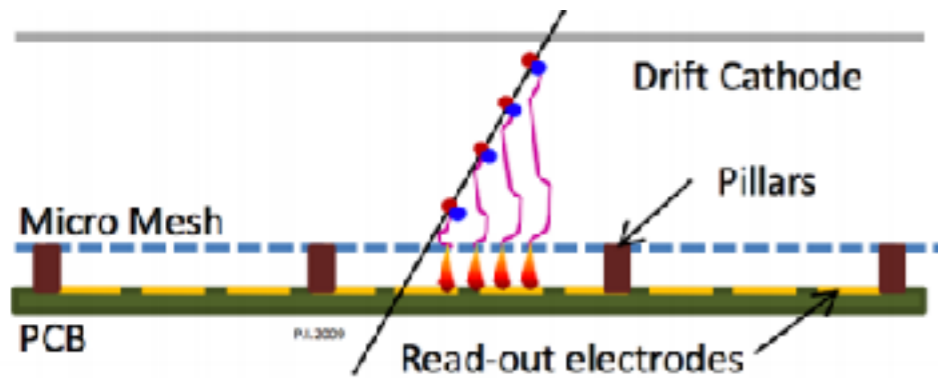
☑ Only **one** amplification gap provides sufficient amplification factor

☑ Resistive anode is needed to avoid hodoscope effect

Ex) $\sigma_{PRF} = 12 \mu\text{m}$
(Typically)



General structure of MICROME GAS



E-field in MICROME GAS

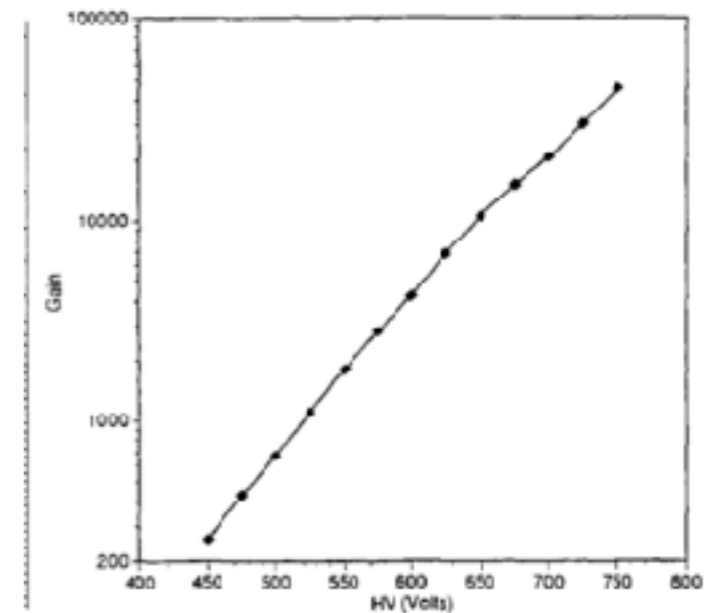


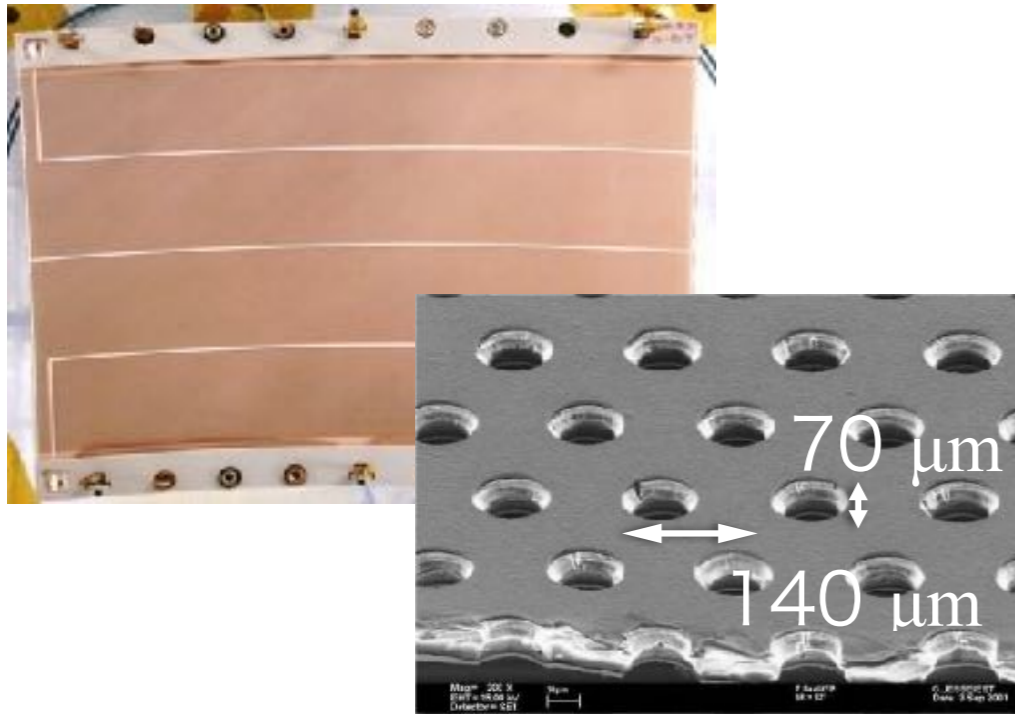
Fig. 9. Variation of gas-gain with anode voltage. The gas mixture was Ar + (10%) CH₄ at atmospheric pressure.

Charge amplification in MICROME GAS

GEM

F.Sauli, NIM A 386(1997)531

Gas Electron Multiplier

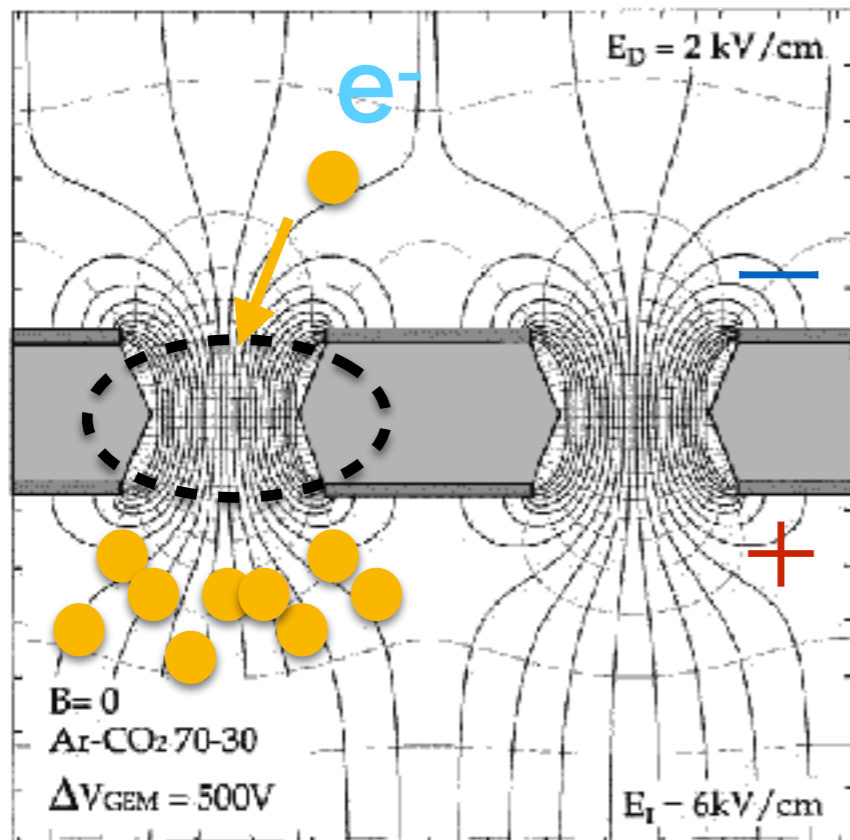


✓ F.Sauli(CERN) proposed in 1997

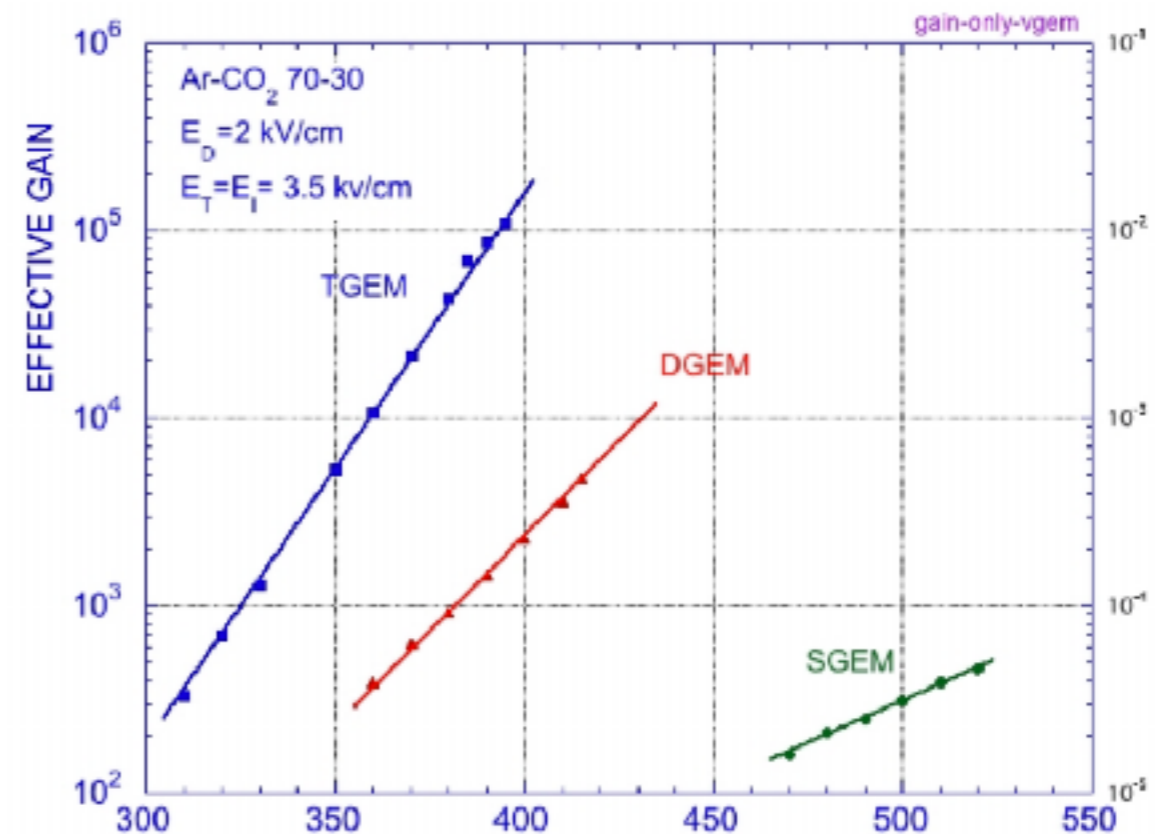
✓ Often used in **multiple** GEM

$\sigma_{\text{PRF}} \sim 300\ \mu\text{m}$

→ **suitable for ordinary pad readout**



E-field in GEM

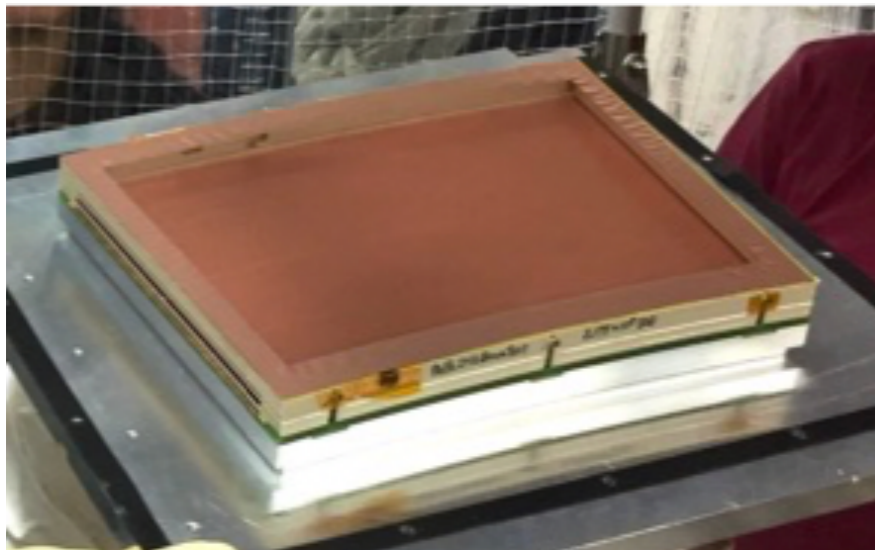


Property of charge amplification in GEM V_{GEM} (V)

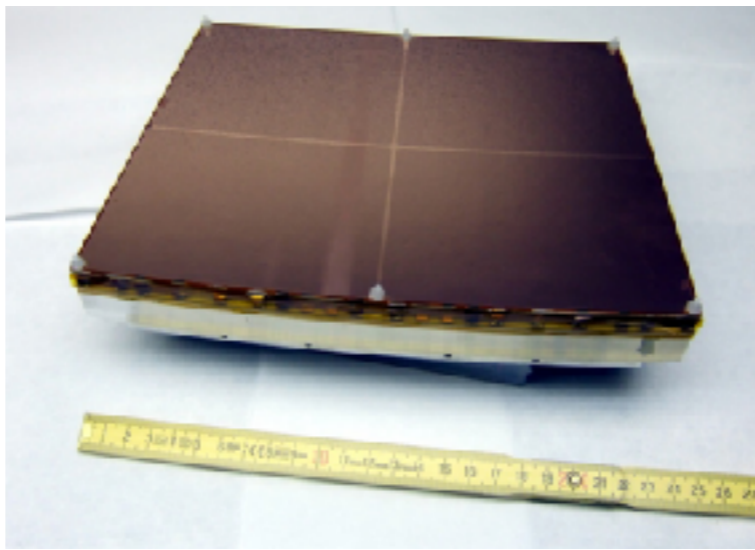
Candidates of readout module - Analog(Pad) readout

GEM

Asian module

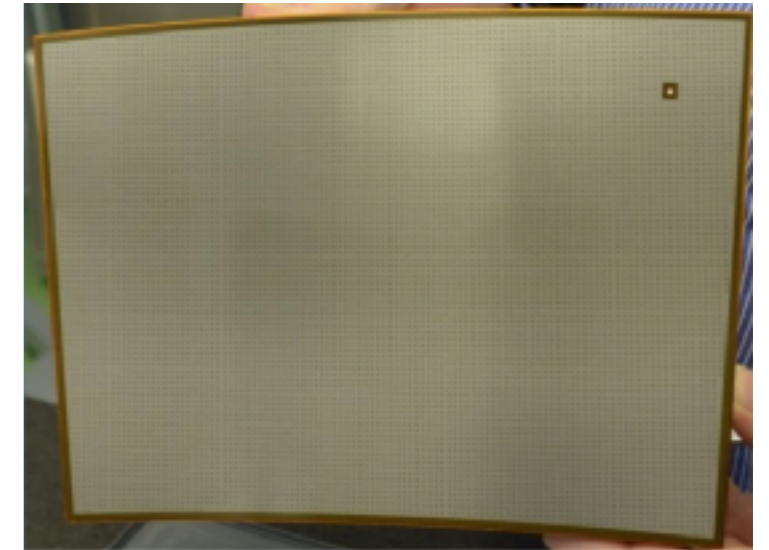


DESY module

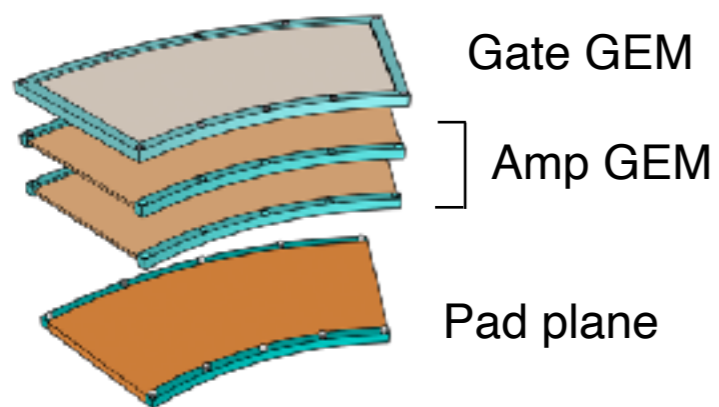


MICROMEKAS

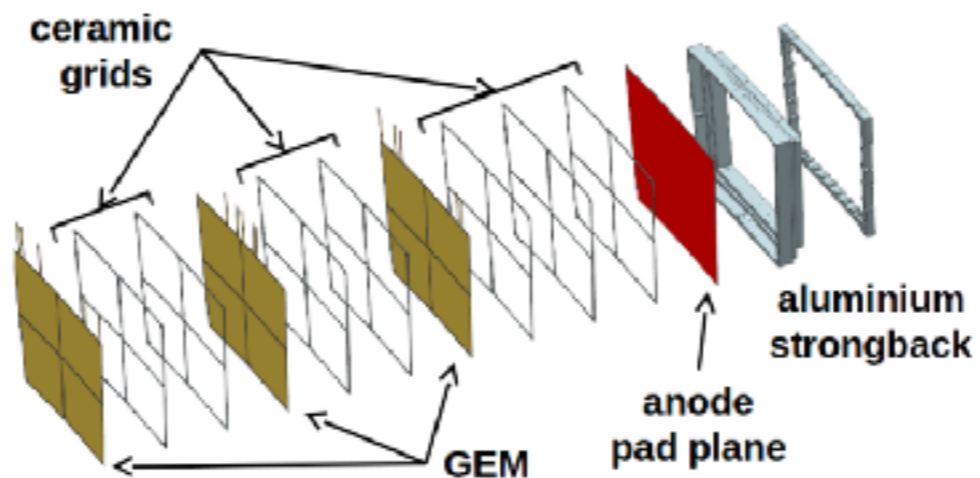
Saclay-Carleton module



Double GEM



Triple GEM

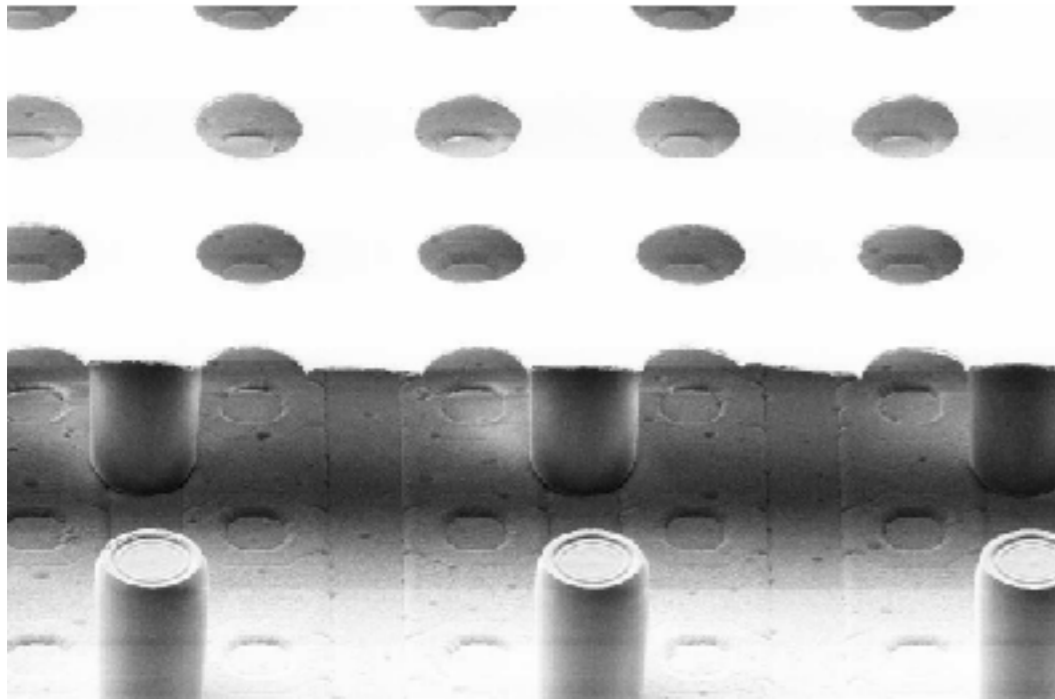


[arXiv:2006.08562](https://arxiv.org/abs/2006.08562)

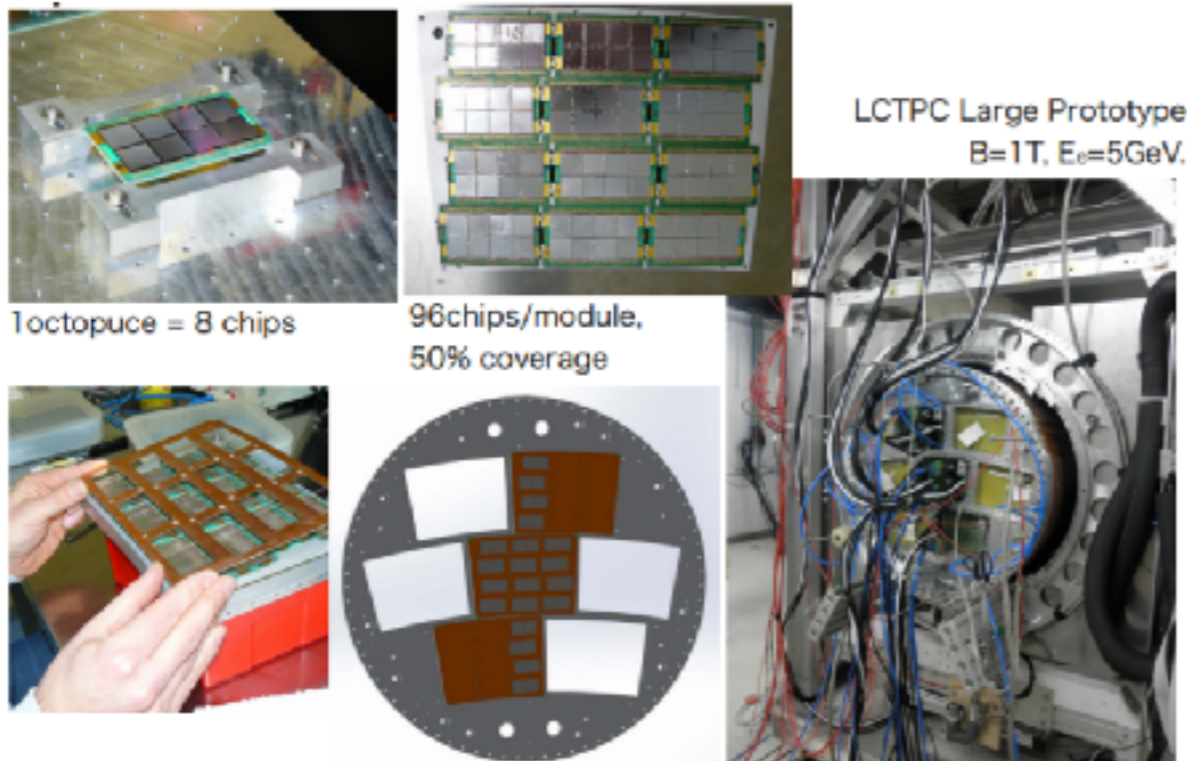
(Resistive anode)

Candidates of readout module - Digital(Pixel) readout

GridPix(TimePix + Protection Layer + Micromegas)



- ☑ Free from gas gain fluctuation effect on spatial resolution
- ☑ Expect 20-30% improvement of position resolution
- ☑ No angular pad effect
- ☑ Theoretically the best but not yet ready for full implementation of a module

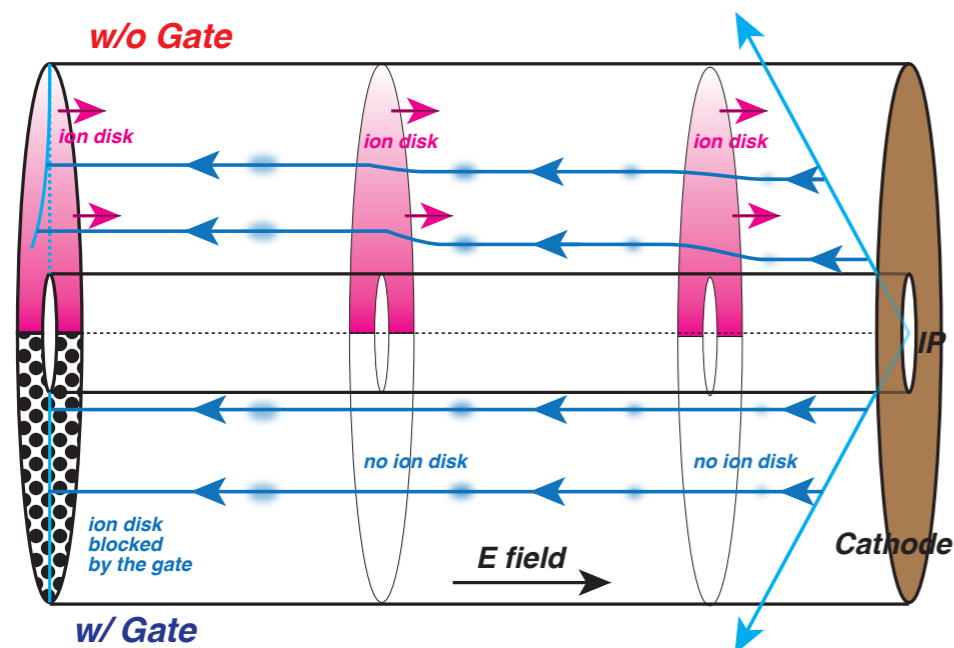
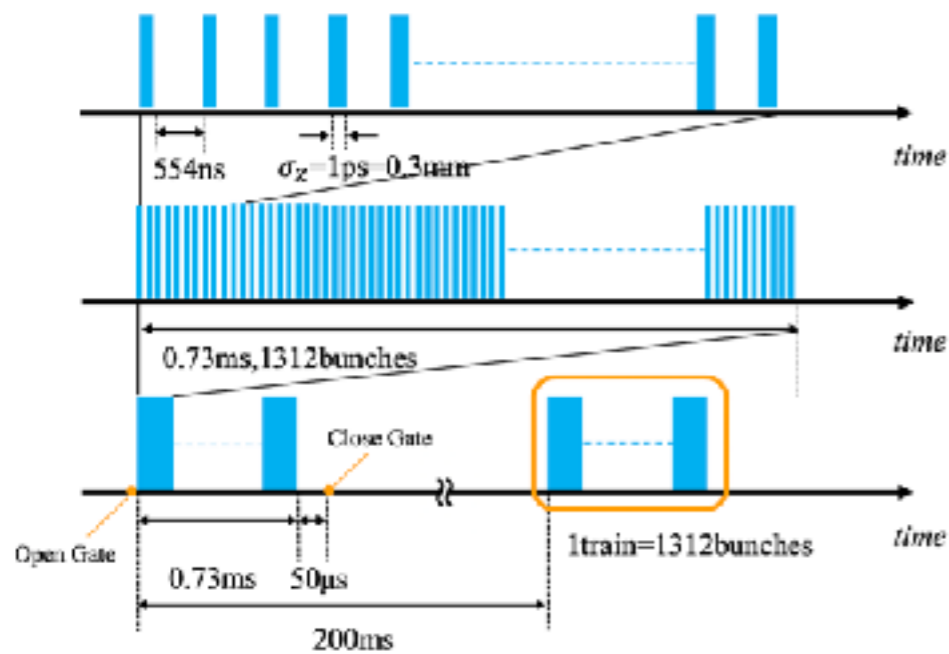


Prototype test

Positive ion backflow

In ionization process, not only electron but also **positive ions** are produced

- Positive ions **flow back in the drift region**
- Making the distortion of E-field
- **Position resolution is worse**



How many are positive ions(disk) in TPC?

Positive ions make **ion disk**(1cm) in 1train

Drift velocity(Iso-C₄H₁₀ ion) : 0.37 cm/s

Drift E-field : 230 V/cm

Distance between trains : 200 ms

Positive ions flow 74 cm forward

Drift length(Max) : 2.2 m

$$\longrightarrow 2.2 \text{ m} / 0.74 \text{ m} \simeq 3$$

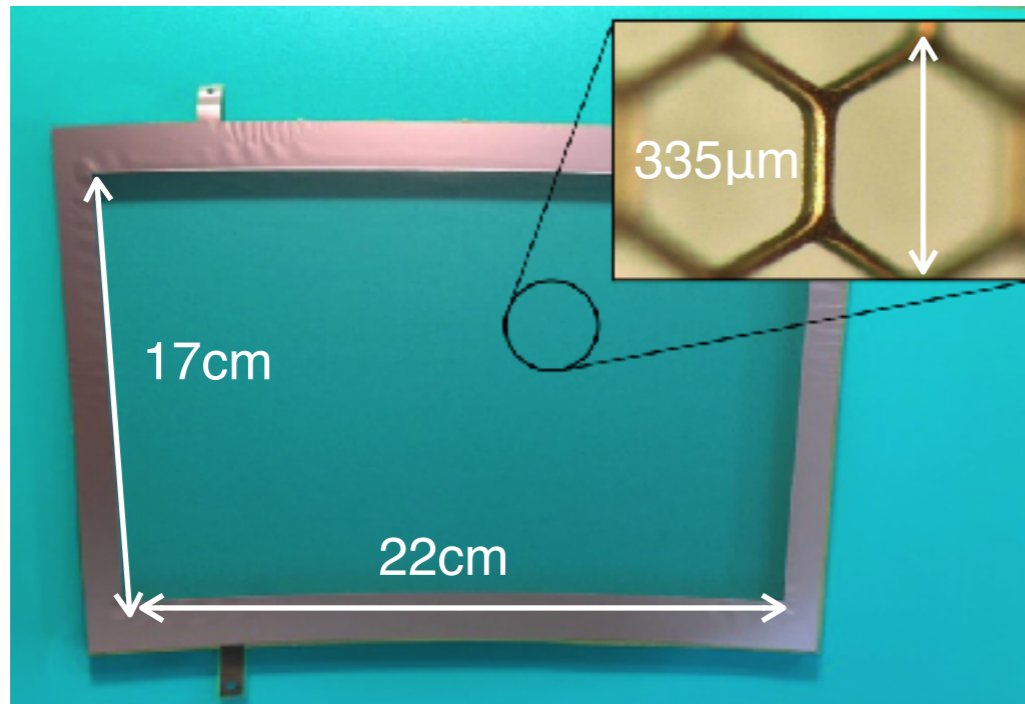
Not ignore

Maximum distortion is 60 μm

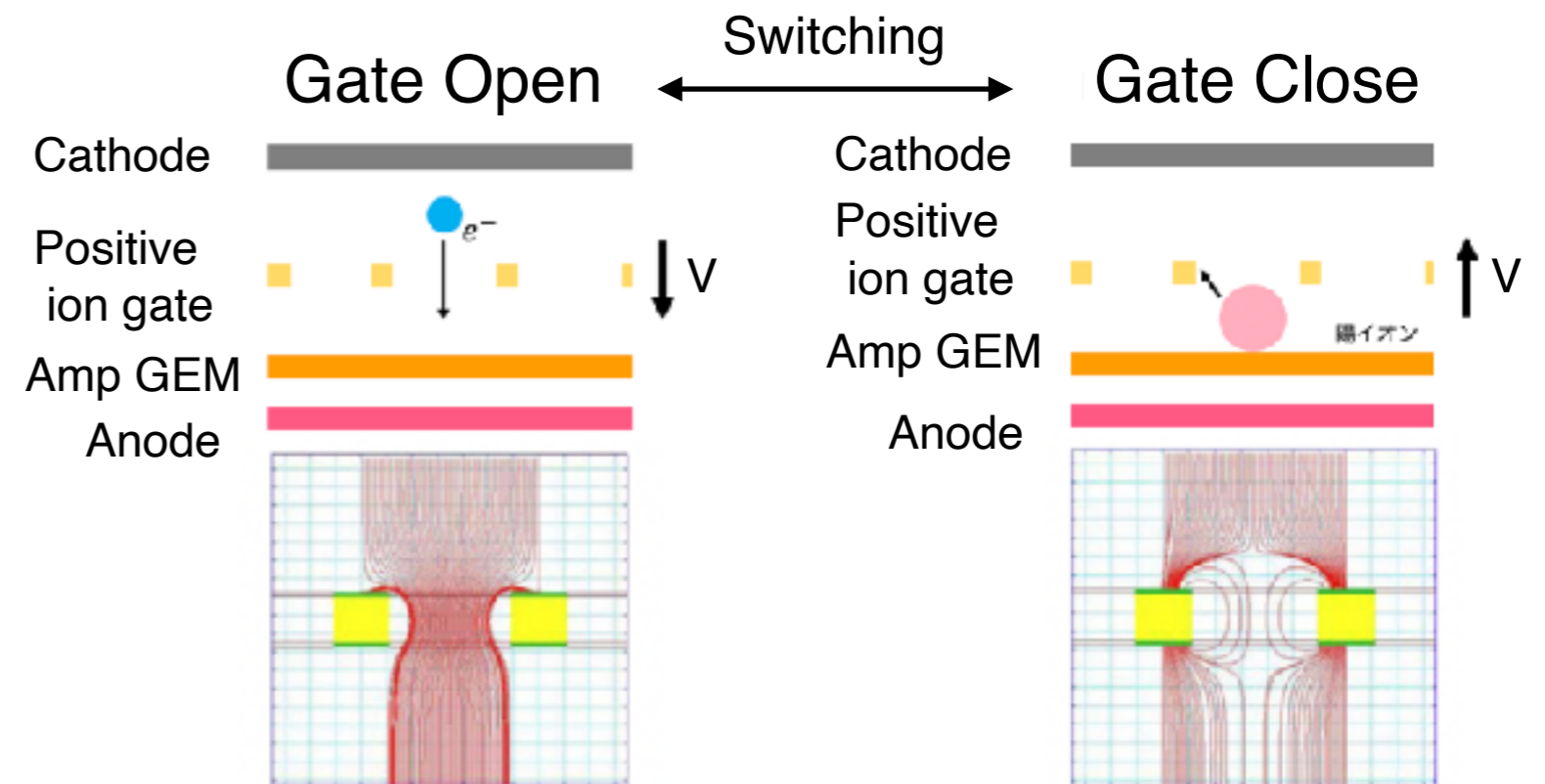
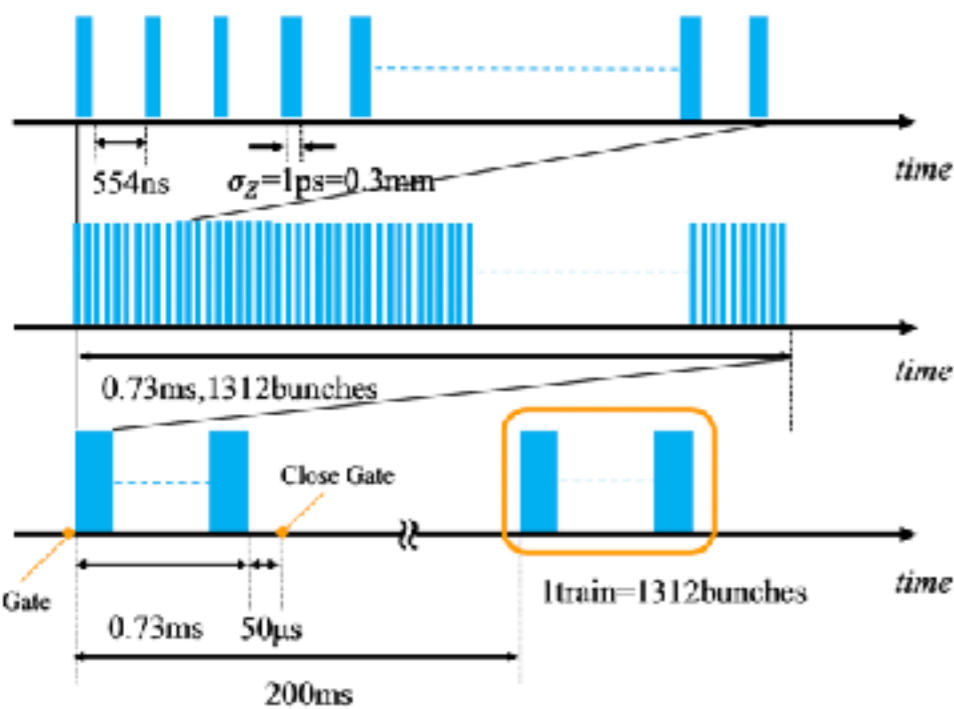
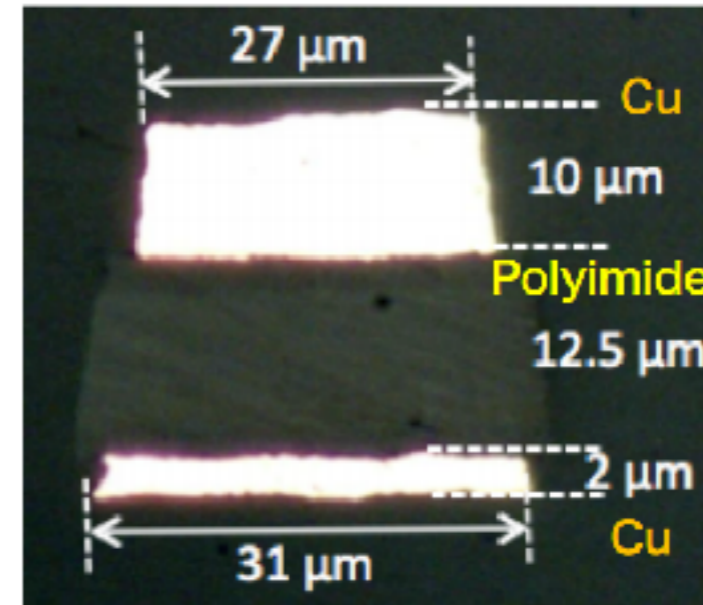
K. Fujii, "Positive Ion Effects (LCTPC collaboration meeting presentation)."

D. Arai, "Ion Problem Report (LCTPC Workpackage Meeting # 145)." 18

Gating Foil



Cross-section plane



Gating foil can keep small distortion in GEM

Summary

☑ What to know in the tracker

“Two 4-vectors” $p^\mu = (E/c, \mathbf{p})$, $x^\mu = (ct, \mathbf{x})$ + Charge

☑ TPC : Time Projection Chamber

- Reconstructing tracks of charged particles in **3-dimension**

Strong points

- ▶ $dE/dx \rightarrow$ Particle ID
- ▶ 200 hits
- ▶ Long-lived particle measurements

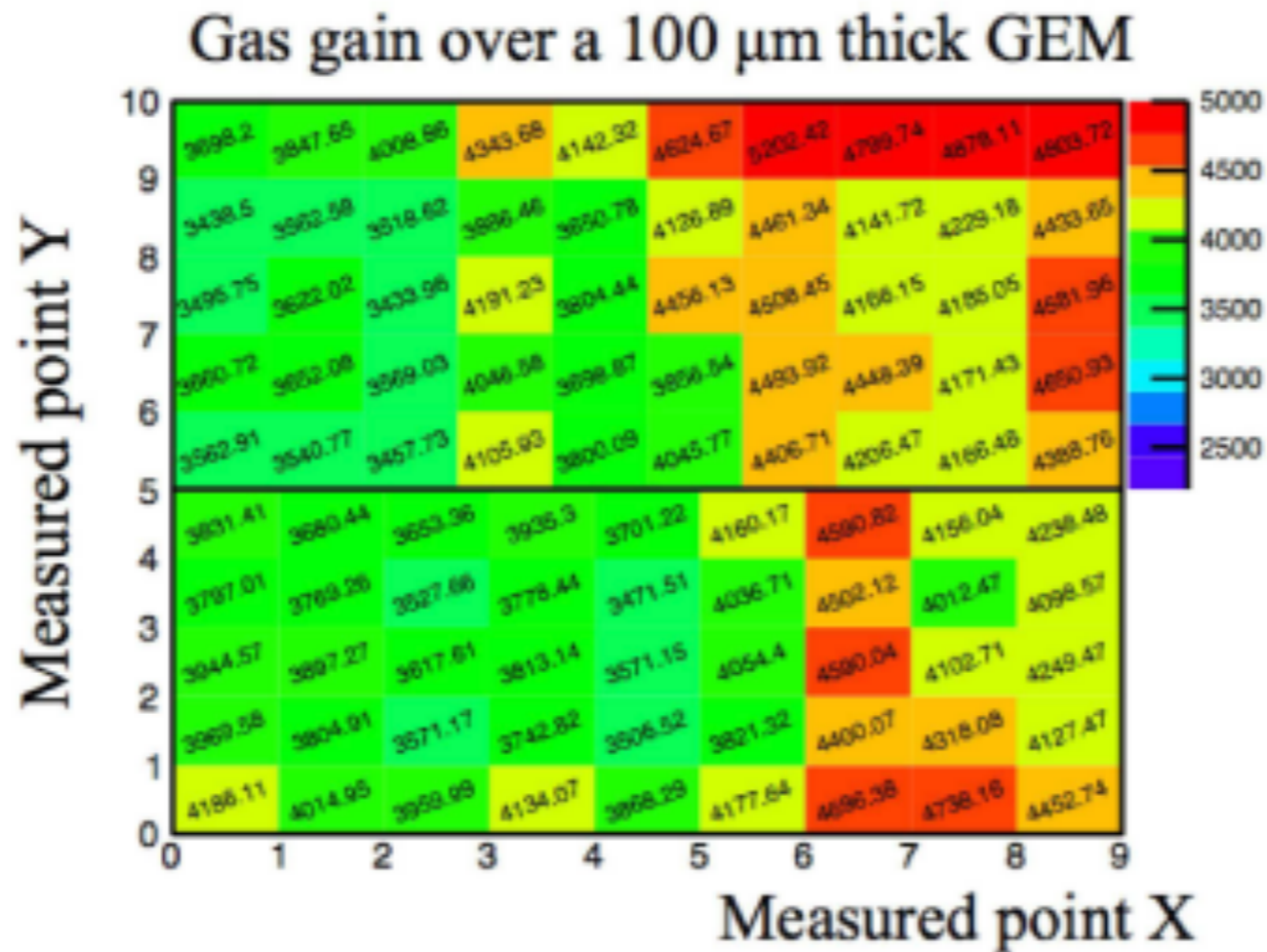
☑ TPC R&D

- Proof of principle : done \rightarrow No show stopper
- Engineering stage : now
New technology ex) Pixel readout

LCTPC Asian group

GEM gain stability

- ☑ Non-uniformity of the gas gain in Asian module

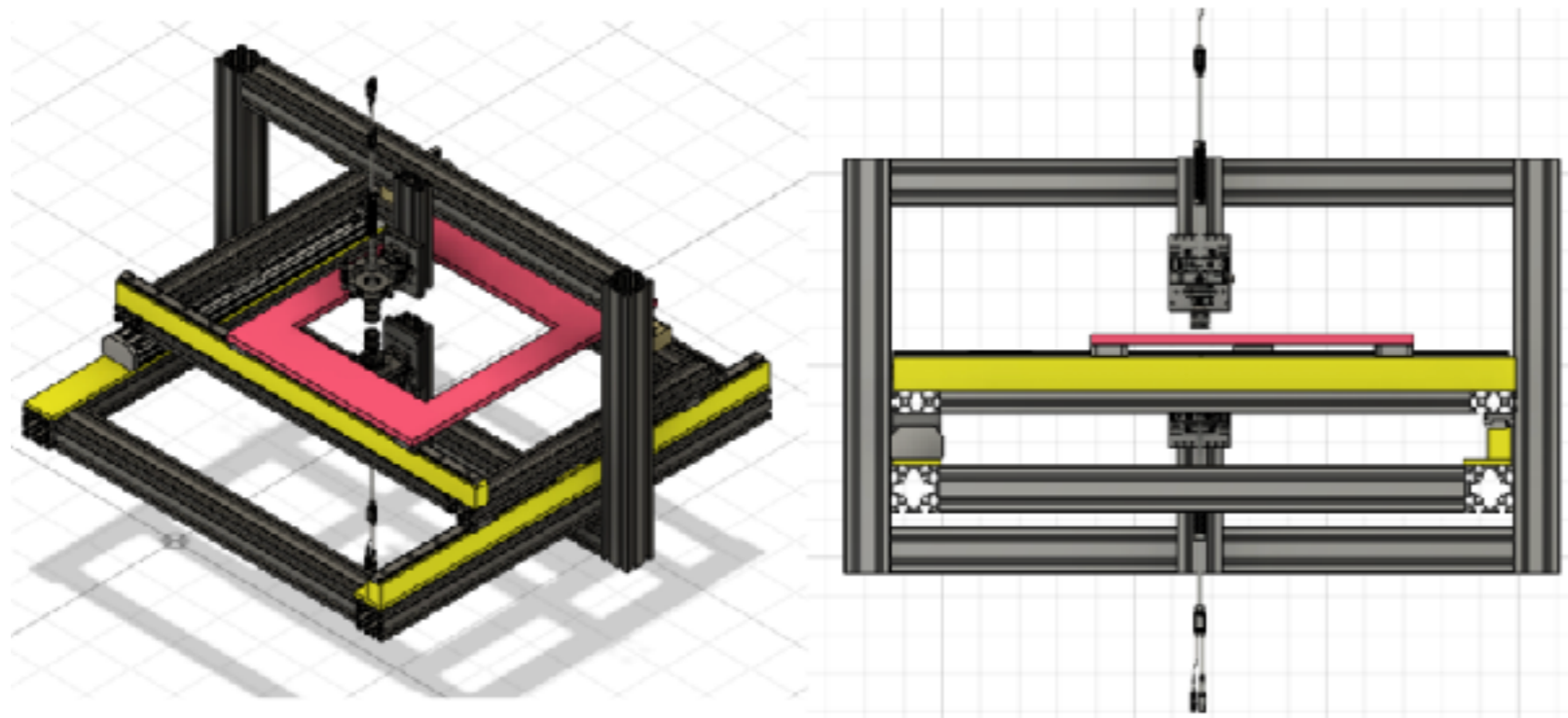


arXiv: 1701.05421

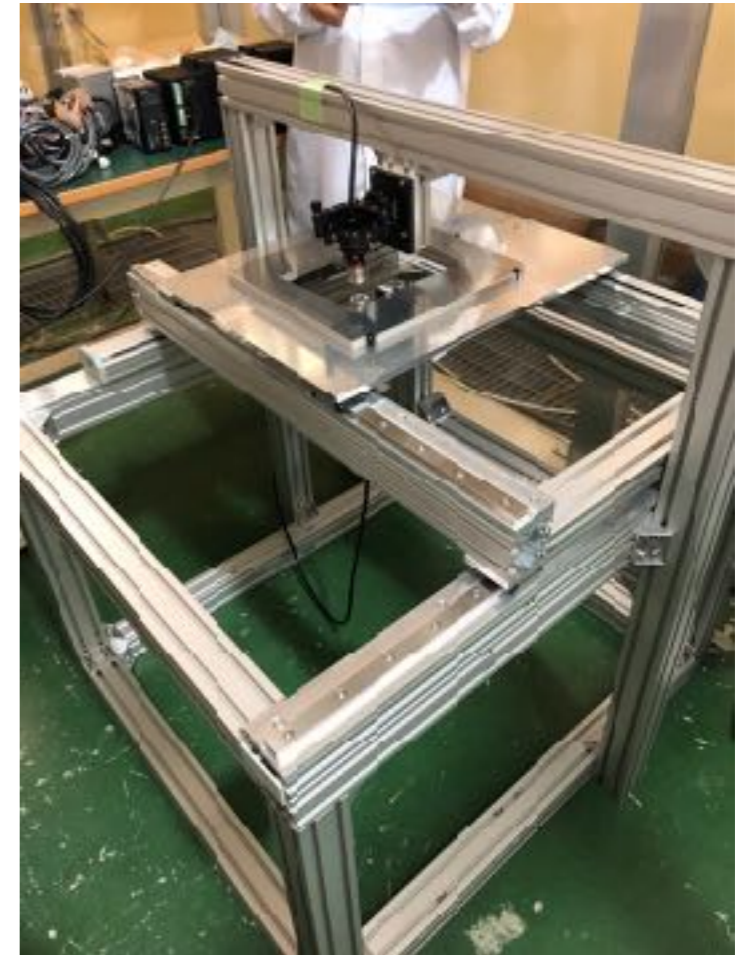
In our simulation study,
the gas gain strongly depends on the thickness of GEM

→ Preparing measurement system of GEM thickness

GEM thickness measurement



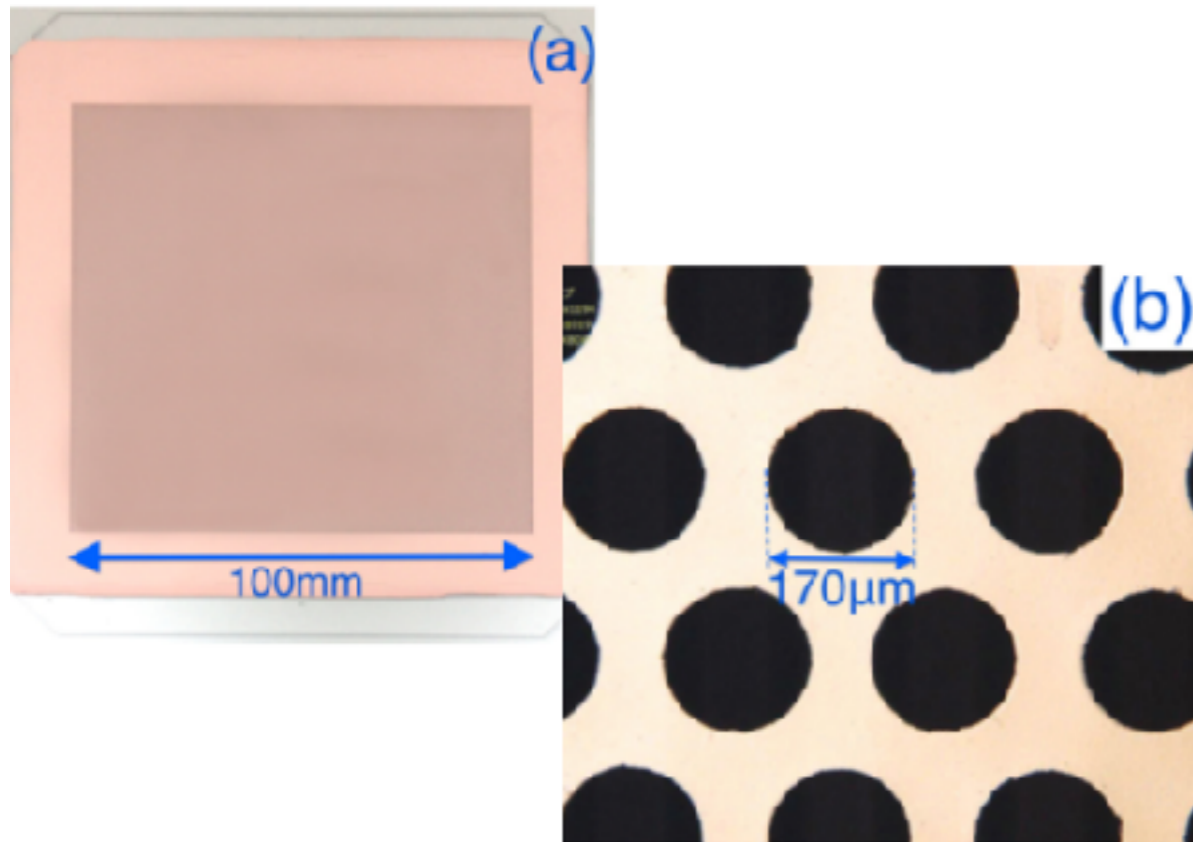
Measurement system



GEM insulation material

Avoid discharge in GEM \longrightarrow New insulation material

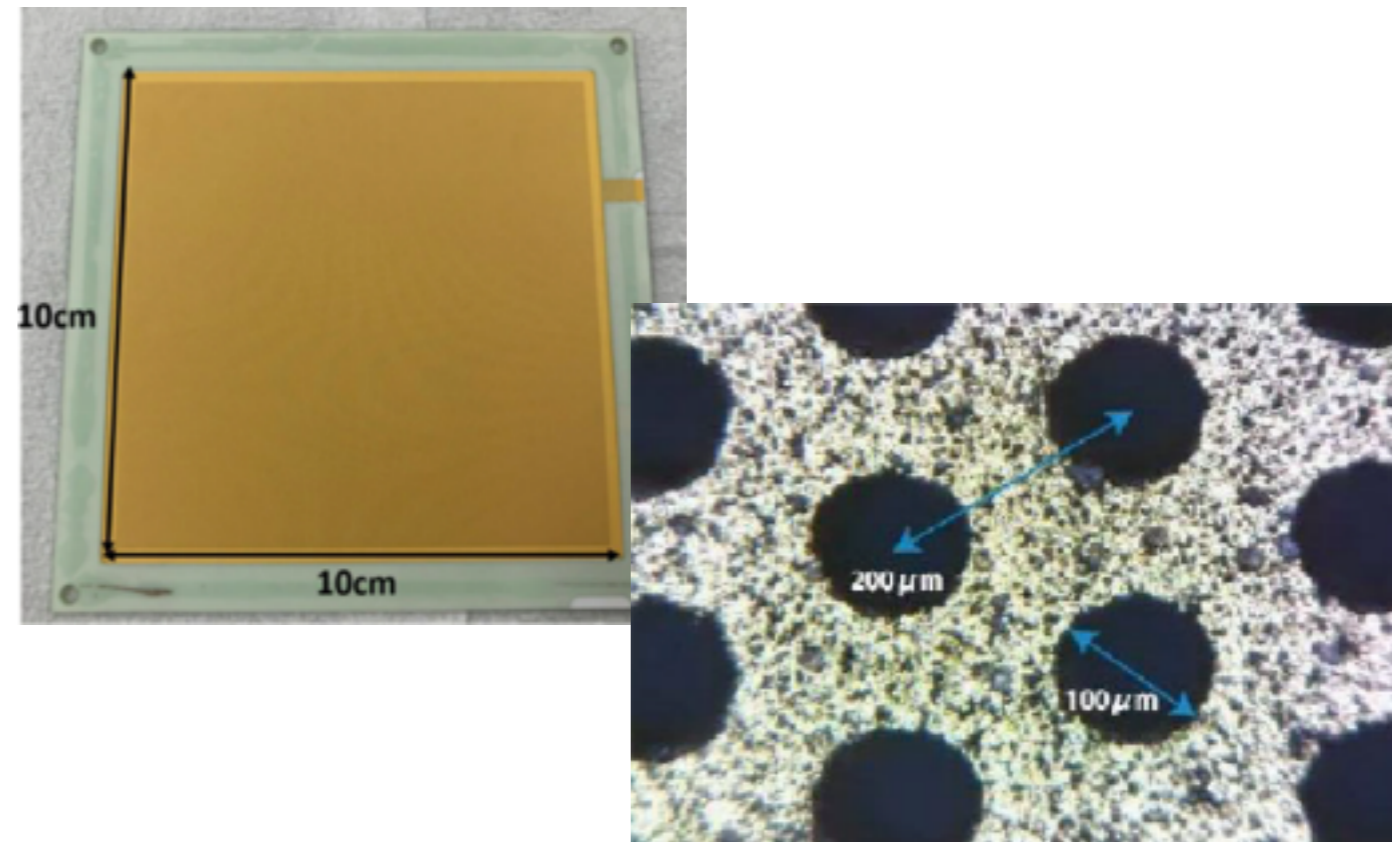
Glass GEM



T.Fujiwara et al 2014 JINST 9 P 11007

Polyimid \rightarrow glass

LTCC



Y.Kato 2020 J.Phys. Conf. Ser. 1498 012010

Low Temperature Co-Fired Ceramics

Back up

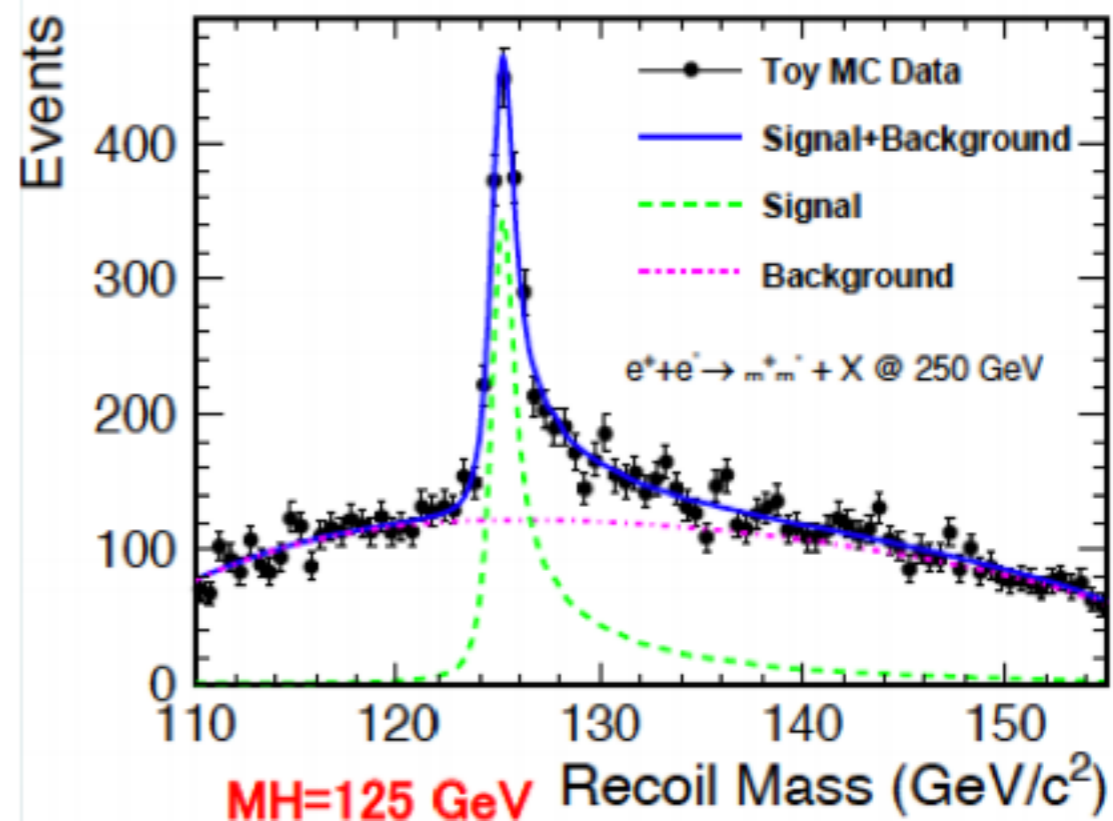
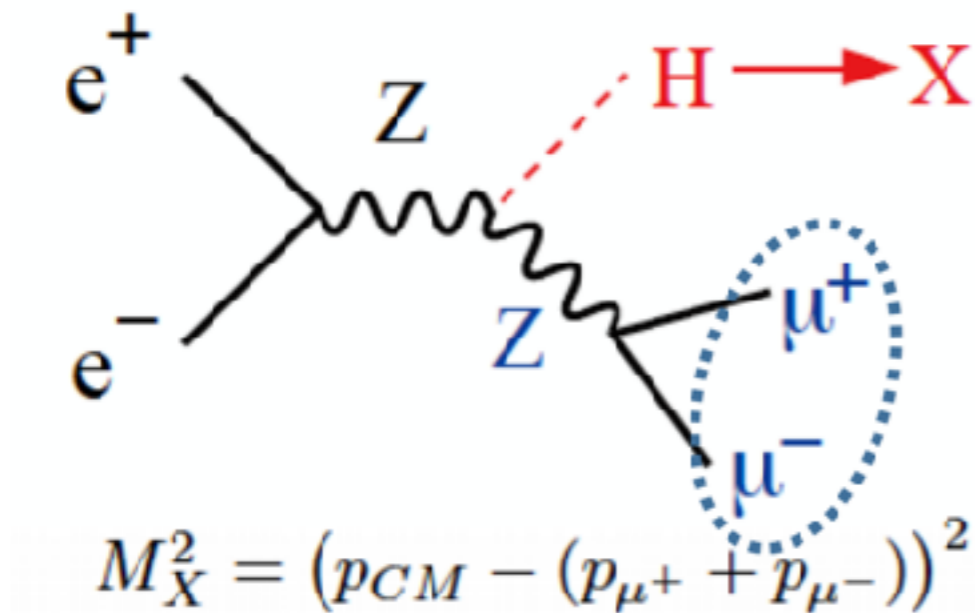
Why is TPC?

(Supplementary documents)

Why we need high momentum resolution?

ILC target : Higgs **precise** measurement

Higgs mass measurement by Recoil mass method



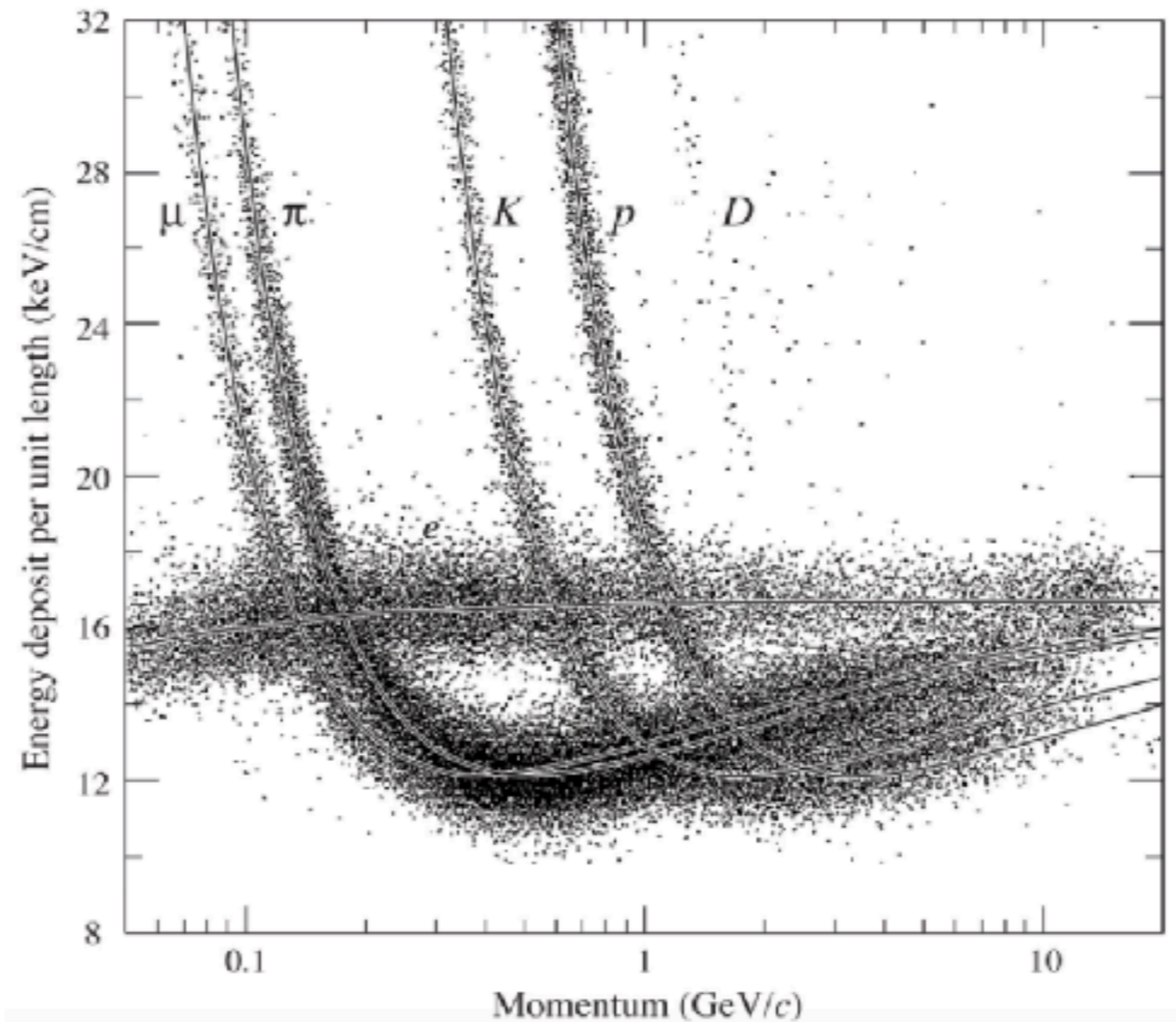
1. Know 4-vector momentum in the initial state in the case e^+e^- collider
2. Measure momentum of μ pair
3. Prove Higgs mass without measure Higgs directly

Recoil mass resolution depends on momentum resolution

—————→ **Essential to high momentum resolution!**

Energy deposit (dE/dx)

Charged particles lost energy when particles pass materials $\longrightarrow dE/dx$



dE/dx for momentum is particle specific

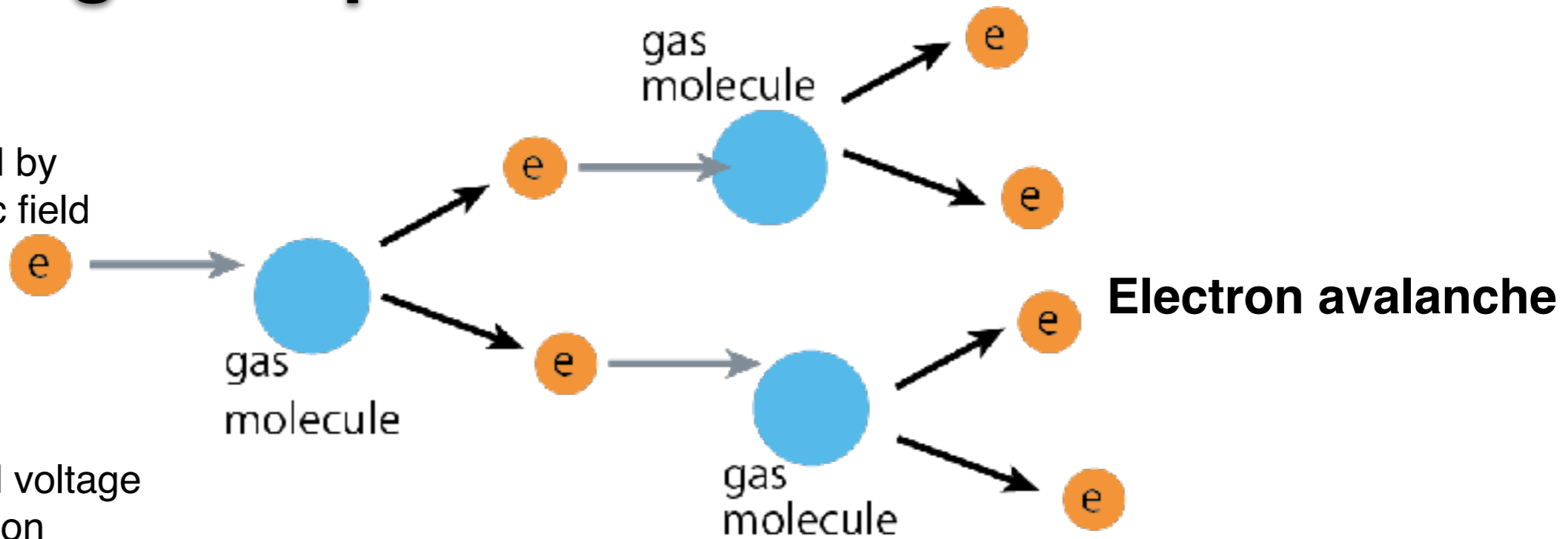
\longrightarrow **Particle identification**

ILD detector requires overall $dE/dx \sim 5\%$

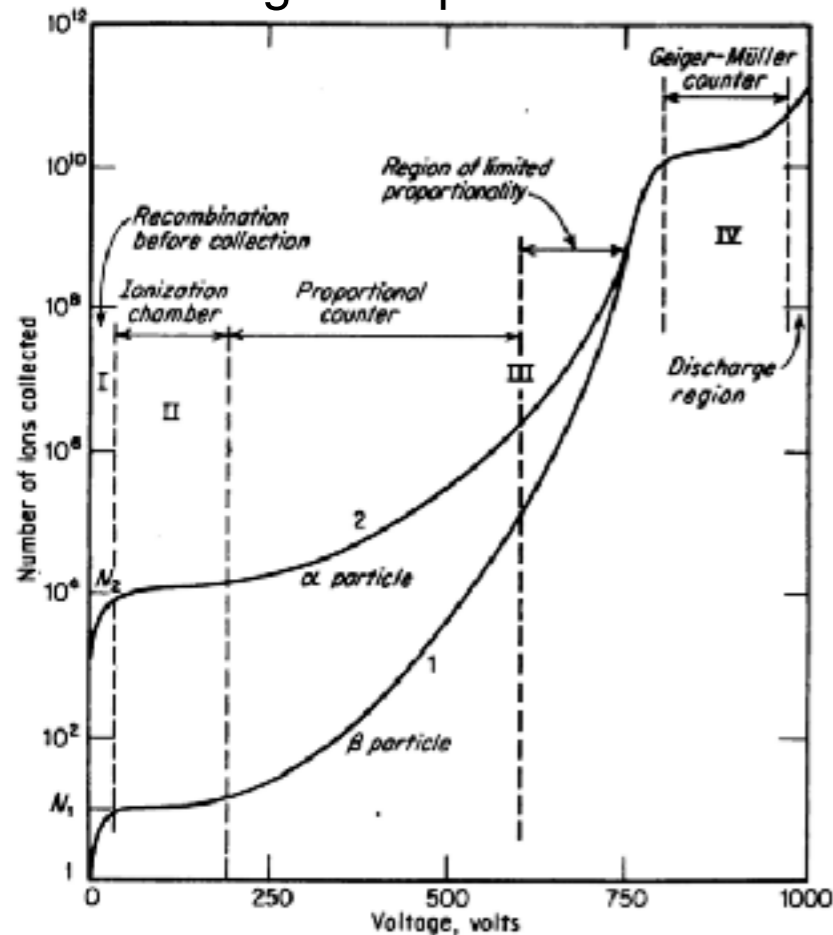
Readout module of TPC (Supplementary documents)

Principle of gas amplification

Accelerated by high electric field



Dependence of applied voltage for gas amplification



The average increase of electrons

$$dn = n\alpha dx$$

Electrons #

Townsend coefficient

$\alpha = \alpha(E(x))$, integrate along electric field line

$$n = n_0 \exp \left[\int_{x_1}^{x_2} \alpha dx \right]$$

n_0 : the number of electrons before gas amplification

Electrons are increased **exponentially**

Electronic transparency on gating foil

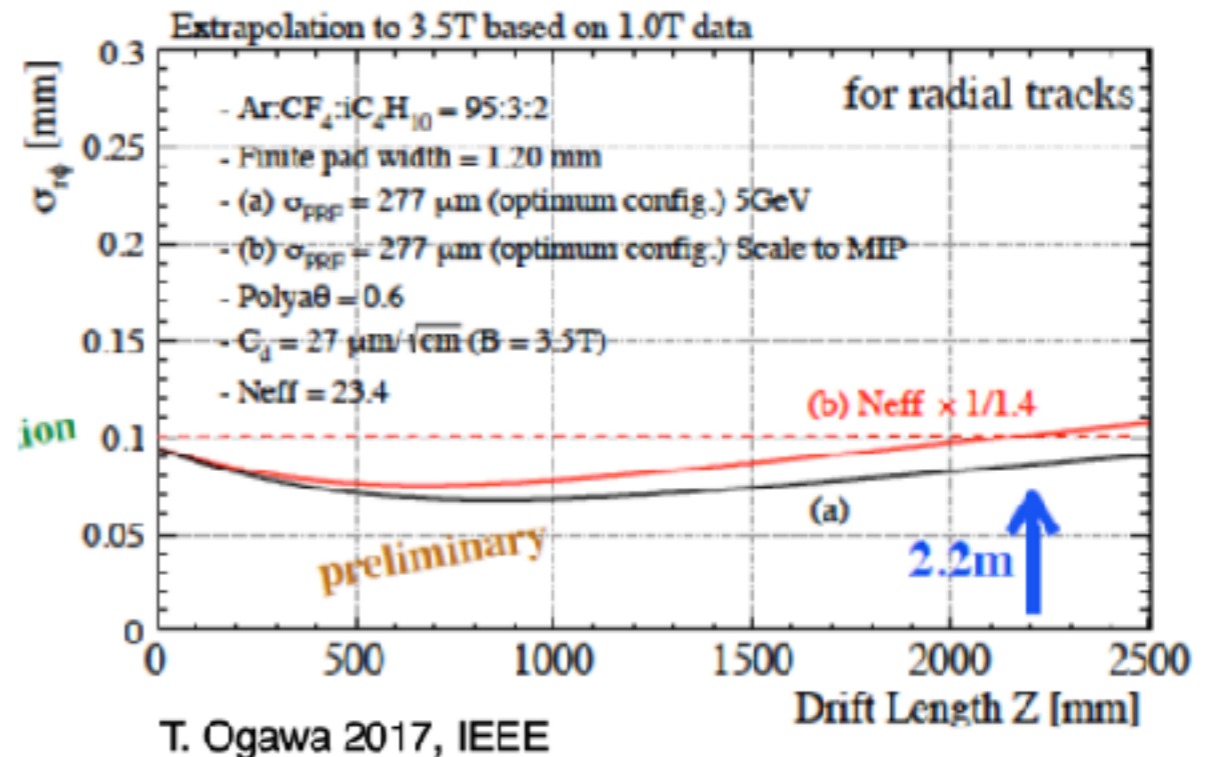
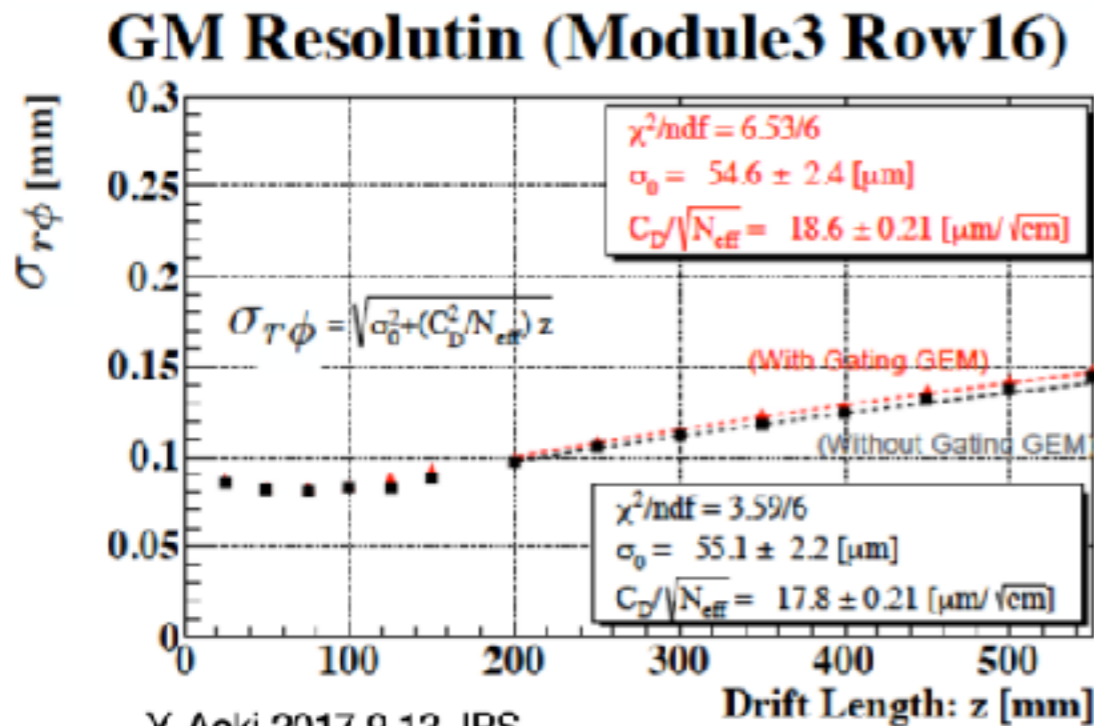
- Gating foil is required
- Preventing positive ions
 - Passing through electrons

→ **Electronic transparency(= Optical openness)** is important

Goal of gate module

Electronic transparency > 80% ← gating foil is 82%
 Rate of blocking ions ~ O(10⁻⁴)

Beam test result with gating foil



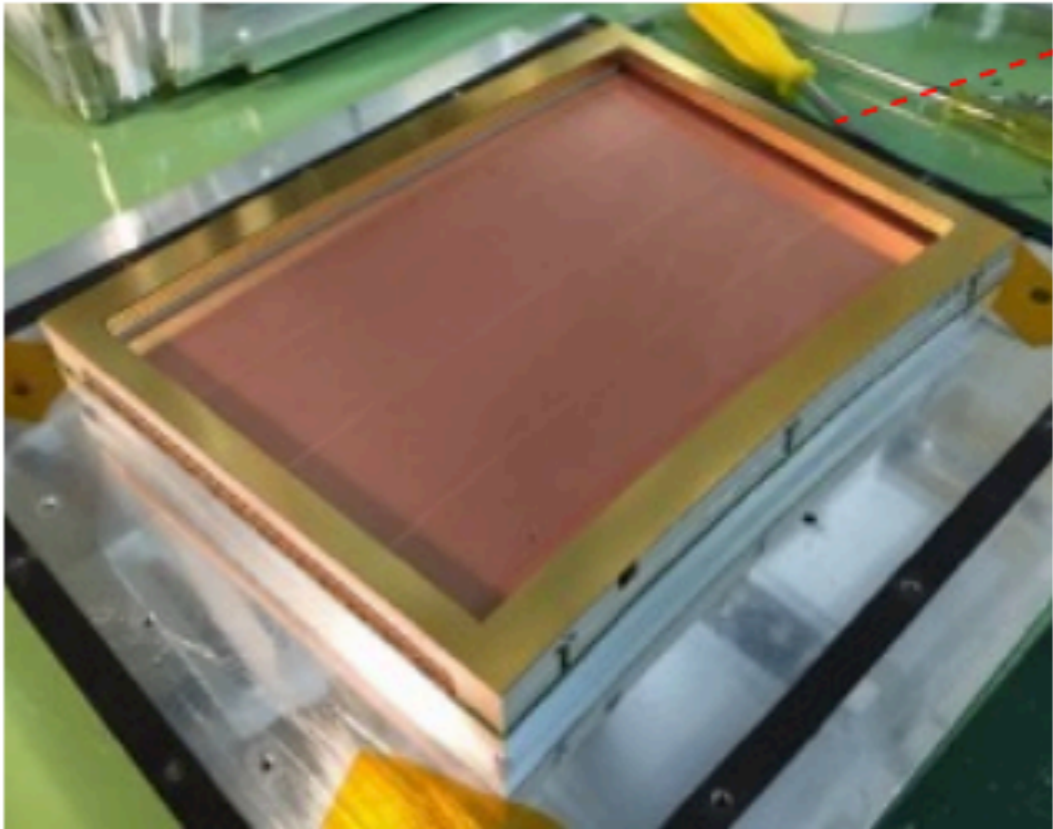
Position resolution achieved 100 μm when we extrapolate to B-field 3.5T and drift length 2.2m

Gating foil (Asian GEM module)

With gating foil

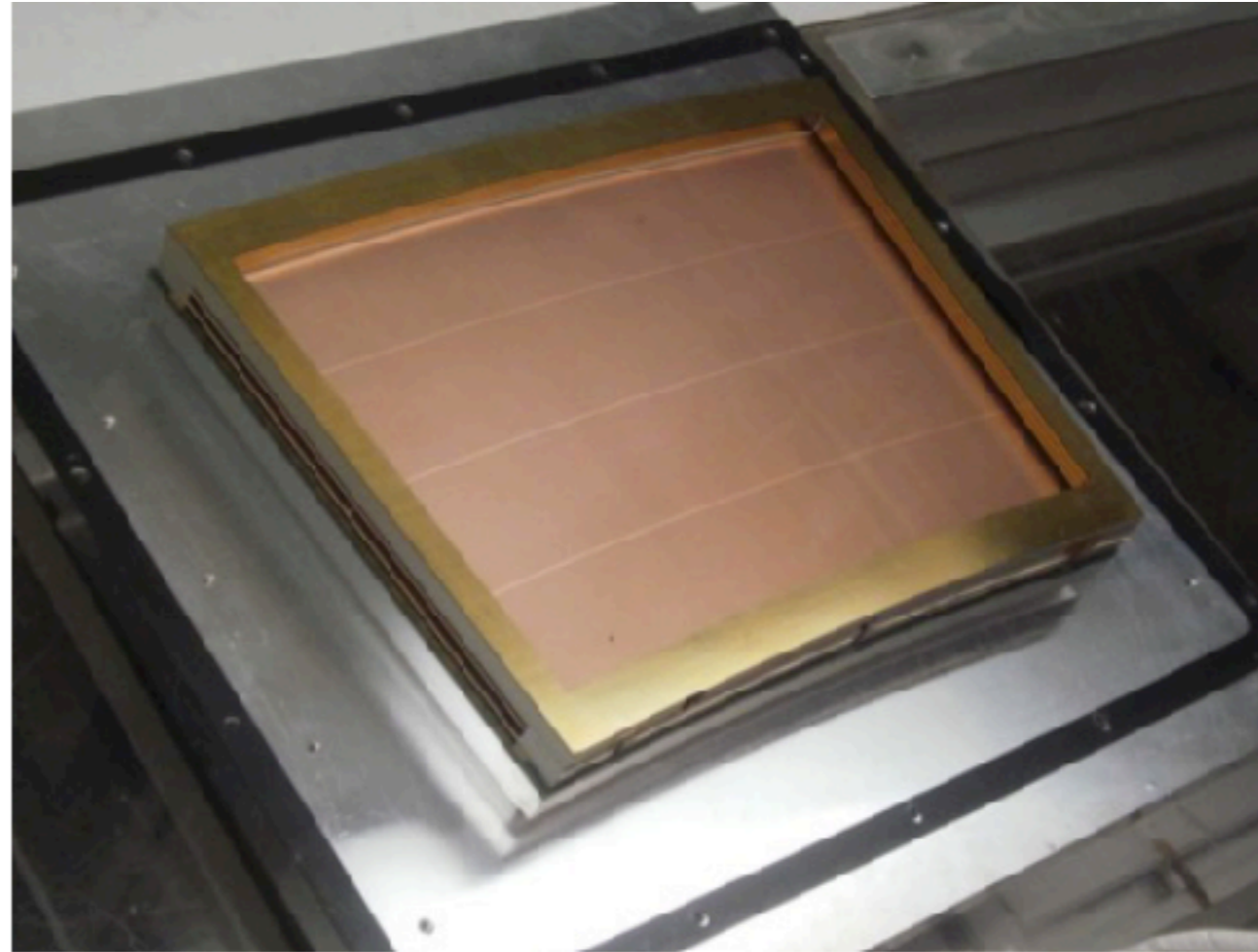
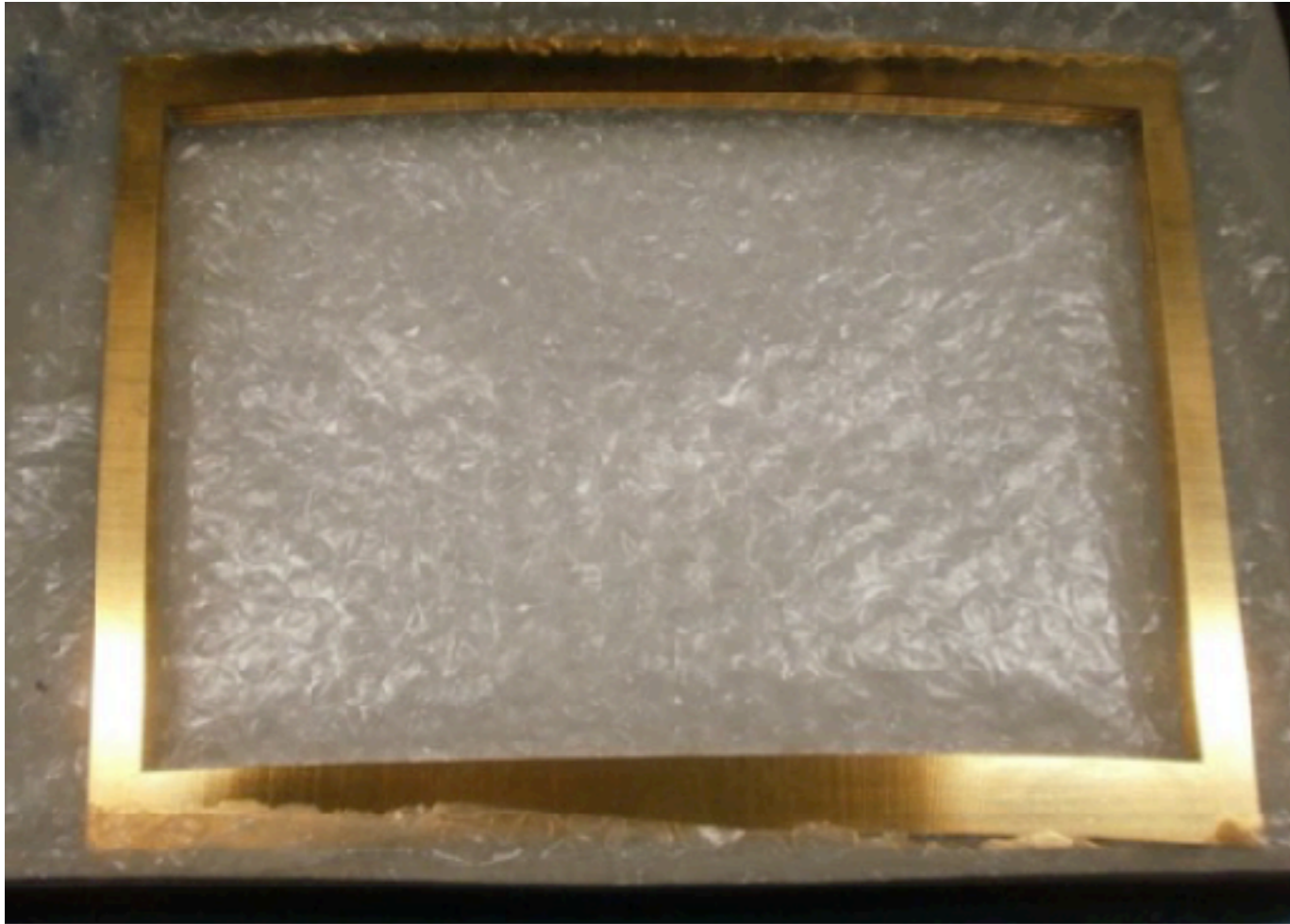


Without gating foil



Field Shaper

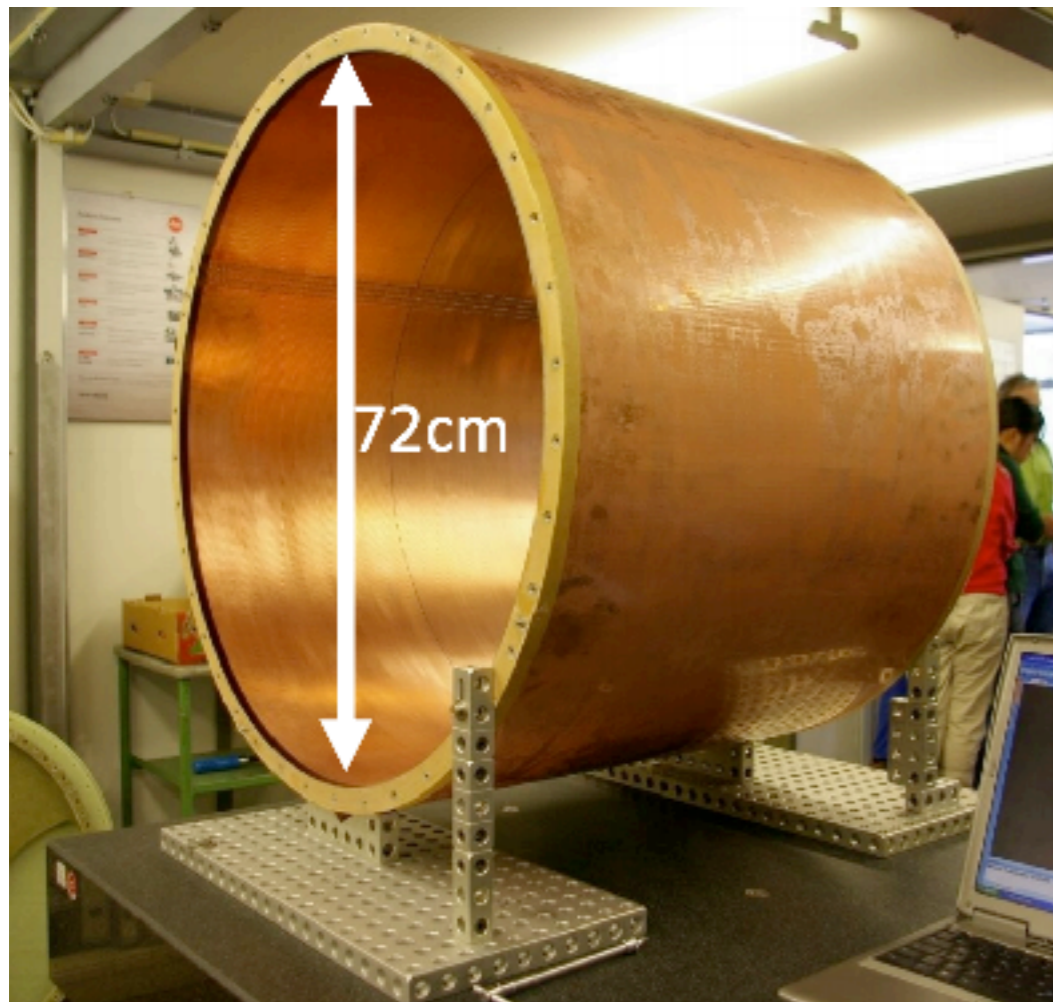
Arrange E-field



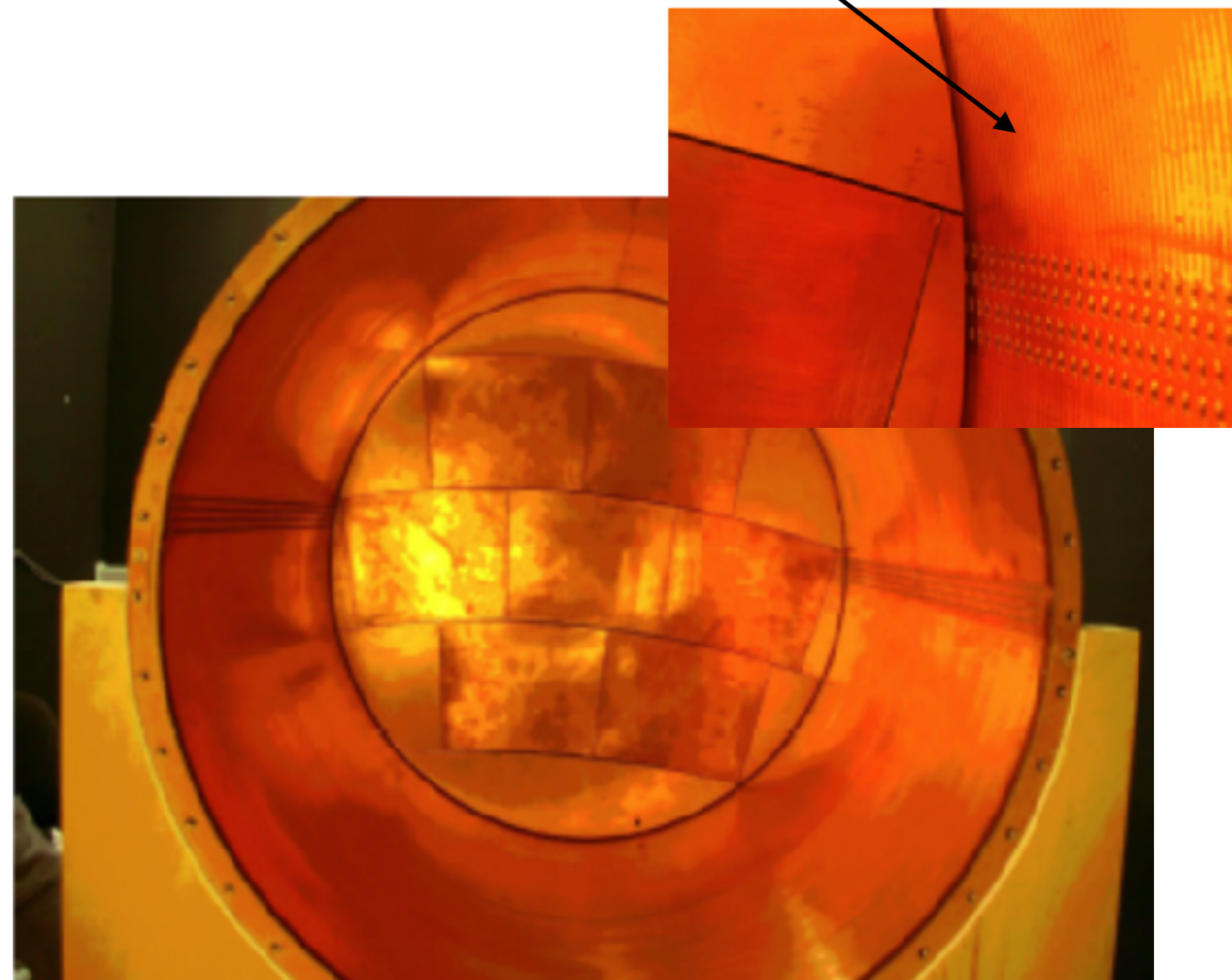
Field Cage

Cylindrical Gas Vessels + Electric field shaper

To ensure the drift field quality, the inner layer of the field cage's wall will be a foil carrying **field strips** with 2.8 mm pitch.



Field cage of large prototype

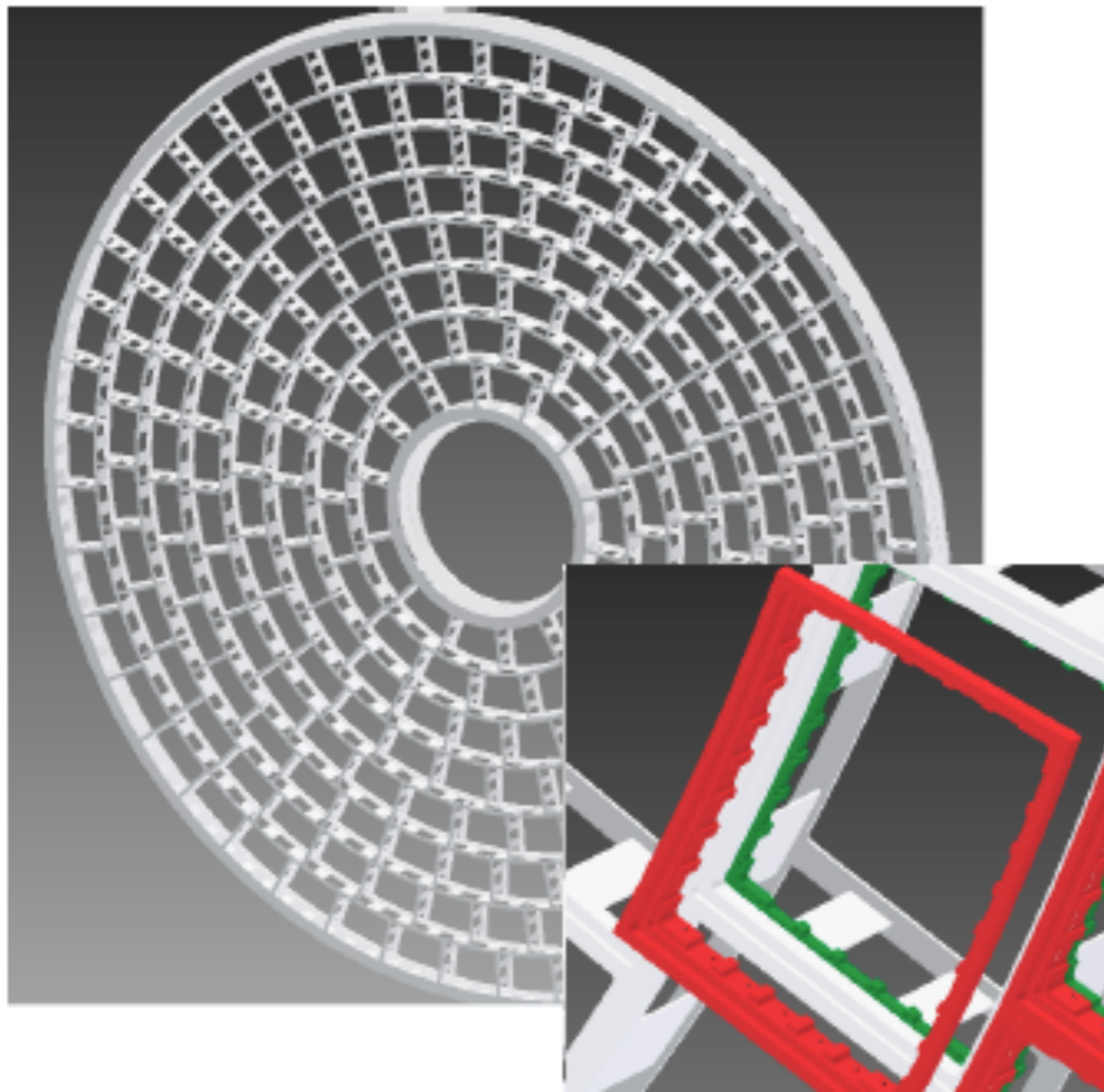


Inside of field cage of large prototype

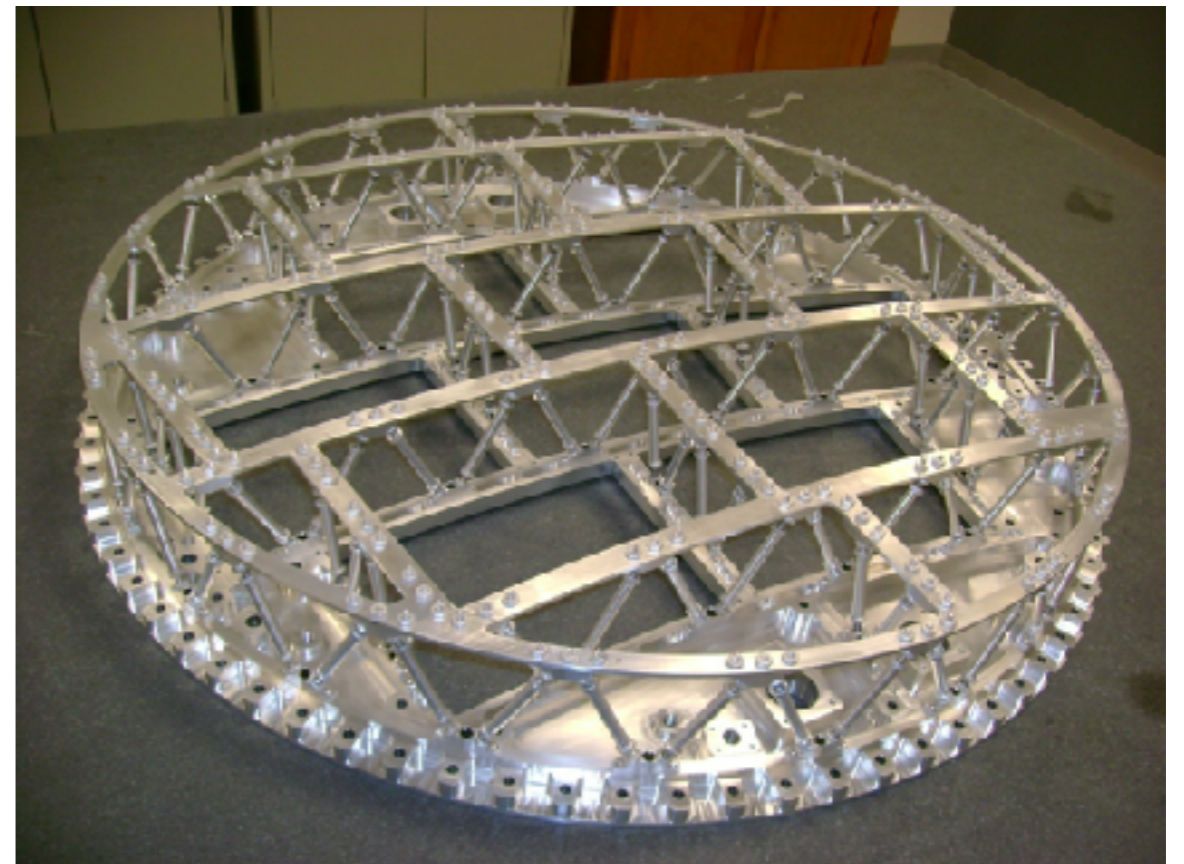
End Plate

Closes the TPC gas volume and supports the modules

“It is important that the endplate is designed to have low mass, while retaining the required mechanical and thermal stability” – ILC TDR vol4



End plate in TDR



End plate in large prototype

Spatial resolution of TPC

Fundamental processes in coordinate measurements

Charged particle $x = \tilde{x}$ **1 Ionizations**

→ Liberation of Electrons

$$\sum_{N=1}^{\infty} P_I(N; \bar{N}) \quad \phi=0 \quad \text{No } \delta\text{-rays}$$

Drift Volume



Drift electrons

1 Drift and Diffusion

Gaussian

$$P_D(\Delta x_i; \sigma_d) = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left[-\frac{1}{2}\left(\frac{\Delta x_i}{\sigma_d}\right)^2\right]$$

$$\sigma_d = C_d \sqrt{z} \quad C_d : \text{Transverse diffusion const.}$$

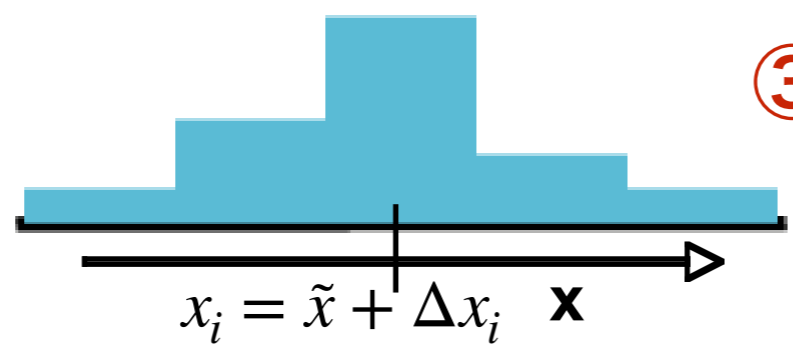
Amplification Gap

2 Amplification and further Diffusion

Readout Pads

$$P_G\left(\frac{G}{\bar{G}}; \theta\right) = \frac{(\theta + 1)^{\theta+1}}{\Gamma(\theta + 1)} \left(\frac{G}{\bar{G}}\right)^{\theta} \exp\left(-(\theta + 1) \left(\frac{G}{\bar{G}}\right)\right)$$

3 Pad Response \bar{G} : Gas gain average



Coordinate

Ionization Statistics

For simplicity, let's first consider the case in which the x-coordinate (arrival point) of each seed electron can be measured exactly

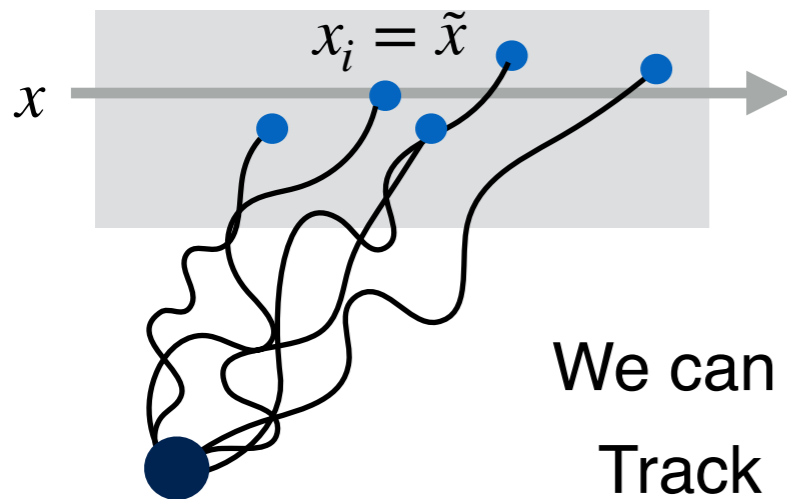
- Pad pitch $\rightarrow 0$ • No fluctuation of gas gain

$$P(\bar{x}; \tilde{x}) = \sum_{N=1}^{\infty} P_I(N; \bar{N}) \prod_{i=1}^N \left(\int_{-\infty}^{+\infty} d\Delta x_i P_D(\Delta x_i; \sigma_d) \right) \delta \left(\bar{x} - \frac{1}{N} \sum_{i=1}^N x_i \right)$$

Ionization statistics

Gaussian diffusion

Ideal readout pad



$$P_D(\Delta x_i; \sigma_d) = \frac{1}{\sqrt{2\pi\sigma_d}} \exp \left[-\frac{1}{2} \left(\frac{\Delta x_i}{\sigma_d} \right)^2 \right]$$

$$\sigma_d = C_d \sqrt{z} \quad C_d : \text{Transverse diffusion const.} \\ z : \text{Drift length}$$

We can assume $\tilde{x} = 0$

Track

$$\langle \bar{x} \rangle := \int d\bar{x} P(\bar{x}) \bar{x} = 0$$

Variance

$$\sigma_{\bar{x}}^2 := \int d\bar{x} P(\bar{x}) \bar{x}^2 = \sigma_d^2 \left\langle \frac{1}{N} \right\rangle =: \sigma_d^2 \frac{1}{N_{eff}}$$

$$N_{eff} := \frac{1}{\langle 1/N \rangle} < \langle N \rangle$$

Spatial resolution depends on
the inverse of the average of its inverse!

Gas gain fluctuation

Now include gas gain fluctuation.

• Pad pitch $\rightarrow 0$

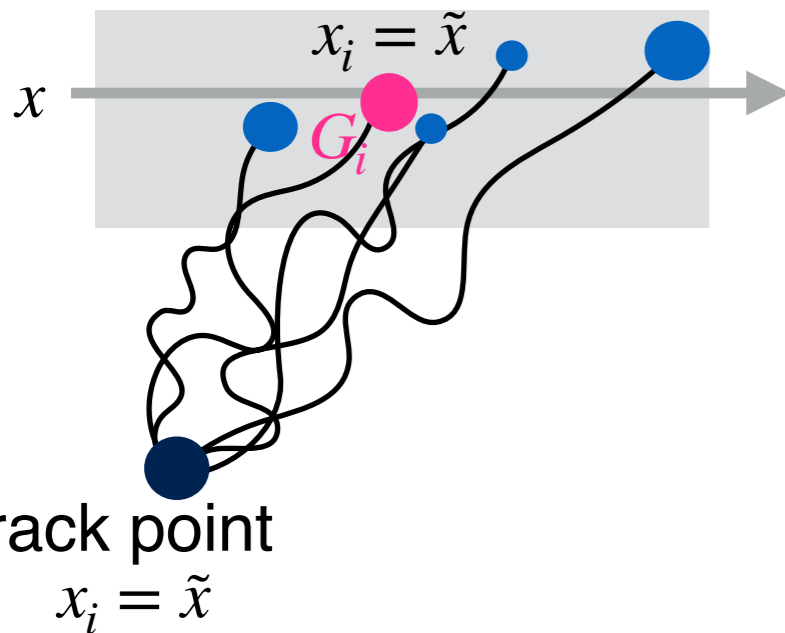
$$P(\bar{x}; \tilde{x}) = \sum_{N=1}^{\infty} P_I(N; \bar{N}) \prod_{i=1}^N \left[\int_{-\infty}^{+\infty} d\Delta x_i P_D(\Delta x_i; \sigma_d) \right]$$

• Gas gain fluctuation G_i

$$\times \int d\left(\frac{G_i}{\bar{G}}\right) P_G\left(\frac{G_i}{\bar{G}}; \theta\right) \delta\left(\bar{x} - \frac{\sum_{i=1}^N G_i x_i}{\sum_{i=1}^N G_i}\right)$$

Gas gain fluctuation

Gain-weighted mean



We can still assume $\tilde{x} = 0$

Track $\langle \bar{x} \rangle := \int d\bar{x} P(\bar{x}) \bar{x} = 0$

Variance $\sigma_{\bar{x}}^2 := \int d\bar{x} P(\bar{x}) \bar{x}^2 \approx \sigma_d^2 \left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle =: \sigma_d^2 \frac{1}{N_{eff}}$

We used $\sum_{i=1}^N G_i \approx N\bar{G}$ (If N is large enough)

$$N_{eff} := \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle \right]^{-1} < \langle N \rangle$$

gas gain fluctuation therefore **further reduces the effective number of electrons**

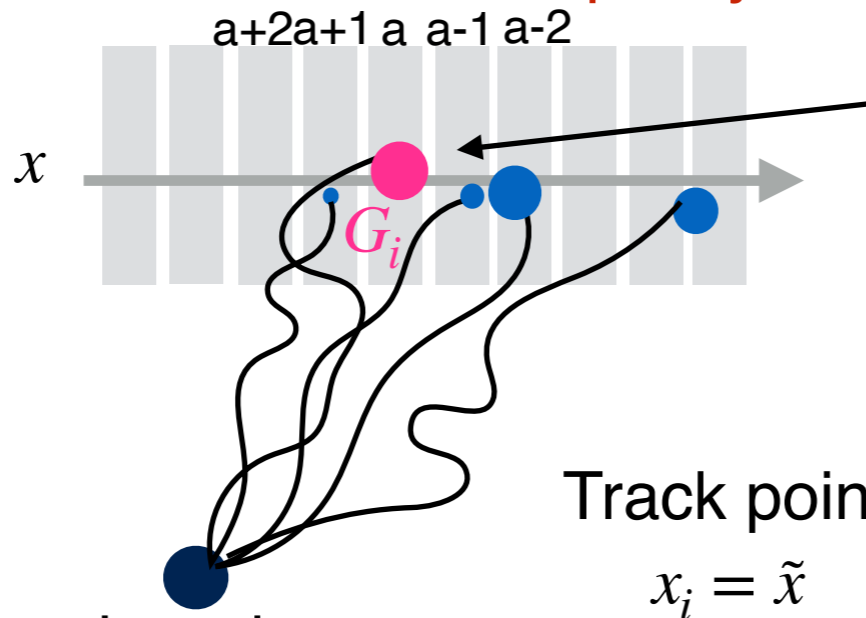
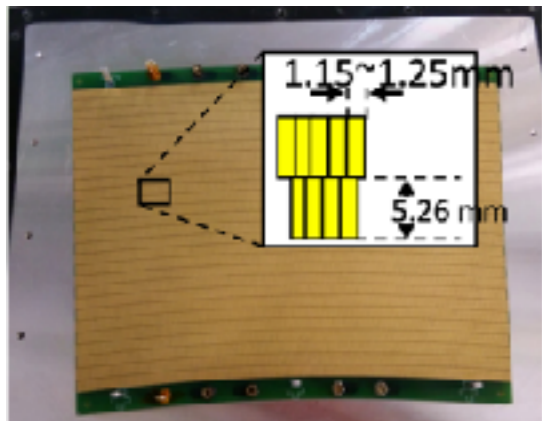
Finite Pad Pitch

Let's finally consider the effect of finite pad pitch!

- Finite pad pitch • Gas gain fluctuation G_i

We can no longer assume $\tilde{x} = 0$ because of breaking of translational invariance

→ Specify the track position relative to the pad center.



$$x_i = \tilde{x} + \Delta x_i$$

Track position

Diffusion $\langle (\Delta x_i)^2 \rangle = \sigma_d^2 = C_d^2 z$

Electric charge on a-th pad

$$Q_a = \sum_i G_i F_a(\tilde{x} + \Delta x_i) + \Delta Q_a$$

Gas gain for i-th seed electron

Normalized pad response function for a-th pad

Electronic noise

$$\langle (\Delta Q_a)^2 \rangle = \sigma_E^2$$

$$\sum_a F_a(\tilde{x} + \Delta x_i) = 1$$

The charge centroid method

Assume pad pitch is w

$$\bar{x} = \sum_a Q_a (aw) / \sum_a Q_a$$

Probability distribution Function for this method

$$P(\bar{x}; \tilde{x}) = \sum_{N=1}^{\infty} P_I(N; \bar{N}) \prod_{i=1}^N \left[\int_{-\infty}^{+\infty} d\Delta x_i P_D(\Delta x_i; \sigma_d) \int d\left(\frac{G_i}{\bar{G}}\right) P_G\left(\frac{G_i}{\bar{G}}; \theta\right) \right] \times \prod_a \left[\int d\Delta Q_a P_E(\Delta Q_a; \sigma_E) \int dQ_a \delta\left(Q_a - \sum_{i=1}^N G_i F_a(x_i) - \Delta Q_a\right) \right] \times \delta\left(\bar{x} - \frac{\sum_a Q_a (aw)}{\sum_a Q_a}\right)$$

Ionization statistics

Gaussian diffusion

Gas gain fluctuation

Electronic noise

Variance of charge centroid

We define $\sigma_{\tilde{x}}^2 := \int_{-1/2}^{1/2} d\left(\frac{\tilde{x}}{w}\right) \int d\bar{x} P(\bar{x}; \tilde{x}) (\bar{x} - \tilde{x})^2$ Average over track position \tilde{x} in the pad pitch w

Integrate over $\Delta Q_a, \Delta x_i, \frac{G_i}{\bar{G}}$ and average over $N \rightarrow \sigma_x^2 = [A] + \frac{1}{N_{eff}} [B] + [C]$

Systematic error of the charge centroid method

$$[A] = \int_{-1/2}^{1/2} d\left(\frac{\tilde{x}}{w}\right) \left(\sum_a (aw) \langle F_a(\tilde{x} + \Delta x) \rangle_{\Delta x} - \tilde{x} \right)^2$$

Purely geometric effect

The diffusion term (Gas gain fluctuation & finite pad pitch)

$$[B] = \int_{-1/2}^{1/2} d\left(\frac{\tilde{x}}{w}\right) \left[\sum_{a,b} (abw^2) \langle F_a(\tilde{x} + \Delta x) F_b(\tilde{x} + \Delta x) \rangle_{\Delta x} - \left(\sum_a (aw) \langle F_a(\tilde{x} + \Delta x) \rangle \right)^2 \right]$$

Asymptotic behavior

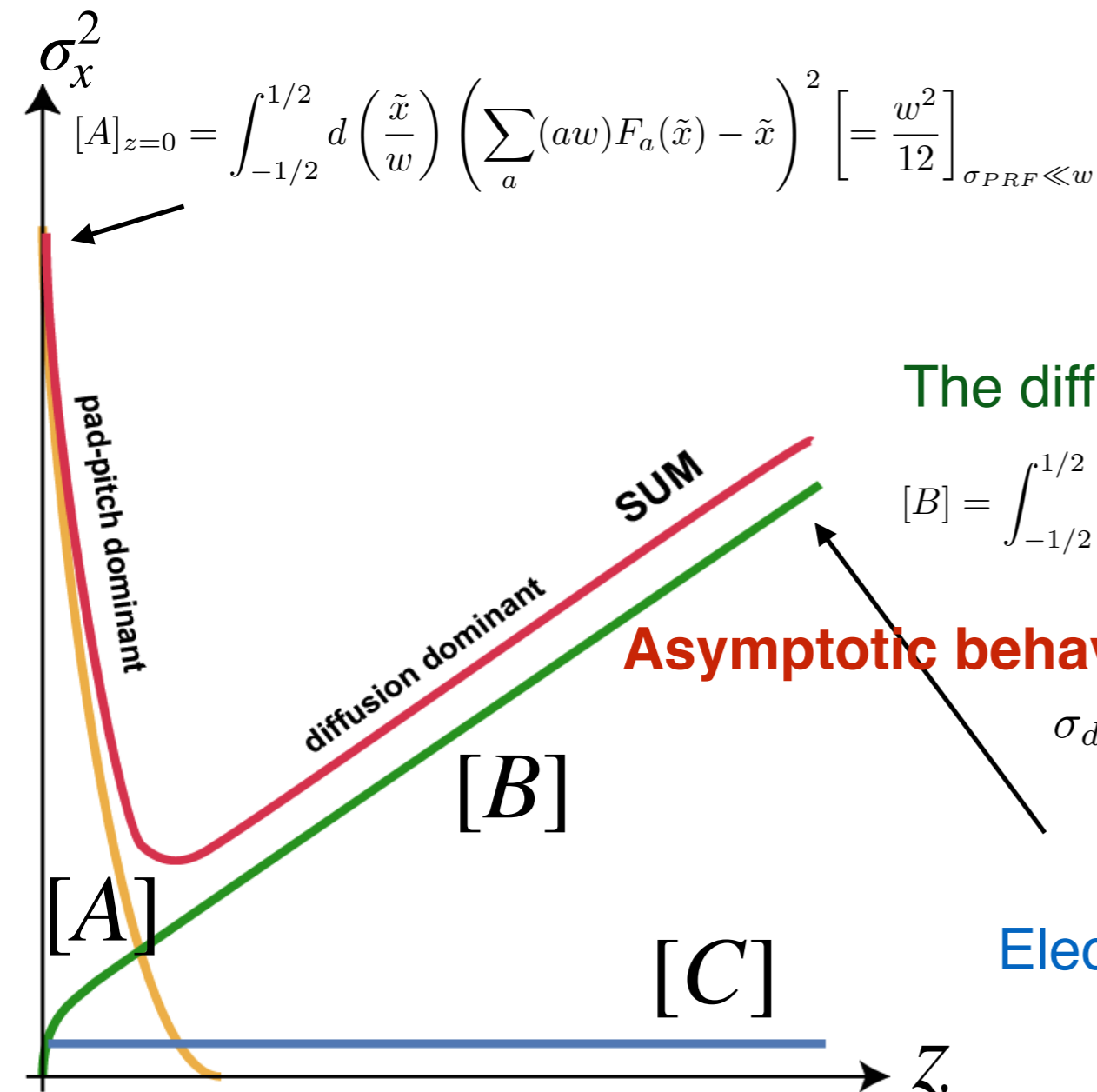
$\sigma_d/w \gg 1$ (long drift distance)

Hodscope effect

$$\sigma_x^2 \approx \frac{1}{N_{eff}} \left[[A]_{z=0} + \sigma_d^2 \right] \left[= \frac{1}{N_{eff}} \left(\frac{w^2}{12} + \sigma_d^2 \right) \right]_{\sigma_{PRF} \ll w}$$

Electric noise

$$[C] = \left(\frac{\sigma_E}{\bar{G}} \right)^2 \left\langle \frac{1}{N^2} \right\rangle_N \sum_a (aw)^2$$



Spatial resolution

$$\sigma_x = \sigma_x(z; w, C_d, N_{eff}, F_a)$$

Drift distance

The number of effective track electrons

Pad pitch

Diffusion const.

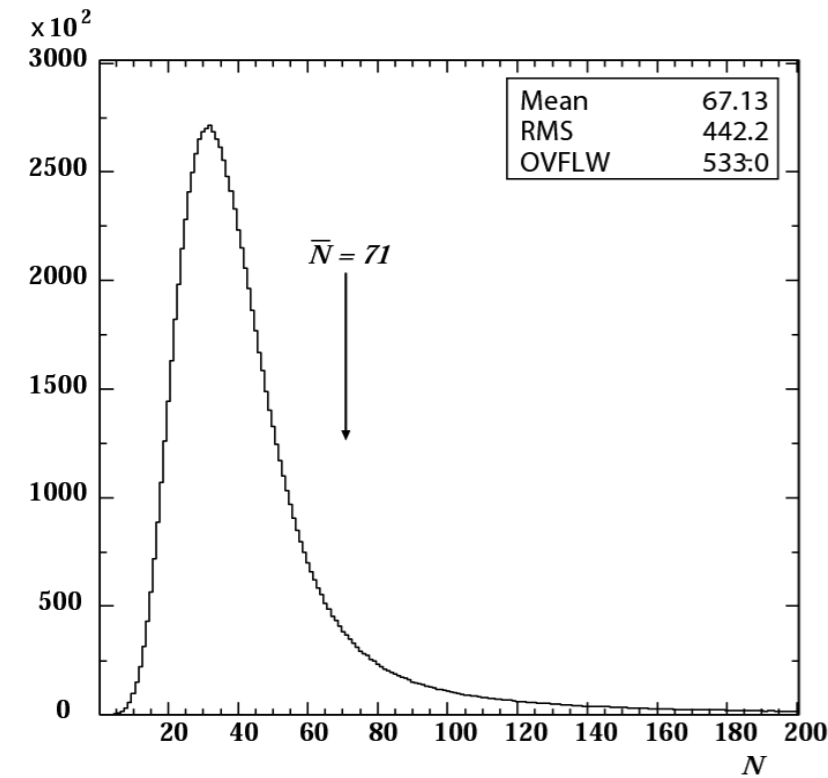
Pad response function

- We can **analytically estimate** the spatial resolution.
- We can improve the spatial resolution based on theoretical basis!

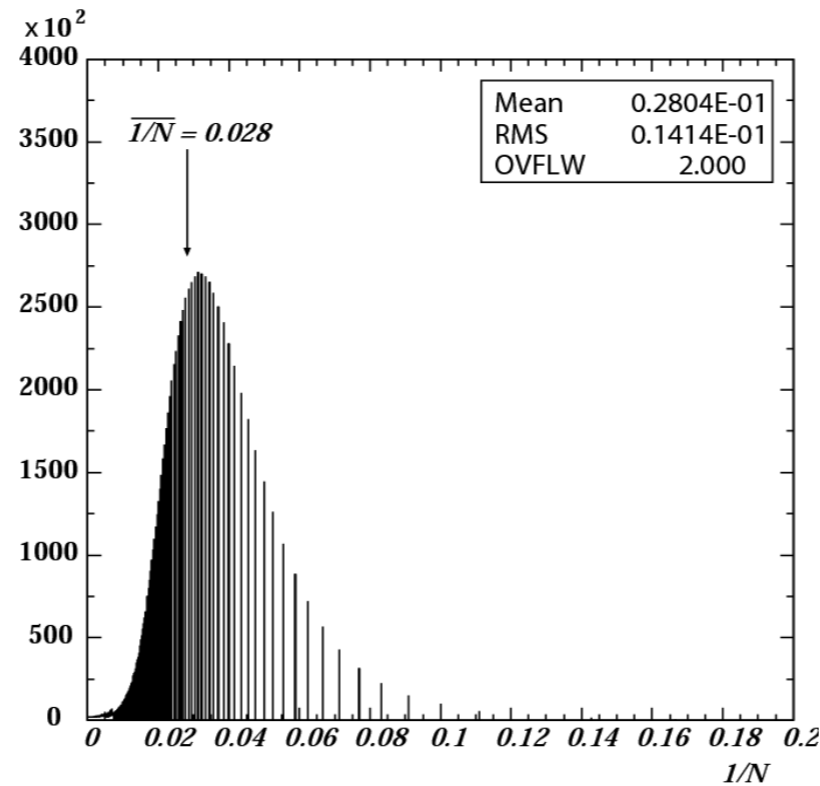
N_{eff} in typical model

4 GeV pion, pad pitch 6mm, pure Ar

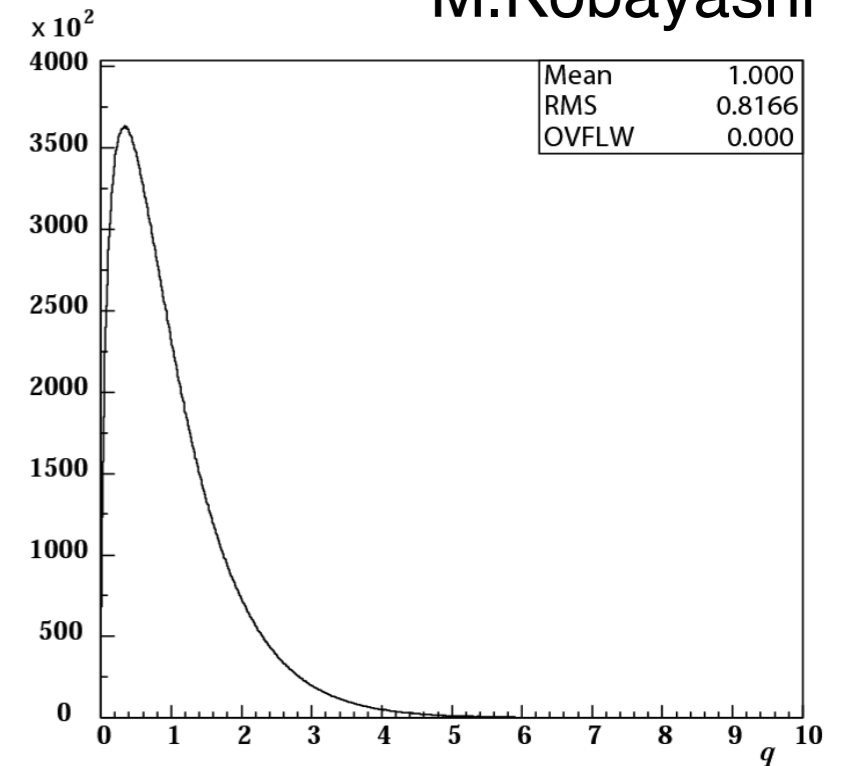
M.Kobayashi



Distribution of N
($\langle N \rangle = 71$)



Distribution of $1/N$
($\langle 1/N \rangle = 0.028$)

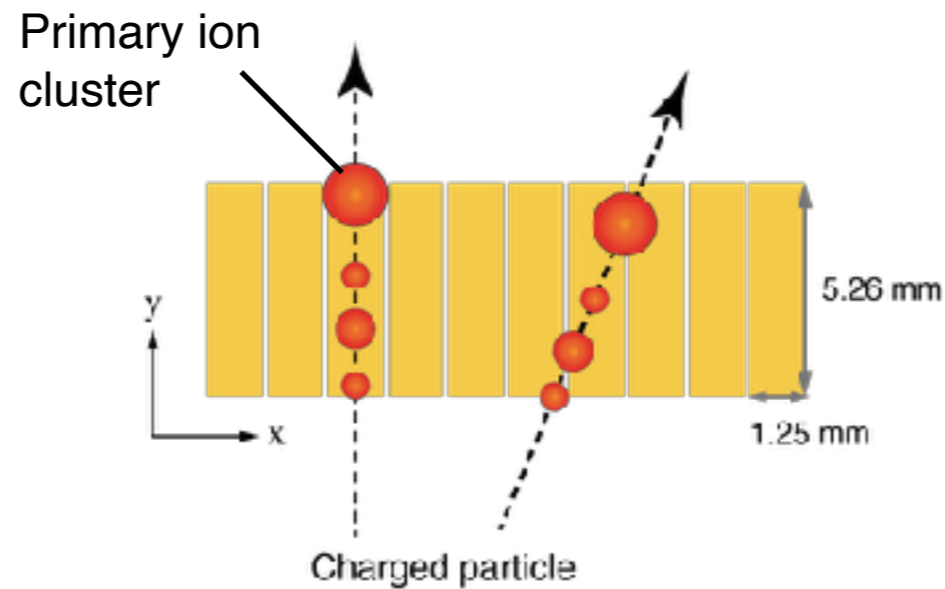


Distribution of Q
($K = 0.67$)

$$\left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle = 1 + \left(\frac{\sigma_G}{\bar{G}} \right)^2 \equiv 1 + K$$

$$N_{eff} := \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle \right]^{-1} = 21 < \frac{1}{\langle 1/N \rangle} = 36 < \langle N \rangle = 71$$

Incident angle effect on the spatial resolution



$$\sigma_x^2(Z; w, L \tan \phi, C_d, N_{eff}, \hat{N}_{eff}, [f]) = [A] + \frac{1}{N_{eff}} [B] + [C] + \frac{1}{\hat{N}_{eff}} [D]$$

Hodoscope effect
(S-shape systematics)
important for the short
drift distance

Effects of diffusion
+ gas gain fluctuation
+ finite pad pitch

Electric noise

Angular Pad effect

$$\frac{1}{\hat{N}_{eff}} [D] = \frac{L^2 \tan^2 \phi}{12 \hat{N}_{eff}} \quad L : \text{Pad row pitch}$$

Analytic expressions for N_{eff} and \hat{N}_{eff}

$$\sigma_x^2(Z; w, L \tan \phi, C_d, N_{eff}, \hat{N}_{eff}, [f]) = [A] + \frac{1}{N_{eff}} [B] + [C] + \frac{1}{\hat{N}_{eff}} [D]$$

The effective number of **electrons**

$$N_{eff} = \left[\left\langle \sum_{i=1}^N \sum_{j=1}^{k_i} \left\langle \left(\frac{G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right)^2 \right\rangle_{G} \right\rangle_{N,k}^{k_i, \sum_{i=1}^N k_i} \right]^{-1} \approx \left[\left\langle \frac{1}{\sum_i k_i} \right\rangle_{N,k} \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle_G \right]^{-1}$$

gas gain fluctuation

electrons

The effective number of **clusters**

$$\hat{N}_{eff} \approx \left[\left\langle \sum_{i=1}^N \left\langle \left(\frac{\sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right)^2 \right\rangle_{G} \right\rangle_{N,k}^{k_i, \sum_{i=1}^N k_i} \right]^{-1} \sim \left[\left\langle \frac{1}{N} \right\rangle_N \left\langle \left(\frac{\hat{G}}{\bar{\hat{G}}} \right)^2 \right\rangle_{k,G} \right]^{-1}$$

cluster size fluctuation
+ gas gain fluctuation

clusters

$\sim N \hat{G}$

Notice

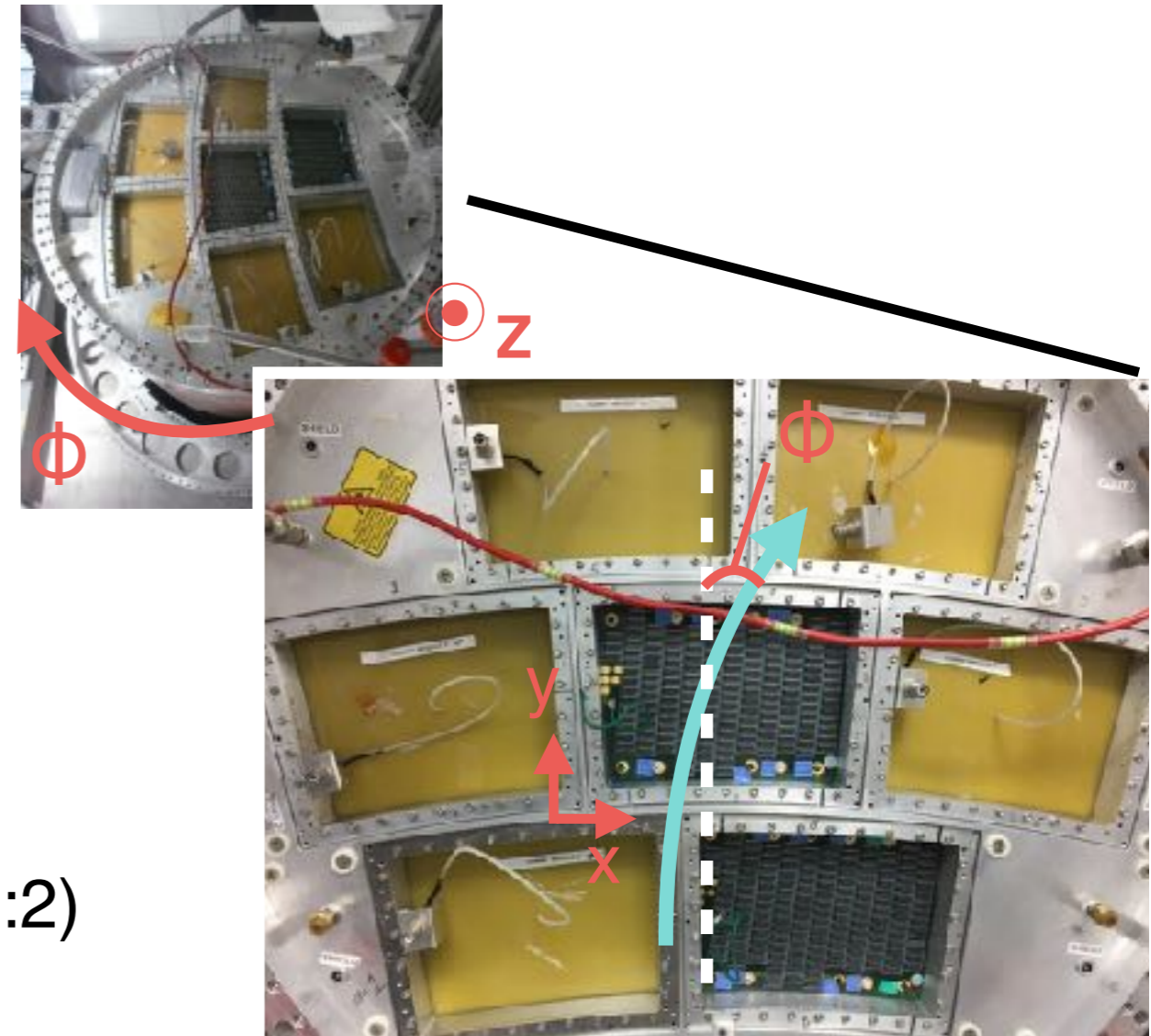
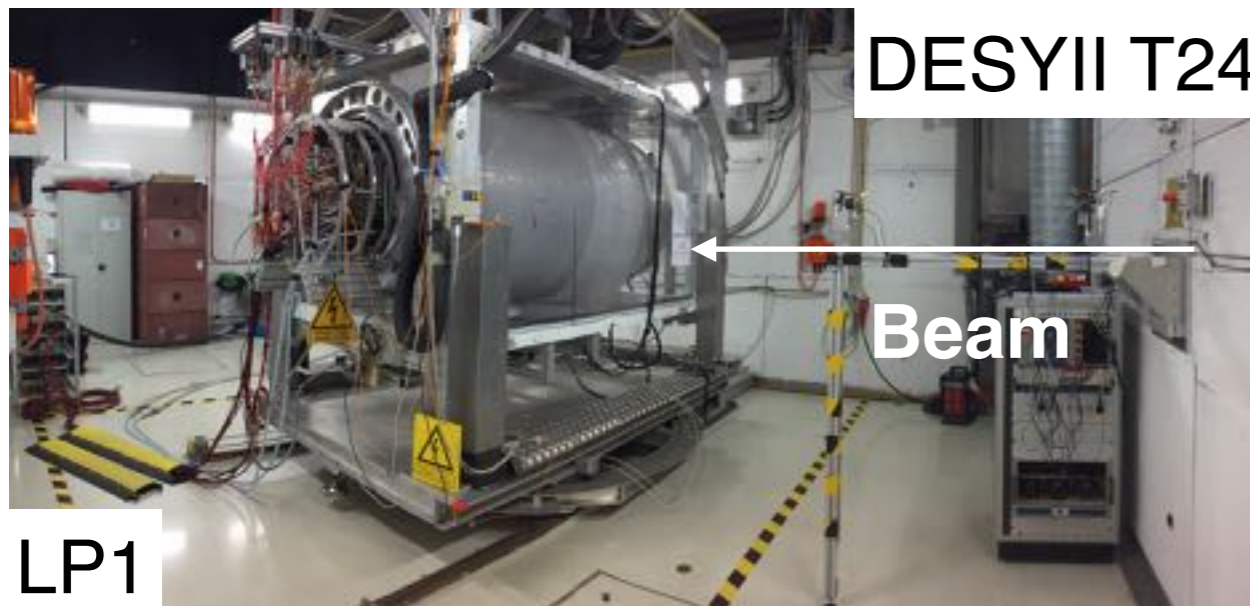
$$\bar{G} \approx \frac{\sum_i \sum_j G_{ij}}{\sum_i k_i} \longrightarrow \hat{N}_{eff} \approx \left[\left\langle \frac{\sum_{i=1}^N k_i^2}{\left(\sum_{i=1}^N k_i \right)^2} \right\rangle + \frac{1}{N_{eff}} - \frac{1}{N_{eff}^{w/o G}} \right]^{-1} \left(N_{eff}^{w/o G} \approx \left[\left\langle \frac{1}{\sum_i k_i} \right\rangle_{N,k} \right]^{-1} \right)$$

w/ gas gain fluctuation w/o gas gain fluctuation

**Beam test of TPC
(Large prototype @DESY)**

Beam test

The original purpose of the beam test was to compare performance of the Asian modules with and without the gating foil.



Set up

- ▶ Electron Beam = 5 GeV
- ▶ $B = 1 \text{ T}$
- ▶ T2K gas (Ar:CF₄:iso-C₄H₁₀ = 95:3:2)
- ▶ Analysis frame work : Marlin TPC
- ▶ 20000evt / 1 run
- ▶ Data set used : $\phi = -20^\circ, 0^\circ, 10^\circ, 20^\circ$
 - ◆ For this analysis we use data taken without the gating foil

