# Feasibility of Bell Inequality violation at ATLAS with flavor entanglement of B meson pairs

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The paper was accepted by PRD ([arXiv:2106.07399]).

# Hidden variable theory

## Einstein's consideration on quantum mechanics

- Quantum Mechanics (QM) is approximation of the complete theory.
- In the complete theory, element of the physical reality (e.g., spin, flavor) is a function of hidden variable λ.

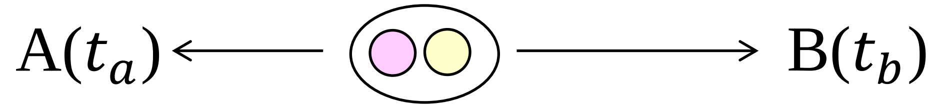
→ Hidden Variable Theory (HVT)

### Premise in HVT

- Locality condition: A measurement on one particle does not influence the other.
- Free will: An experimenter has freedom to choose a measurement condition.

Bell developed formula which HVT must satisfy.

# Bell inequality




PDF (Probability Density Function) of A($t_a$)B($t_b$)

- Expectation value of A($t_a$)B($t_b$): $C(t_a, t_b) = \sum_{A,B} AB\,P_{t_a t_b}(A, B)$

- In HVT, (A, B) is a function of hidden variable λ:

$$P_{t_a t_b}(A, B) = \int P_{t_a}(A, \lambda) P_{t_b}(B, \lambda) P(\lambda) d\lambda$$

Free will

Locality condition

$$|C(t_a, t_b)| = \left| \int \boxed{\sum_A AP(t_a, \lambda)}\ \boxed{\sum_B BP(t_b, \lambda)} P(\lambda)\, d\lambda \right| \leqq 1$$

$\leqq 1$    $\leqq 1$    $\leqq 1$

**Bell inequality**

$$|S| = |C(a, b) + C(a', b) + C(a, b') - C(a', b')| \leqq 2$$

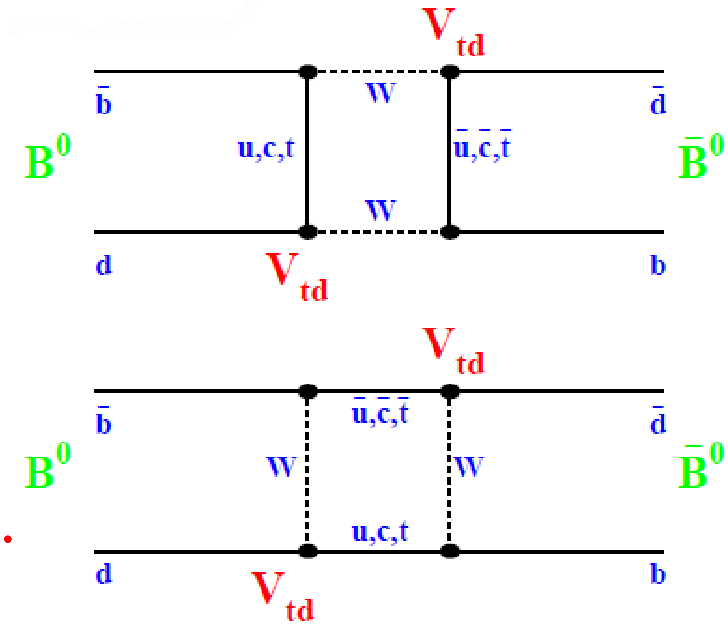
If HVT is correct, |S|≦2.

# Flavor mixing in B meson

- $B^0$ meson has mass ($B_H$, $B_L$) and flavor eigenstates ($B^0$, $\bar{B}^0$), and they are expressed by each other:

$$|B^0\rangle = \frac{|B_H\rangle + |B_L\rangle}{\sqrt{2}},\ |\bar{B}^0\rangle = \frac{|B_H\rangle - |B_L\rangle}{\sqrt{2}}$$

(Assuming CP conservation)



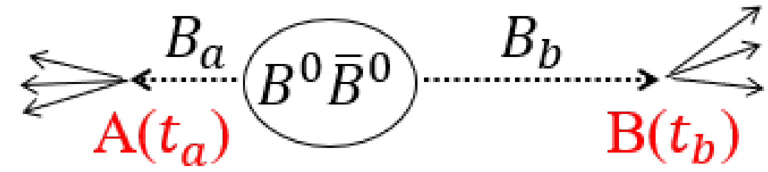

- $B^0$ and $\bar{B}^0$ are mixed during time evolution.

($\Delta M = M_H - M_L$, $\Gamma = \Gamma_H = \Gamma_L$)

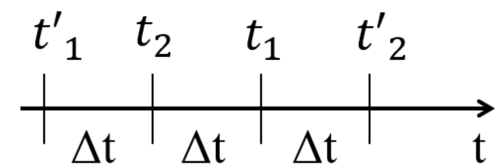
$$\mathrm{P}(B^0 \rightarrow B^0, \mathrm{t}) = \frac{e^{-\Gamma t}}{2}(1 + \cos \Delta M t)$$

$$\mathrm{P}(B^0 \rightarrow \bar{B}^0, \mathrm{t}) = \frac{e^{-\Gamma t}}{2}(1 - \cos \Delta M t)$$

# Flavor correlation



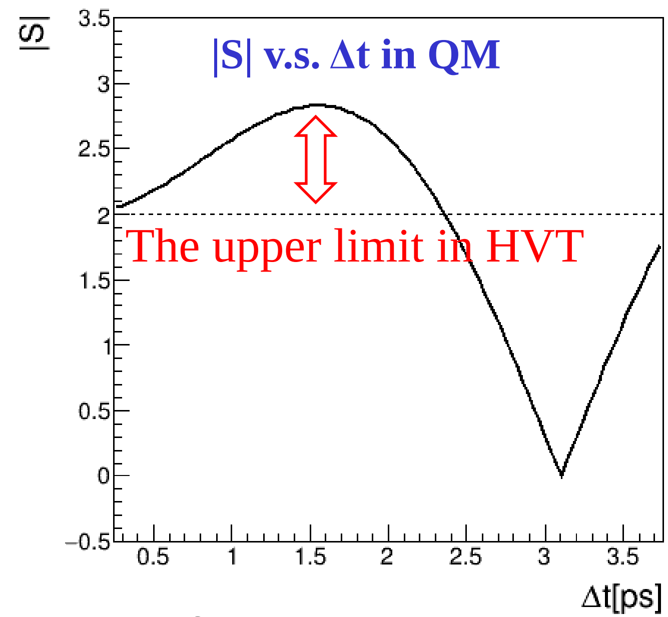

Expectation value of flavor pairs (A, B) at decay times ($t_{a/b}$) of $B_{a/b}$ :




$$C^Q(t_a, t_b) = \sum_{A,B} AB P^Q_{t_b\, t_b}(A, B)$$

$$= \frac{N^{B^0B^0}_{(t_a,t_b)} + N^{\bar{B}^0\bar{B}^0}_{(t_a,t_b)} - N^{B^0\bar{B}^0}_{(t_a,t_b)} - N^{\bar{B}^0B^0}_{(t_a,t_b)}}{N^{B^0B^0}_{(t_a,t_b)} + N^{\bar{B}^0\bar{B}^0}_{(t_a,t_b)} + N^{B^0\bar{B}^0}_{(t_a,t_b)} + N^{\bar{B}^0B^0}_{(t_a,t_b)}}$$

$$= -\cos(\Delta M \Delta t) \leftarrow t_a - t_b$$




Defining $t_2 - t'_1 = t_1 - t_2 = t'_2 - t_1 = \Delta t$,

$$|S^Q(\Delta t)| = |C^Q(t_1, t_2) + C^Q(t'_1, t_2) + C^Q(t_1, t'_2) - C^Q(t'_1, t'_2)|$$

$$\quad -\cos(\Delta M\Delta t) \quad -\cos(\Delta M\Delta t) \quad -\cos(\Delta M\Delta t) \quad -\cos(3\Delta M\Delta t)$$

$$= |-3\cos(\Delta M\Delta t) + \cos(3\Delta M\Delta t)|$$

In QM, |S| can be $2\sqrt{2}$ at the maximum. → QM violates Bell inequality.

# Previous experiments

[PLA373 (2008) 39-44]

## B meson (Belle) [arXiv:0310192 (2003)]

- A measurement only for Δt, where $(t_a, t_b)$ was not measured separately.
- $|S|$ has the maximum value of 4 and includes that of QM.

## K meson (CPLEAR) [PLB422, 339-348]

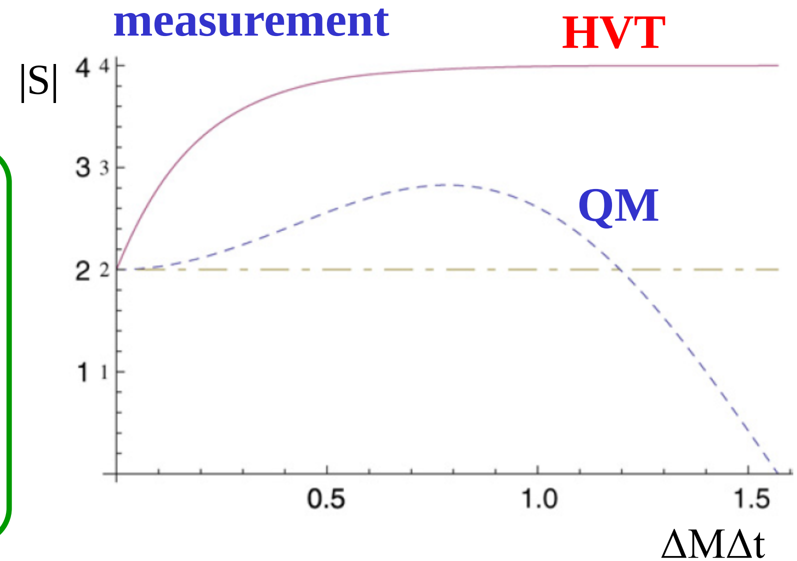
A measurement only for $C^Q(t_a, t_b)$ and is not Bell test.

## Both experiments

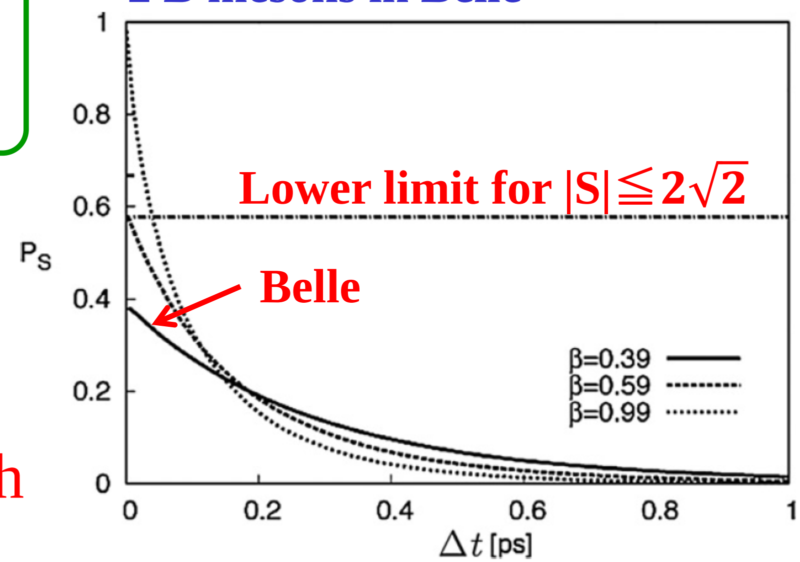
Most of 2 mesons are not space-like and the locality is not ensured.

The previous experiments were not enough for Bell test on the flavor entanglement.

**|S| in QM and HVT only with Δt measurement**

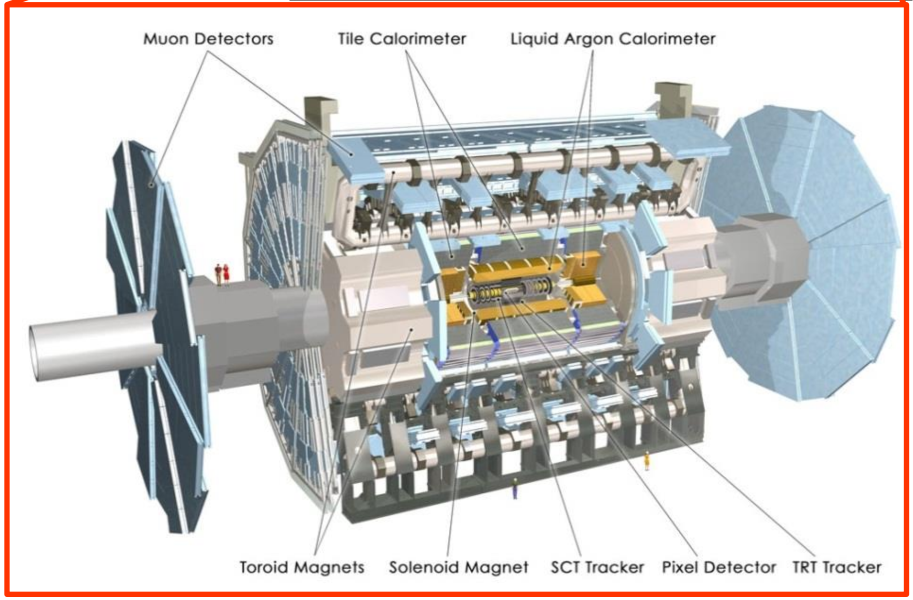



**Fraction of space-like events of 2 B mesons in Belle**



# ATLAS experiment

- High energy experiment using proton-proton collisions at LHC (Large Hadron Collider).
- The experiment started in 2010 and discovered Higgs boson in 2012.

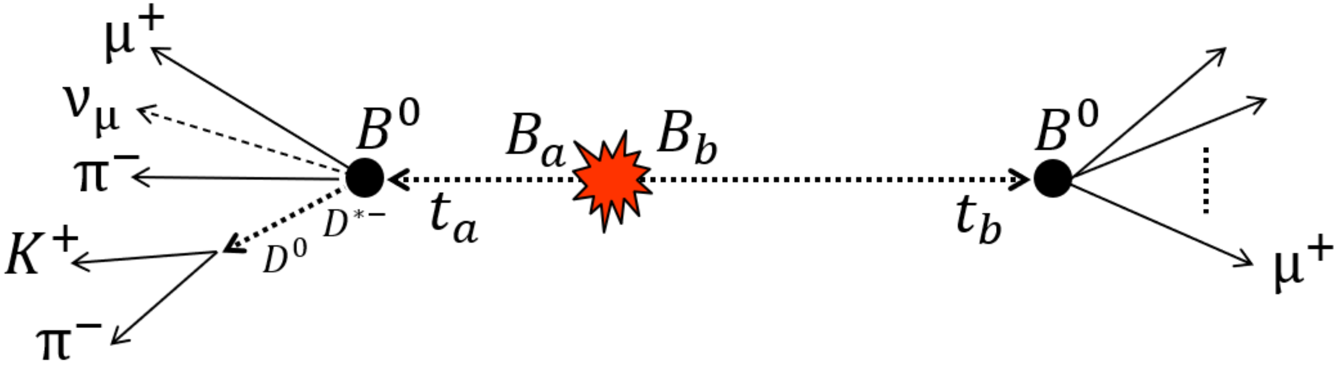



| | Period | Integrated luminosity |
|---|---|---|
| Run1 | 2010-12 | 5.1fb-1 @7TeV<br>21.3 fb-1 @8 TeV |
| Run2 | 2015-18 | 149 fb-1 @13 TeV |
| Run3 | 2021-24 | 180 fb-1 @~14 TeV |

- The instantaneous luminosity reached more than twice of LHC design value ($2.2 \times 10^{34}$ $cm^{-2}s^{-1}$) and # of pile-up ($\mu$) became <60.
- Except for usual pp collisions, data are taken in low-$\mu$ operation ($\mu$~1) and heavy ion collisions.

# Simulation study

- The simulation was performed for truth level study with PYTHIA8.245.
- The low-μ (μ~1) operation with 1 $fb^{-1}$ of data was assumed.
- 「$B^0 \rightarrow D^{*-}\mu^+\nu$ ($D^{*-} \rightarrow D^0\pi^-$, $D^0 \rightarrow K^+\pi^-$)」 events were selected.
  - B meson flavor can be identified with charge sign of decay objects.
  - The decay time ($t_a$, $t_b$) can be measured, reconstructing vertex with $\mu^+$ and $\pi^-$ from $D^{*-}$ decay ($\sigma_t$~0.11 ps)
- The similar selection criteria used in 「$D^{*+}\mu^-X$」 cross-section measurement at ATLAS was used [Nucl. Phys. B 864 (2012) 341-381].

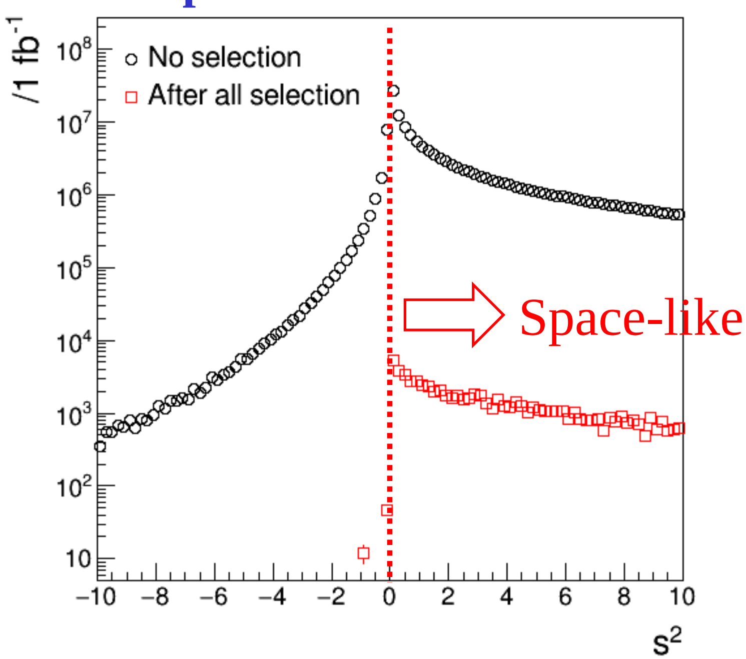


# Sensitivity to Bell inequality violation

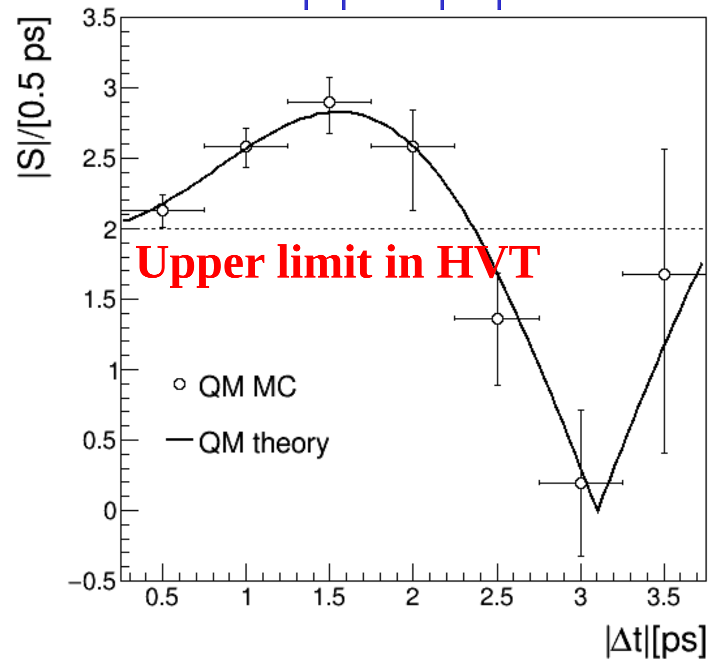
- More than 99% B meson pairs are space-like in ATLAS.
- |S(1.5±0.25 ps)| = 2.89 ± 0.17 (stat) ± -0.13 (syst)
  - |S| has the maximum value at 1.55ps and deviation from 2 can be detected with 4.2σ significance.

⇨ **The first Bell test on B meson flavor is possible!**

**Proper distance distribution**




**|S| v.s. |Δt|**



# Summary & conclusions

- Bell inequality provides the upper limit of correlation between two particles that HVT should satisfy.
- Entangled state of two meson flavors in QM violates Bell inequality.
- The previous experiments on meson flavor entanglement were inconclusive for Bell inequality violation only with $\Delta t$ measurement without satisfying the locality condition.
- Our simulation study concluded that Bell test on B meson flavor is possible at ATLAS experiment.
  - Bell inequality violation can be tested with 4.2 $\sigma$ precision.
- We organized analysis group in ATLAS and aim the measurement during Run 3 (2022-24).
- The paper was accepted and will be published in PRD ([arXiv:2106.07399]).

# Backup

# Q&A in Bell test with meson flavor

Q1 |S| has the maximum value at 1.55 ps. Is the oscillation frequency is too long with respect to lifetime of $B_d^0$ (1.5 ps)?

A1 No problem since $C^Q(t_a, t_b)$ is normalized by # of events at $(t_a, t_b)$.

Q2 HVT assumes free will of experimenter in a measurement but decay of $B_d^0$ is determined by nature. Can it be assumed as free will?

A2 We assumed that $B_d^0$ decay randomly and it corresponds to free will of the particle. Also in Aspect experiment, a random generator is used to operate the detector and it is assumed as free will.

# Possible loophole in Bell test

- Free will loophole

  This study is assumed that the decay of a particle happened randomly with particle's will.

- Efficiency loophole

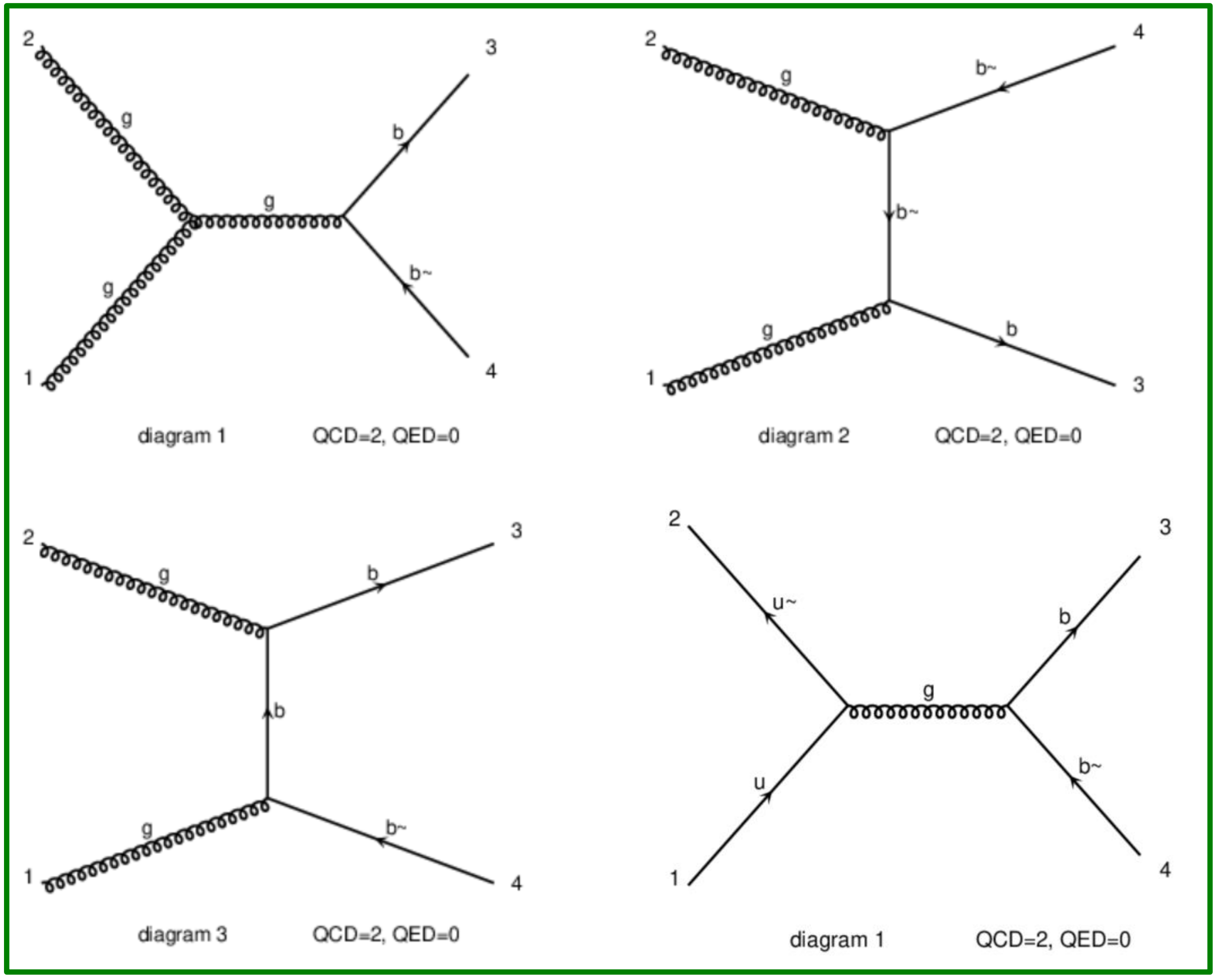
  About 82.8% ($2\sqrt{2} - 2$) of efficiency is necessary to close the efficiency loophole. Since the efficiency in this study is only 2%, fair sampling assumption is assumed.

- Locality loophole

  2 particles decay randomly and they are space-like, therefore, locality loop hole is closed successfully.

# $b\bar{b}$ events in proton-proton collision

① $gg/q\bar{q} \to b\bar{b}$




**Cross-section of "$gg/q\bar{q} \to b\bar{b}$" events decaying into "$D^*\mu\nu$"**

| | Cross-sec. [nb] |
|---|---|
| $B^0\bar{B}^0$ | 4.5 |
| $B^{*0}\bar{B}^{*0}$ | 21.8 |
| $B^0\bar{B}^{*0}$ | 19.8 |
| Total | 46.2 |

② $b\bar{b}$ from「$gg/q\bar{q} \to jj$」

# Bell inequality in flavor correlation (1)

PDF of $B_{a/b}$ with flavor (A, B) at their decay times ($t_{a/b}$):

\# of events with flavor (A, B) at the decay time of $B_{a/b}(t_a, t_b)$

$$P^{Q}_{t_a,t_b}(A,B) = \frac{N^{AB}_{(t_a,t_b)}}{N^{B^0 B^0}_{(t_a,t_b)} + N^{\bar{B}^0 \bar{B}^0}_{(t_a,t_b)} + N^{B^0 \bar{B}^0}_{(t_a,t_b)} + N^{\bar{B}^0 B^0}_{(t_a,t_b)}}$$

$$= \frac{1}{4}\left(1 - AB\cos(\Delta M \Delta t)\right)$$

$\Delta t$: $t_a - t_b$


$$C^{Q}(t_a,t_b) = \sum_{A,B} AB\,P^{Q}_{t_b\,t_b}(A,B)$$

$$= \frac{N^{B^0 B^0}_{(t_a,t_b)} + N^{\bar{B}^0 \bar{B}^0}_{(t_a,t_b)} - N^{B^0 \bar{B}^0}_{(t_a,t_b)} - N^{\bar{B}^0 B^0}_{(t_a,t_b)}}{N^{B^0 B^0}_{(t_a,t_b)} + N^{\bar{B}^0 \bar{B}^0}_{(t_a,t_b)} + N^{B^0 \bar{B}^0}_{(t_a,t_b)} + N^{\bar{B}^0 B^0}_{(t_a,t_b)}}$$

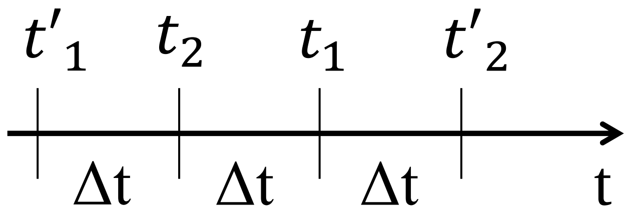
$$= -\cos(\Delta M \Delta t)$$

# Bell inequality in flavor correlation (2)

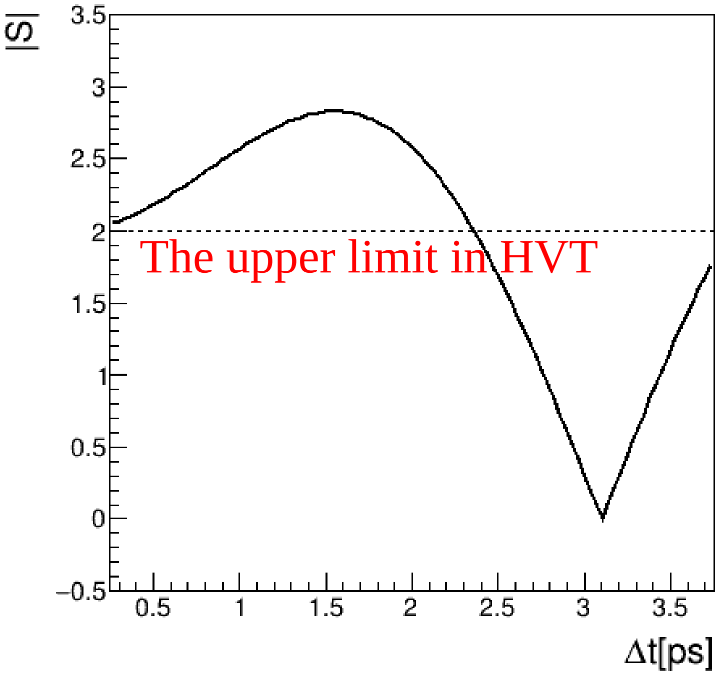
Defining $t_2 - t'_1 = t_1 - t_2 = t'_2 - t_1 = \Delta t$,

$$|S^Q(\Delta t)| = |\underbrace{C^Q(t_1, t_2)}_{-\cos(\Delta M \Delta t)} + \underbrace{C^Q(t'_1, t_2)}_{-\cos(\Delta M \Delta t)} + \underbrace{C^Q(t_1, t'_2)}_{-\cos(\Delta M \Delta t)} - \underbrace{C^Q(t'_1, t'_2)}_{-\cos(3\Delta M \Delta t)}|$$

$$= |-3\cos(\Delta M \Delta t) + \cos(3\Delta M \Delta t)|$$




|S| as a function of Δt in QM



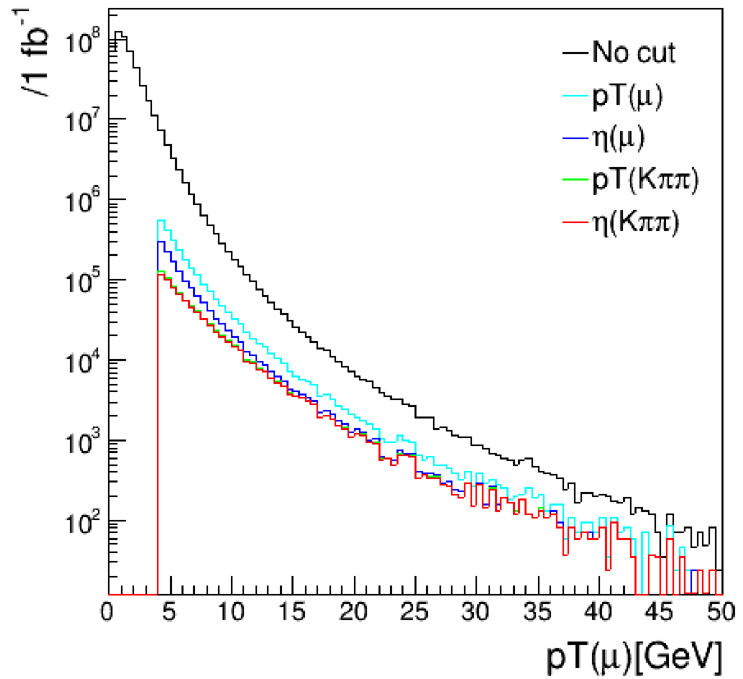

In QM, |S| has the maximum value of $2\sqrt{2}$ at Δt = 1.55 ps. → **QM violates Bell inequality**

- $\Delta M = 3.334 \times 10^{-10}$ MeV

# Acceptance cut

- Selection criteria used in 「$D^{*+}\mu^{-}X$」 analysis at ATLAS was assumed [Nucl. Phys. B 864 (2012) 341-381].
- 「pT(μ)>6 GeV」 was modified to 「> 4 GeV」
- 「pT($K^{\pm}\pi^{\mp}\pi^{\mp}$) > 4.5 GeV」 was modified to 「>3 GeV」

**μ-pT distribution in 「$B^0 \to D^{*-}\mu^{+}\nu$」**




| Cut | Total acceptance ($A$) | $\sigma \times A$ (pb) |
|---|---:|---:|
| No cut | 1.0 | 247,611 |
| pT(μ) > 4 GeV | $5.03 \times 10^{-3}$ | 1,246 |
| $\lvert\eta(\mu)\rvert < 2.4$ | $2.79 \times 10^{-3}$ | 690 |
| pT($K^{\pm}\pi^{\mp}\pi^{\mp}$) > 3 GeV | $1.56 \times 10^{-3}$ | 385 |
| $\lvert\eta(K^{-}\pi^{+}\pi^{+})\rvert < 2.5$ | $1.49 \times 10^{-3}$ | 369 |

# Event selection

| | Efficiency | Comment |
|---|---|---|
| Track reconstruction ($\varepsilon_{reco}$) | 0.483 | From「$D^{*+}\mu^{-}X$」analysis |
| Trigger ($\varepsilon_{trigger}$) | 0.429 | $(0.819 \times 0.8)^2$ was assumed.<br>• 0.819 is efficiency for single-μ trigger with pT>6GeV. |

| Selection criteria ($\varepsilon_{selection}$) | Total eff. | Comment |
|---|---|---|
| pT > 1 GeV for $\pi^+/K^-$ in $D^0$ candidates | 0.510 | |
| pT > 250 MeV for $\pi^+$ from $D^{*+}$ | 0.452 | |
| •\|m($K^-\pi^+$) – m($D^0$)\| < 64 MeV (pT($K^-\pi^+\pi^+$) > 12 GeV, \|η($K^-\pi^+\pi^+$)\| > 1.3)<br>•\|m($K^-\pi^+$) – m($D^0$)\| < 40 MeV elsewhere | 0.209 | Assume $\sigma^2$ cut (0.46) |
| 2.5 GeV < m($D^{*+}\mu^-$) < 5.4 GeV | 0.097 | Assume $\sigma^2$ cut (0.46) |

$\varepsilon_{total}$ $(\varepsilon_{reco} \times \varepsilon_{trigger} \times \varepsilon_{selection})$ = 0.020

$(\sigma \times A) \times \varepsilon_{total} \times$ L(pb$^{-1}$)=7.4L(pb$^{-1}$) events → 7.4k events with 1fb$^{-1}$.

# Background & systematic errors

## Background

- Contamination of $B^0\bar{B}^0$ originated from different gluons is less than 0.1% (negligible)

- BG contamination in 「$D^{*+}\mu^-$X」 analysis was 「6.8±0.26%」, which is taken into account in this study.
  - 6.2% is combinatorial BG (e.g.:「c → $D^{*+}$X」 and 「$\bar{c}$ → $\mu^-$X'」)

## Systematic errors on |S|

- BG contamination
  - Shifting the entries in one Δt bin by 0.26%, the maximum shift in |S| was adopted as the systematic error of BG contamination (most conservative evaluation).
  - BG should be smaller for μ~1 since 「$D^{*+}\mu^-$X」 analysis used data with μ>2.

- Δt resolution
  - Evaluated by fluctuating Δt with the resolution ($0.11\sqrt{2}$ps) 1000 times.

# Decay time resolution in ATLAS

- Refer to ΔΓ measurement "$B^0 \to J/\psi K_s$, $B^0 \to J/\psi K^*$" in ATLAS
  - [JEP06(2016)081](more detail is in[ATL-COM-PHYS-2015-170])
- Decay length ($L^B_{prop}$ = ct) is calculated as follows:
  - $x^{J/\psi}$: Decay position of "J/ψ→μμ"

$$L^B_{\text{prop}} = \frac{(x^{J/\psi} - x^{\text{PV}})p^B_x + (y^{J/\psi} - y^{\text{PV}})p^B_y}{(p^B_{\text{T}})^2} m_{B^0} = \frac{\vec{d} \cdot \vec{k}_\perp}{\beta_\perp \gamma_\perp}$$

$\sigma(L^B_{prop})$ = 34 μm

⇨ $\sigma(t) = \frac{\sigma(L^B_{prop})}{c} = 0.11$ ps

# ATLAS v.s. Belle

## Decay length/time measurement

- 2 $B^0$ mesons travel along the z-axis in Belle.

  → The production position of $B^0$ is unknown due to beam size in the z-direction. Only Δz and Δt can be measured.

- 2 $B^0$ mesons can go to xy direction in ATLAS, production and decay position of an individual $B^0$ mesons can be measured.

## Resolution of decay length/time

- Belle : σ(Δz) ~ 100 um, σ(Δt) ~ 1.2 ps
- ATLAS : $\sigma(L^{B}_{prop} = ct)$ ~ 34 um, σ(t) = 0.11 ps