

Bottom-production and bottom-fragmentation at the ILC

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Outline

- 1 *b*-Quark Pairs: Experiment
- 2 *b*-Quark Pairs: Theory
- 3 *B*-Hadron Production: Experiment
- 4 *B*-Hadron Production: Theory

b-Quark-Pair Production at LEP and SLC

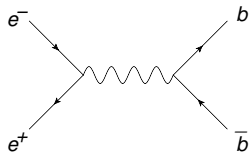
- Process considered:

$$e^+e^- \rightarrow b\bar{b}$$

- $$\frac{d\sigma}{d\cos\theta}(e_R^+e_L^- \rightarrow b\bar{b}) = M_{LL}(1 + \cos\theta)^2 + M_{LR}(1 - \cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e_L^+e_R^- \rightarrow b\bar{b}) = M_{RR}(1 + \cos\theta)^2 + M_{RL}(1 - \cos\theta)^2$$

- At LEP: unpolarised beams
 \Rightarrow Only forward-backward asymmetry
- At SLC: polarised electron beam
 \Rightarrow Forward-backward-left-right asymmetry
- Beam polarisation disentangles contributions



b-Quark-Pair Production at LEP and SLC

- The asymmetries can be used to measure $\sin^2 \theta_{\text{eff}}^{\text{lept}}$
See also the talk by A. Irlles
- 3σ tension between LEP and SLC
 \Rightarrow Sign of BSM physics?
- Many BSM particles couple differently to *t*/*b* than to other quarks
- Right-handed particles are more strongly affected (small SM couplings)

Examples: Peskin, Yoon (arXiv:1811.07877)
Hosotani et al. (arXiv:1705.05282)
Djouadi et al. (hep-ph/0610173)

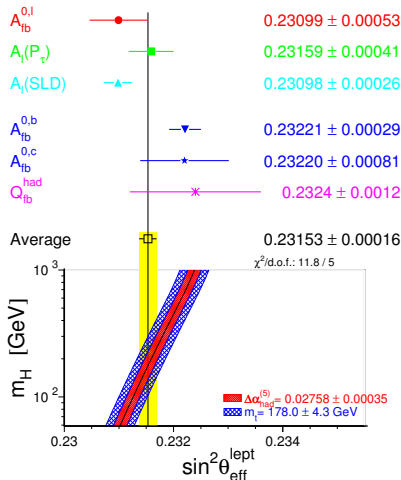


Image credit: hep-ex/0509008

b-Quark-Pair Production at ILC

- At ILC: both beams polarised
 ⇒ Enhanced cross section
- Errors expected to be one order of magnitude smaller
 ⇒ Resolution of the LEP/SLC tension
- Strong exclusion limits on BSM models
- GigaZ for factor 10 reduction of the error on g_{Rb}
 ⇒ Modification of Z-boson couplings or new resonances?

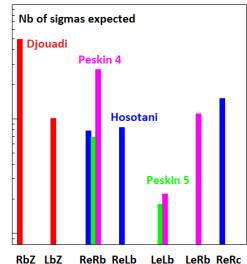
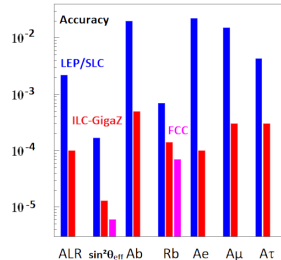


Image credit: arXiv:1905.00220

$e^+e^- \rightarrow b\bar{b}$: Theory Prospects

- $e^+e^- \rightarrow q\bar{q}$ theoretically very similar for all q
- For $b\bar{b}$: non-zero mass (typically only at one-loop) and Wbt -coupling
- Many ingredients (Z/γ -exchange, box diagrams, ISR, etc.)
- Focus here on three pseudo-observables: R_b , $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and $\sin^2 \theta_{\text{eff}}^b$
- Current theory precision better than experimental one
- Significant work needed on theory side to match precision of the ILC

	$\delta R_b [10^{-5}]$	$\delta \sin^2 \theta_{\text{eff}}^{\text{lept}} [10^{-6}]$	$\delta \sin^2 \theta_{\text{eff}}^b [10^{-6}]$
Current exp.	66	160	16000
Current th.	10	45	53
Expected exp.	15	13	1500
3-loop EW th.	5	15	-
Dom. 4-loop th.	<3	<7	-

Table values:
[arXiv:1809.01830](https://arxiv.org/abs/1809.01830),
[arXiv:1901.02648](https://arxiv.org/abs/1901.02648),
[arXiv:1906.08815](https://arxiv.org/abs/1906.08815),
[arXiv:1908.11299](https://arxiv.org/abs/1908.11299)

Theory Status and Prospects for $\sin^2 \theta_{\text{eff}}^{\text{lept}}$

- $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ not specific to $b\bar{b}$, but central to motivation
- $\sin^2 \theta_{\text{eff}}^{\text{lept}} = (1 - M_W^2/M_Z^2)(1 + \Delta\kappa)$
- Full two-loop corrections ($\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$) known
Djouadi and Verzegnassi ('87), Djouadi ('88), Kniehl et al. ('88), Awramik et al. ('04, '06),
Hollik et al. ('05, '07)
- Three-loop ($\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha^2\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$), as well as
four-loop $\mathcal{O}(\alpha\alpha_s^3)$, corrections known in the large- m_t limit
Avdeev et al. ('94), Chetyrkin et al. ('95, '95), van der Bij et al. ('00), Faisst et al. ('03),
Schröder and Steinhauser ('05), Chetyrkin et al. ('06), Boughezal and Czakon ('06)
- Recently: leading fermionic three-loop corrections at $\mathcal{O}(\alpha^3)$
and $\mathcal{O}(\alpha^2\alpha_s)$
Chen and Freitas ('20, '21)

Theory Status and Prospects for R_b

- $R_b = \sigma(e^+e^- \rightarrow b\bar{b})/\sigma(e^+e^- \rightarrow \text{hadrons})$
- Full two-loop corrections ($\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$) known
Freitas ('14), Dubovyk et al. ('18), references on previous slide
- Partial three- and four-loop corrections as on the previous slide
- Final-state radiation through $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha_s^4)$, including bottom-mass corrections through $\mathcal{O}(\alpha_s^3)$
Kataev ('92), Chetyrkin et al. ('94), Baikov et al. ('08)
- Non-factorisable contributions at $\mathcal{O}(\alpha\alpha_s)$
*Fleischer et al. ('92), Buchalla and Buras ('93), Chetyrkin et al. ('93), Degrandi ('93),
Czarnecki and Kühn ('96), Harlander et al. ('97)*

Diagram Statistics for *Z*-Decays

$Z \rightarrow b\bar{b}$	1 loop	2 loops	3 loops
Number of topologies	1	14 $\xrightarrow{(A)}$ 7 $\xrightarrow{(B)}$ 5	211 $\xrightarrow{(A)}$ 84 $\xrightarrow{(B)}$ 51
Number of diagrams	15	2383 $\xrightarrow{(A,B)}$ 1074	490 387 $\xrightarrow{(A,B)}$ 120 472
Fermionic loops	0	150	17 580
Bosonic loops	15	924	102 892
Planar / non-planar	15 / 0	981/133	84 059/36 413
QCD / EW	1 / 14	98 / 1016	10 386/110 086
$Z \rightarrow e^+e^-, \dots$			
Number of topologies	1	14 $\xrightarrow{(A)}$ 7 $\xrightarrow{(B)}$ 5	211 $\xrightarrow{(A)}$ 84 $\xrightarrow{(B)}$ 51
Number of diagrams	14	2012 $\xrightarrow{(A,B)}$ 880	397 690 $\xrightarrow{(A,B)}$ 91 472
Fermionic loops	0	114	13104
Bosonic loops	14	766	78 368
Planar / non-planar	14 / 0	782/98	65 487/25 985
QCD / EW	0 / 14	0 / 880	144/91 328

Table taken from [arXiv:1809.01830](https://arxiv.org/abs/1809.01830)

B-Hadron Production at Past e^+e^- -Colliders

- Inclusive *B*-hadron production: $e^+e^- \rightarrow B + X$
- Studied at ALEPH, OPAL, DELPHI and SLD at the *Z*-pole
[hep-ex/0106051](#), [hep-ex/0210031](#), [arXiv:1102.4748](#), [hep-ex/0202031](#)

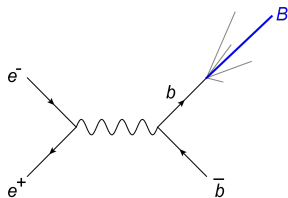
- To this day the best constraints on bottom-fragmentation

- Consider weakly-decaying *B*-hadrons

- Most experiments do not distinguish between mesons and baryons (exception: ALEPH)

- Main observable of interest: *B*-hadron energy (normalised to beam energy: x_B)

- Either x_B -spectrum or additionally first 5 moments (DELPHI)



B-Hadron Production at Past e^+e^- -Colliders

- Four different techniques used
- All measurements compatible
- Total errors $\mathcal{O}(5\%)$
- $3\sigma_{\text{stat}} \approx \sigma_{\text{syst}}$
- Exception: ALEPH, where $\sigma_{\text{stat}} \approx \sigma_{\text{syst}}$ (low reconstruction efficiency)

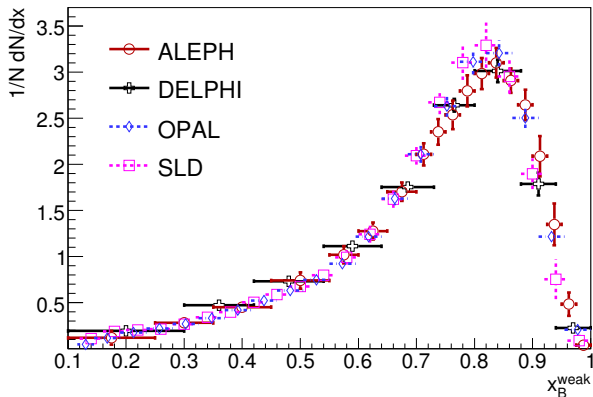


Image credit: arXiv:1102.4748

B-Hadron Production at the ILC

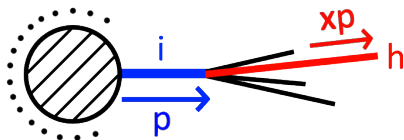
- $\sigma_{\text{stat,LEP1}} \sim \mathcal{O}(1\%) \Rightarrow \sigma_{\text{stat,ILC-GigaZ}} \sim \mathcal{O}(0.1\%)$
- Systematic error dominated by energy smearing and *B*/*D*-physics modelling
- ILD energy resolution an order of magnitude better than LEP
 \Rightarrow Error reduced by factor $\mathcal{O}(10)$
- Physics modelling highly analysis-dependent
 \Rightarrow Would need a dedicated simulation/analysis!
- Higher statistics and energy resolution
 \Rightarrow Smaller bins? $x \sim 1$ theoretically interesting
- Separation of *B*-mesons (90%) and *B*-baryons (10%)?

Heavy-Quark Fragmentation

- Use fragmentation functions $D_{i \rightarrow h}(x)$ to describe production of hadrons:

$$d\sigma_h(p_h) = \int_0^1 \frac{dx}{x} \sum_{i=b, \bar{b}, g, u, d, \dots} d\sigma_i \left(p_i = \frac{p_h}{x} \right) D_{i \rightarrow h}(x)$$

- Fragmentation functions (FFs) non-perturbative objects
 \Rightarrow Must be fitted to data
- Fully analogous to PDFs
- No parton showers needed



Heavy-Quark Fragmentation

- For heavy-flavoured hadrons, e.g. $h = B$ ($m_b \gg \Lambda_{\text{QCD}}$):
can describe production of heavy quark perturbatively
 \Rightarrow Can split FFs into perturbative (PFFs) and non-perturbative (NPFFs) parts

Mele and Nason ('91)
- Need to fit only one NPFF instead of n_{partons} FFs
- Like PDFs, scale dependence predicted by theory (time-like splitting functions)
 \Rightarrow Can be used to resum $\ln p_{T,B}/m_b$
- Large- x region enhanced by large logarithms (soft radiation)
 \Rightarrow Resummation needed for correct description of large x

B-Hadron Production: Theory Status

- The NNLO PFFs have been known for 17 years

Melnikov and Mitov ('04), Mitov ('04)

- The NNLO time-like splitting functions are also known

Mitov, Moch and Vogt ('06), Moch and Vogt ('07), Almasy, Vogt and Moch ('11), Chen et al. ('20)

- The (massive) NNLO coefficient functions known for over 20 years

Rijken and van Neerven ('96), Bernreuther et al. ('97), Brandenburg and Uwer ('97), Rodrigo ('96, '97),

Rodrigo et al. ('97, '99), Nason and Oleari ('97), Oleari ('98))

- Up to N^3LL large- x resummation

Fickinger et al. ('16)

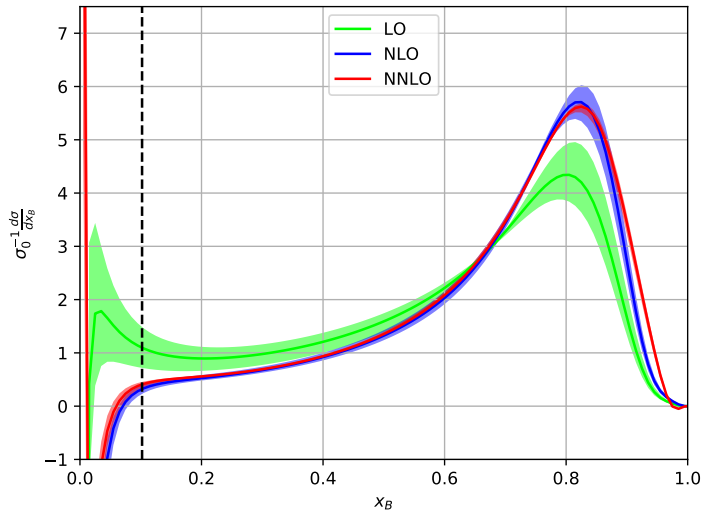
- Partial N^3LO and N^4LO results for space-like splitting functions
(time-like: large- N_c non-singlet quark-to-quark contribution at N^3LO)

Davies et al. ('16), Moch et al. ('17), Moch et al. ('18), Herzog et al. ('19), Moch et al. ('17))

B-Hadron Production: Theory Status

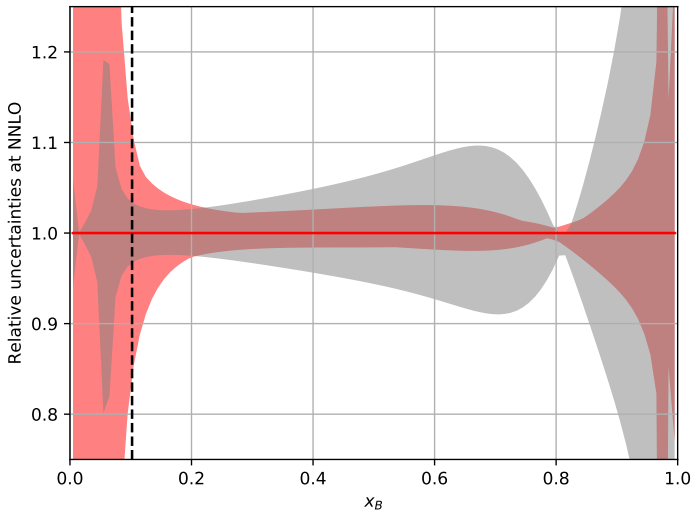
- Per-mille precision at the ILC
⇒ Need to include EW corrections
- One-loop EW corrections to coefficient functions for $e^+e^- \rightarrow b\bar{b}g$
Maina et al. ('02), Maina et al. ('04), Carloni-Calame et al ('08)
- Other partial corrections obtainable from other results (e.g. R_b)
- Full $\mathcal{O}(\alpha\alpha_s)$ coefficient functions not yet published
- So far, only effects of ISR considered
Cacciari, Nason and Oleari ('05)
- Need studies on EW corrections to heavy-quark fragmentation

B-Hadron Production at the ILC: Predictions

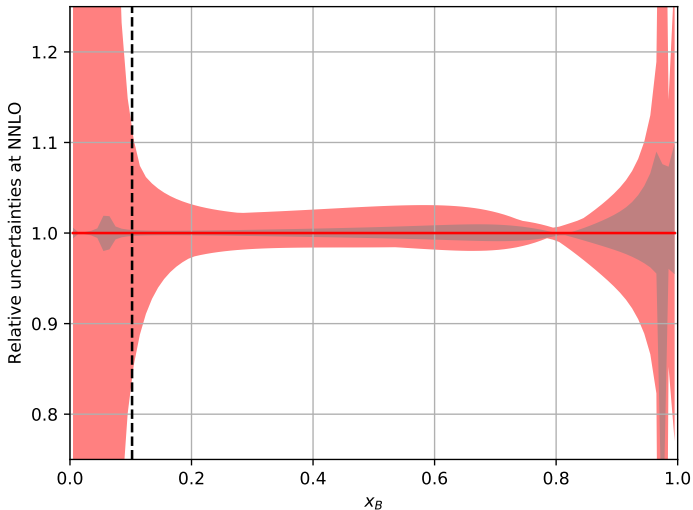


Coefficient functions: Rijken and van Neerven ('96). FFs: Fickinger et al. ('19); Czakon, TG, Mitov and Poncelet ('21). 17/23

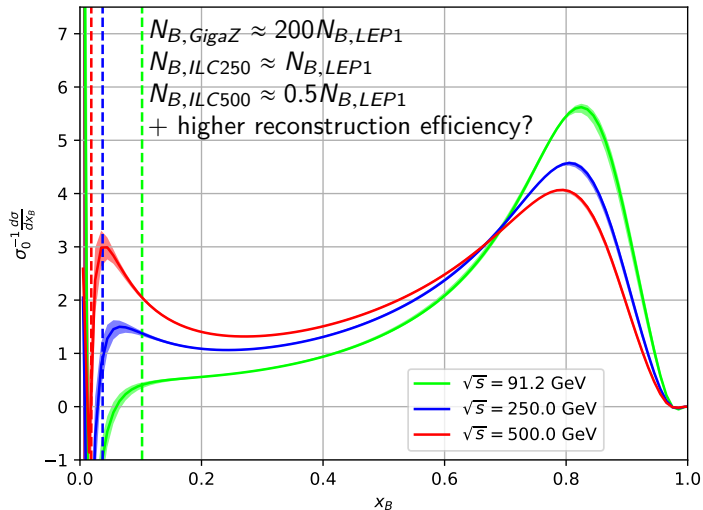
B-Hadron Spectrum: NNLO QCD Vs. LEP/SLC Precision



B-Hadron Spectrum: NNLO QCD Vs. ILC Precision



B-Hadron Production at the ILC: Predictions

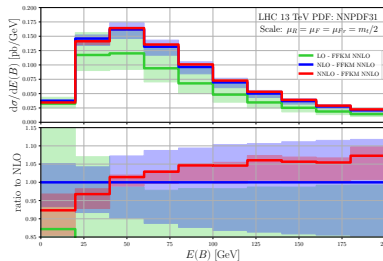
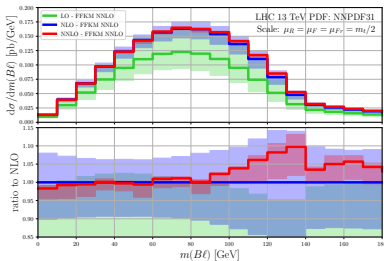


Top-Pair Events with B-hadrons at the LHC

- Numerical framework for NNLO calculations with fragmentation recently completed

Czakon, TG, Mitov and Poncelet ('21)

- Applicable to any process without further modifications
- First application: $pp \rightarrow t(\rightarrow B + X)\bar{t}$ at the LHC



B-Hadron Production at the ILC: Prospects

- Aside from improved precision:
 - Can afford smaller bins
⇒ More detailed studies of large- x behaviour
 - Higher statistics and better charge determination
⇒ Separation of different hadrons (B^+ , B^- , B^0/\bar{B}^0 , Λ_b , ...)
 - Multiple collision energies
⇒ Study scale evolution within a single process/machine
 - Higher energies ⇒ Probe smaller x (small- x resummation?)
 - Study multiple final states
($e^+e^- \rightarrow B + X$, but also e.g. $e^+e^- \rightarrow t(\rightarrow B + X)\bar{t}$)
 - Study multiple spectra of a single process: x_B , but also $\cos\theta_B$ and $p_{T,B}$
⇒ When does the fragmentation formalism (not) work? Corrections?
- Tests of non-perturbative predictions (moments of NPFF)

Conclusions and Outlook

$e^+e^- \rightarrow b\bar{b}$ at the ILC:

- Precision of related EW observables improved by factor 5 – 30
- Resolution of long-standing 3σ discrepancy
- Need to improve theory by at least one order in perturbation theory

B-hadrons at the ILC:

- Much needed update of experimental precision
- Next perturbative order needed to match ILC precision
- Much more detail available (see previous slide)
⇒ Detailed studies of QCD factorisation and non-perturbative effects
- Any NNLO cross section readily calculable with new framework

B -Hadron Production at Hadron Colliders

- Fragmentation functions are universal
 \Rightarrow Extract FFs from e^+e^- -data, make predictions for hadron colliders
- Status 2001: differential CDF measurement factor 3 higher than NLO QCD
- Problem with theory? Problem with experiment? New physics?

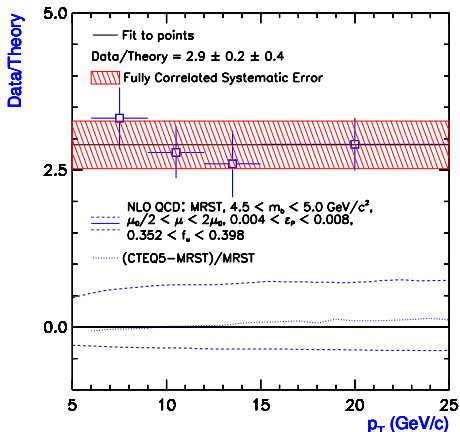


Image credit: hep-ph/0111359

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B-Hadron Production at Hadron Colliders

- Several updates to theory predictions:

Cacciari and Nason (2002), Cacciari et al. (2003)

- Resummation of $\ln p_T/m_b$ at NLL accuracy
- Merging with fixed-order massive predictions at low p_T (FONLL)
- Consistent extraction and application of NPFF
- Fitting the first few moments of the x_B -spectrum
- Large- x resummation through NLL

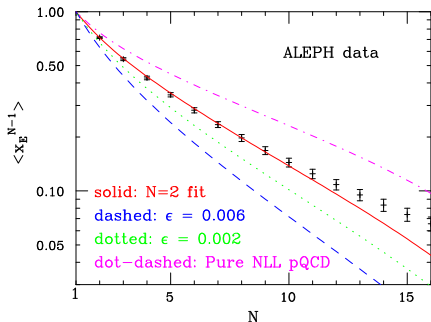


Image credit: hep-ph/0204025

B-Hadron Production at Hadron Colliders

- After theory improvements:
 $\text{data/theory} = 1.7 \pm 0.5$ (exp.) ± 0.5 (th.)
- New data from CDF (2003) and ZEUS (2003, e^-p -collisions) further reduced discrepancy

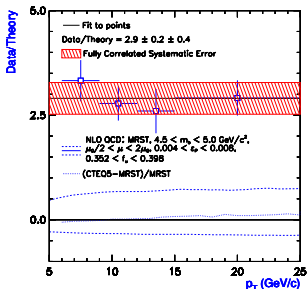


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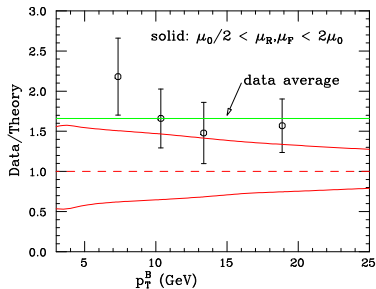


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