
QCD aspects of Top Quark Pair Production in e^+e^- in the Continuum

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fdk Π Doktoratskolleg
Particles and Interactions

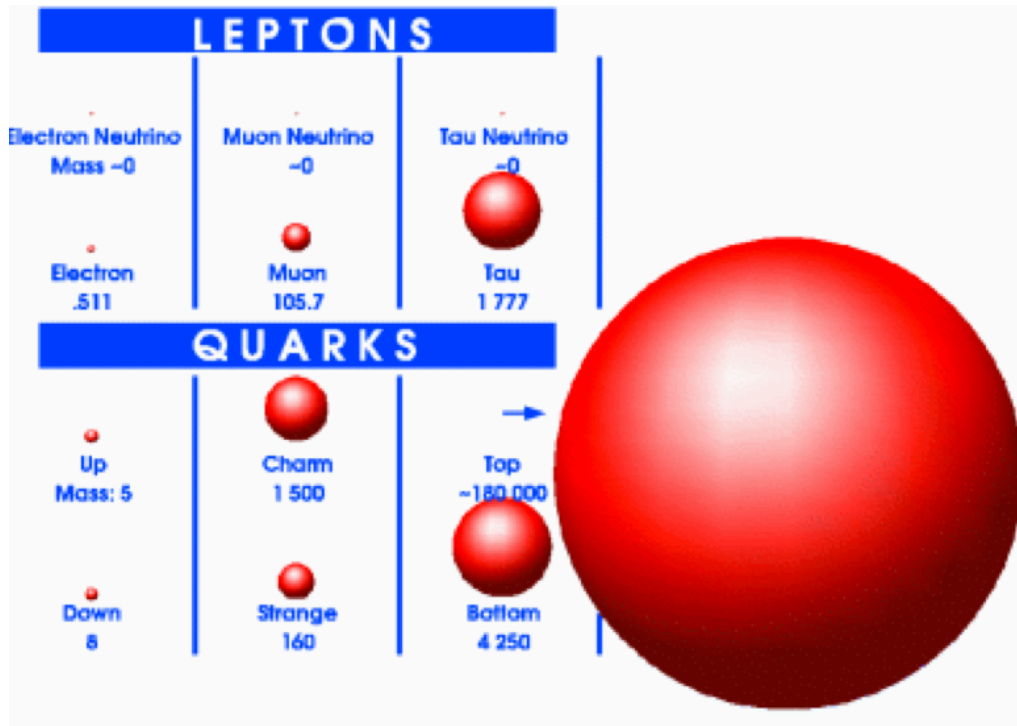


FWF
Der Wissenschaftsfonds.

Outline

- Fixed-order calculations
- Total cross section
- Top mass schemes and couplings
- Associated top pair threshold
Threshold – continuum matching
- Boosted top factorization
- MC top mass problem and direct reconstruction
- Conclusions

.. not just the heaviest SM particle



- Top quark: heaviest known particle
 - Most sensitive to the mechanism of mass generation
 - Peculiar role in the generation of flavor.
 - Top might not be the SM-Top, but have a non-SM component.
 - Top as calibration tool for new physics particles (SUSY and other exotics)
 - Top production major background in new physics searches
 - One of crucial motivations for New Physics
- Very special physics laboratory: $\Gamma_t \gg \Lambda_{\text{QCD}}$
 - Top treated a particle: p_T , spin, σ_{tot} , $\sigma(\text{single top})$, $\sigma(\text{tt}+X)$,... $\rightarrow q \gg \Gamma_t$
 - Quantum state sensitive low-E QCD and unstable particle effects: m_t , endpoint regions $\rightarrow q \sim \Gamma_t$
 - Multiscale problem: p_T , m_t , Γ_t , Λ_{QCD} , . . . (depends on resolution of observable)

Status on FO Calculations

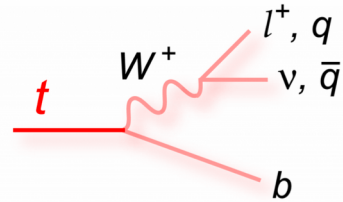
Stable Tops:

$$\sigma_{t\bar{t}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right]$$

- Total inclusive cross section known to $O(\alpha_S^2)$ (FO-num)
- Total inclusive cross section known to $O(\alpha_S^3)$ (FO-Pade)
- NLO EW corrections (FO)
- Full differential $t\bar{t}$ $O(\alpha_S^2)$ (subtractions)

Kühn, Chetyrkin, Steinhauser, AHH,... '96
 Maier, Marquard.. '17
 AHH, Mateu Zebarjad '08
 Kiyo, Maier etal '09, Greynat etal '09
 Fleischer, Leike, Riemann, Wertenbach '03
 Gao, Zhu'16
 Chen, Dekkers, Heisler, Bernreuther '16

Top Decay (NWA):

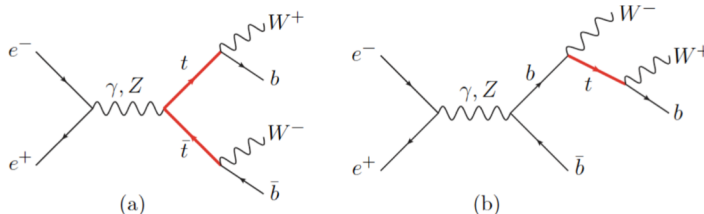


- Total decay rate $O(\alpha_S^2)$ (FO)
- Fully differential $O(\alpha_S^2)$ (FO subtractions)
- NLO EW corrections

Charnecki etal '10
 Gao, Li '12
 Bruchseifer, Caola, Melnikov '13
 '90s

“Off-shell” Top quarks: → essential for reconstructed top invariant mass, endpoint decay spectra

- Full off-shell $e^+e^- \rightarrow WWbb$ $O(\alpha_S)$ (FO)



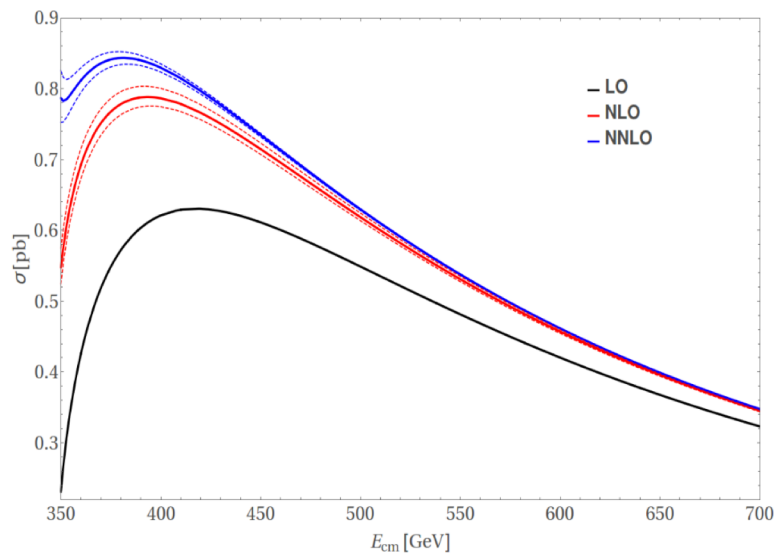
Guo, Ma, Zhang, Wang '08
 MadGraph5@NLO, WHIZARD, ...
 Standard now

Status on FO Calculations

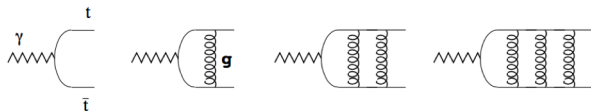
Is this good enough? In general not!

Example: total $t\bar{t}$ cross section

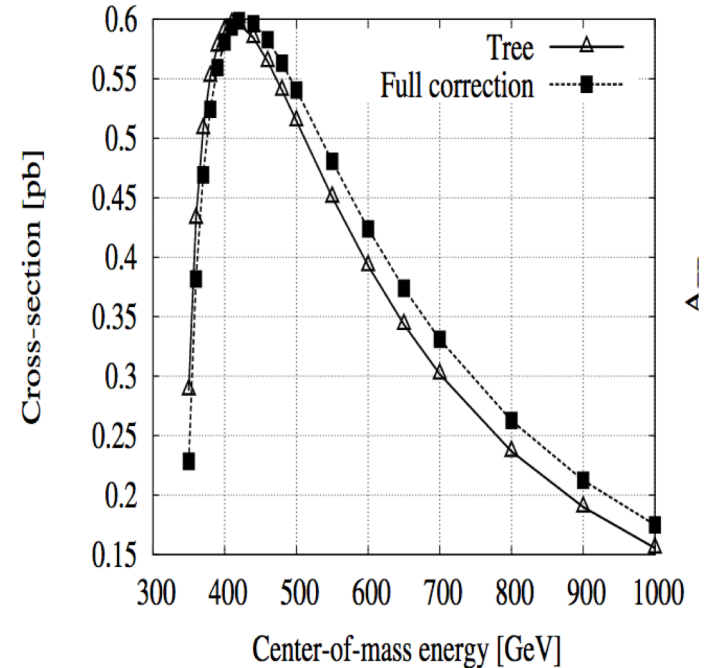
Chen, Dekkers, Heisler, Bernreuther '16



- Huge correction at threshold $E_{cm} \approx 2m_t$
- **Coulomb effects** $\sim (\alpha_S/v)^n$
- Resummation mandatory (very well developed)



Fleischer, Leike, Riemann, Werthenbach '03



- **EW Sudakovs logarithms** for very large energies: $\log(E_{cm}/m_t)$
- Fixed-order fine for FCC-ee
- Problematic above 1 TeV
(No complete code exists yet)

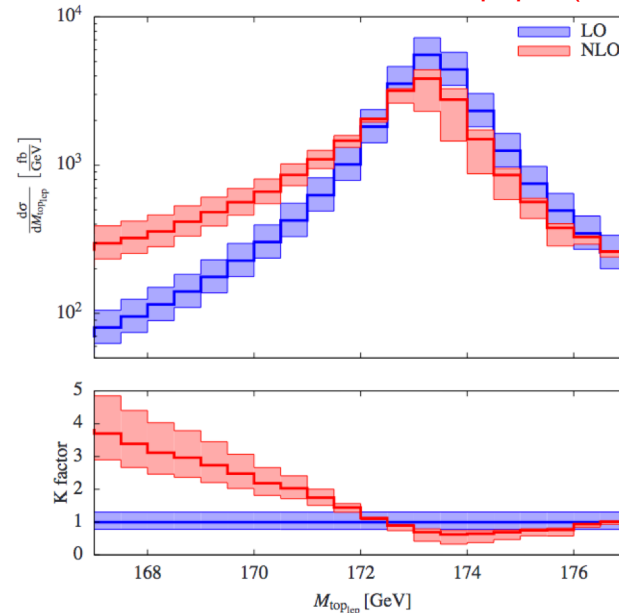
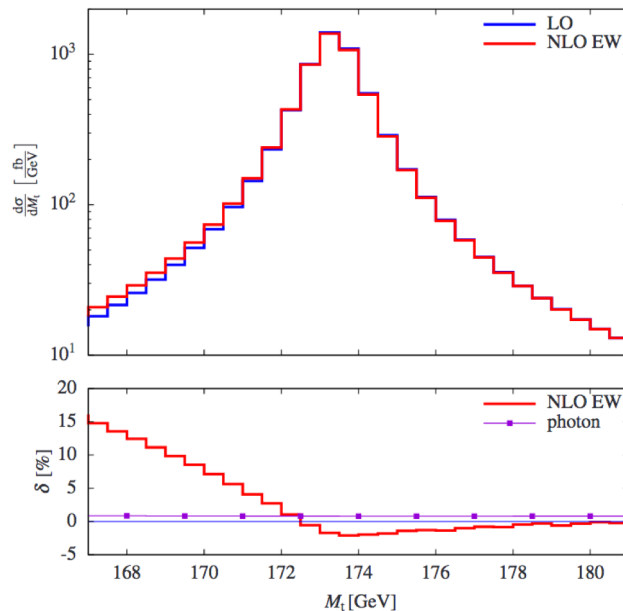
Status on FO Calculations

Is this good enough? In general not (endpoint, off-shell, ... regions)

Example: 'reconstructed' top invariant mass

Pellen, Denner '17

From an LHC paper (sorry for that..)



- Scale variation does not cover perturbative uncertainty.
- Fixed-order not sufficient, e.g. large QCD logs $\log(m_t/\Gamma_t)$
- **Combined QCD/electroweak resummation** mandatory (not worked out yet, but possible using knowledge from flavor physics)
- **Hadronization effects** large at threshold and resonances

Update on $R(e^+e^- \rightarrow t\bar{t})$ at $O(\alpha_s^3)$

Dehnadi, Mateu, Stahlhofen, Widl, AHH to appear

$$\Pi(z) = \Pi^{(0)}(z) + \left(\frac{C_F \alpha_s}{\pi}\right) \Pi^{(1)}(z) + \left(\frac{\alpha_s}{\pi}\right)^2 \Pi^{(2)}(z) + \left(\frac{\alpha_s}{\pi}\right)^3 \Pi^{(3)}(z) + \dots \quad z = \frac{q^2}{4m_t^2}$$

- Reconstruction of $\Pi^{(3)}$ from threshold ($z=1$), low-energy ($z=0$), high-energy ($z \gg 1$) results using Padé approximations.

$$\begin{aligned} \Pi_{\text{low}}^{(n)}(z) &= \sum_{i=1}^{n_{\text{low}}} C_i z^i \\ \Pi_{\text{high}}^{(n)}(z) &= \sum_{i=0}^{n_{\text{high}}} \sum_{j=0}^m D_{ij} \frac{1}{z^i} \log^j(-4z) \\ \Pi_{\text{threshold}}^{(n)}(z) &= \sum_{i=1-k}^{n_{\text{thres}}-k} \sum_{j=0}^m K_{ij} (1-z)^{\frac{i}{2}} \log^j(1-z) \end{aligned}$$

$$\Pi^{(3)}(z) \sim \Pi_{\text{ansatz}}^{(3)}(z) + P^{[N/W]}(w(z))$$

Maier, Maierhofer, Marquard

$$P^{[N/M]}(\omega) = \frac{c_0 + c_1 \omega + \dots + c_N \omega^N}{1 + c_{N+1} \omega + \dots + c_{N+M} \omega^M}$$

- Input: $n_{\text{low}}=4$, $n_{\text{high}}=2$, $n_{\text{thres}}=3$ (full), $n_{\text{thres}}(\log(v))$

$$w(z) = \frac{1 - \sqrt{1-z}}{1 + \sqrt{1-z}}$$

Maier, Maierhofer, Marquard,
Chetyrkin, Kühn, Sturm,
Boghezal, Czakin, Schutzmeier

NNLO NRQCD

NNLL vNRQCD

Manohar, Stewart, AHH,
Stahlhofen '00, '13

z-plane mapped onto w-unit circle

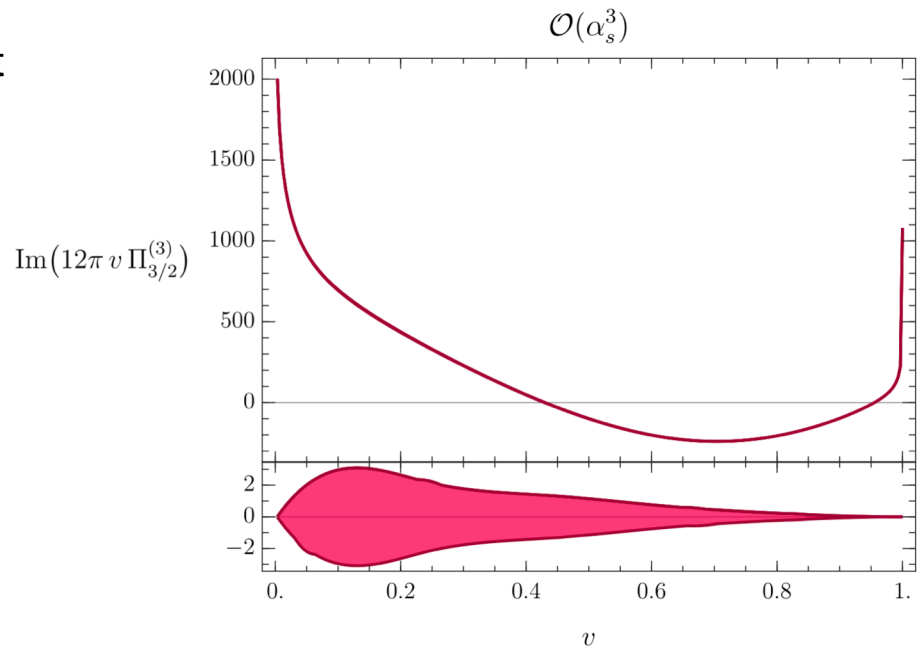
NEW!

Update on $R(e^+e^- \rightarrow t\bar{t})$ at $O(\alpha_s^3)$

Dehnadi, Mateu, Stahlhofen, Widl, AHH to appear

$$\Pi(z) = \Pi^{(0)}(z) + \left(\frac{C_F \alpha_s}{\pi}\right) \Pi^{(1)}(z) + \left(\frac{\alpha_s}{\pi}\right)^2 \Pi^{(2)}(z) + \left(\frac{\alpha_s}{\pi}\right)^3 \Pi^{(3)}(z) + \dots \quad z = \frac{q^2}{4m_t^2}$$

- Use exact result for $\Pi^{(2)}$ as benchmark for Padé construction to examine convergence when endpoint information is added.
- Uncertainty in $\Pi^{(3)}$ reduced from percent by one order of magnitude to permille precision for energies $Q \lesssim 400$ GeV
- Exactly known from the practical perspective.
- Scale variation dominant error for QCD total cross section at $O(\alpha_s^3)$



Top Quark Mass Schemes

- High precision demands to take into account the properties of mass schemes and that one picks an adequate scheme
- Very well understood: $O(\alpha_S^4)$ results! Marquard, Smirnov, Smirnov, Steinhauser'15
- Pole mass m_t^{pole} not adequate for some high-precision applications due to a renormalon ambiguity:
 - $\Delta m_t^{\text{pole}} = 110 \text{ MeV}$ Beneke, Nason, etal '16
 - $\Delta m_t^{\text{pole}} = 250 \text{ MeV}$ AHH, Lepenik, Preisser '17
- Pole ambiguity arises because linear IR effects absorbed into the mass
- Ambiguity-free masses only absorb effects above their renormalization scale μ (“short-distance masses”): $m_t(\mu) \leftarrow \mu = \text{dynamical scale of the process}$

Top Quark Mass Schemes

- Most popular **short-distance mass schemes**:

MSbar:
$$m_t^{\text{pole}} - \bar{m}_t(\mu) = \frac{4}{3} \left(\frac{\alpha_s(\mu)}{\pi} \right) \bar{m}_t(\mu) + \dots$$

Meaningful for
 $\mu > m_t$

$$\frac{d}{d \ln \mu} \bar{m}_t(\mu) = -\bar{m}_t(\mu) \left(\frac{\alpha_s(\mu)}{\pi} \right) + \dots$$

Threshold masses: kinetic

1S

Bigi, Shifmann, Uraltsev '97

AHH, Ligeti, Manohar '98

PS

Beneke '98

RS

Pineda '01

Constructed from
ttbar threshold
and B physics
observables,
renormalon study

MSR:
$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = \frac{4}{3} \left(\frac{\alpha_s(R)}{\pi} \right) R + \dots$$

AHH, Jain, Scimemi, Stewart '08

$$\frac{d}{d \ln R} m_t^{\text{MSR}}(R) = -\frac{4}{3} R \left(\frac{\alpha_s(R)}{\pi} \right) + \dots$$

Meaningful for
 $R < m_t$

- MSbar+MSR: Consistent flavor number dependent RG evolution with threshold matching
- MSR “interpolates” between pole mass and MSbar mass

Top Quark Mass Schemes

- Theoretical precision achievable for short-distance masses:
10-20 MeV for all heavy quarks (QCD only!)

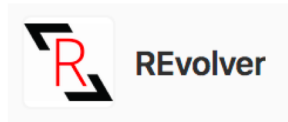
- Software tools:

Rundec/Crundec (QCD only)

Herren et al. 1703.03751

- Collections of fixed-order conversion routines (C++, Mathematica)
- RG-evolution for $\overline{\text{MS}}$ masses only
- MSR mass not supported

REvolver (QCD only)



AH, Lepenik, Mateu 2102.01085

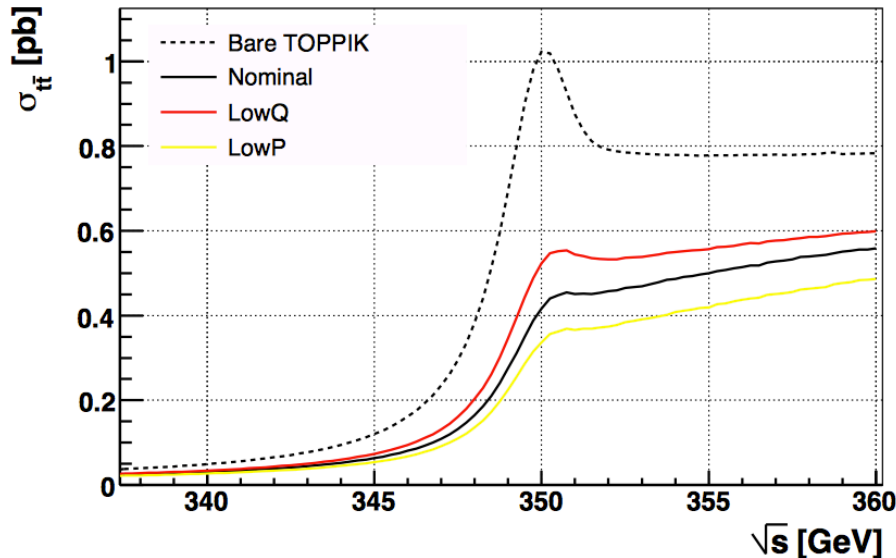
- Core concept, physics scenarios (C++ library & Python, Mathematica interfaces)
- RG-evolution for all mass relations including lighter flavor threshold corrections
- MSR mass supported
- Fast and exact solutions for RGE running
- Pole mass diagnostic routines

Electroweak corrections partly known, but not generally available in tools !

Top Threshold

Top pair total inclusive cross section:

$$\sigma(e + e^- \rightarrow t\bar{t} + X) \text{ at } E_{cm} \approx 2m_t$$



Principle: m_t from $\sigma_{tt}(m_t)$

Advantages:

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood (renormalons, summations)
- ▷ Top decay protects from non-pert effects

Crucial difference to top pairs at LHC

- Remnant of a topionium resonance (“postronium of QCD”): $R_{\text{bind}} = m_t \alpha_s \sim 30 \text{ GeV}$
- Crucial to control e^+e^- luminosity spectrum
- Binding energy about twice the top quark width:
- Can be calculated in pQCD (nonrelativistic expansion)
- Non-resonant effects very small, little background

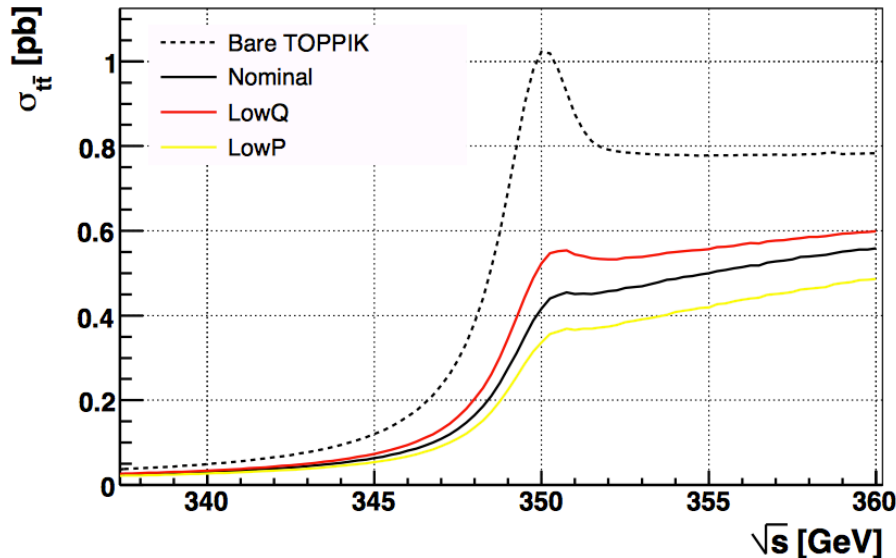
$$E_{\text{bind}} \approx \frac{\alpha_s^2 m_t}{2} \approx 2\Gamma_t$$

Top Threshold

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Principle: m_t from $\sigma_{tt}(m_t)$



Advantages:

- ▷ count number of $t\bar{t}$ events
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- ▷ physics well understood (renormalons, summations)
- ▷ Top decay protects from non-pert effects

Crucial difference to top pairs at LHC

- The only observable known where a threshold structure with resolution $\ll 1$ GeV is generated by QCD dynamics at much larger scale: $R_{\text{bind}} = m_t \alpha_S \sim 30$ GeV
- Color singlet state protects from non-perturbative effects.

We could not be more lucky!

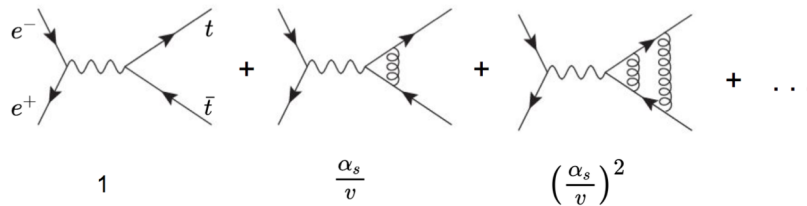


Unfortunately no such observable at the LHC !



Top Threshold

- Coulomb resummations
- Finite Width effects are leading order
- NRQCD effective field theory counting ($\alpha_s \sim v$)



- Total cross section at NNLO (FO in $\alpha_s \sim v$)
- Total cross section NNLO+NNLL (sum $\ln(\alpha_s) \sim \ln(v)$)
- Total cross section NNNLO
- Non-resonant EW effects NNLL
- Non-resonant EW effects NNNLO_{partial}
- Top p_t 3-momentum distribution NNLO
- Full differential: NLO+(N)LL in Whizard MC

AHH, Beneke, Melnikov, Nagano, Ota, Penin, Pivovarov, Signer, Smirnov, Sumion, Teubner, Yakovlev, Yekhovskiy '01

AHH, Stahlhofen, '13

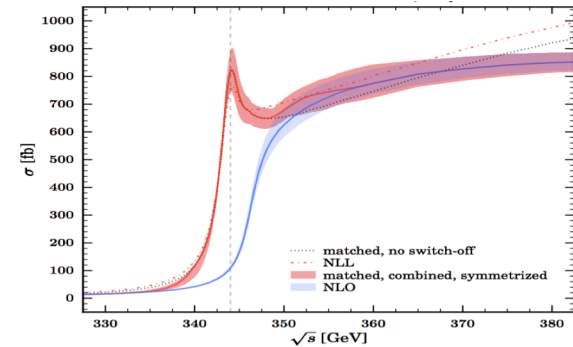
Beneke, Kiyo, Marquard, Piclum, Steinhauser '13

AHH, Reisser, Ruiz-Femenia '04, '10

Beneke, Maier, Rauh, Ruiz-Femenia '17,

AHH, Teubner '00

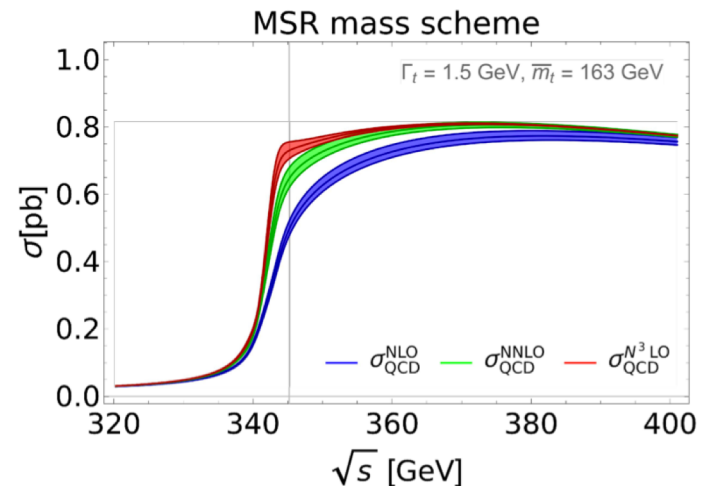
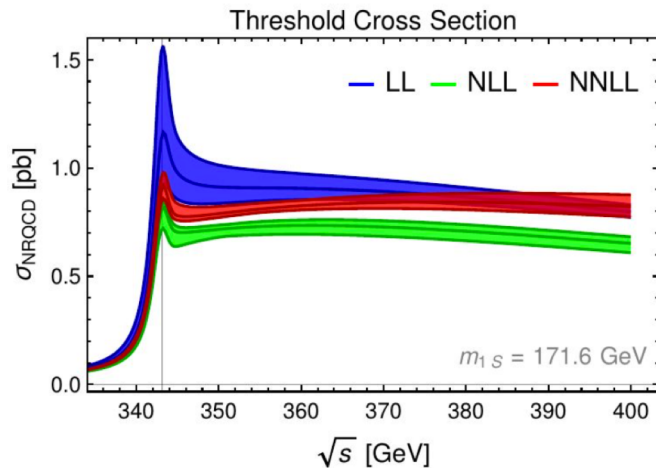
Chokoufe, AHH, Kilian, Reuter, Stahlhofen, Teubner, Weiss '17



Total cross section in very good shape.

Threshold Continuum Matching

- Total $t\bar{t}+X$ cross section:
Combination of the of NNNLO FO and NNLO+NNLL threshold cross section



- Threshold cross section does not provide any good description above threshold region
- Fixed-order cross section sufficient for $Q \gtrsim 400 \text{ GeV}$
- Not straightforward to obtain a good description in the region between 350 and 400 GeV
- No natural smooth interpolation between continuum and threshold

Threshold Continuum Matching

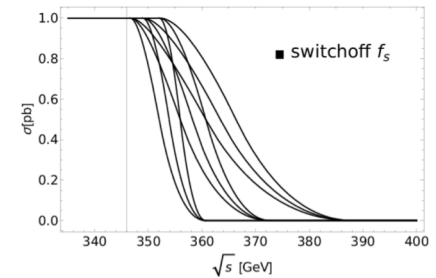
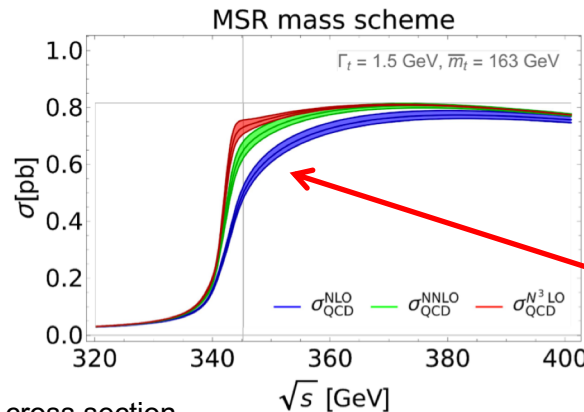
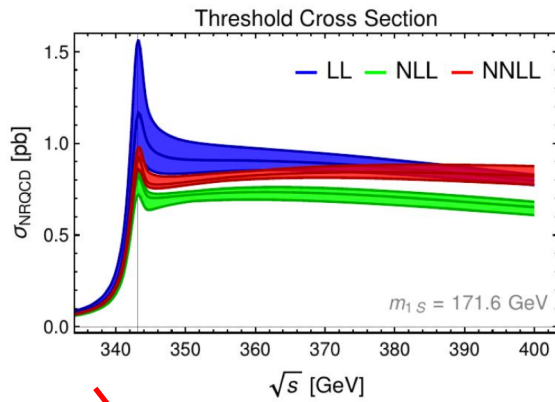
Dehnadi, Mateu, Stahlhofen, Widl, AHH to appear

- Matched $t\bar{t}+X$ cross section in QCD:

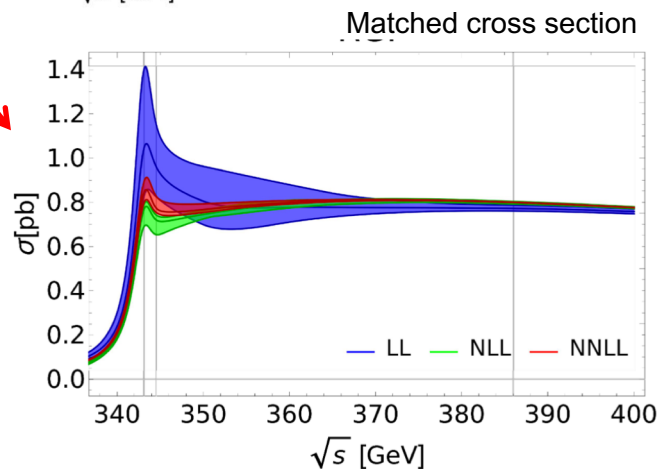
Combination of the of NNNLO FO and NNLO+NNLL threshold cross section

$$\sigma_{\text{matched}} = \sigma_{\text{QCD}} + (\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}) \cdot f_s$$

Switchoff- function



FO Coulomb singularities tamed in MSR mass scheme



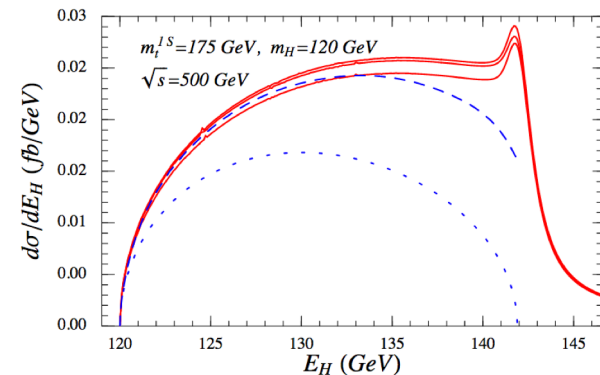
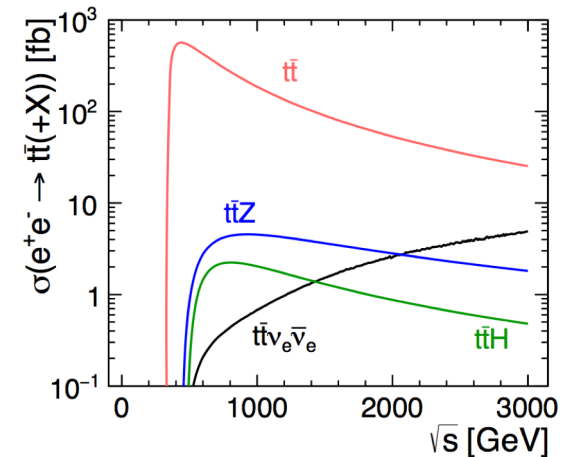
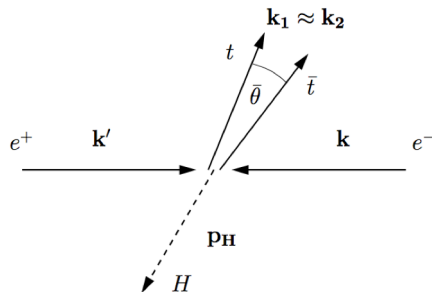
- Use of R-dependent MSR mass crucial to good matching.
- Dependence on switch function smaller than scale-dependence

Associated Top Threshold Physics (I)

- A future e^+e^- collider with many associated $t\bar{t}$ thresholds
- Technology exists to extend $t\bar{t}$ threshold machinery to them, but much less event

$t\bar{t} + H$:

- NLO QCD Dawson, Reina '17,
 - NLO EW corrections Dener, et al., Belanger, et al. You, et al '03,
 - NLL threshold enhancement Farrell, AHH '05
- Kinematic threshold enhancement reaching far into the continuum region for associated $t\bar{t}$ production, enhances cross section
 - Similar situation: $t\bar{t} + Z$



\sqrt{s} [GeV]	m_H [GeV]	$\sigma(\text{Born})$ [fb]	$\sigma(\alpha_s)$ [fb]	$\sigma(\text{NLL})$ [fb]	$\frac{\sigma(\text{NLL})}{\sigma(\text{Born})}$	$\frac{\sigma(\text{NLL})}{\sigma(\alpha_s)}$	$\frac{\sigma(\text{NLL}) _{\beta < 0.2}}{\sigma(\alpha_s)_{\beta < 0.2}}$
500	120	0.151	0.263	0.357(20)	2.362	1.359	1.78

Farrell, AHH '05

Associated Top Threshold Physics (II)

tt + γ :

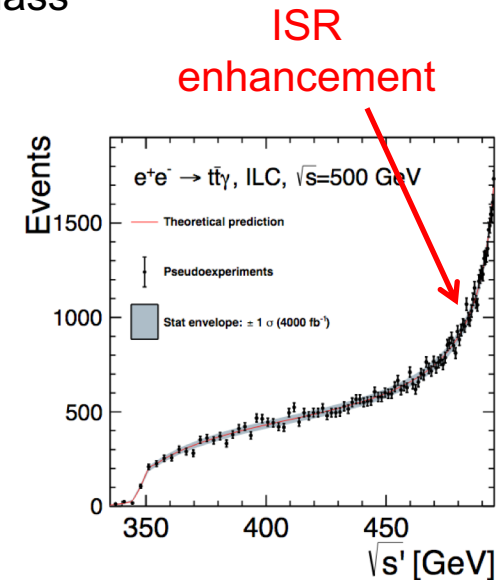
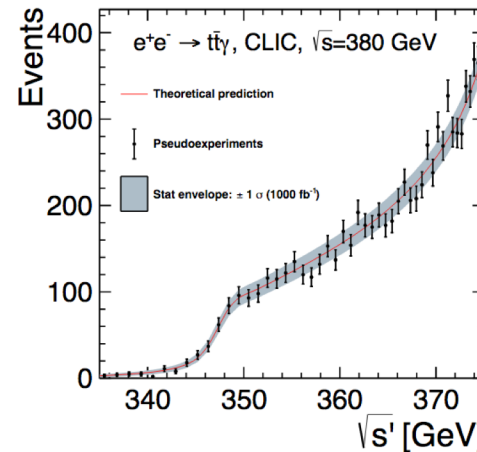
Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

- Radiative return to the tt threshold allows for top threshold top mass measurements at higher energies.

$$\frac{d\sigma_{t\bar{t}\gamma}}{d\cos\theta d\sqrt{s'}} = 2g(x, \theta) \sqrt{\frac{1-2x}{s}} \frac{\alpha_{em}}{\pi} \sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}},$$

$$s' = s \left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$



- Matched threshold (NNLL+NNLO)-continuum (NNNLO) cross section essential
- Realistic simulation experimental analysis
- Statistics dominated

cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

Associated Top Threshold Physics

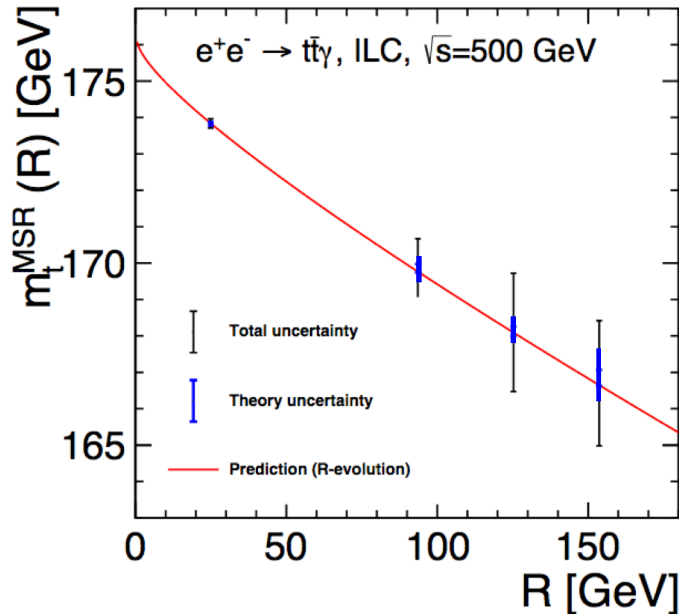
tt + γ :

Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

- Running MSR mass measurements

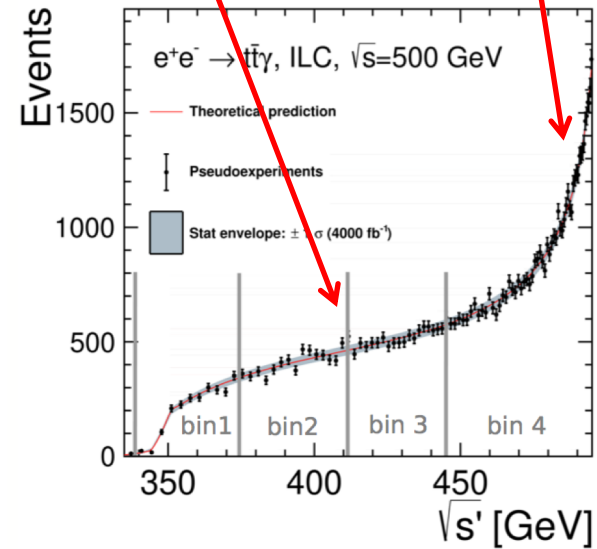
$$\frac{d\sigma_{t\bar{t}\gamma}}{d\cos\theta d\sqrt{s'}} = 2g(x, \theta) \sqrt{\frac{1-2x}{s}} \frac{\alpha_{\text{em}}}{\pi} \sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}}, \quad s' = s \left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$



Probes top mass sensitivity at scales $m_f \nu < m_t$

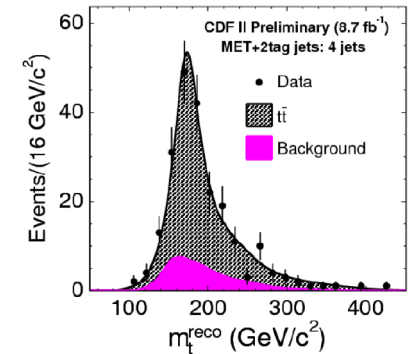
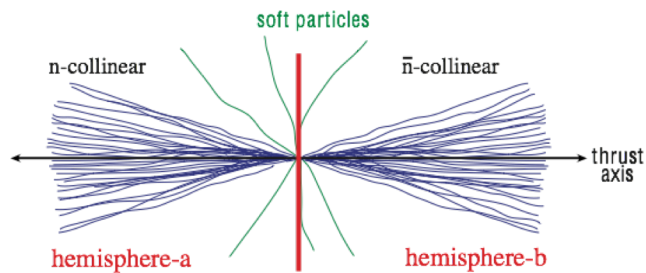
ISR enhancement



Factorization for Boosted Top Quarks

Basic event structure:

- Top and antitop boosted back-to-back: $M_{tt} \gg m_t$
- Top jets in the resonance region: $M_{J,top} \sim m_t$
- [LHC: central top jets: $|\eta_J| \lesssim 1$ (beam separation)]
- [Veto on additional hard (gluon) jet]



- Top and ant-top decays separated (\rightarrow factorization)
- Soft radiation between jets not sensitive to top decay (top-bottom collinear color line)



One systematic avenue to make (QCD+ew) resummed + hadron level predictions for top decay sensitive observables.

- Single-top treatment of top and anti-top decays
- Clean separation of scales: p_T (or E_{cm}) $\gg m_t \gg \Gamma_t \gg \Lambda_{QCD}$

- Factorized cross section at hadron level for e^+e^-

$$\sigma \sim H_Q \times H_m \times B \otimes D \otimes S$$

“hard”
“mass mode”
“jet”
“decay”
“soft”

Factorization for Boosted Top Quarks

Stable Tops:

from on-shell top parton momenta



Ferrogli, Pecjak, Yang, '12

Pejcak, Scott, Wang, Yang, '16

- Systematic resummation of QCD logarithms $\log(M_{t\bar{t}}/m_t)$
- Applies for inclusive (hard) differential distributions ($x \rightarrow 1$)

$$\frac{d\hat{\sigma}_{ij}^{\text{NLP}}}{dM_{t\bar{t}} d\Theta} = \frac{16\pi^2 \alpha_s^2(\mu_r)}{M_{t\bar{t}}^5} \sqrt{\frac{M_{t\bar{t}} + 2m_t}{2M_{t\bar{t}}}} \sum_{\alpha} c_{ij,\alpha}(\cos\theta_t) \times H_{ij,\alpha}(z, M_{t\bar{t}}, Q_T, Y, \mu_r, \mu_f) J^{\alpha}(E).$$

“Off-shell” Tops: \rightarrow “top state” = (top + u.collinear radiation)_{color singlet}

- Systematic resummation of QCD logarithms $\log(M_{t\bar{t}}/m_t)$, $\log(m_t/\Gamma_t)$ Fleming, AH, Mantry, Stewart '07
- Applies for event-shape-type observables (e.g. top jet mass, 2-jettiness)
- **Hadron level** prediction: non-perturbative effects through factorization

$$\left(\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hem}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(l^+, l^-, \mu)$$

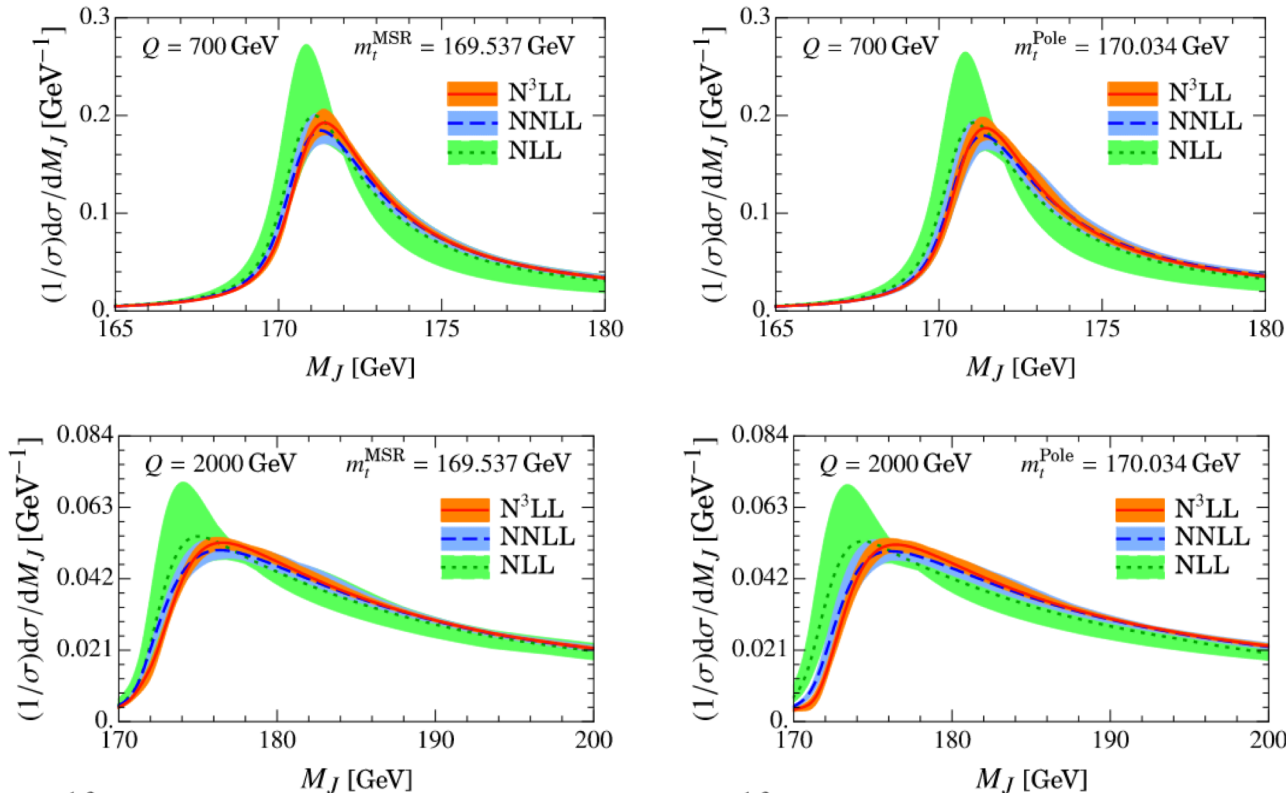
Bachu, Ahm Mateu, Pathak, Stewart '20

- Systematic treatment of groomed jet masses AH, Mantry, Pathak, Stewart '17 + '19
- Provides tests of MC generators \rightarrow MC top mass calibration Butenschön etal '16

2-Jettiness Distribution at NNNLL

Bachu, Mateu, Pathak, Stewart, AHH '20

- Top resonance distribution
- NNLO fixed order hard, jet, soft functions
- NNNLL RG evolution
- Combined SCET and bHQET factorization
- LO Finite lifetime effects (no NWA!)
- Perturbative uncertainty: $\sim 4\%$
- Cancellation between pole mass and large-angle soft radiation renormalons at $Q \sim 1$ TeV (soft and mass effects very similar)

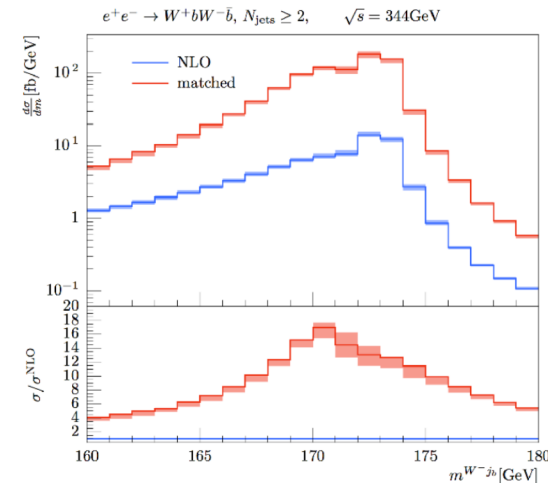
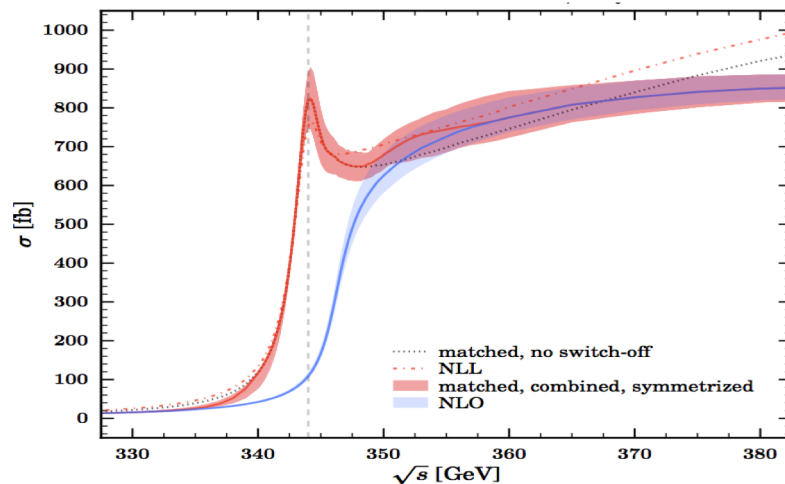


Top Threshold

MC for Differential Cross Sections at threshold:

Bach, Nejad, AH, Kilian, Reuter '17

- $\text{NLO}_{\text{FO}} + \text{NLL}_{\text{threshold}}$ implementation in Whizard
- Whizard threshold implementation does NOT contain NLL ultrasoft effects !
Therefore $\text{NLO}_{\text{FO}} + \text{NLL}_{\text{threshold}}$ only for total cross section, $\text{NLO}_{\text{FO}} + \text{LL}_{\text{threshold}}$ otherwise.



- Ultrasoft non-factorizable corrections still have to be added
- Important: state-of-the-art parton showers do not provide correct LL QCD resummation!

Top Mass from Direct Reconstruction

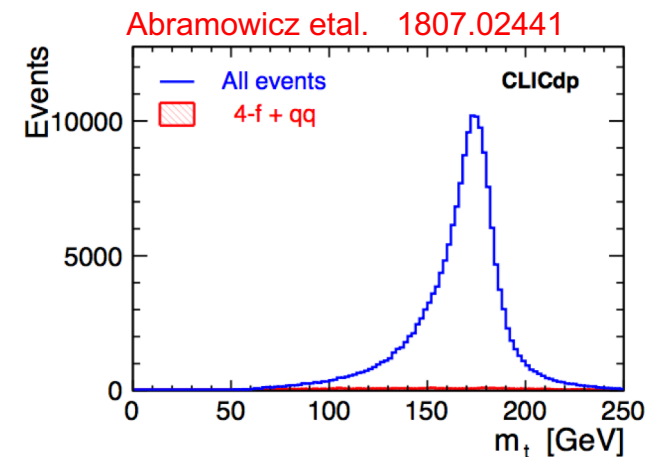
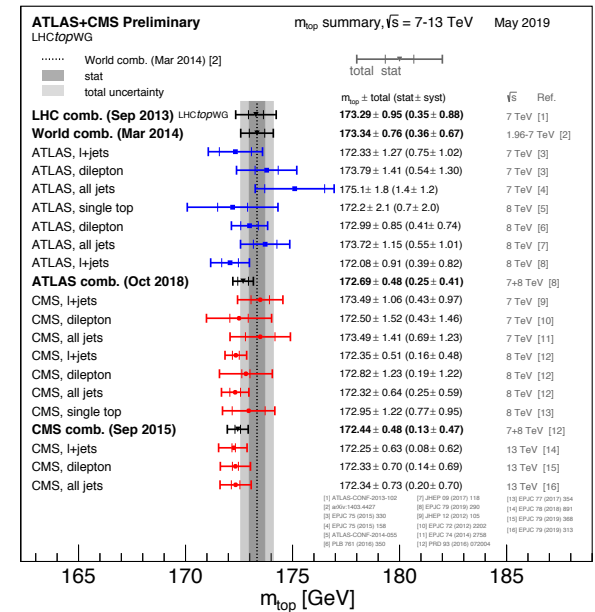
- Direct mass measurements (template or matrix element fits) are the most precise method to determine the top mass at the LHC
- Variables (M_{lb} , m_t^{reco}) cannot be described by FO computation and are described completely by parton shower and hadronization dynamics in Monte-Carlo generators.
- Because MC have limited (observable dependent) precision the measured top mass m_t^{MC} cannot be a priori assigned to a particular mass scheme.

"MC Top Quark Mass Problem"

- The situation is not different at a lepton collider, but the systematic uncertainties are much smaller.
- CLIC simulation study: m_t^{reco} template fit $E_{cm} = 380$ GeV

$$(\Delta m_t^{MC})^{stat} \sim 30 \text{ MeV} \quad (\Delta m_t^{MC})^{syst} \sim 50 \text{ MeV}$$

Competitive with threshold measurements.



Top Mass from Direct Reconstruction

Why bother given that we have the top threshold?

- For lepton collision is it much easier to understand the MC top mass interpretation problem and we can use the consistency with the threshold mass measurements as a benchmark to improve the intrinsic precision of MC generators and make them into much more reliable tools.

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}}$$



pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup

analyzed for e^+e^- collisions
Plätzer, Samitz, AHH 1807.06617



Non-perturbative contribution:

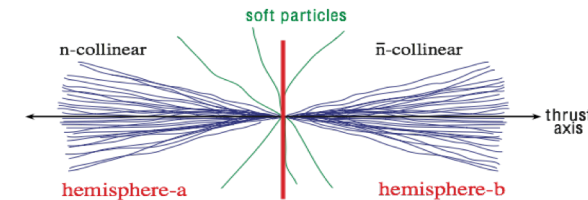
- Effects of hadronization model
- Depends on parton shower setup

Work in progress
Plätzer, Samitz, AHH

Top Mass from Direct Reconstruction

Plätzer, Samitz, AHH '18

- Analytic parton-level analysis of QCD factorization calculation (NLL') and the Herwig angular-ordered parton shower for the 2-jettiness τ_2 distribution for boosted top pair production in the NWA



- Herwig shower is NLL precise for τ_2 .
- Definition of generator mass can be computed by comparison to NLL' QCD calculation.
- Generator mass $m_t^{\text{CB}}(Q_0)$ depends on the shower cut $Q_0=1.25$ GeV.

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3}Q_0\alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

$$m_t^{\text{MSR}}(Q_0) - m_t^{\text{CB}}(Q_0) = 120 \pm 70 \text{ MeV}$$

$$m_t^{\text{pole}} - m_t^{\text{CB}}(Q_0) = 480 \pm 260 \text{ MeV}$$

- First step of a general long-term project (work in progress, progress expected)
- Universality for top decay sensitive observables still to be demonstrated
- Result shows that the question is very relevant also for LHC.

Conclusions

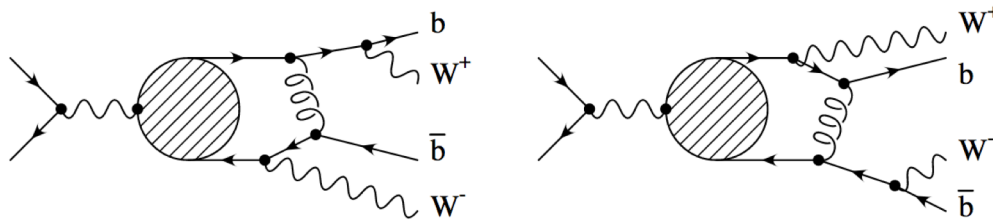
- Many useful calculations and tools already exist.
- By the time a future LC comes into operations, however, many additional theoretical developments are needed to take full advantage of the high level of precision expected in the experimental measurements.
- Total cross section: QCD contributions in rather good shape, electroweak corrections partly known, but not available in tools.
- Analytic predictions for realistic differential cross sections available only for very few distributions.
- Boosted top quarks are ideal to make (QCD+ew) resummed and hadron level predictions that can be used for experimental analysis or as a tool to test other less precise tools such as Monte-Carlo event generators.
- The top quark Monte-Carlo mass problem can be more easily resolved for a LC than for the LHC.

Backup Slides

Top Threshold

Differential Cross Sections:

- Has not received much attention in the past, but important to correctly simulate experimental cuts
- Very (!) hard problem due to ultrasoft ($E \lesssim \Gamma_t$) gluon exchange between the top quarks and their decay products. They cancel in the fully inclusive cross section



Melnikov, Yakovlev '93

- Non-factorizable effects possible due to selection cuts (**size unknown, but likely not large**)

Effects increase the more restrictive cuts are.

Small for generous (wide) cuts

Contribute at NLL/NLO order for differential cross section.

- Theoretically hard due to existence of Coulomb form factor that is defined in the non-relativistic limit only (usual subtraction techniques known from NLO-revolution do not apply)

AHH, Reisser, Ruiz-Femenia '10

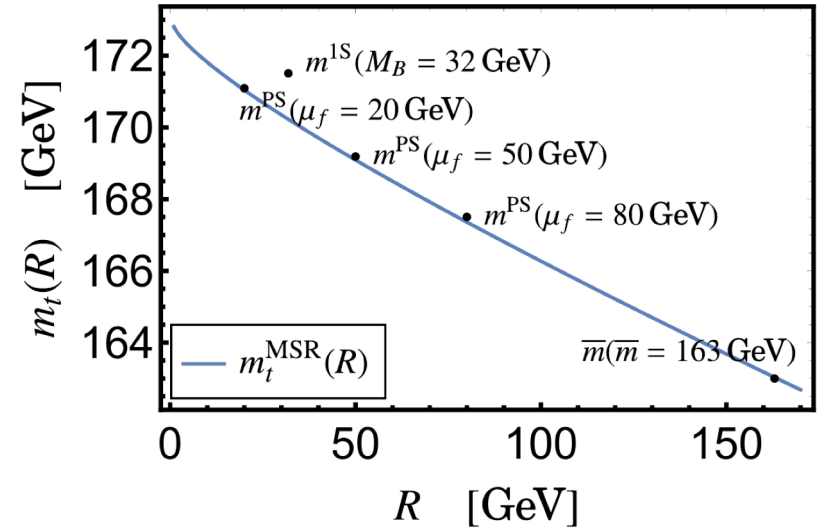
Top Quark Mass Schemes

Has RGE linear in R and numerically very close to threshold masses at their respective scale.

MSR mass is generalization of the MSbar mass for scales $R < m_t$.

Conversion precision between all short-distance mass better than 30 MeV.

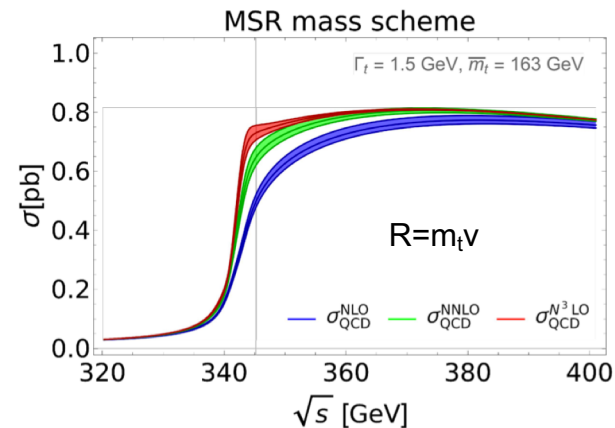
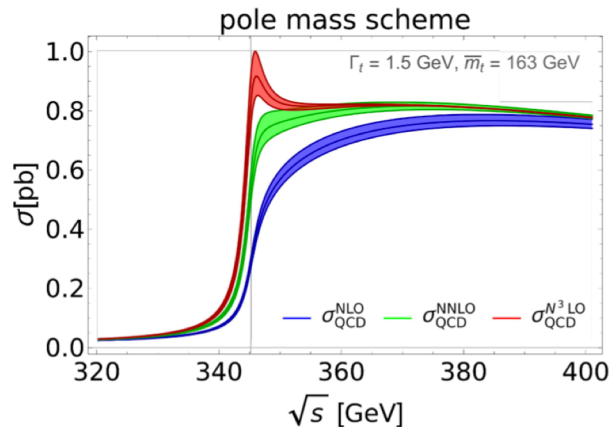
AHH, Jain, Scimemi, Stewart, Lepenik, Preisser '17



Improved perturbative behavior by choosing appropriate scale R

e.g. total inclusive $t\bar{t}$ FO cross section

Widl, AHH to appear



Status of Top Mass Determinations at the LHC

- NLO matched MC:

- MadGraph5_aMC@NLO
- POWHEG

→ common: set $m_t^{\text{MC}} = m_t^{\text{pole}}$ in NLO-matched MCs
(when m_t^{pole} is used for the hard NLO MEs)

→ elevates the first hard emission
($p_{\text{trans}} \gtrsim 10 \text{ GeV}$) to NLO precision

→ diff. cross sections dominated by soft and
collinear radiation not improved:
 m_t^{MC} has same meaning as for
unmatched MC

→ observables used for direct top mass
not improved by NLO-matching

Alwall etal. '14

Alioli, Nason, Oleari, Re '10

