

Fermion Pair Production at the ILC in GUT Inspired Gauge-Higgs Unification

Department of Physics, Kyushu University
Naoki Yamatsu

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This talk is mainly based on Ref. [1, [Phys.Rev.D102\(2020\)015029](#)]
collaborated with Shuichiro Funatsu (CCNU), Hisaki Hatanaka,
Yutaka Hosotani (Osaka U.), Yuta Oriyasa (Czech Tech. U.)
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Short summary of this talk

We discuss fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired gauge-Higgs unification (GHU).

- (1) Due to the Z' bosons coupling to quarks and leptons with large parity violation, several observables in $e^-e^+ \rightarrow f\bar{f}$ ($= \mu^- \mu^+, \dots$) processes have large polarization dependence.
- (2) Deviations from the SM are testable for the Kaluza-Klein (KK) mass scale up to ~ 15 TeV in the early stage of ILC.

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Gauge-Higgs unification (GHU) and Z' bosons

Gauge Higgs unification (GHU)

- Higgs boson as the zero mode of the 5th dimensional component of the 5D gauge field [2–7, Y.Hosotani'83;A.T.Davies,A.McLachlan'88;H.Hatanaka et al'98;...]

$SU(3) \times SO(5) \times U(1)$ GHU in the 5D Randall-Sundrum warped space (GHU A-model) [8–20, K.Agashe et al.'05;A.D.Medina et al.'07;Y.Hosotani et al.'08;...]

- Phenomenologies at EW scale are almost the same as the SM [12, 13, 15, 16, S.Funatsu et al.'13-16;...] for $\theta_H \lesssim 0.11$, $m_{\text{KK}} \gtrsim 9\text{TeV}$;

E.g., Higgs couplings of quarks, leptons, W and Z bosons are approximately the SM values times $\cos \theta_H$; the deviation is about 1% for $\theta_H \simeq 0.1$.

- KK excited neutral vector bosons $\gamma^{(n)}$, $Z^{(n)}$, $Z_R^{(n)}$ ($n \geq 1$) can be identified with Z' bosons. ($\gamma^{(1)}$, $Z^{(1)}$, $Z_R^{(1)}$ have KK scale masses $O(m_{\text{KK}})$.)

GHU A-model at e^-e^+ colliders

[17, 21–25, S.Funatsu et al.'17; J.Yoon, M.E.Peskin'18; S.Funatsu'19; F.Richard'18; A.Irles et al'20]

- Z' bosons couple to quarks and leptons with large parity violation.
Right-handed quarks and leptons have rather large couplings to Z' boson.
- ⇒ Amplitudes of $e^-e^+ \rightarrow f\bar{f}$ ($= \mu^-\mu^+, \dots$) have large polarization dependence.
- ⇒ Signal strength of cross section and forward-backward (FB) asymmetry becomes larger by using initial polarized e^- and e^+ states.
- ⇒ The masses up to about 10TeV can be explored at 250 GeV ILC with 250 fb^{-1} data. [17, 22, 23, S.Funatsu et al.'17; J.Yoon, M.E.Peskin'18; S.Funatsu'19]

In the talk, we discuss fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired GHU (B-model).

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GUT inspired GHU (GHU B-model)

GUT inspired GHU (B-model) [18–20, S.Funatsu et al.'19;...].

- Matter content inspired by $SO(11)$ GHGUT ($SO(11) \supset SU(3) \times SO(5) \times U(1)$) [26, 27, Y.Hosotani, N.Yamatsu'15;...].
- Quarks and leptons belong to spinor **4** or singlet **1** of $SO(5)$ in the B-model. (They belong to vector **5** of $SO(5)$ in the GHU A-model.)
- Electro-weak symmetry breaking pattern is the same as that in the A-model

$$SO(5) \times U(1)_X \xrightarrow{BC_s} \underbrace{SO(4)}_{SU(2)_L \times \cancel{SU(2)_R}} \times U(1)_X \xrightarrow{\theta_H \neq 0} U(1)_{EM}.$$

- First KK photon $\gamma^{(1)}$, $Z^{(1)}$, $Z_R^{(1)}$ have about KK scale masses.

$SU(3) \times SO(5) \times U(1)$ GHU: A- and B-models

	B-model (=GUT inspired)	A-model
Quark Lepton	$(\mathbf{3}, \mathbf{4})_{\frac{1}{6}}$ $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}^+$ $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}^-$ $(\mathbf{1}, \mathbf{4})_{-\frac{1}{2}}$	$(\mathbf{3}, \mathbf{5})_{\frac{2}{3}}$ $(\mathbf{3}, \mathbf{5})_{-\frac{1}{3}}$ $(\mathbf{1}, \mathbf{5})_0$ $(\mathbf{1}, \mathbf{5})_{-1}$
Dark fermion	$(\mathbf{3}, \mathbf{4})_{\frac{1}{6}}$ $(\mathbf{1}, \mathbf{5})_0^+$ $(\mathbf{1}, \mathbf{5})_0^-$	$(\mathbf{1}, \mathbf{4})_{\frac{1}{2}}$
Brane fermion	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{3}, [\mathbf{2}, \mathbf{1}])_{\frac{7}{6}, \frac{1}{6}, -\frac{5}{6}}$ $(\mathbf{1}, [\mathbf{2}, \mathbf{1}])_{\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}}$
Brane scalar	$(\mathbf{1}, \mathbf{4})_{\frac{1}{2}}$	$(\mathbf{1}, [\mathbf{1}, \mathbf{2}])_{\frac{1}{2}}$

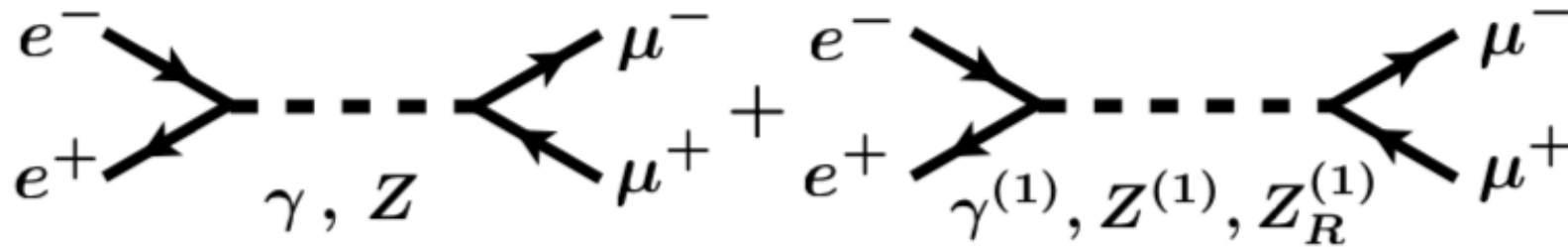
B-model = the $SO(11)$ GUT inspired model [18–20, S.Funatsu et al.'19-20];

A-model = the (non-GUT inspired) previous model [12–17, S.Funatsu et al.'13-17].

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Observables in $e^-e^+ \rightarrow \{V_i\} \rightarrow f\bar{f}$ ($f\bar{f} = \mu^-\mu^+, \dots$)



Observables	(Symbol)	Unpolarized	Polarized
Cross sections	$(\sigma^{f\bar{f}})$	Yes	Yes
FB asymmetries	$(A_{FB}^{f\bar{f}})$	Yes	Yes
LR asymmetries	$(A_{LR}^{f\bar{f}})$	No	Yes
LR FB asymmetries	$(A_{LR,FB}^{f\bar{f}})$	No	Yes

Forward-backward (FB) asymmetries, left-right (LR) and LR FB asymmetries are studied in e.g., Refs. [28–32, B.Schrempp et al'88;D.C.Kennedy et al'89;SLD Collaboration'94-95].

Polarized e^-e^+ to $f\bar{f}$ production

s -channel process of $e^-e^+ \rightarrow f\bar{f}$ expressed by 4 quantities $Q_{e_X f_Y}$ ($X, Y = L, R$).

$$Q_{e_X f_Y} = \sum_i \frac{g_{V_i e}^X g_{V_i f}^Y}{(s - m_{V_i}^2) + im_{V_i} \Gamma_{V_i}}, \quad P_{\text{eff}} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}},$$

Obs.	Linear combination of observed amplitudes (Overall factor is omitted, $\sqrt{s} \gg m_f$)
σ	$(1 - P_{\text{eff}})(Q_{e_L f_L} ^2 + Q_{e_L f_R} ^2) + (1 + P_{\text{eff}})(Q_{e_R f_R} ^2 + Q_{e_R f_L} ^2)$
A_{FB}	$(1 - P_{\text{eff}})(Q_{e_L f_L} ^2 - Q_{e_L f_R} ^2) + (1 + P_{\text{eff}})(Q_{e_R f_R} ^2 - Q_{e_R f_L} ^2)$
A_{LR}	$ Q_{e_L f_L} ^2 + Q_{e_L f_R} ^2 - Q_{e_R f_R} ^2 - Q_{e_R f_L} ^2$
$A_{LR,FB}$	$ Q_{e_L f_L} ^2 - Q_{e_L f_R} ^2 - Q_{e_R f_R} ^2 + Q_{e_R f_L} ^2$

In the following, we will show the results of $e^-e^+ \rightarrow \mu^-\mu^+$ process with polarized e^-e^+ , $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$ at $\sqrt{s} = 250$ GeV with 250 fb^{-1} data in the GHU B-model.

[Setup] Parameter set GHU(B)

Name	θ_H [rad.]	m_{KK} [TeV]	z_L	k [GeV]	$m_{\gamma(1)}$ [TeV]	$\Gamma_{\gamma(1)}$ [TeV]	$m_{Z(1)}$ [TeV]	$\Gamma_{Z(1)}$ [TeV]	$m_{Z_R(1)}$ [TeV]	$\Gamma_{Z_R(1)}$ [TeV]
B ^L	0.10	11.0	2.0×10^8	6.9×10^{11}	8.7	2.1	8.7	4.8	8.4	0.60
B	0.10	13.0	3.9×10^{11}	1.6×10^{15}	10.2	3.3	10.2	7.8	10.0	0.82
B ^H	0.10	15.0	2.7×10^{15}	1.3×10^{19}	11.7	4.9	11.7	11.8	11.5	1.3

f	g_{Zf}^L	g_{Zf}^R	$g_{Z(1)f}^L$	$g_{Z(1)f}^R$	$g_{Z_R(1)f}^L$	$g_{Z_R(1)f}^R$	$g_{\gamma(1)f}^L$	$g_{\gamma(1)f}^R$
e	-0.3058	0.2629	-1.7621	-0.0584	-1.0444	0	-2.7587	0.1071
μ	-0.3058	0.2629	-1.6778	-0.0584	-0.9969	0	-2.6268	0.1071
τ	-0.3058	0.2629	-1.6218	-0.0584	-0.9652	0.0001	-2.5391	0.1070

- The parameter set GHU(B) $(\theta_H, m_{KK}) = (0.10, 13\text{TeV})$ is the benchmark in our calculation. (Coupling constants in units of $g_w = e/\sin\theta_W^0$.)

We first fix the values of θ_H and m_{KK} , and then determine the other parameters. (For more information, see Ref. [1, S.Funatsu et al.'20].)

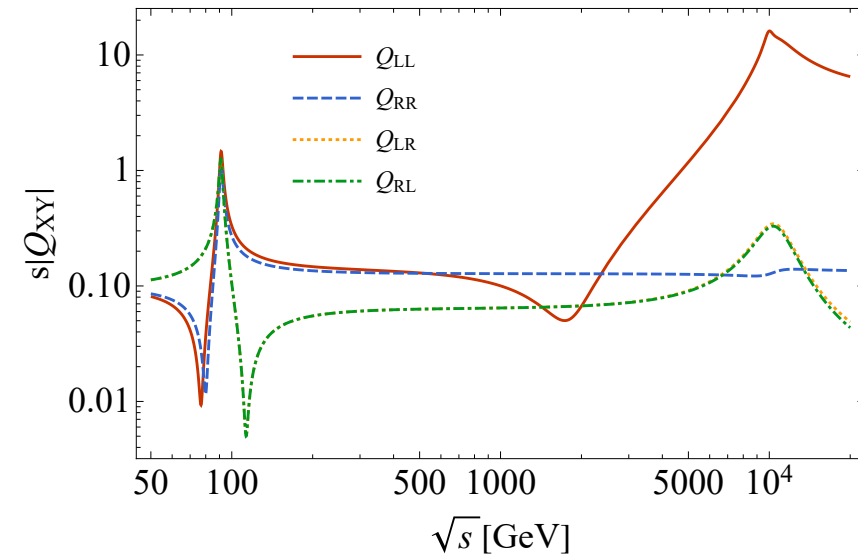
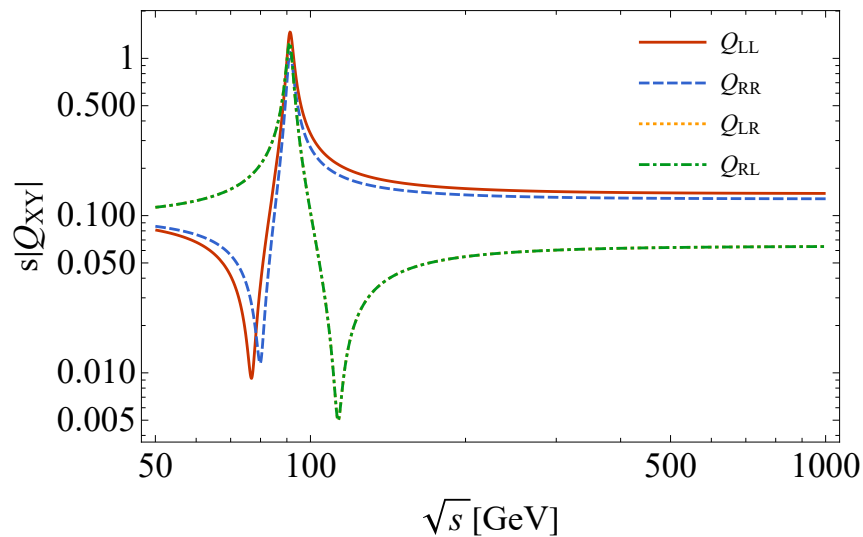
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[Calculation:1/5] Amplitude

$$Q_{e_X f_Y} = \sum_{V_i} \frac{g_{V_e}^X g_{V_f}^Y g_w^2}{(s - m_V^2) + im_V \Gamma_V}, \quad V_i = \begin{cases} \gamma, Z, & \text{for SM,} \\ \gamma, Z, \gamma^{(1)}, Z^{(1)}, Z_R^{(1)} & \text{for GHU} \end{cases} \cdot$$

$e^- e^+ \rightarrow \mu^- \mu^+$



- The amplitude $s|Q_{XY}|$ ($X, Y = L, R$) vs \sqrt{s} [GeV] for the SM (left figure) and the GHU (B) (right figure) are shown, Q_{XY} stands for $Q_{e_X f_Y}(e^- e^+ \rightarrow \mu^- \mu^+)$.

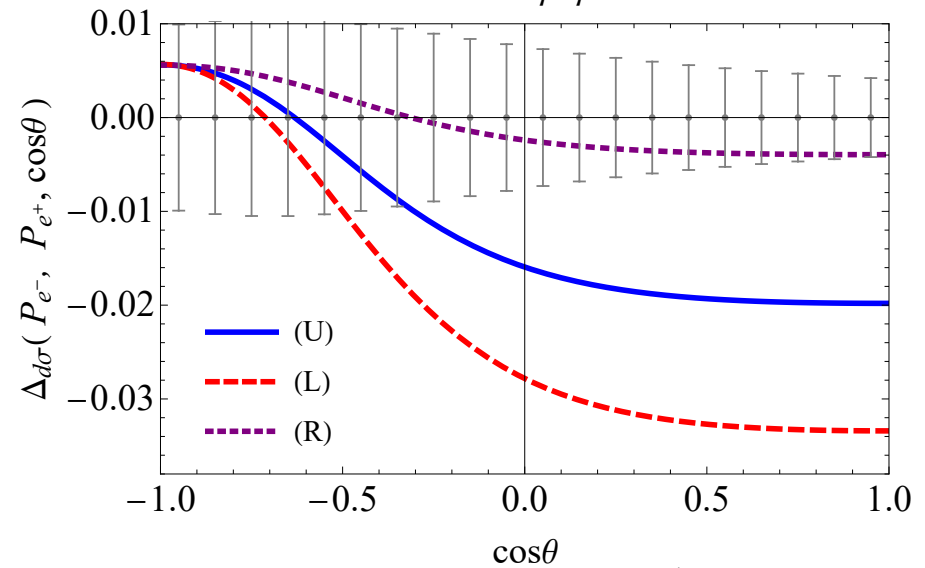
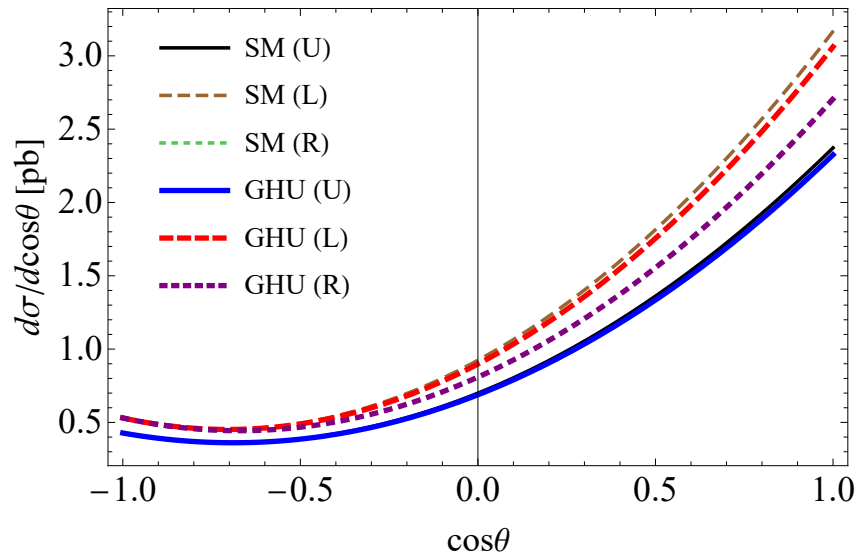
[Calculation:2/5] Differential cross section

$$\frac{d\sigma_{LR}^{f\bar{f}}}{d\cos\theta}(\cos\theta) \stackrel{m_f=0}{=} \frac{s}{32\pi} \left\{ (1+\cos\theta)^2 |Q_{e_L f_L}|^2 + (1-\cos\theta)^2 |Q_{e_L f_R}|^2 \right\},$$

$$\frac{d\sigma_{RL}^{f\bar{f}}}{d\cos\theta}(\cos\theta) \stackrel{m_f=0}{=} \frac{s}{32\pi} \left\{ (1+\cos\theta)^2 |Q_{e_R f_R}|^2 + (1-\cos\theta)^2 |Q_{e_R f_L}|^2 \right\}.$$

$e^-e^+ \rightarrow \mu^- \mu^+$

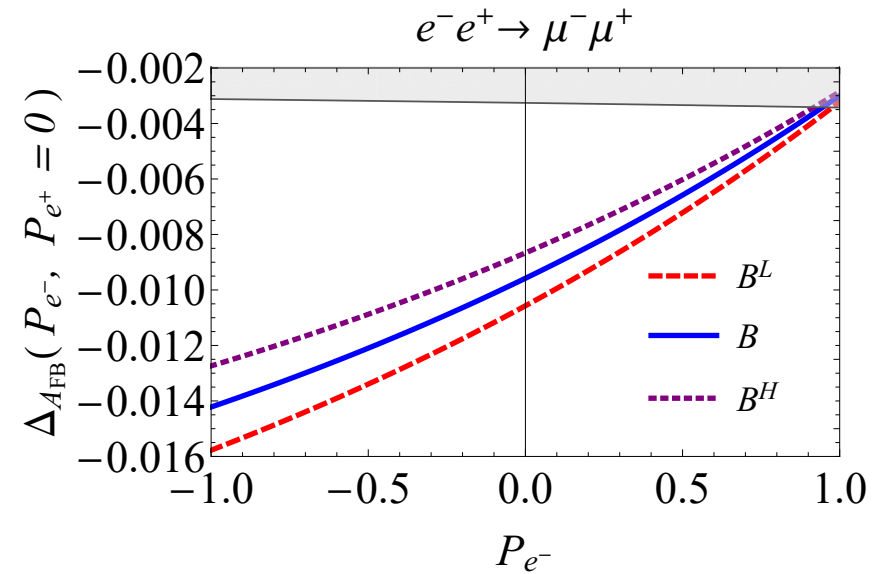
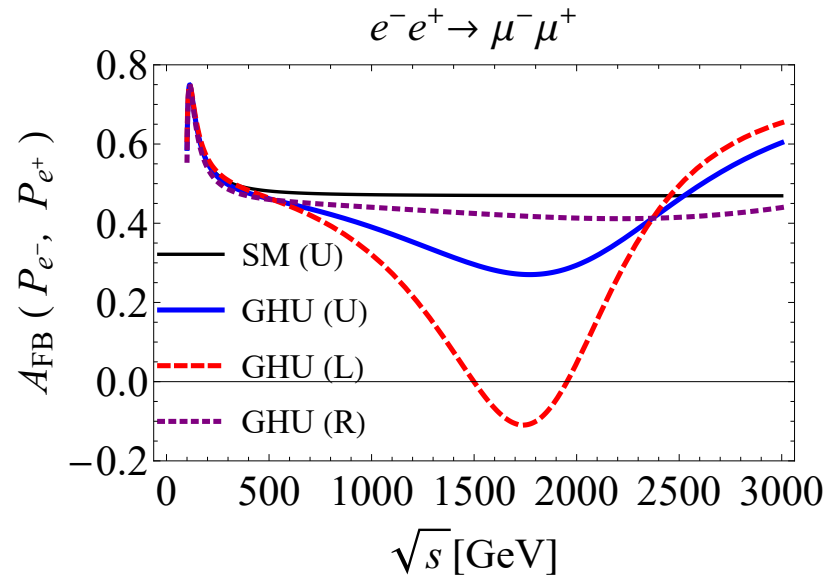
$e^-e^+ \rightarrow \mu^- \mu^+$



- $(P_{e^-}, P_{e^+}) = (0, 0)(U), (-0.8, +0.3)(L), (+0.8, -0.3)(R); \sqrt{s} = 250 \text{ GeV}, 250 \text{ fb}^{-1}$

[Calculation:3/5] Forward-backward asymmetry

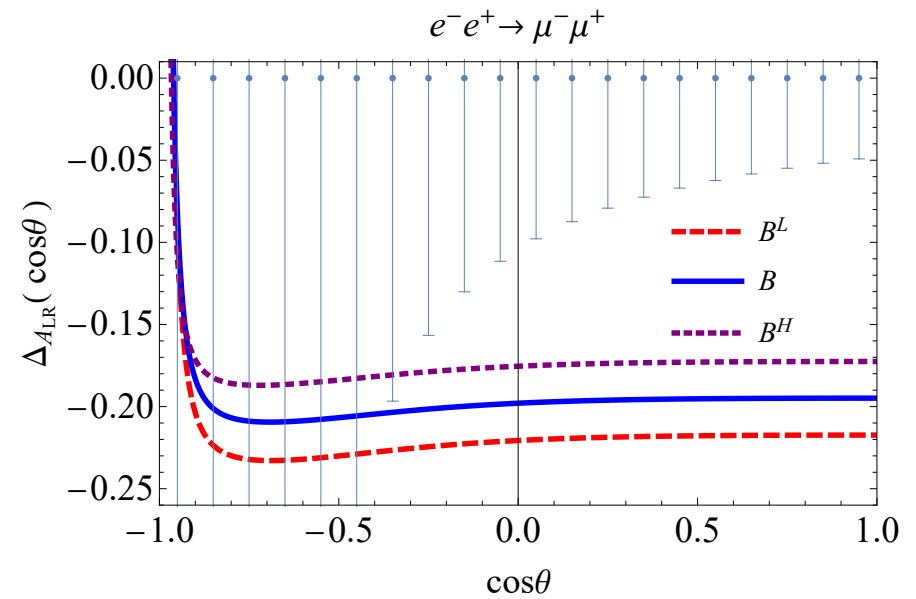
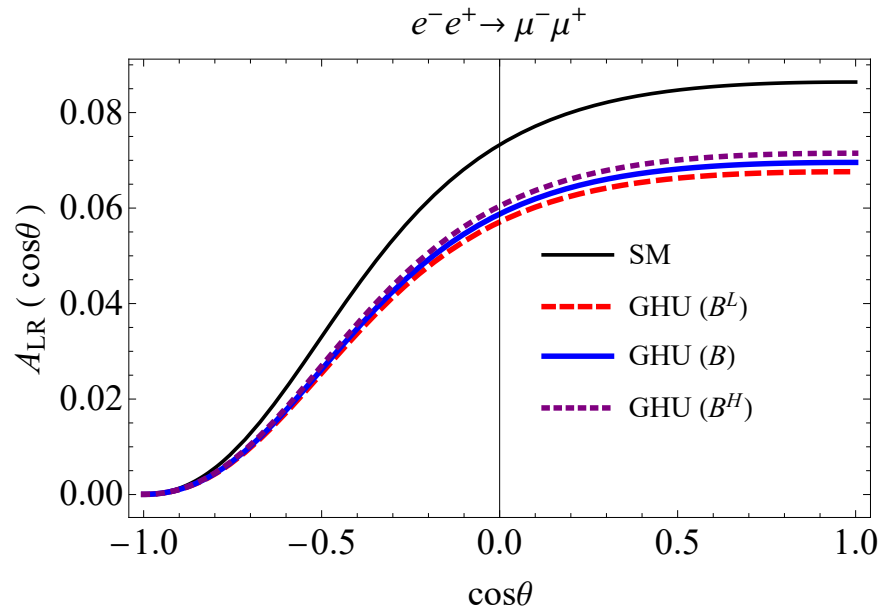
$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \frac{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) - \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) + \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}.$$



- (left figure) $(P_{e^-}, P_{e^+}) = (0, 0)(U), (-0.8, +0.3)(L), (+0.8, -0.3)(R)$;
 (right figure) GHU $\{(B^L), (B), (B^H)\} \leftrightarrow m_{KK} = \{11, 13, 15\} \text{ TeV}; \sqrt{s} = 250 \text{ GeV}, 250 \text{ fb}^{-1}$.

[Calculation:4/5] Left-right asymmetry

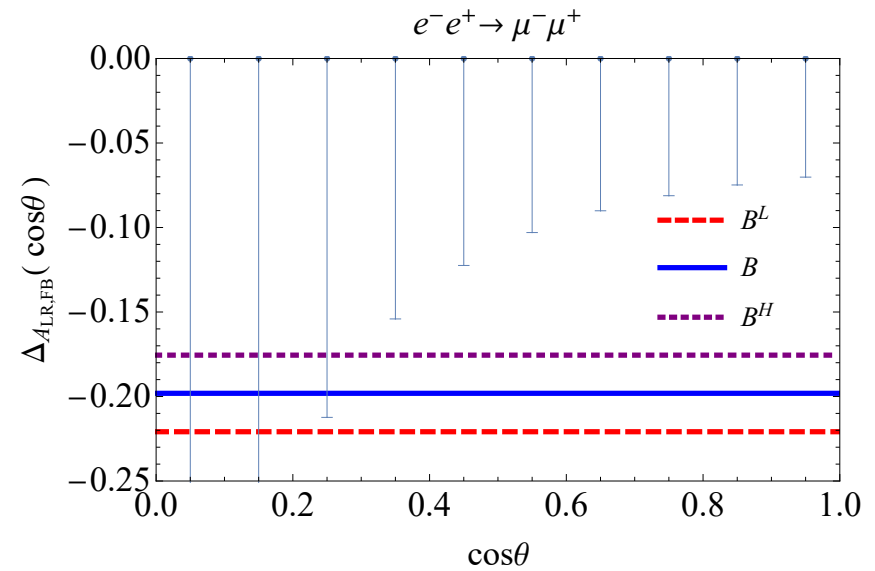
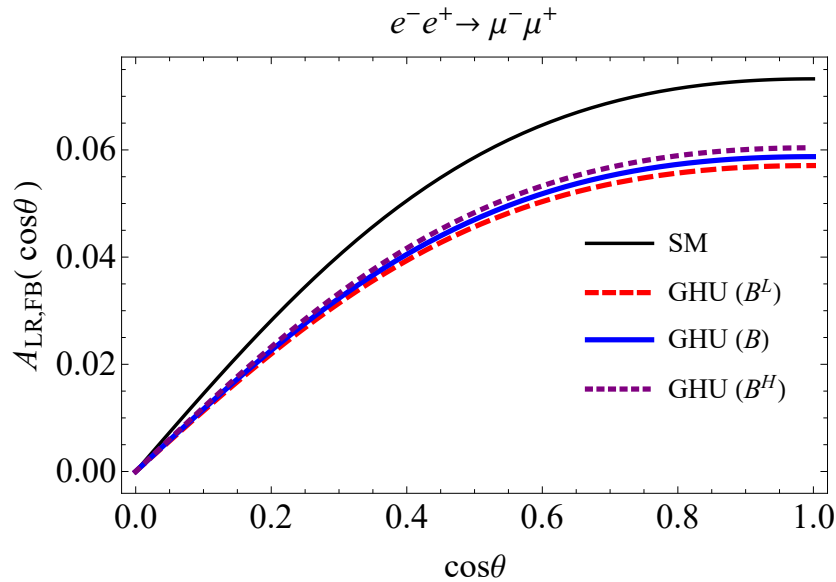
$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{\sigma_{LR}^{f\bar{f}}(\cos\theta) - \sigma_{RL}^{f\bar{f}}(\cos\theta)}{\sigma_{LR}^{f\bar{f}}(\cos\theta) + \sigma_{RL}^{f\bar{f}}(\cos\theta)}.$$



- GHU $\{(B^L), (B), (B^H)\} \leftrightarrow m_{KK} = \{11, 13, 15\} \text{ TeV}; \sqrt{s} = 250 \text{ GeV}, 250 \text{ fb}^{-1}; (P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$

[Calculation:5/5] LR FB asymmetries

$$A_{LR,FB}^{f\bar{f}}(\cos\theta) = \frac{\left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}} \right] (\cos\theta) - \left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}} \right] (-\cos\theta)}{\left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}} \right] (\cos\theta) + \left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}} \right] (-\cos\theta)} \stackrel{m_f \rightarrow 0}{\propto} \frac{2 \cos\theta}{1 + \cos^2\theta}$$



- GHU $\{(B^L), (B), (B^H)\} \leftrightarrow m_{KK} = \{11, 13, 15\} \text{ TeV}; \sqrt{s} = 250 \text{ GeV}, 250 \text{ fb}^{-1}$

Summary

We discussed fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired gauge-Higgs unification (GHU B-model).

- (1) Due to the Z' bosons coupling to quarks and leptons with large parity violation, several observables in $e^-e^+ \rightarrow f\bar{f}$ ($= \mu^- \mu^+, \dots$) processes have large polarization dependence.
- (2) Deviations from the SM are testable for the Kaluza-Klein (KK) mass scale up to ~ 15 TeV in the early stage of ILC.

[Observable:1/6] Differential cross section

Differential cross section for $e^-e^+ \rightarrow (V_i) \rightarrow f\bar{f}$:

$$\frac{d\sigma^{f\bar{f}}}{d\cos\theta}(P_{e^-}, P_{e^+}, \cos\theta) \propto (1 - P_{\text{eff}}) \underbrace{\frac{d\sigma_{LR}^{f\bar{f}}}{d\cos\theta}(\cos\theta)}_{e_L^- e_R^+ \rightarrow (V_i) \rightarrow f\bar{f}} + (1 + P_{\text{eff}}) \underbrace{\frac{d\sigma_{RL}^{f\bar{f}}}{d\cos\theta}(\cos\theta)}_{e_R^- e_L^+ \rightarrow (V_i) \rightarrow f\bar{f}},$$

For $e_L^- e_R^+ \rightarrow f\bar{f}$ and $e_R^- e_L^+ \rightarrow f\bar{f}$ ($\sqrt{s} \gg m_f$),

$$\frac{d\sigma_{LR}^{f\bar{f}}}{d\cos\theta}(\cos\theta) \underset{\sim}{\propto} (1 + \cos\theta)^2 |Q_{e_L f_L}|^2 + (1 - \cos\theta)^2 |Q_{e_L f_R}|^2,$$

$$\frac{d\sigma_{RL}^{f\bar{f}}}{d\cos\theta}(\cos\theta) \underset{\sim}{\propto} (1 + \cos\theta)^2 |Q_{e_R f_R}|^2 + (1 - \cos\theta)^2 |Q_{e_R f_L}|^2,$$

$$P_{\text{eff}} := \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}}, \quad Q_{e_X f_Y} := \sum_i \frac{g_{V_i e}^X g_{V_i f}^Y}{(s - m_{V_i}^2) + im_{V_i} \Gamma_{V_i}}.$$

[Observable:2/6] Total cross section

Total cross section for $e^-e^+ \rightarrow (V_i) \rightarrow f\bar{f}$:

$$\sigma_{\text{tot}}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \int_{-\cos\theta_{\text{max}}}^{+\cos\theta_{\text{max}}} \frac{d\sigma^{f\bar{f}}}{d\cos\theta}(P_{e^-}, P_{e^+}, \cos\theta) d\cos\theta$$

$$\underset{\sim}{\propto} (1 - P_{\text{eff}})\sigma_{LR}^{f\bar{f}} + (1 + P_{\text{eff}})\sigma_{RL}^{f\bar{f}}.$$

Total cross section for $e_L^-e_R^+ \rightarrow f\bar{f}$ and $e_R^-e_L^+ \rightarrow f\bar{f}$ ($\sqrt{s} \gg m_f$):

$$\sigma_{LR}^{f\bar{f}} \underset{\sim}{\propto} |Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2, \quad \sigma_{RL}^{f\bar{f}} \underset{\sim}{\propto} |Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2.$$

[Observable:3/6] Forward-backward asymmetry

Forward-backward asymmetry for $e^-e^+ \rightarrow (V_i) \rightarrow f\bar{f}$:

$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \frac{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) - \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) + \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})},$$

Forward/backward cross section for $\sigma_{F/B}^{f\bar{f}}(P_{e^-}, P_{e^+})$ ($\sqrt{s} \gg m_f$)

$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) \simeq \frac{3B_1 - B_2}{4B_1 + B_2},$$

$$B_1 = (1 - P_{\text{eff}}) |Q_{e_L f_L}|^2 + (1 + P_{\text{eff}}) |Q_{e_R f_R}|^2,$$

$$B_2 = (1 - P_{\text{eff}}) |Q_{e_L f_R}|^2 + (1 + P_{\text{eff}}) |Q_{e_R f_L}|^2.$$

[Observable:4/6] Left-right (LR) asymmetry

LR asymmetry for $e^-e^+ \rightarrow (V_i) \rightarrow f\bar{f}$ [29, 33, B.Schrempp et al.'88;...]:

$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{\sigma_{LR}^{f\bar{f}}(\cos\theta) - \sigma_{RL}^{f\bar{f}}(\cos\theta)}{\sigma_{LR}^{f\bar{f}}(\cos\theta) + \sigma_{RL}^{f\bar{f}}(\cos\theta)}.$$

Observable LR asymmetry:

$$A_{LR}^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) = \frac{\sigma^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) - \sigma^{f\bar{f}}(-P_{e^-}, -P_{e^+}, \cos\theta)}{\sigma^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) + \sigma^{f\bar{f}}(-P_{e^-}, -P_{e^+}, \cos\theta)}.$$

The relations between the above two asymmetries :

$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{1}{P_{\text{eff}}} A_{LR}^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta).$$

[Observable:5/6] Left-right (LR) asymmetry

LR asymmetry for $e^-e^+ \rightarrow (V_i) \rightarrow f\bar{f}$ [29, 33, B.Schrempp et al.'88;...]:

$$A_{LR}^{f\bar{f}} = \frac{\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}}}{\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}}}.$$

For $\sqrt{s} \gg m_f$,

$$A_{LR}^{f\bar{f}} \simeq \frac{[|Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2] - [|Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2]}{[|Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2] + [|Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2]}.$$

[Observable:6/6] LR FB asymmetry

LR FB asymmetry [28–32, D.C.Kennedy et al.'89;SLD Collaboration'94'95]:

$$A_{LR,FB}^{f\bar{f}}(\cos\theta) = \frac{\left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}} \right] (\cos\theta) - \left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}} \right] (-\cos\theta)}{\left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}} \right] (\cos\theta) + \left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}} \right] (-\cos\theta)}.$$

For $\sqrt{s} \gg m_f$,

$$A_{LR,FB}^{f\bar{f}}(\cos\theta) \approx \frac{2 \cos\theta \left[|Q_{e_L f_L}|^2 - |Q_{e_L f_R}|^2 \right] - \left[|Q_{e_R f_R}|^2 - |Q_{e_R f_L}|^2 \right]}{1 + \cos^2\theta \left[|Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2 + |Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2 \right]}.$$

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