

CP–Violating Invariants in the SMEFT

Emanuele Gendy Abd El Sayed
DESY & Universität Hamburg



Universität Hamburg

DER FORSCHUNG | DER LEHRE | DER BILDUNG

Based on: Q. Bonnefoy, E.G., C. Grojean, J. Ruderman: 2112.03889

Motivation

- Explore CP–Violation beyond the Standard Model
- Define a flavor–independent formalism for CPV
- Count the independent new sources of CP–violation at order $1/\Lambda^2$ in the EFT expansion

Outline

- Review (pt. 1): CP–Violation in the Standard Model
- Review (pt. 2): the Standard Model Effective Field Theory
- CPV in the SMEFT: no way out of collectivity
- Counting CP–odd flavor–invariants at order $1/\Lambda^2$
- Next steps and Summary

CP–Violation in the Standard Model

In the Electroweak sector, CP violation is encoded in the CKM matrix

$$\begin{aligned}
 \mathcal{L}_{\text{mix}} &= \frac{e}{\sqrt{2} \sin \theta_w} \left[\bar{u}_L V W^+ d_L + \bar{d}_L V^\dagger W^- u_L \right] \\
 &= \frac{e}{\sqrt{2} \sin \theta_w} \left[W_\mu^+ \bar{u} V \gamma^\mu \left(\frac{1 - \gamma_5}{2} \right) d + W_\mu^- \bar{d} V^\dagger \gamma^\mu \left(\frac{1 - \gamma_5}{2} \right) u \right]
 \end{aligned}$$

Under CP:

$$\mathcal{L}_{\text{mix}} \rightarrow \frac{e}{\sqrt{2} \sin \theta_w} \left[W_\mu^+ \bar{u} (V^\dagger)^T \gamma^\mu \left(\frac{1 - \gamma_5}{2} \right) d + W_\mu^- \bar{d} V^T \gamma^\mu \left(\frac{1 - \gamma_5}{2} \right) u \right]$$

so a complex CKM matrix breaks CP

Taken from: Matthew D. Schwartz, “Quantum Field Theory and the Standard Model”

CP–Violation in the Standard Model

CP–Violation must thus have a flavor–independent meaning. In the SM, this is provided by the Jarlskog Invariant

$$J_4 \equiv \text{Im Tr} \left[Y_u Y_u^\dagger, Y_d Y_d^\dagger \right]^3 = 6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)\mathcal{J}$$

← mass degeneracies!

where $\mathcal{J} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin(\delta_{\text{CKM}})$

In the standard parametrization

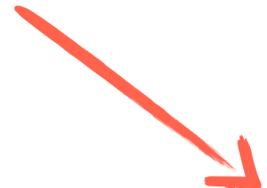
$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta_{\text{CKM}}} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta_{\text{CKM}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\text{CKM}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CKM}}} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_{\text{CKM}}} & c_{13}c_{23} \end{pmatrix}$$

CP–Violation in the Standard Model

CP in the Standard Model is conserved **iff** $J_4 = 0$



No CPV if $N_f = 2$



CP breaking is a collective effect!

The Standard Model Effective Field Theory

The Standard Model is generally intended as the renormalizable part of a larger description, that includes the effects from heavy resonances that cannot be produced on-shell (assuming no new light degrees of freedom).

Deviations from the dimension-4 SM are parametrized via higher dimensional, gauge invariant operators, built with SM field

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_{n \geq 5} \frac{c_n}{\Lambda^{n-4}} \mathcal{O}^{(n)}$$

We will focus on operators of dimension 6 in the Warsaw basis ([B. Grzadkowski et al. arXiv:1008.4884](#))

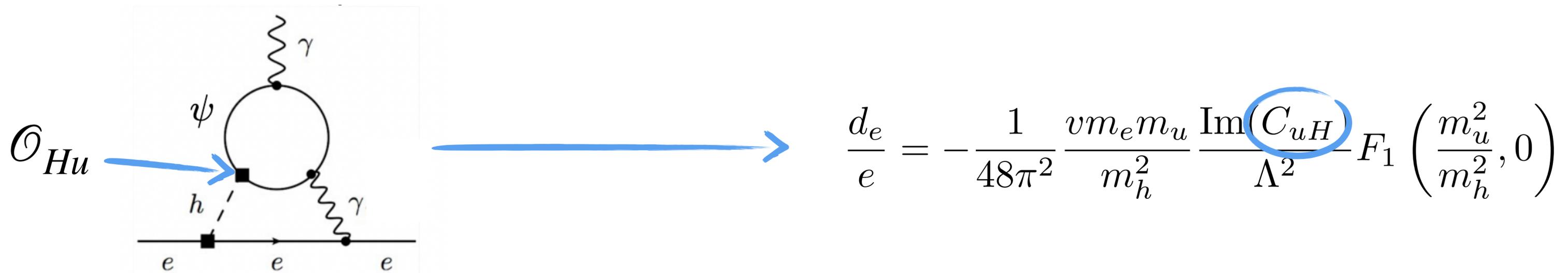
CP-Violation in the SMEFT

As we know from the SM, the presence of phases alone does not necessary imply CPV.

Take a SMEFT with just one generation and only turn on the modified Yukawa \mathcal{O}_{uH} operator

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2} |H|^2 \bar{Q}_L u_R \tilde{H}$$

After EWSB this operator produces a correction to the electron EDM via a Barr-Zee type diagram



CP-Violation in the SMEFT

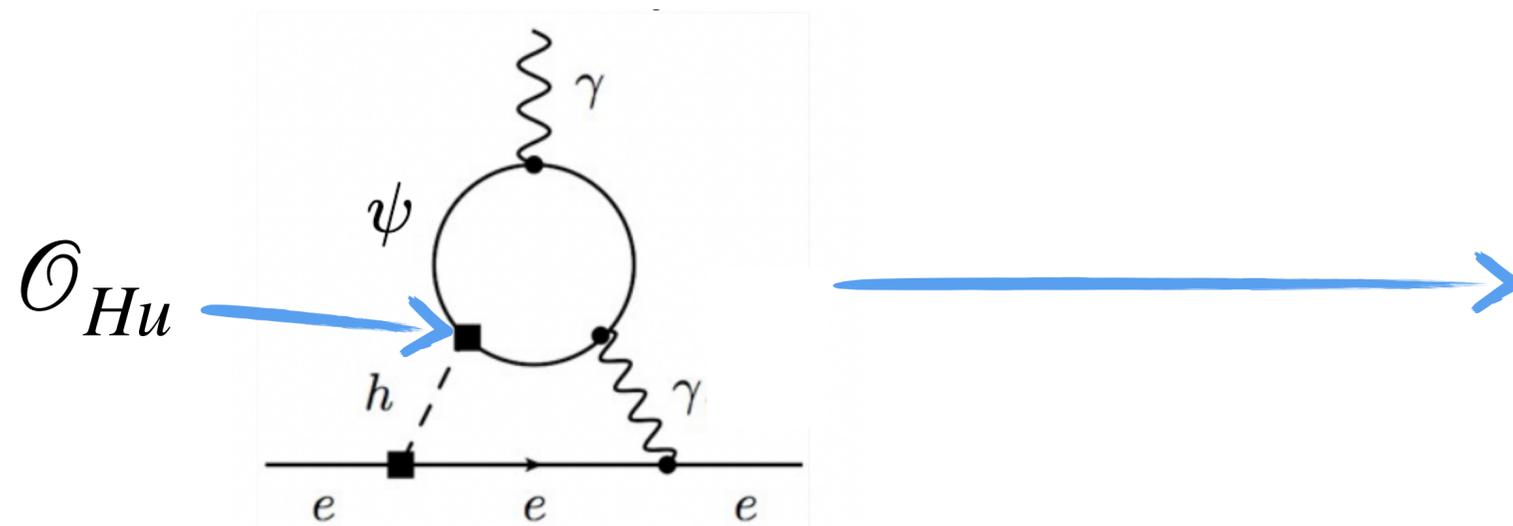
- One could argue that this phase can be removed

redefining $u_R \rightarrow e^{-i \arg(C_{Hu})} u_R$

- However, it will pop up again in the mass term

$\mathcal{L} \supset -m \bar{u}_L u_R$

- A careful evaluation yields



$$\frac{d_e}{e} = -\frac{1}{48\pi^2} \frac{vm_e}{m_h^2} \frac{\text{Im}(m_u^* C_{uH})}{\Lambda^2} F_1\left(\frac{|m_u|^2}{m_h^2}, 0\right)$$

Rephasing invariant!

CP–Violation in the SMEFT

Which phases are physical for 3 flavors? When does the SMEFT break CP?

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \dots \Rightarrow |\mathcal{A}^{(4)}|^2 + 2\text{Re} \left(\mathcal{A}^{(4)} \mathcal{A}^{(6)*} \right)$$

Conserves CP iff $J_4 = 0$

Conserves CP iff $J_4 = 0$ & ???=0

More precisely, what are the order parameters of CP–Violation in the SMEFT?

CP-odd invariants

Given a SMEFT dimension-6 operator containing fermions, we can build a set of CP-odd flavor invariants by giving it spurionic transformation properties.

For example, turning on only $\mathcal{O}_{HQ}^{(1)} = C_{HQ,mn}^{(1)} \left(H^\dagger i \overleftrightarrow{D}_\mu H \right) \bar{Q}_m \gamma^\mu Q_n$

Hermitian 3x3 matrix \rightarrow 3 phases

$$L_1^{HQ(1)} = \text{ImTr}(Y_u Y_u^\dagger Y_d Y_d^\dagger C_{HQ}^{(1)})$$

$$L_2^{HQ(1)} = \text{ImTr}((Y_u Y_u^\dagger)^2 (Y_d Y_d^\dagger)^2 C_{HQ}^{(1)})$$

$$L_3^{HQ(1)} = \text{ImTr}(Y_u Y_u^\dagger Y_d Y_d^\dagger (Y_u Y_u^\dagger)^2 (Y_d Y_d^\dagger)^2 C_{HQ}^{(1)})$$

CP is conserved **iff** $J_4 = L_1^{HQ(1)} = L_2^{HQ(1)} = L_3^{HQ(1)} = 0$

CP-odd invariants

How many conditions?

| | Type of op. | # of ops | # real | # im. | # CP-odd invariants |
|-----------|-----------------|----------|--------|-------|---------------------|
| bilinears | Yukawa | 3 | 27 | 27 | 21 |
| | Dipoles | 8 | 72 | 72 | 60 |
| | current-current | 8 | 51 | 30 | 21 |
| | all bilinears | 19 | 150 | 129 | 102 |
| 4-Fermi | LLLL | 5 | 171 | 126 | 54 |
| | RRRR | 7 | 255 | 195 | 126 |
| | LLRR | 8 | 360 | 288 | 174 |
| | LRRL | 1 | 81 | 81 | 27 |
| | LRLR | 4 | 324 | 324 | 216 |
| | all 4-Fermi | 25 | 1191 | 1014 | 597 |
| | all | | 1341 | 1143 | 699 |

The number of independent linear CP-odd invariants is smaller than the number of new phases!

CP-odd observables at $\mathcal{O}(1/\Lambda^2)$

Working at $\mathcal{O}(1/\Lambda^2)$ reduces the number of CP-violating parameters. Let us start from the up-basis

$$Y_u = \text{diag}(y_u, y_c, y_t) \quad Y_d = V_{\text{CKM}} \text{diag}(y_d, y_s, y_b) \quad Y_e = \text{diag}(y_e, y_\mu, y_\tau)$$

In the lepton sector, this choice breaks the $U(3)_L \times U(3)_e$ of the free Lagrangian down to the $U(1)^3$ described by the transformation

$$(L, e) \rightarrow \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3})(L, e)$$

This has to be a symmetry of all observables.

At dimension 6, operators containing leptons are charged under this symmetry, e.g.

$$\mathcal{O}_{He} = \frac{1}{\Lambda^2} C_{He,mn} (H^\dagger i \overleftrightarrow{D}_\mu H) \bar{e}_m \gamma^\mu e_n \longrightarrow C_{He,mn} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{12}^* & c_{22} & c_{23} \\ c_{13}^* & c_{23}^* & c_{33} \end{pmatrix} \xrightarrow{U(1)^3} \begin{pmatrix} c_{11} & c_{12} e^{i(\delta_2 - \delta_1)} & c_{13} e^{i(\delta_3 - \delta_1)} \\ c_{12}^* e^{-i(\delta_2 - \delta_1)} & c_{22} & c_{23} e^{i(\delta_3 - \delta_2)} \\ c_{13}^* e^{-i(\delta_3 - \delta_1)} & c_{23}^* e^{-i(\delta_3 - \delta_2)} & c_{33} \end{pmatrix}$$

Off-diagonal coefficients are charged under such $U(1)^3$, so at $\mathcal{O}(1/\Lambda^2)$ no invariant containing them can be built

CPV in the SM: a collective effect

In the Standard Model, the smallness of the phenomenological parameters from all three generations conspire to produce a non-zero but small J_4

$$J_4 \equiv \text{Im Tr} \left[Y_u Y_u^\dagger, Y_d Y_d^\dagger \right]^3 = 6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)\mathcal{J}$$

Using the Wolfenstein parametrization

$$Y_u = \text{diag}(a_u \lambda^8, a_c \lambda^4, a_t \lambda^0)$$

$$Y_d = V_{\text{CKM}} \text{diag}(a_d \lambda^7, a_c \lambda^4, a_b \lambda^3)$$

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



$$J_4 \approx \lambda^{36}$$

with $\lambda \approx 0.2$, $a_i = \mathcal{O}(1)$

CPV in the SM: a collective effect

This large suppression carries over to the physical observables, e.g. the electron EDM d_e

$$\frac{d_e}{e} \propto m_c^2 m_s^2 \mathcal{J} < 10^{-34} \text{cm}$$

(Pospelov and Ritz, 1311.5537)

(some mass suppression can be lifted by the dynamics)

$$|d_e^{\text{max}}| = 1.1 \times 10^{-29} e \text{ cm at } 90\% \text{ C. L.}$$

(ACME collaboration)

CPV in the SMEFT: a collective effect

In the SMEFT, CPV still exhibits a collective nature, but the suppression does not need to be the same. E.g.

$$\mathcal{O}_{HQ}^{(1)} = C_{HQ,mn}^{(1)} \left(H^\dagger i \overleftrightarrow{D}_\mu H \right) \bar{Q}_m \gamma^\mu Q_n \longrightarrow \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} A a_b^2 a_t^2 \text{Im} C_{HQ,23}^{(1)} \lambda^8 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(\lambda^9)$$

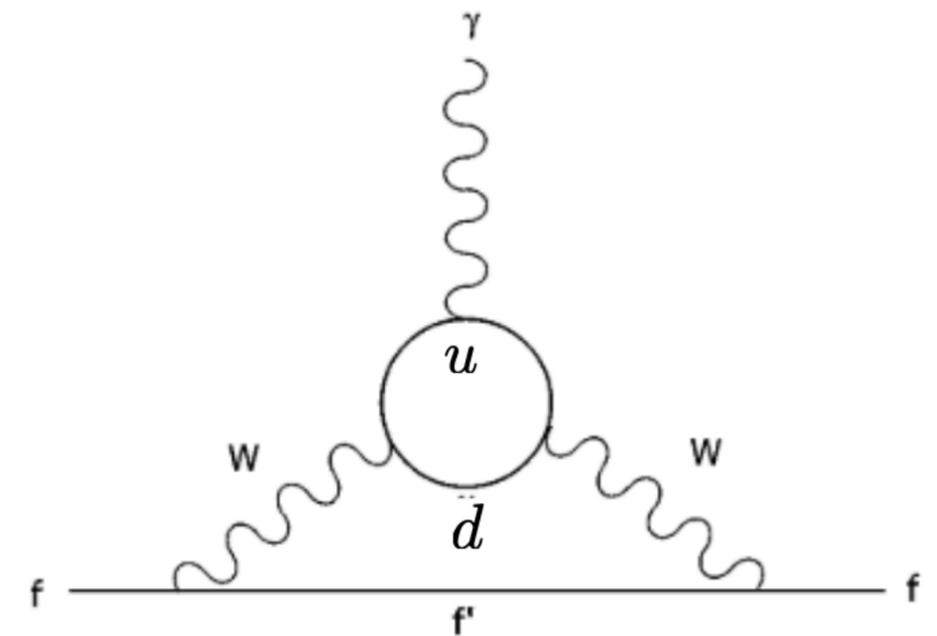
Specific ansatz for the flavor structure can change the suppression

$$\text{MFV} \longrightarrow L_1 \sim \lambda^{16}$$

CPV in the SMEFT: a collective effect

As for the SM, this suppression is carried over to observables, modulo some mass suppression lifted by the dynamics. Again for the EDM

$$\mathcal{L} \supset \frac{C_{Hud,mn}}{\Lambda^2} i \tilde{H}^\dagger D_\mu H \bar{u}_{R,m} \gamma^\mu d_{R,n} \quad \longrightarrow$$



Invariant

$$\frac{d_e}{e} = \sum_{i,j} \text{Im}(V_{CKM,ij} C_{Hud,ij}^*) F(m_{u_i}^2, m_{d_i}^2) \sim \text{Im}[C_{Hud,32}^*] \lambda^7 - \text{Re}[C_{Hud,32}^*] \sin \delta_{CKM} \lambda^9$$

Next steps

- Study the interference of dimension-6 CP-even coefficients with the SM CP-odd phase

$$\frac{d_e}{e} = \sum_{i,j} \text{Im}(V_{\text{CKM},ij} C_{Hud,ij}^*) F(m_{u_i}^2, m_{d_i}^2) \sim \text{Im}[C_{Hud,32}^*] \lambda^7 - \text{Re}[C_{Hud,32}^*] \sin \delta_{\text{CKM}} \lambda^9$$

- Study the suppression of invariants in realistic flavor models

- Match to UV models

Summary

- CPV in the SMEFT inherits the collective nature of CPV in the SM
- Additional CP breaking contained in the coefficients of fermionic higher dimensional operators can be consistently captured by CP-odd linear flavor-invariants
- Not all new phases contained in the operator coefficients break CP at order $\mathcal{O}(1/\Lambda^2)$. Using invariants straightforwardly provides the correct counting
- The invariants can be used to check the suppression of CPV coming from SMEFT operators, connect to UV models and more...