Physics potential with radiative return events at ILC250

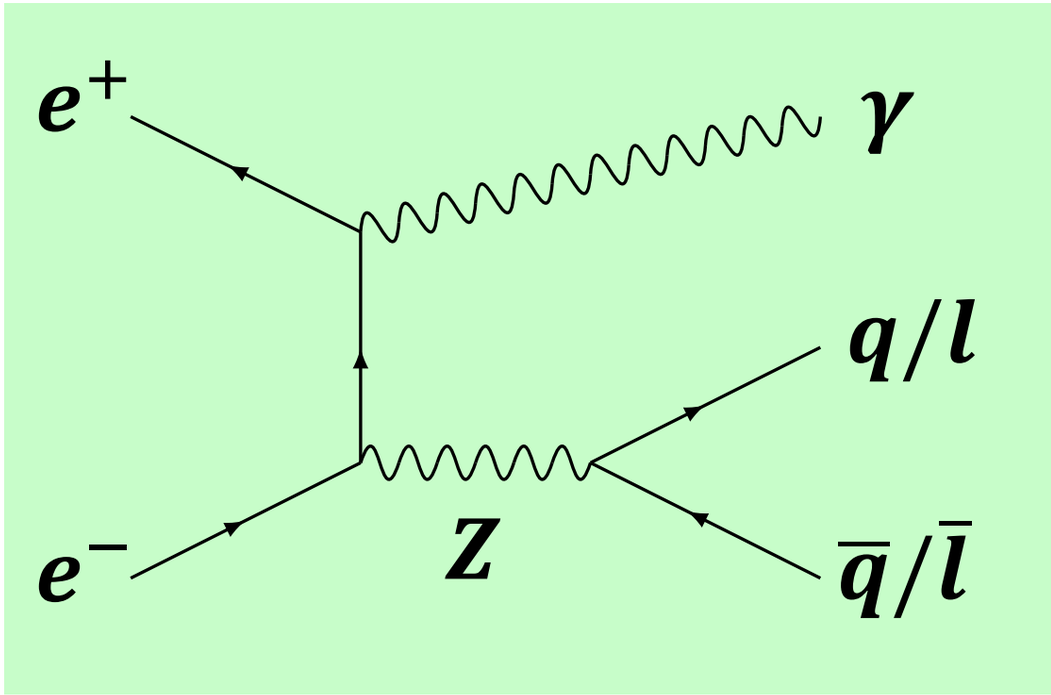
Takahiro Mizuno

Introduction

Potential of radiative return events at ILC250

- The useful observables for the SMEFT global fit include EWPOs e.g. left-right polarization asymmetry A_{LR} of the Z-pole cross section.
  We can perform precise measurement of A_{LR} using radiative return **$e^+e^- \rightarrow \gamma Z$ process**.

- This process is also useful to reduce the uncertainties of the photon and jet energy scales.
  (I will briefly introduce the data-driven calibration method we developed.)



Physics at ILC 250

- Primary physics aim of ILC 250: measuring the **coupling constants between the Higgs boson and various other standard model (SM) particles**
- The coupling constants can deviate from their SM values because of possible Beyond the Standard Model (BSM) effects.
- The size of corrections to the SM predictions for the Higgs boson couplings:

$$a\frac{m_H^2}{M^2}$$

M: new particle mass [GeV]
m_H: 125 [GeV]
a: coefficients of order 1

→ Need to measure the couplings with the precision ~1%

SM effective field theory (SMEFT)

- Lorentz invariant and SU(2) × U(1) gauge invariant dim-6 operators

$$\frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) \quad \frac{c_T}{2v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) \quad \frac{c_6\lambda}{v^2}(\Phi^\dagger\Phi)^3$$

$$\frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}W^{a\mu\nu} \quad \frac{4gg' c_{WB}}{m_W^2}\Phi^\dagger t^a \Phi W^a_{\mu\nu}B^{\mu\nu}$$

$$\frac{g'^2 c_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} \quad \frac{g^3 c_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho W^{c\rho\mu}$$

$$i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu L) \quad 4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu t^a L)$$

$$i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{e}\gamma_\mu e)$$

ILC, Higgs related part

(CP-Conserving)

- The useful observables for the SMEFT global fit include not only those from the reactions that directly involve the Higgs boson, but also those from Electroweak Precision Observables (EWPOs) for W and Z bosons.
  For example, left-right polarization asymmetry A_{LR} of the Z-pole cross section is important.

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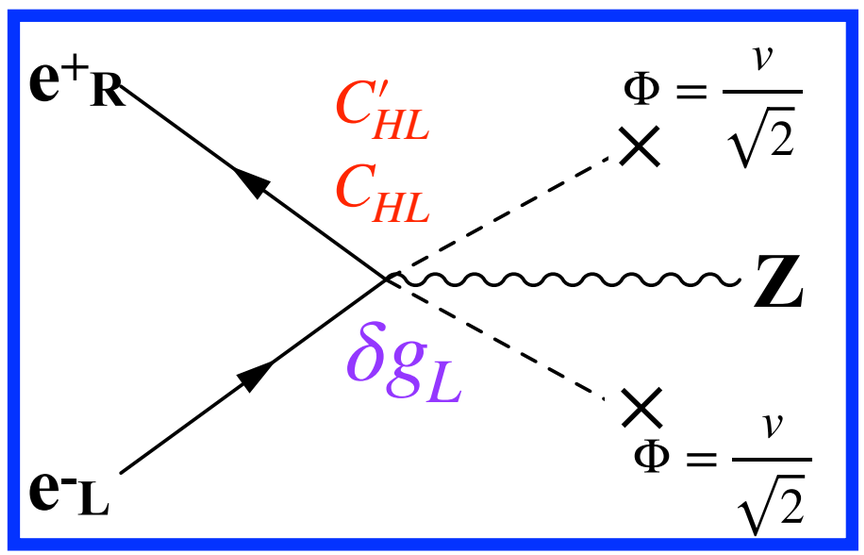
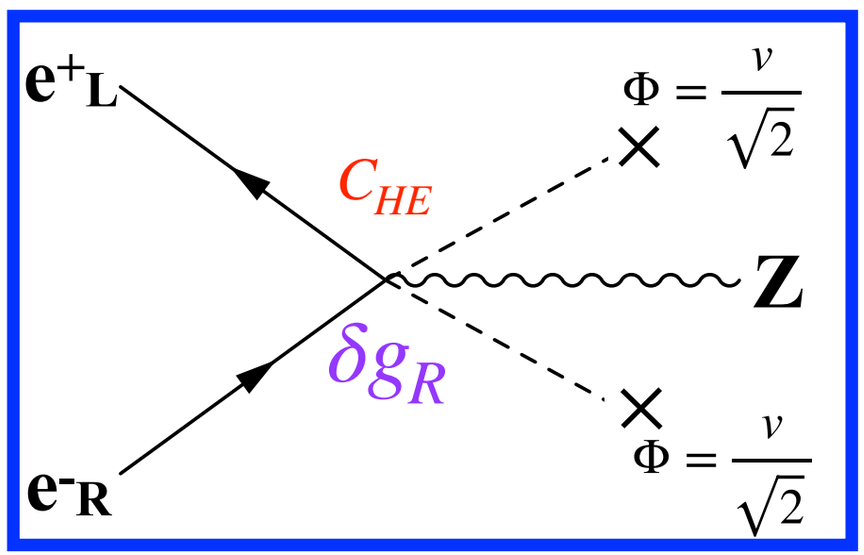
  For example, left-right polarization asymmetry A_{LR} of the Z-pole cross section is important.

$$\Delta\mathcal{L} = i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D^\mu}\Phi)(\bar{L}\gamma_\mu L) + 4i\frac{c\prime_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D^\mu}\Phi)(\bar{L}\gamma_\mu t^a L) + i\frac{\bar{c}_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D^\mu}\Phi)(\bar{e}\gamma_\mu e)$$

$(vZ^\mu v)$ $(vt^a Z^\mu v)$ $(vZ^\mu v)$





$$g_L = \frac{g}{c_W}[(-\frac{1}{2} + s_W^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}(c_{HL} + c\prime_{HL}) - s_W c_W \delta Z_{AZ}]$$

$$g_R = \frac{g}{c_W}[(+s_W^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}c_{HE} - s_W c_W \delta Z_{AZ}]$$

$$\delta Z_Z = c_W^2(8c_{WW}) + 2s_W^2(8c_{WB}) + \frac{s_W^4}{c_W^2}(8c_{BB})$$

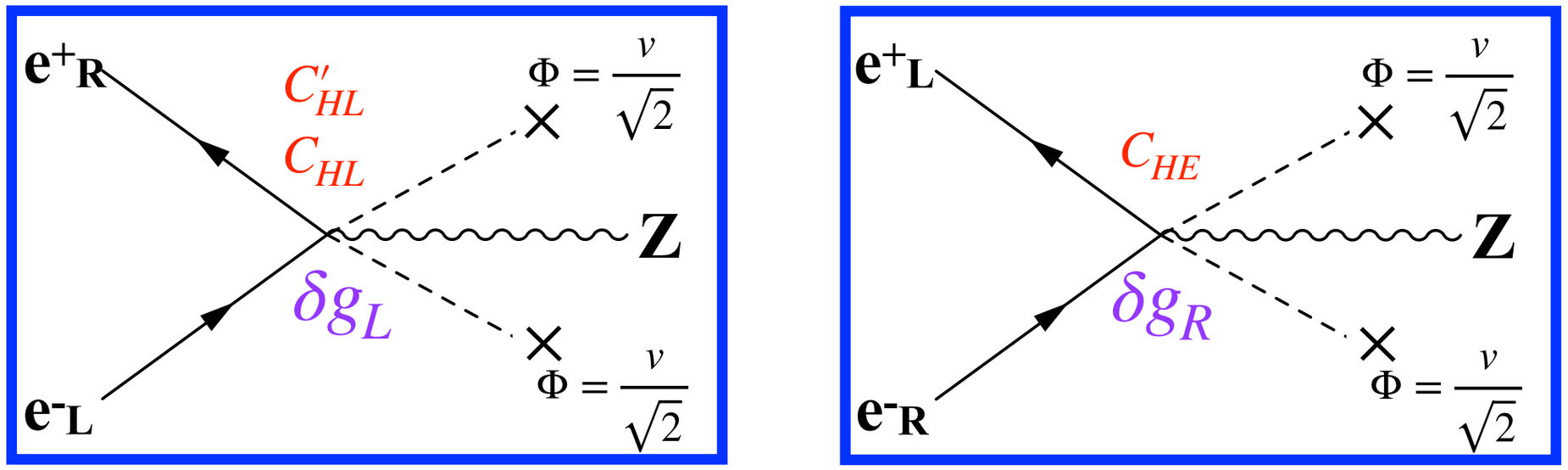
$$\delta Z_{AZ} = s_W c_W((8c_{WW}) - (1 - \frac{s_W}{c_W^2})(8c_{WB}) - \frac{s_W}{c_W^2}(8c_{BB})$$

$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2(\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

$$\Delta\mathcal{L} = i\frac{c_{HL}}{v^2}\boxed{(\Phi^\dagger \overleftrightarrow{D^\mu}\Phi)}(\bar{L}\gamma_\mu L) + 4i\frac{c\prime_{HL}}{v^2}\boxed{(\Phi^\dagger t^a \overleftrightarrow{D^\mu}\Phi)}(\bar{L}\gamma_\mu t^a L) + i\frac{\bar{c}_{HE}}{v^2}\boxed{(\Phi^\dagger \overleftrightarrow{D^\mu}\Phi)}(\bar{e}\gamma_\mu e)$$

$(vZ^\mu v)$ $(vt^a Z^\mu v)$ $(vZ^\mu v)$



Current best measurement (SLC)

A_{LR}= 0.1514 ± 0.0019 (statistic error) ± 0.0011(systematic error)

Need full detector simulation of radiative return events to check how much error we have at ILC 250

$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

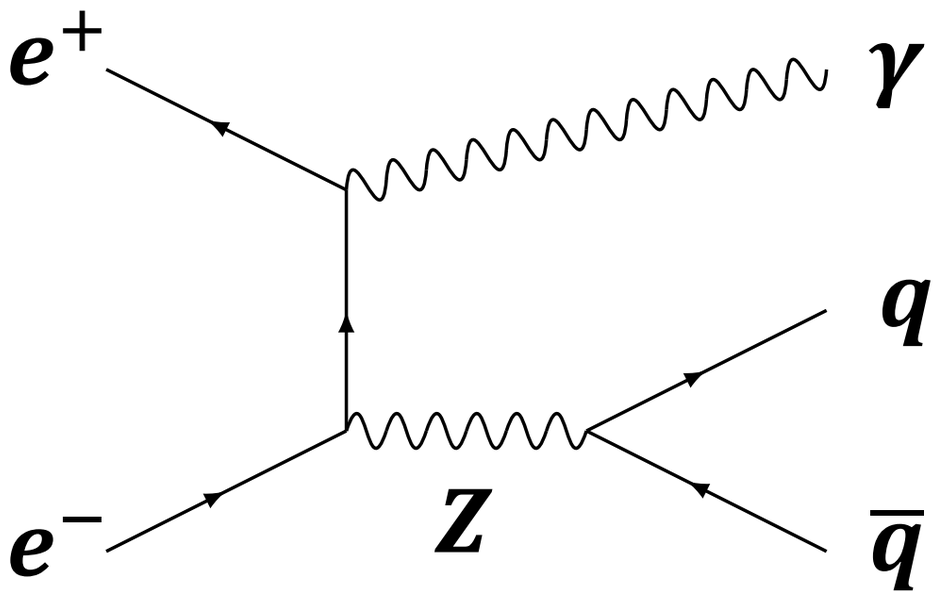
$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2(\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

Simulation Setup

Full simulation

- E_{CM}= 250 GeV
- $\int \mathscr{L} dt$ =900 fb^{-1} for each of the 2 polarization combinations
- $\sin^2\theta_W = 0.22225$.

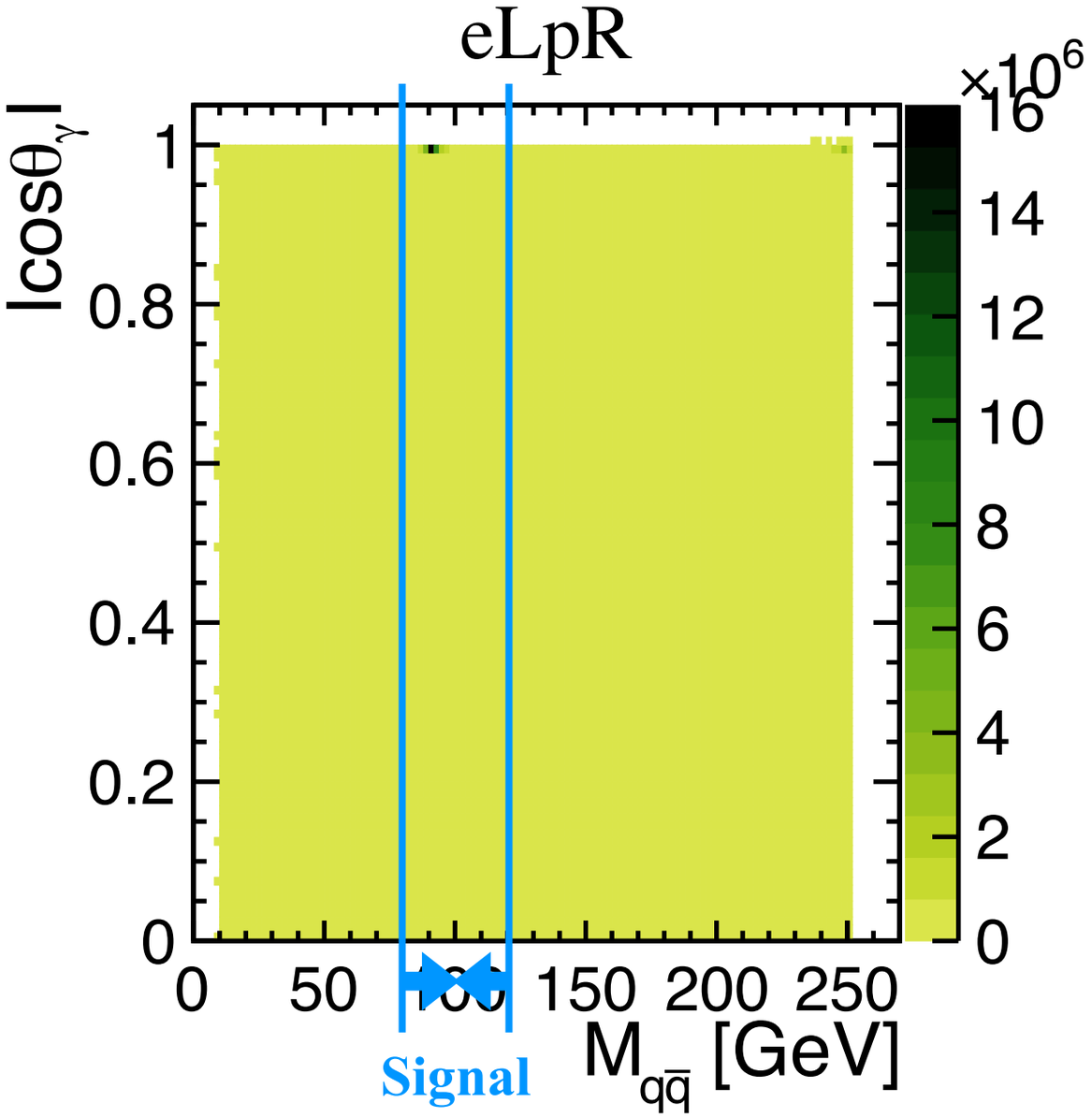
Signal: $e^+e^- \rightarrow Z\gamma \rightarrow 2 \text{ jets} + \gamma$

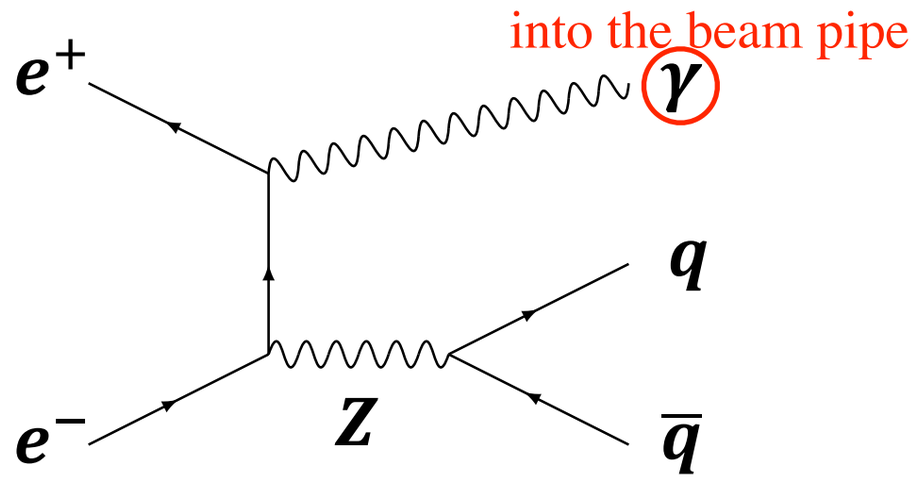


Background: All the events only with 2 jets in the final state

Signal event definition

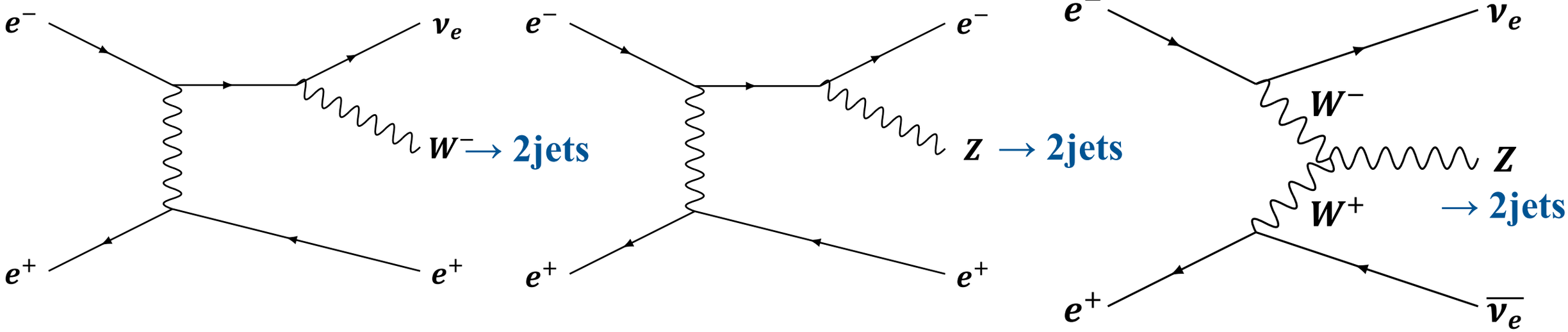
Signal event: radiative return with photon escaping into beam pipe

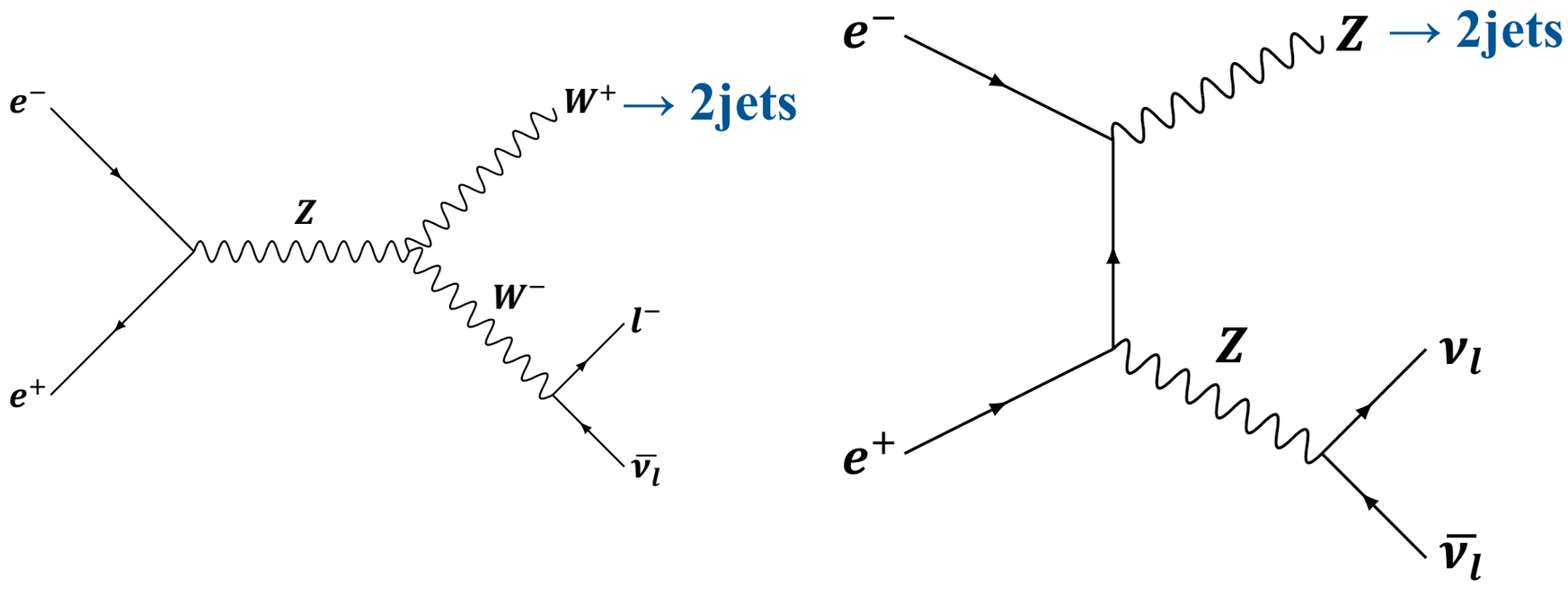




Signal: 80 GeV $< M_{q\bar{q}} <$ 120 GeV

Major backgrounds





Event selection

Cut 1 $N_{\gamma(E>50\,\mathrm{GeV})} = 0$

Cut 2 $120\,\mathrm{GeV} < E_{vis} < 160\,\mathrm{GeV}$

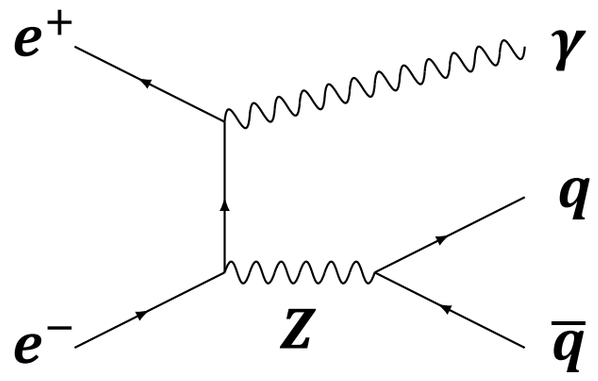
Cut 3 $|\cos\theta_{2j}| > 0.95$

Cut 4 $N_{J1}^{charged} + N_{J2}^{charged} > 4$

Cut 5 $N_{J1}^{total} + N_{J2}^{total} > 10$

Cut 6 $50\,\mathrm{GeV} < M_{2j} < 160\,\mathrm{GeV}$

Cut 7 $\cos\theta_{12} > -0.99$ or $\frac{E_{J1}-E_{J2}}{E_{J1}+E_{J2}} > 0.5$



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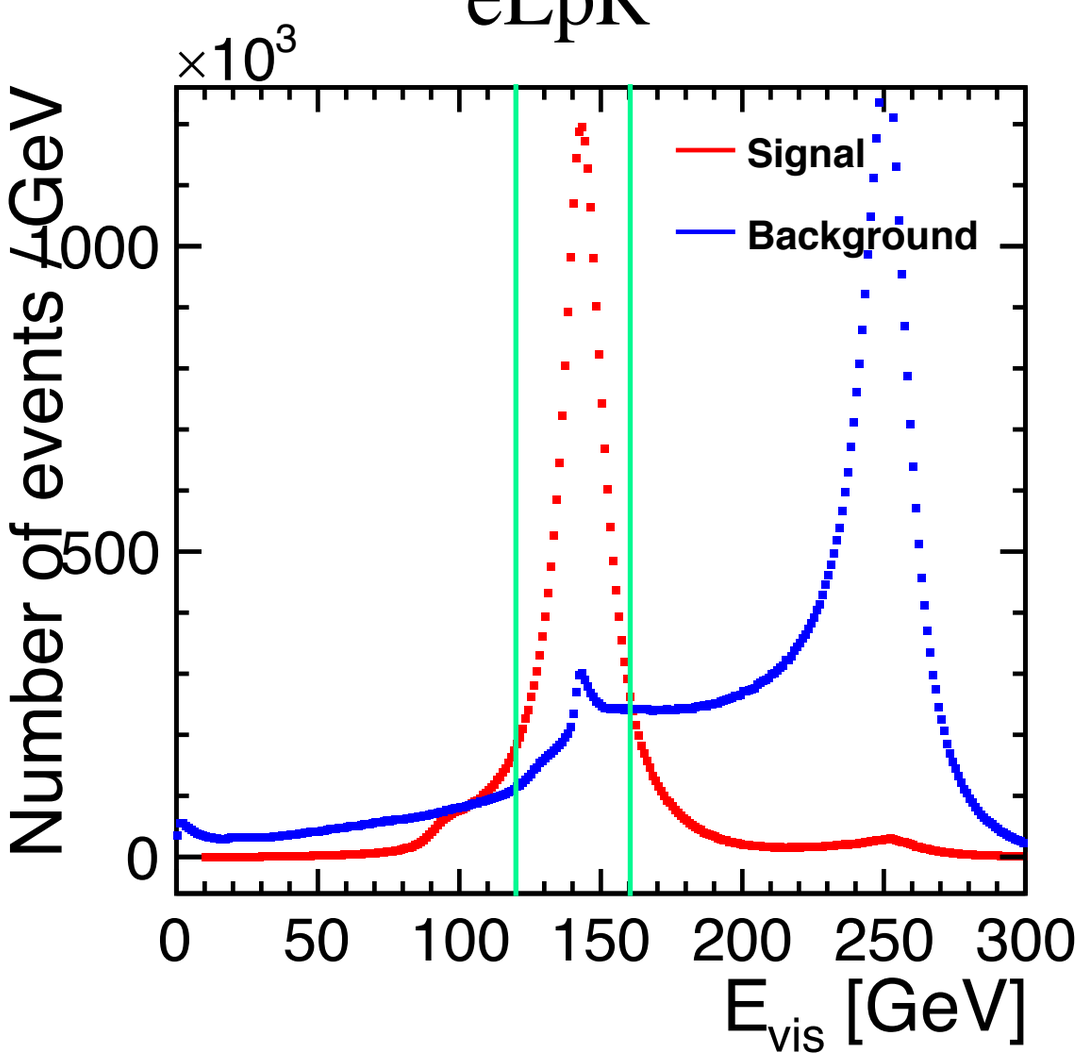

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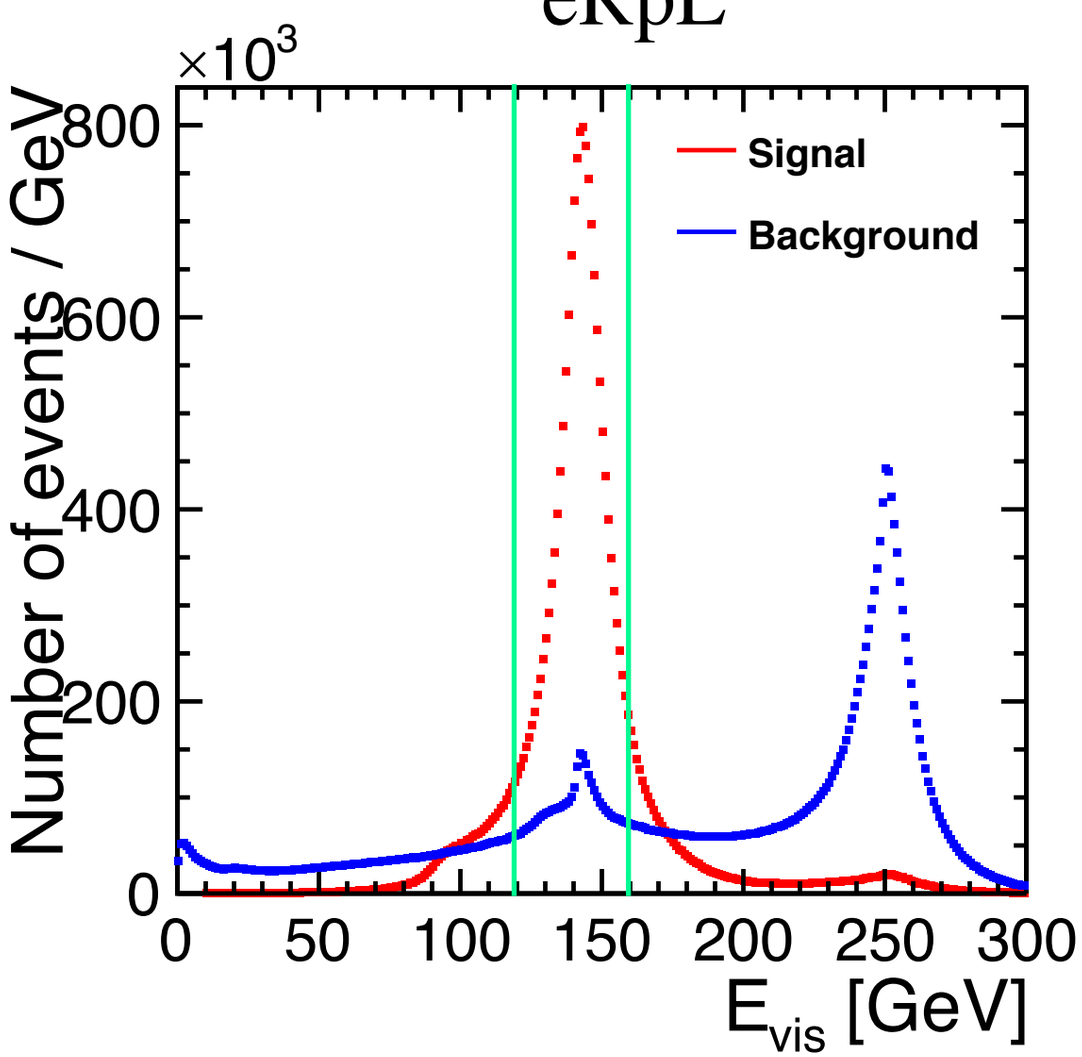

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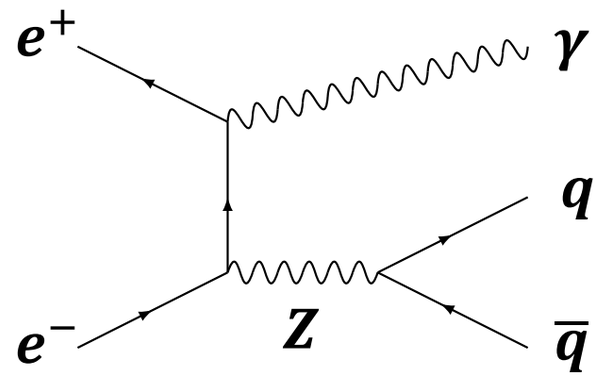

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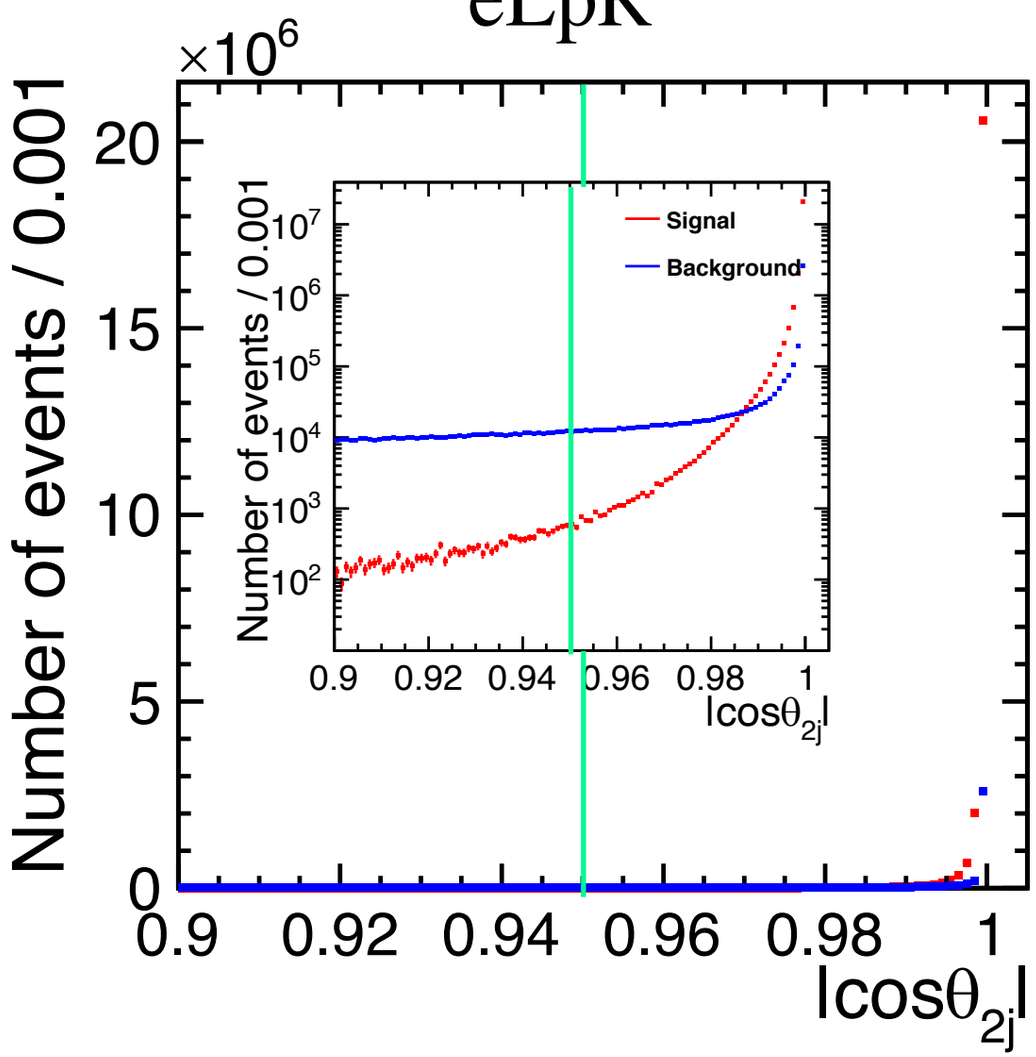

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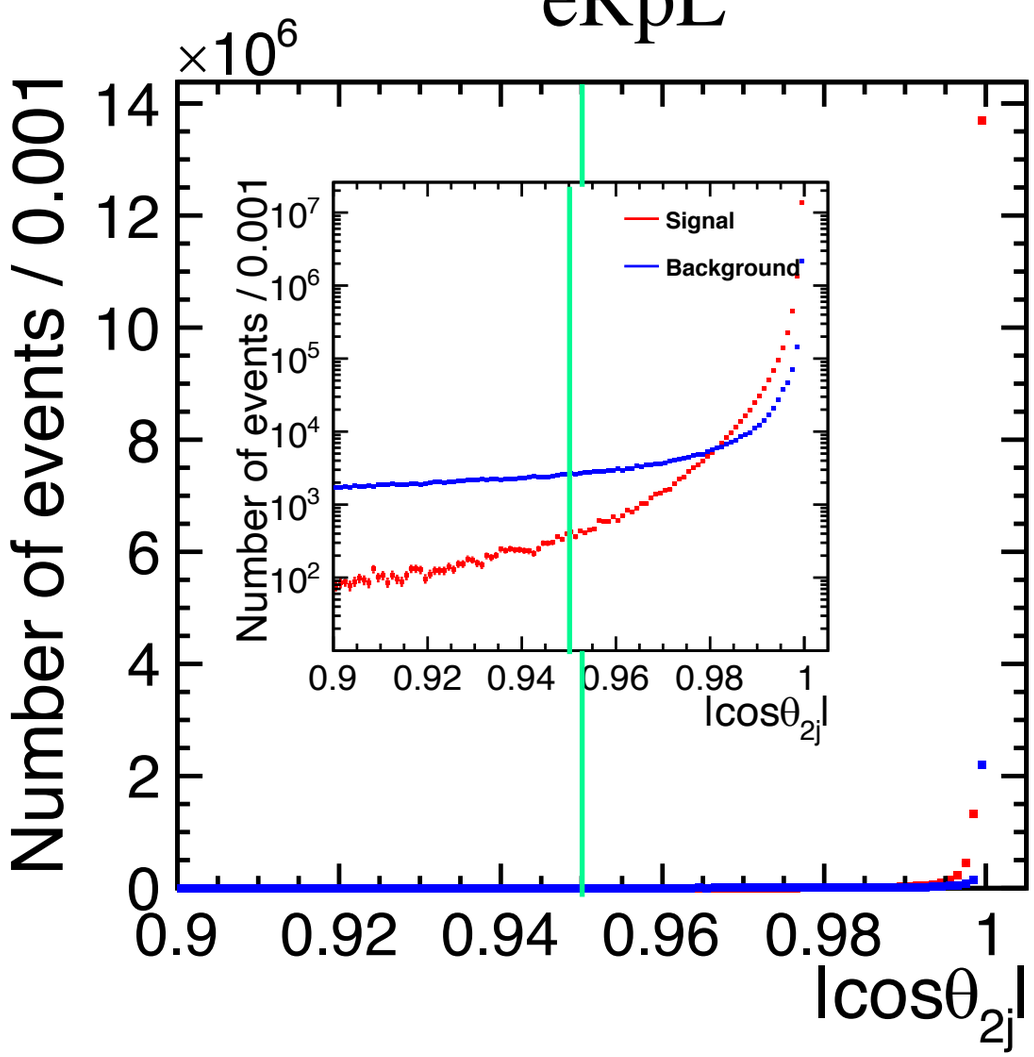

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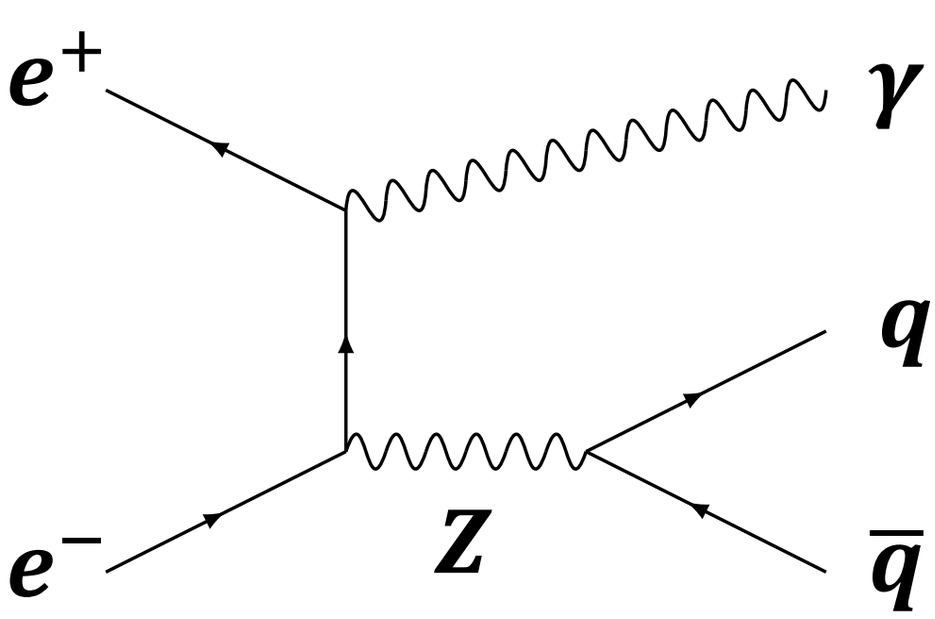

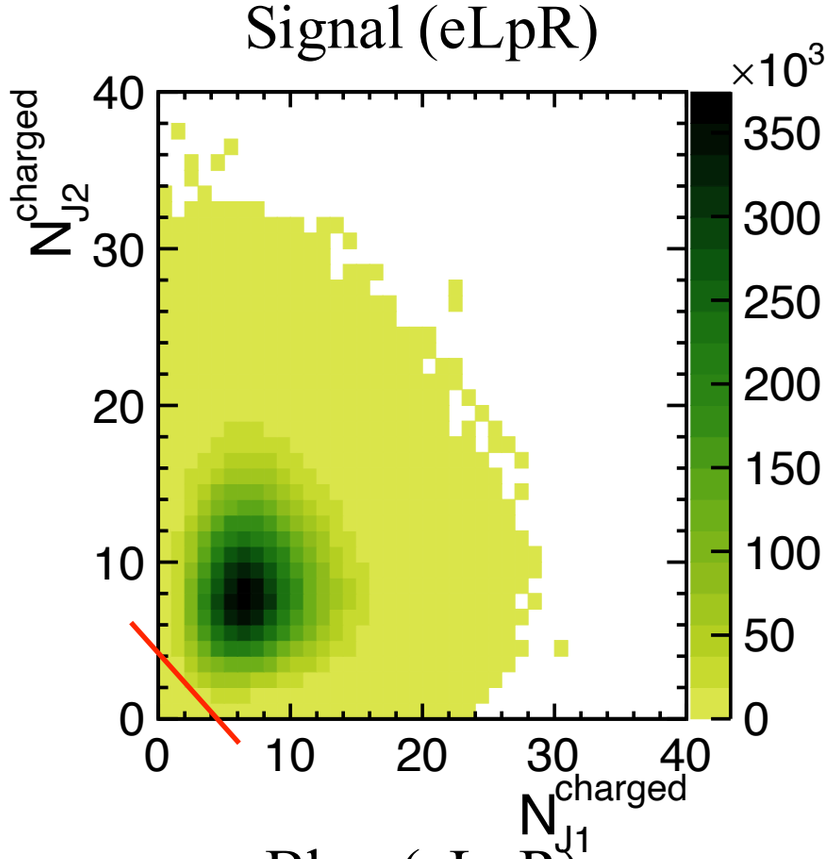

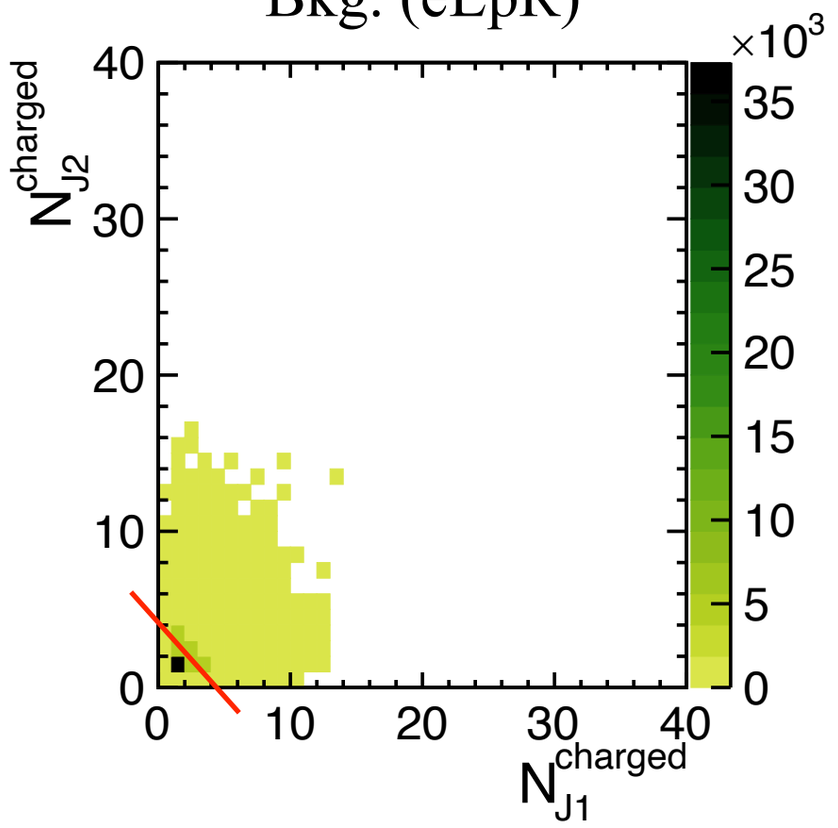

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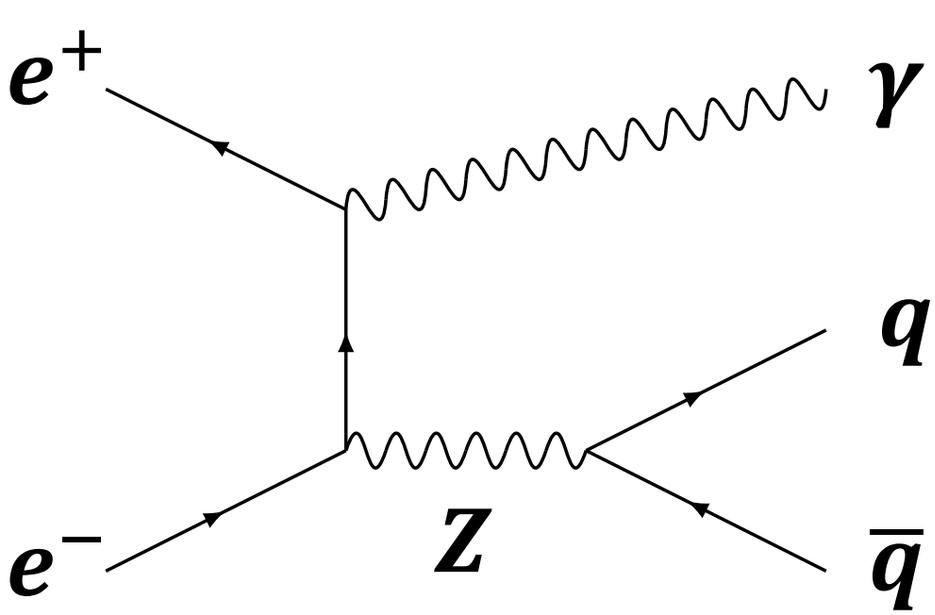

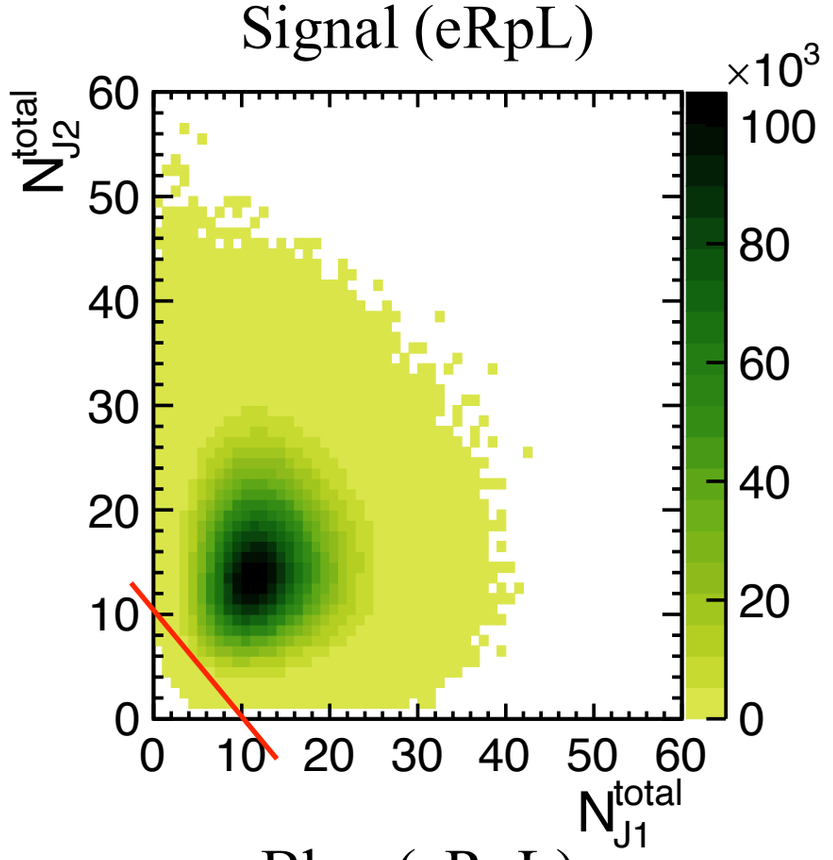

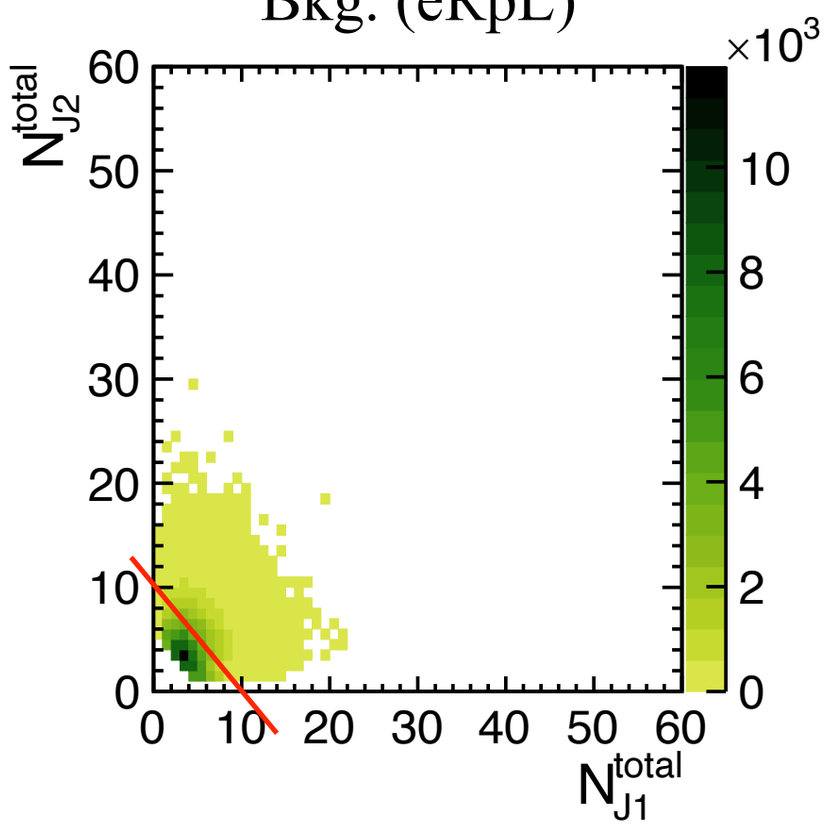

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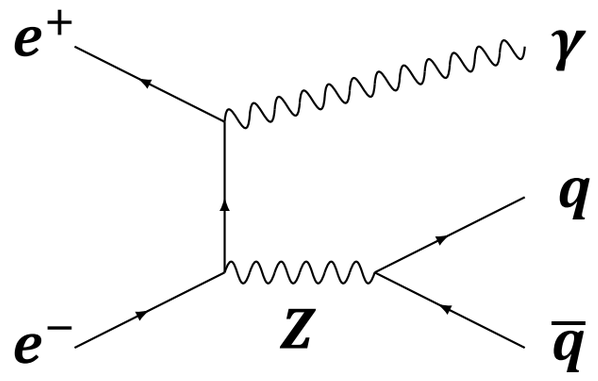

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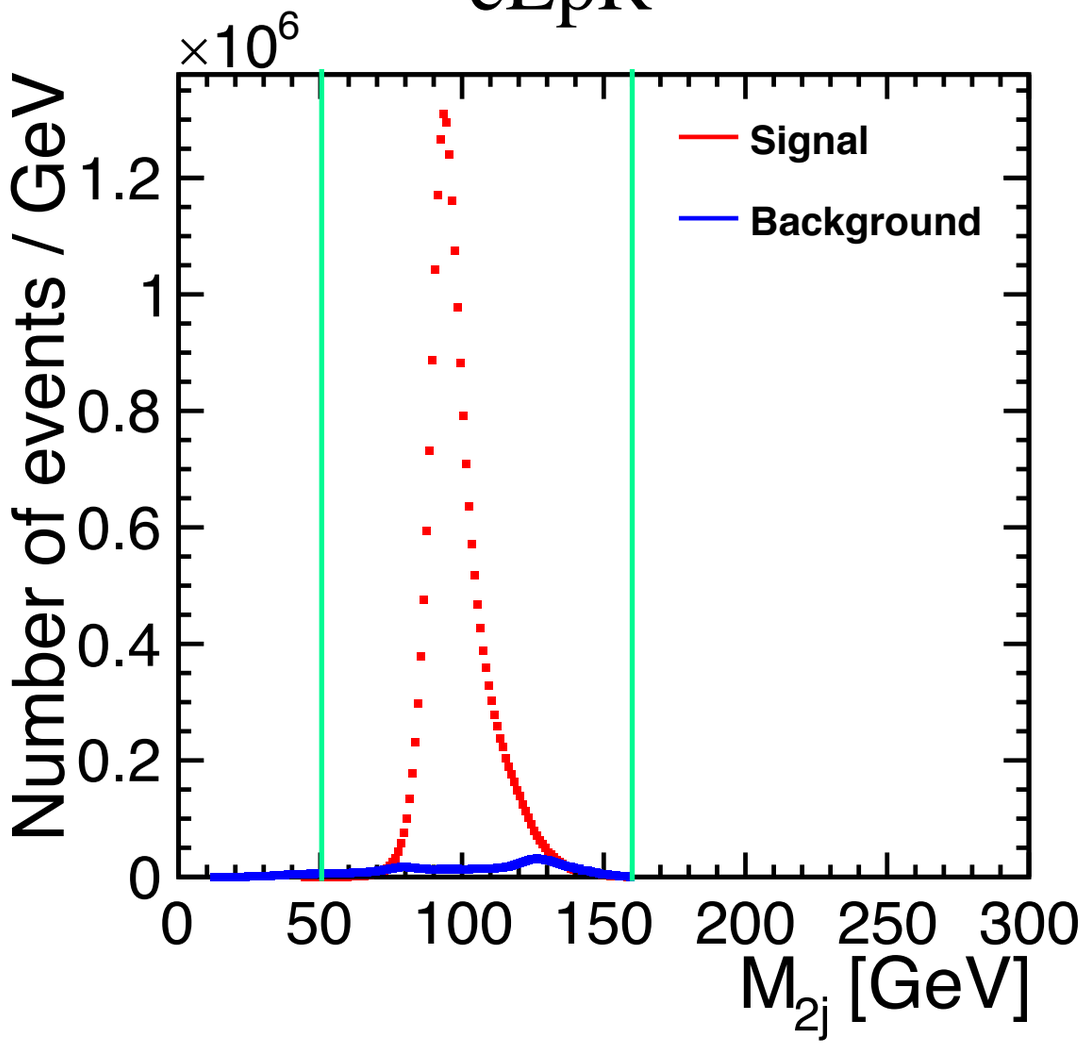

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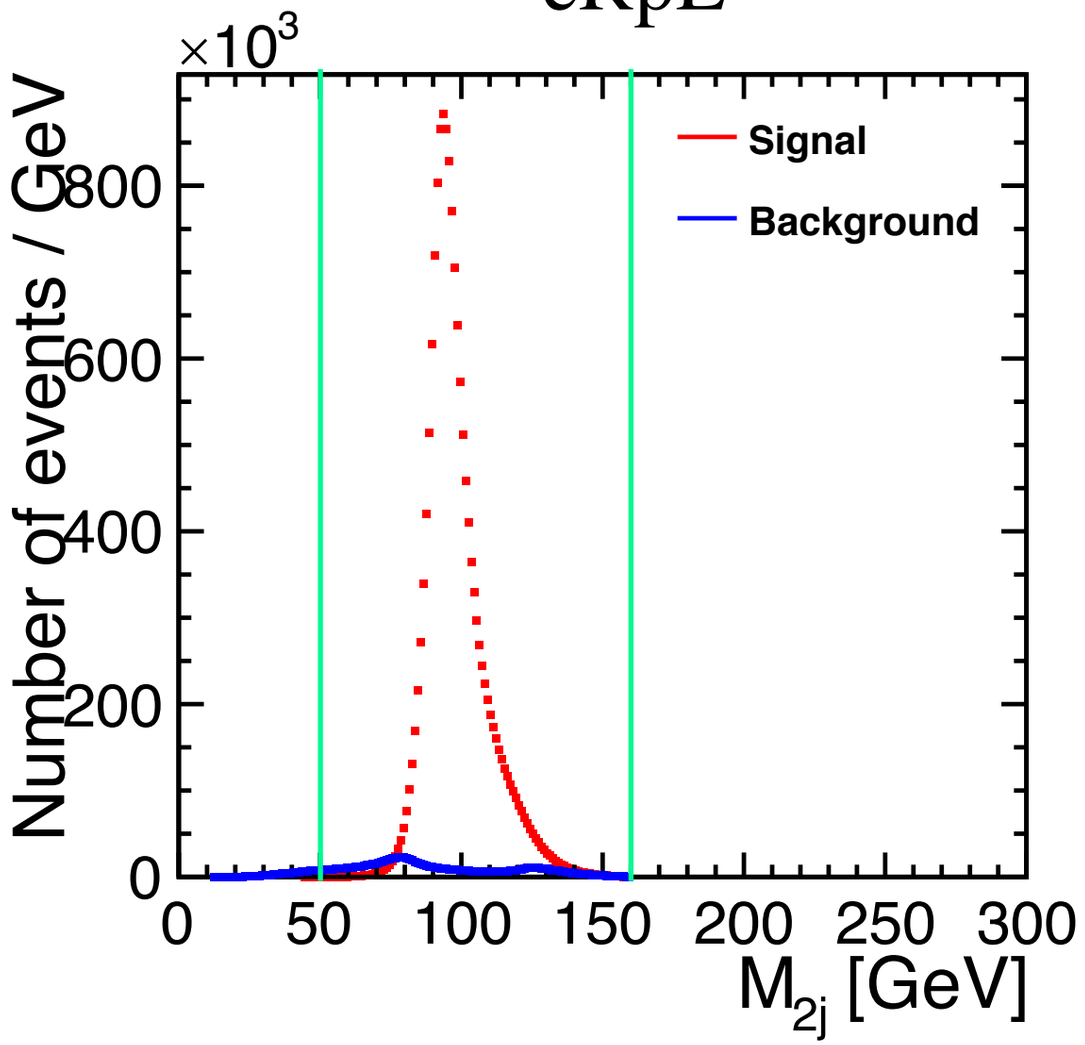

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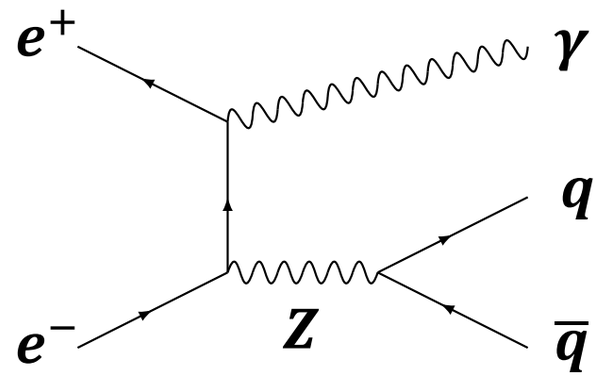

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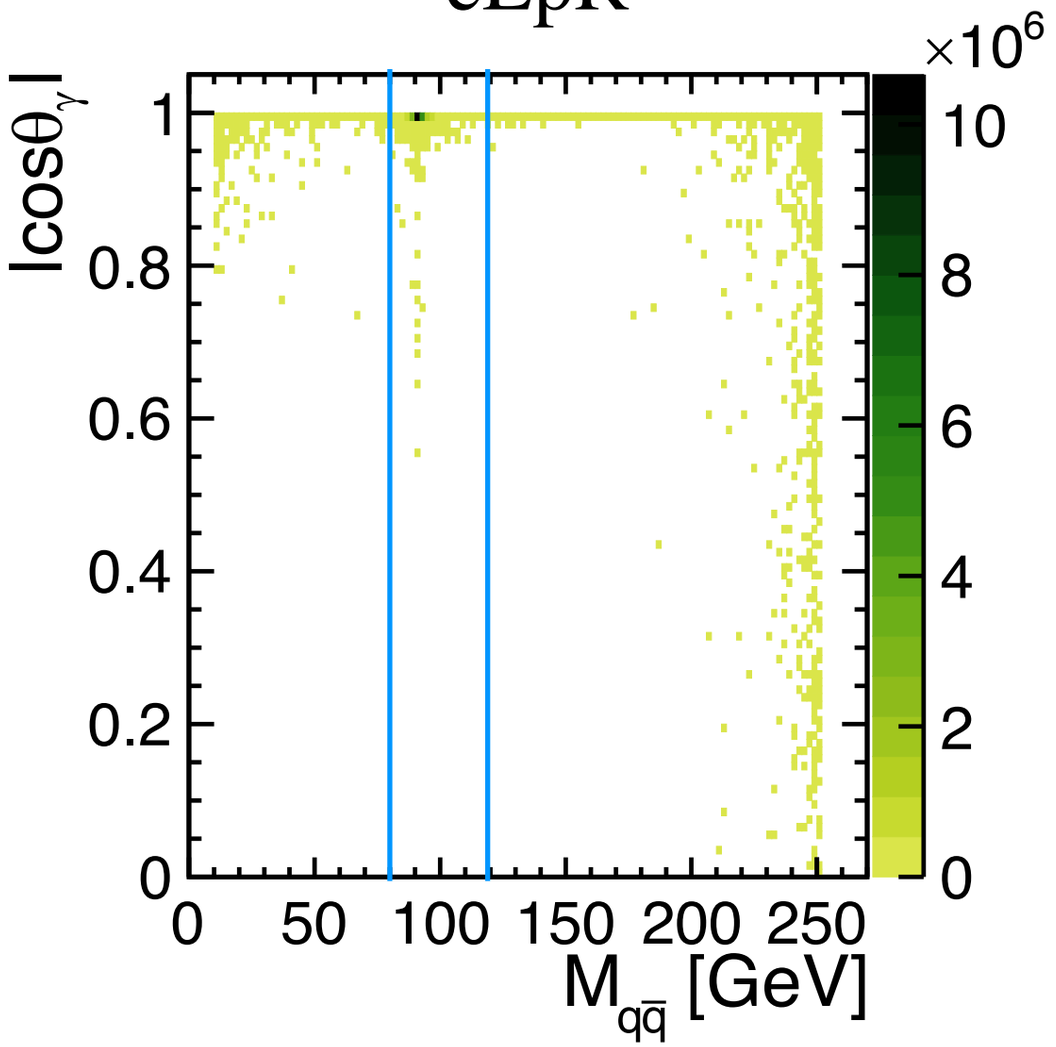

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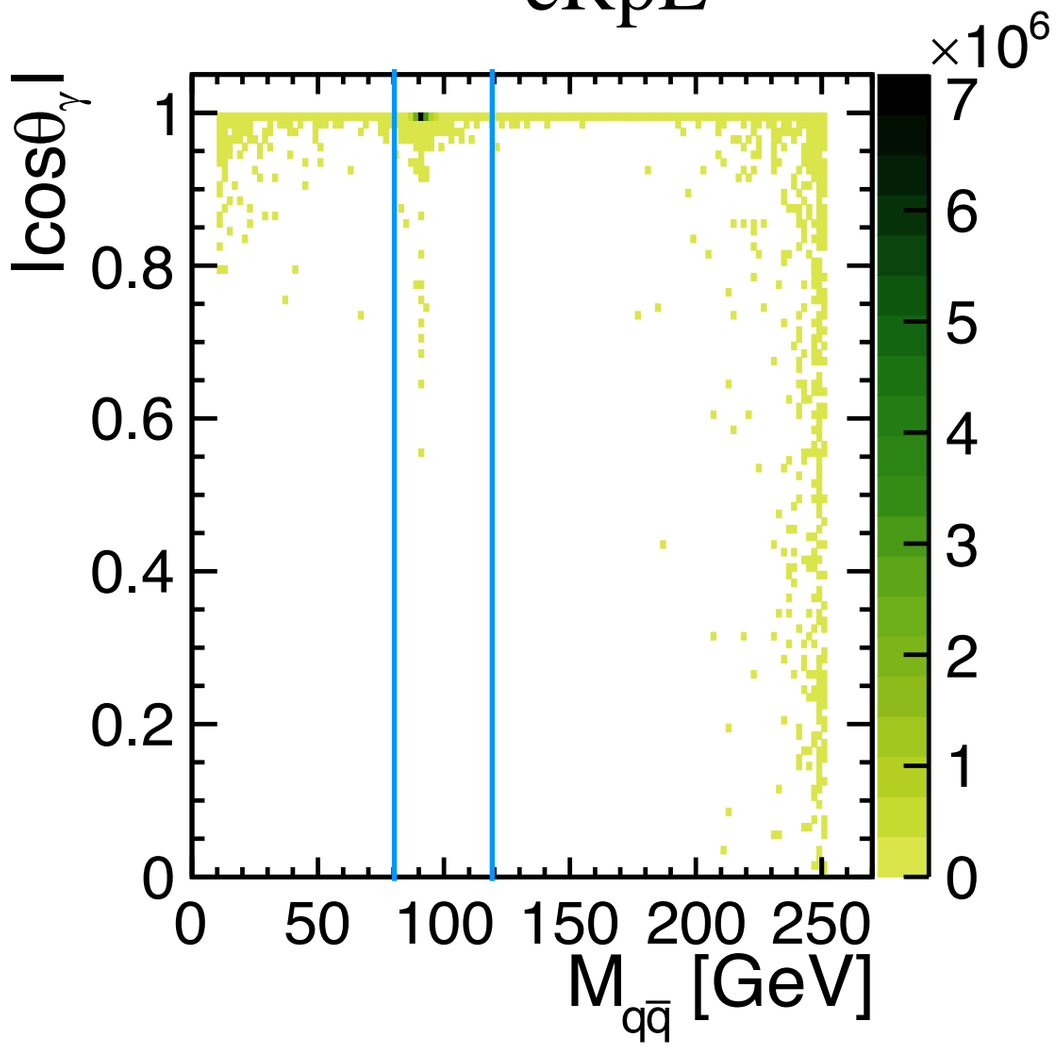

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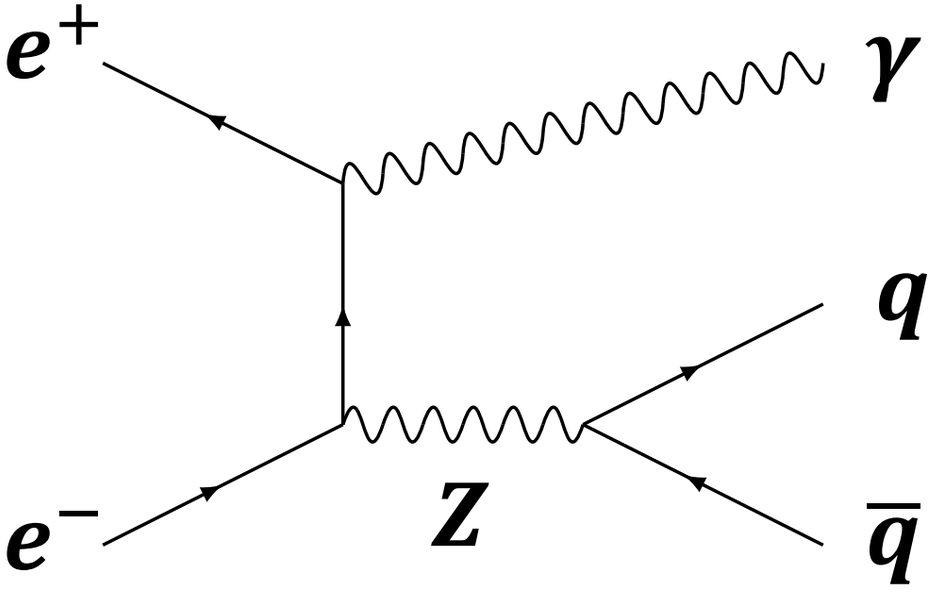
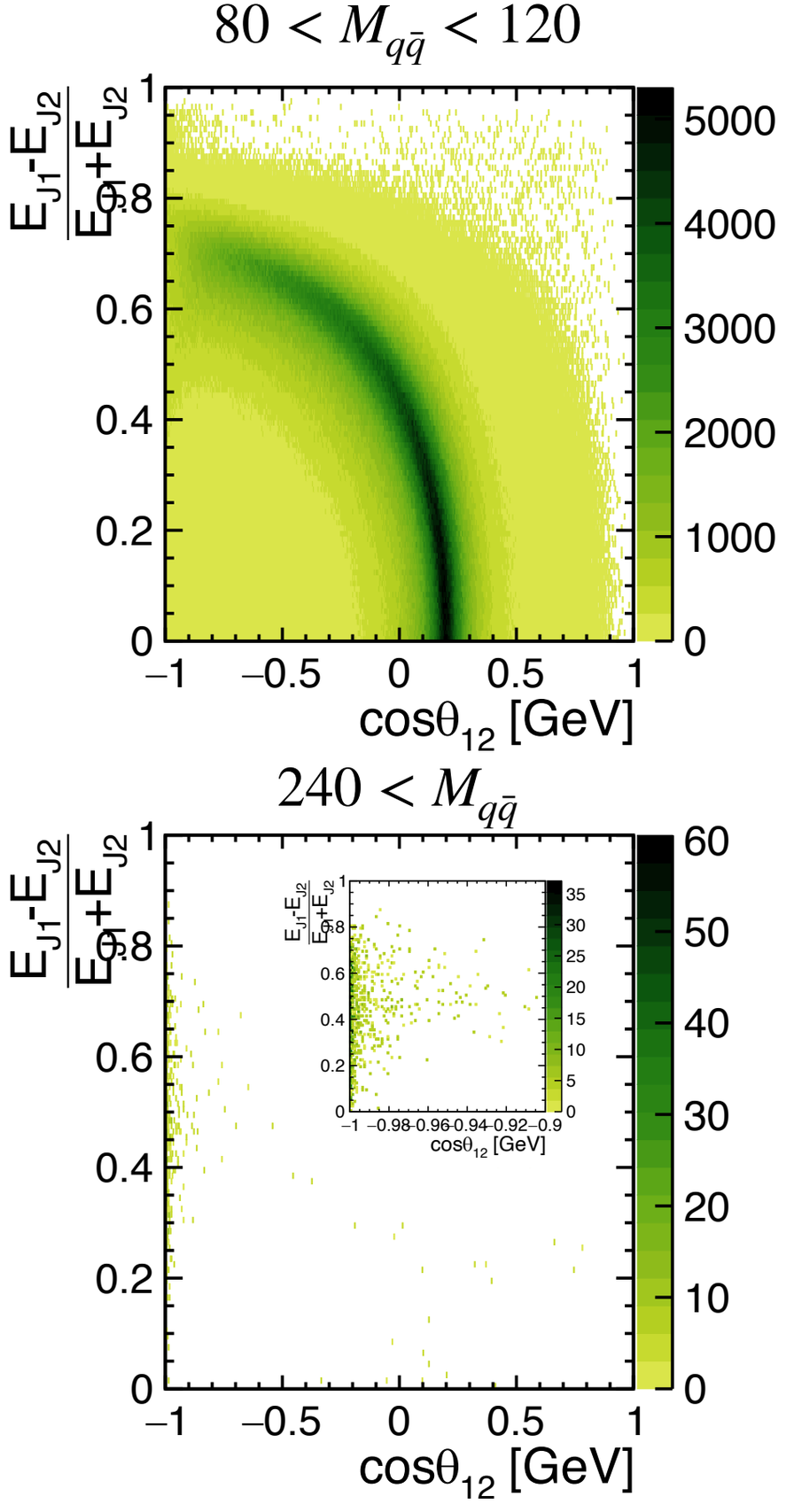




Cut Table

$(-0.8, +0.3)$ polarization, assuming $\int Ldt = 900\,\mathrm{fb}^{-1}$.

$\times 10^6$ events	Signal	Signal (Core)	2f_l	4f_l	4f_sl	4f_h	2f_h	Bkg. Total
Expected	46.0	32.5	12.7	9.34	17.2	15.1	23.6	78.1
Cut 1	32.7	31.1	10.1	5.96	16.0	14.8	21.6	68.3
Cut 2	24.6	24.4	2.55	1.46	3.22	0.00422	1.09	8.32
Cut 3	24.5	24.4	1.93	0.366	0.526	0.00352	1.04	3.87
Cut 4	24.4	24.3	0.299	0.0574	0.523	0.00352	1.00	1.88
Cut 5	24.3	24.2	0.0651	0.0102	0.520	0.00352	0.977	1.58
Cut 6	24.2	24.2	0.0571	0.00807	0.470	0.00210	0.694	1.23
Cut 7	24.2	24.1	0.0534	0.00647	0.463	0.00204	0.682	1.21

Efficiency = 0.52678 ±0.00017 **B/S= 0.0499 for (-0.8, +0.3)**

$(+0.8, -0.3)$ polarization, assuming $\int Ldt = 900\,\mathrm{fb}^{-1}$.

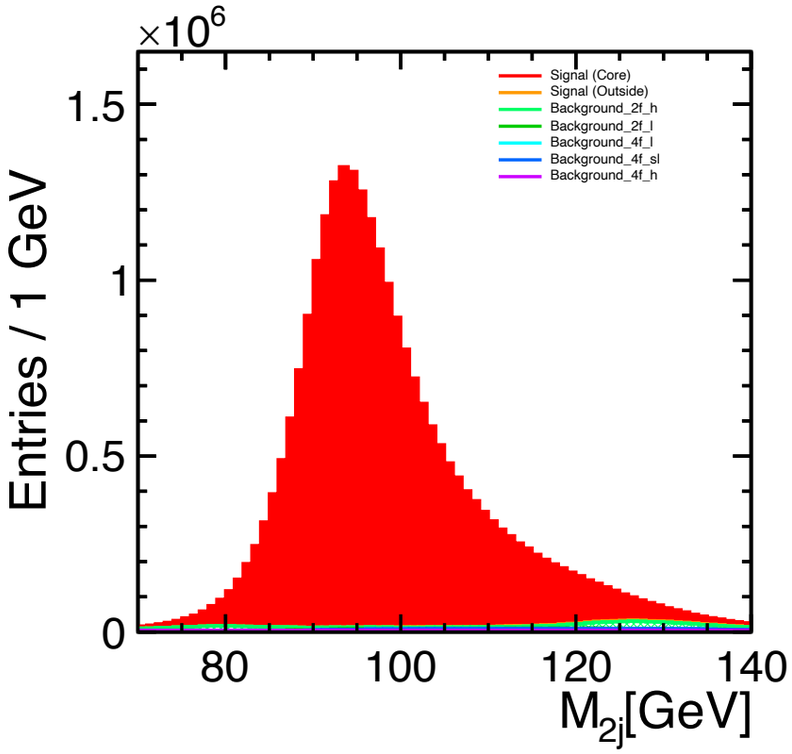
$\times 10^6$ events	Signal	Signal (Core)	2f_l	4f_l	4f_sl	4f_h	2f_h	Bkg. Total
Expected	30.5	21.6	9.84	5.50	2.56	1.41	10.6	29.9
Cut 1	21.7	20.6	7.77	2.33	1.86	1.38	9.37	22.7
Cut 2	16.3	16.2	1.83	0.378	0.370	0.00137	1.04	3.62
Cut 3	16.3	16.2	1.37	0.259	0.106	0.00124	1.03	2.77
Cut 4	16.2	16.1	0.212	0.0357	0.104	0.00124	0.985	1.34
Cut 5	16.2	16.1	0.0454	0.00603	0.102	0.00124	0.958	1.11
Cut 6	16.1	16.0	0.0396	0.00468	0.0934	0.000986	0.616	0.754
Cut 7	16.1	16.0	0.0372	0.00320	0.0900	0.000967	0.609	0.740

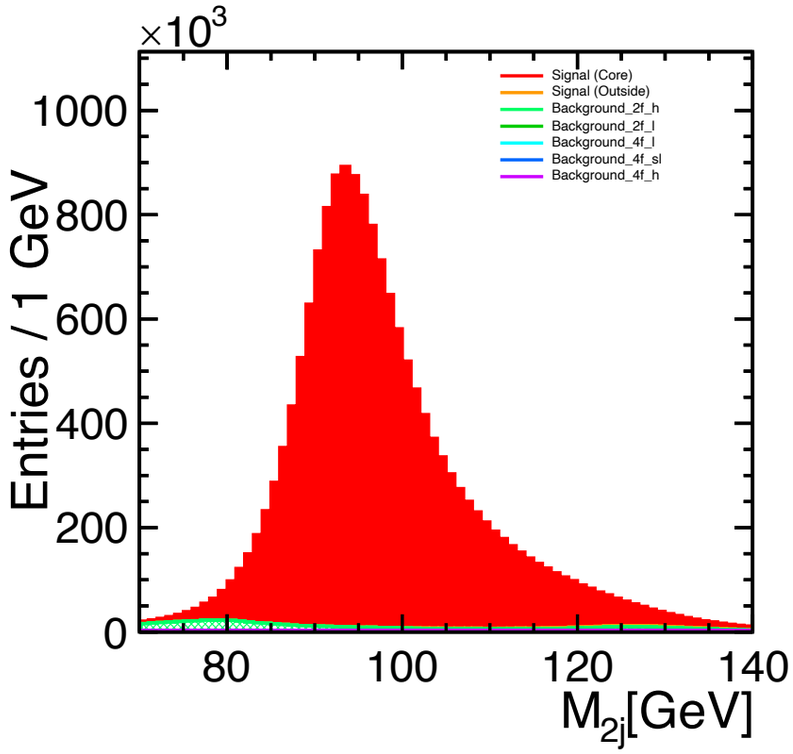
Efficiency = 0.52715 ±0.00016 **B/S= 0.0461 for (+0.8, -0.3)**

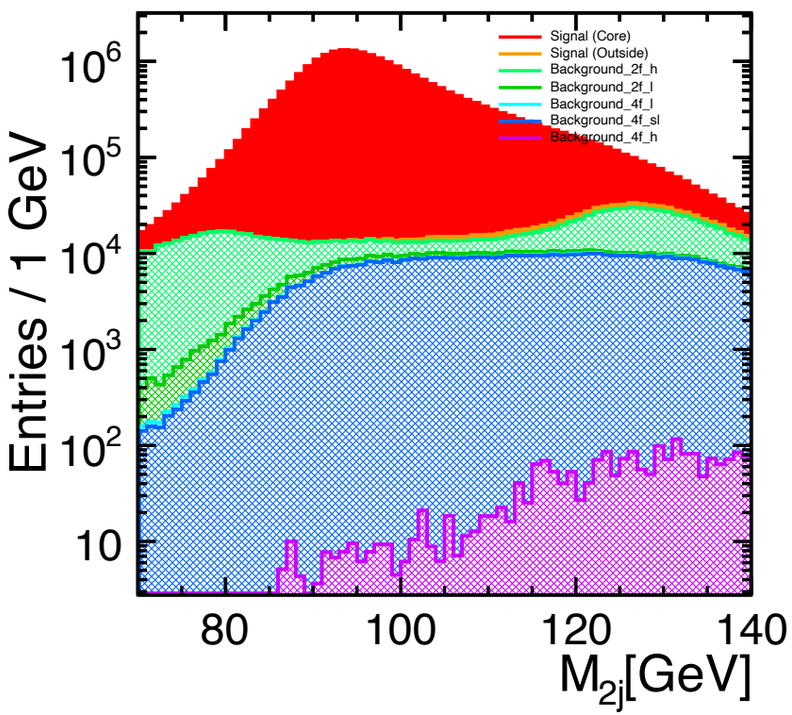
Final Event Sample

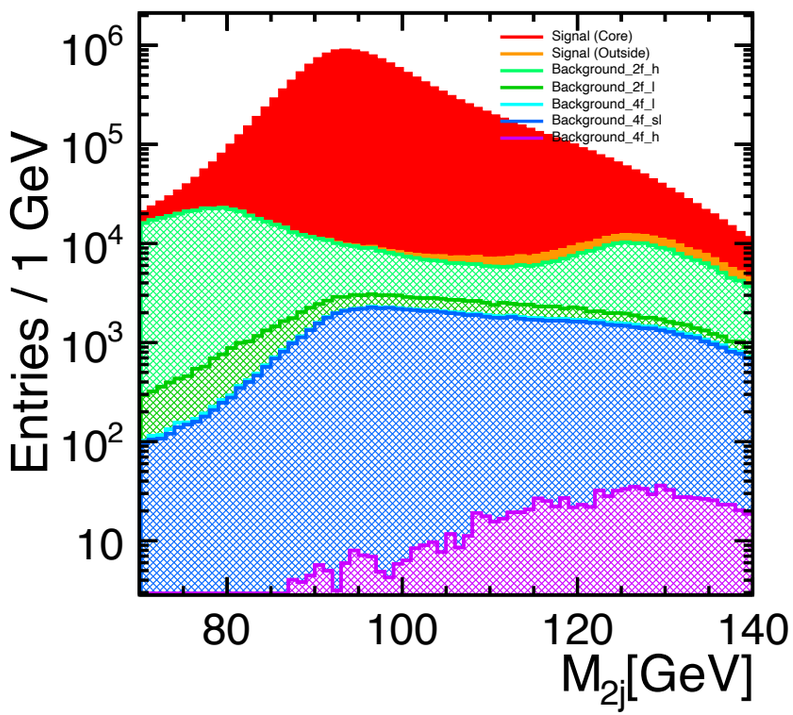
$(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$ $(P_{e^-}, P_{e^+}) = (+0.8, -0.3)$

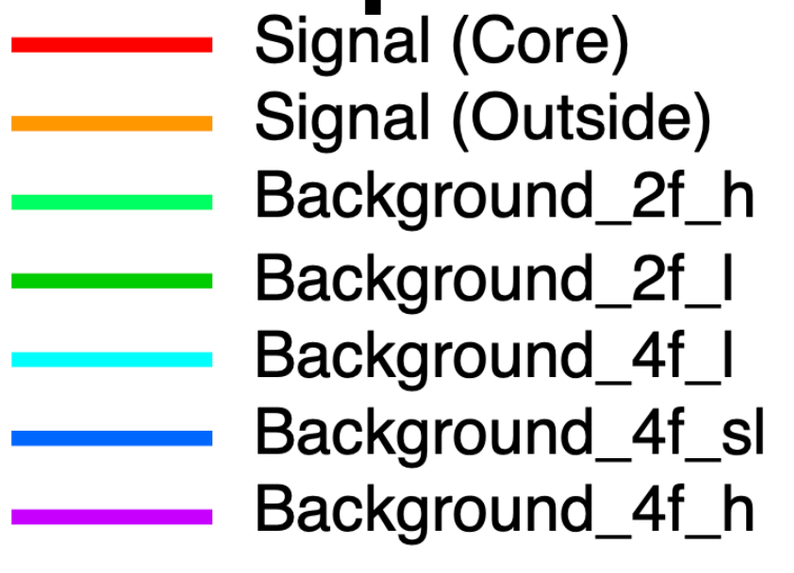
$|\cos\theta_{\gamma(\text{truth})}| > 0.999$











B/S is indeed very good.

Estimation of A_{LR} precision (1)

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}.$$ L/R : 100% polarization

$$A_{LRobs} \equiv \frac{\sigma_{-+} - \sigma_{+-}}{\sigma_{-+} + \sigma_{+-}}$$ -/+ : Polarization at ILC

$$\sigma_{-+} = \frac{1}{4}(1 + |P_-|)(1 + |P_+|)\sigma_L + \frac{1}{4}(1 - |P_-|)(1 - |P_+|)\sigma_R$$

$$\sigma_{+-} = \frac{1}{4}(1 - |P_-|)(1 - |P_+|)\sigma_L + \frac{1}{4}(1 + |P_-|)(1 + |P_+|)\sigma_R$$

$$A_{LR} = A_{LRobs}\frac{1 + |P_-||P_+|}{|P_-| + |P_+|} = A_{LRobs} \times f$$

The error of the A_{LR} can be expressed as

$$\left(\frac{\Delta A_{LR}}{A_{LR}}\right)^2 = \left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 + \left(\frac{\Delta f}{f}\right)^2$$

Estimation of A_{LR} precision (2)

Assume $\Delta|P_-|$ and $\Delta|P_+|$ are independent, then

$$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{|P_-|(1+|P_+|)(1-|P_+|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_-|}{|P_-|}\right)^2 + \left(\frac{|P_+|(1+|P_-|)(1-|P_-|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_+|}{|P_+|}\right)^2$$

As for the error of A_{LRobs}, defining

$$N_{-+} = \eta_{-+} L_{-+} \sigma_{-+} \qquad \alpha \equiv L_{-+}\eta_{-+}$$

$$N_{+-} = \eta_{+-} L_{+-} \sigma_{+-}, \qquad \beta \equiv L_{+-}\eta_{+-},$$

N: number of events

η: selection efficiency

L: integrated luminosity

$$A_{LRobs} = \frac{\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}}{\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}},$$

Correlated parts of the error of α and β cancel in A_{LRobs}.

-> Δα and Δβ below only refer to uncorrelated parts.

$$\left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 = \left(\frac{2\left(\frac{N_{-+}}{\alpha}\right)\left(\frac{N_{+-}}{\beta}\right)}{\left(\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}\right)\left(\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}\right)}\right)^2 \left(\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta N_{-+}}{N_{-+}}\right)^2 + \left(\frac{\Delta N_{+-}}{N_{+-}}\right)^2\right)$$

Results

If errors of η, L, and polarization are negligible,

$A_{LR} = 0.22810 \pm$ **0.00018** (stat)

If we add polarization error $\Delta f/f =$ **0.001**,

Absolute error of $A_{LR} = 0.00022$

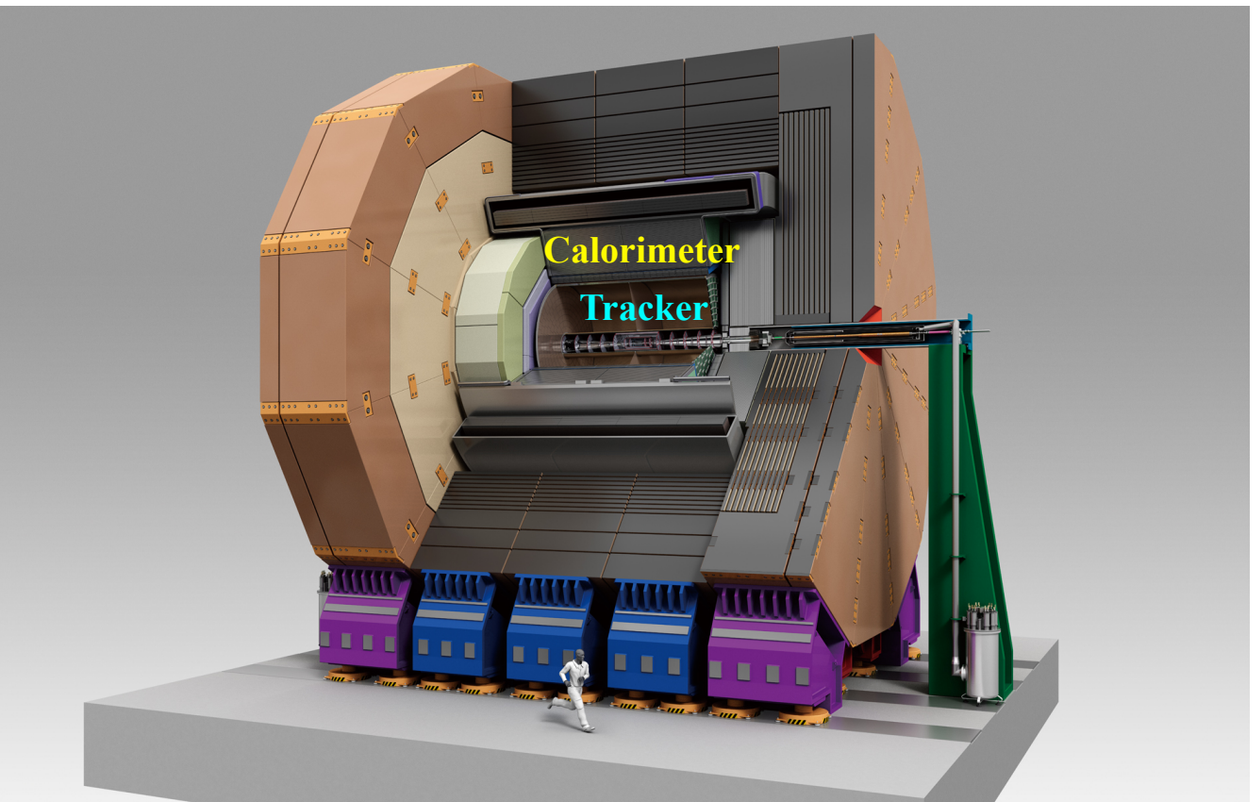
If $\Delta\alpha/\alpha = \Delta\beta/\beta$ (uncorrelated) = **0.00016 (i.e. 0.016%)**,
Total syst. error = **0.00017** <comparable to the statistical error>

Absolute error = 0.00025
8.8 times better precision than that from the SLC (0.00219).

Most of the systematic error will be canceled out we think. The dominant systematic error would be polarization uncertainty.

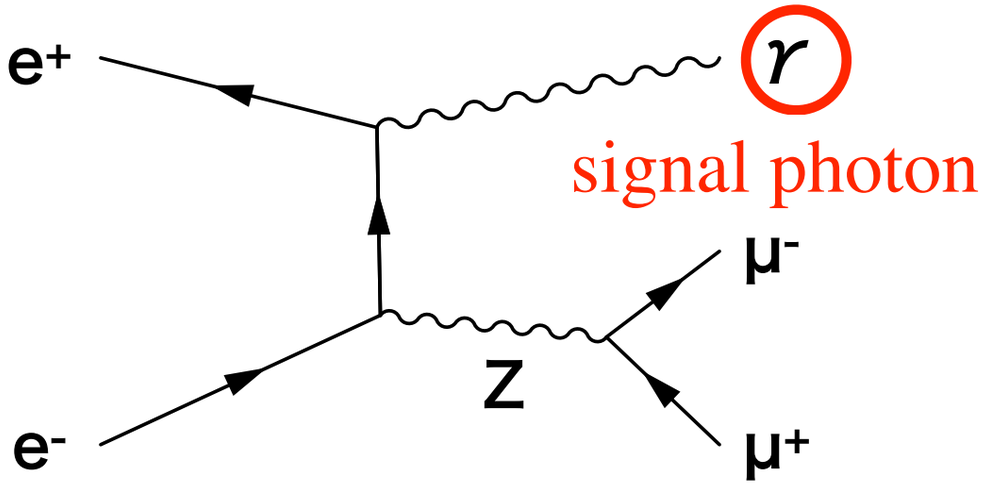
International Large Detector (ILD)

- To take full advantage of the ILC, a high performance detector system is essential.
- **PFA (Particle Flow Analysis)**: reconstructing every individual particle created in the event taking the best available measurement for each given type: **tracker** for **charged particles**, **calorimeter** for **neutral particles**

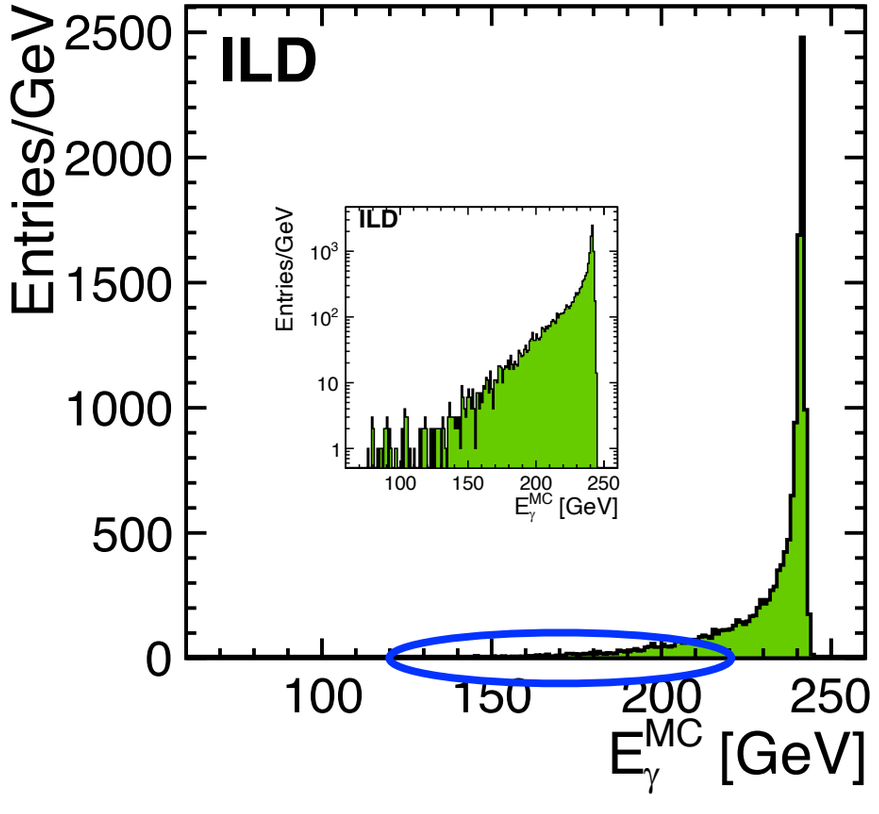


- **PFO (Particle Flow Object)**: reconstructed particle with PFA
- In order to reduce the uncertainties of the photon and jet energy scales, we developed data-driven calibration method using **$e^+e^- \rightarrow \gamma Z$ process**.

Photon Energy Reconstruction Method



Direction Angle
θ: polar angle
φ: azimuthal angle



E_{CM} = 500 GeV

$\int \mathscr{L} dt$ =1600 fb^{-1} for each polarization combinations:

eLpR : (e^-, e^+) = (-80%, +30%)

eRpL :(+80%, -30%)

Angular Method (Ang. Method)
Using (θ_{μ^-},θ_{μ^+},θ_γ,φ_{μ^-},φ_{μ^+},φ_γ,E_{μ^-},E_{μ^+}) -> Determine (E_γ,E_{ISR})

$$\begin{cases} E_{\mu^-} + E_{\mu^+} + E_\gamma + |P_{ISR}| = 500 \\ E_{\mu^-} \sin\theta_{\mu^-} \sin\phi_{\mu^-} + E_{\mu^+} \sin\theta_{\mu^+} \sin\phi_{\mu^+} + E_\gamma \sin\theta_\gamma \sin\phi_\gamma = 0 \end{cases}$$

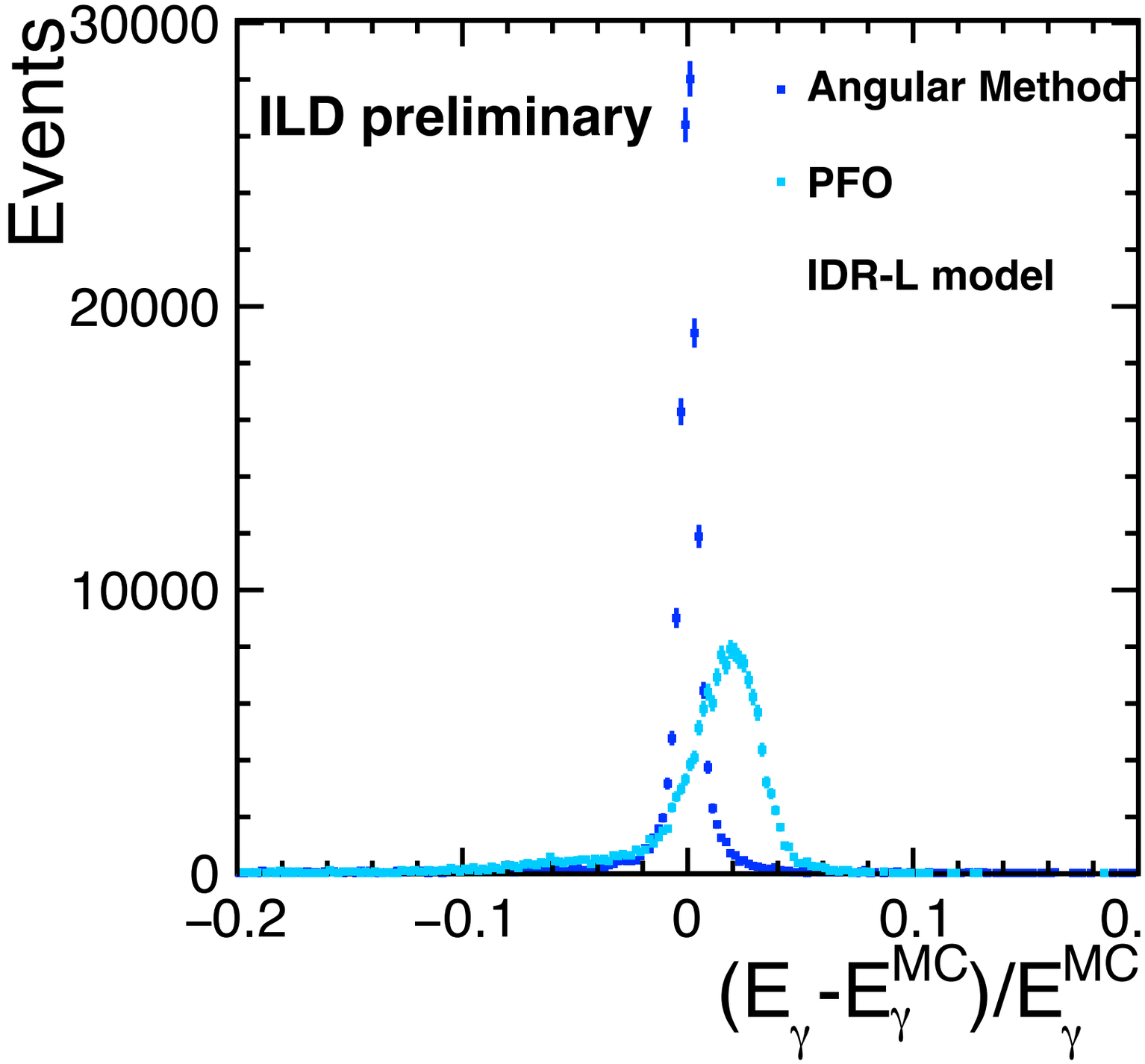
ISR photon: **additional** unseen photon

Advantage: photon energy can be determined without calculating P_{ISR}.

Potential drawback: singularities at $\sin\theta_\gamma$ or $\sin\varphi_\gamma$=0

Relative bias in photon energy

$$\frac{E - E_{(MCtruth)}}{E_{(MCtruth)}}$$



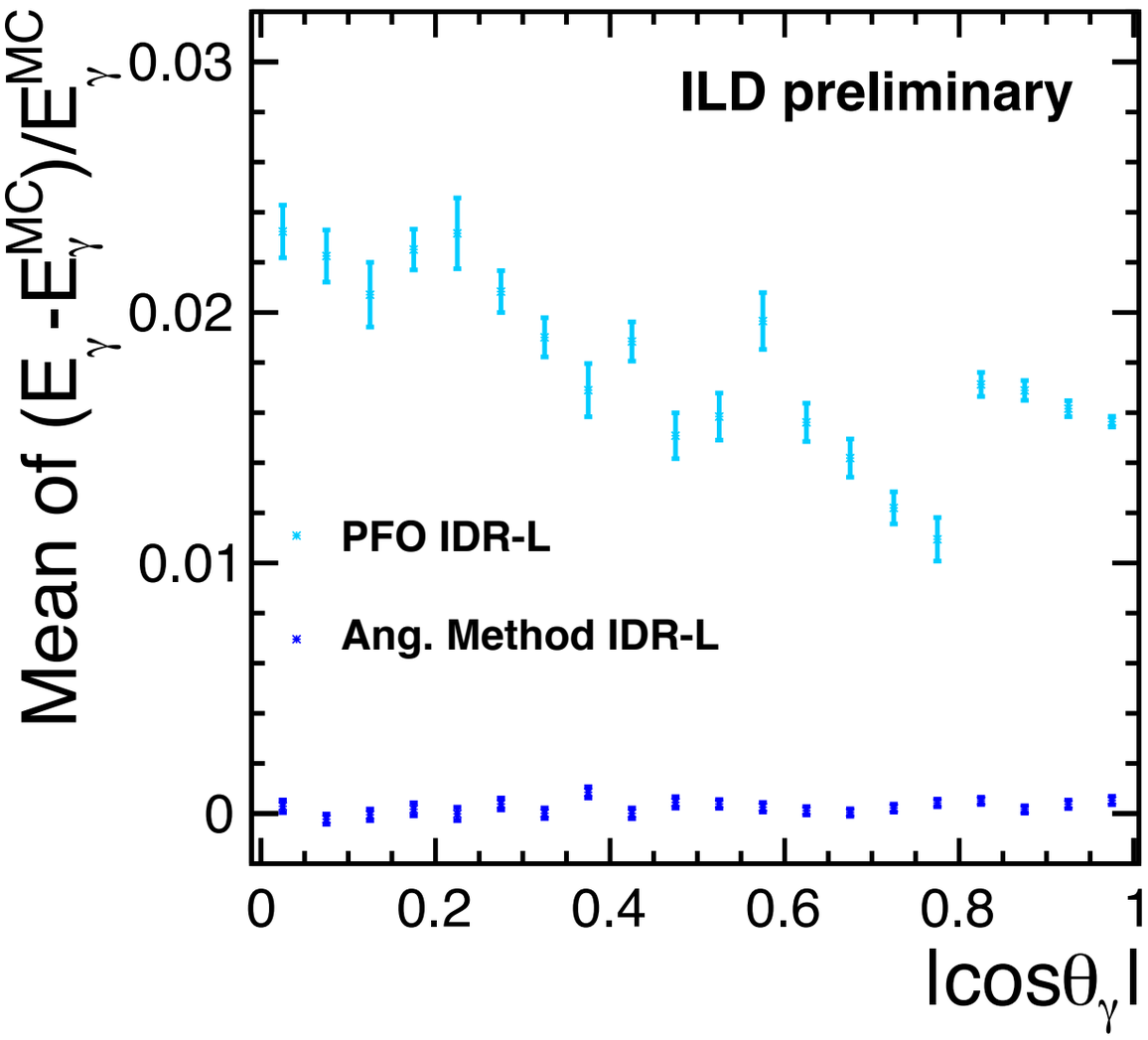
Samples:
$|M(\mu^+\mu^-)-91.2| < 10$ GeV
Large ILD model

Ang. Method is better than PFO:

- **sharper peak**
- **less bias**

Determination of calibration factor

Relative energy bias



PFO has significant bias and its $|\cos\theta|$ dependence.

Ang. Method has almost no bias with no $|\cos\theta|$ dependence.

-> Calculate calibration factor f bin by bin

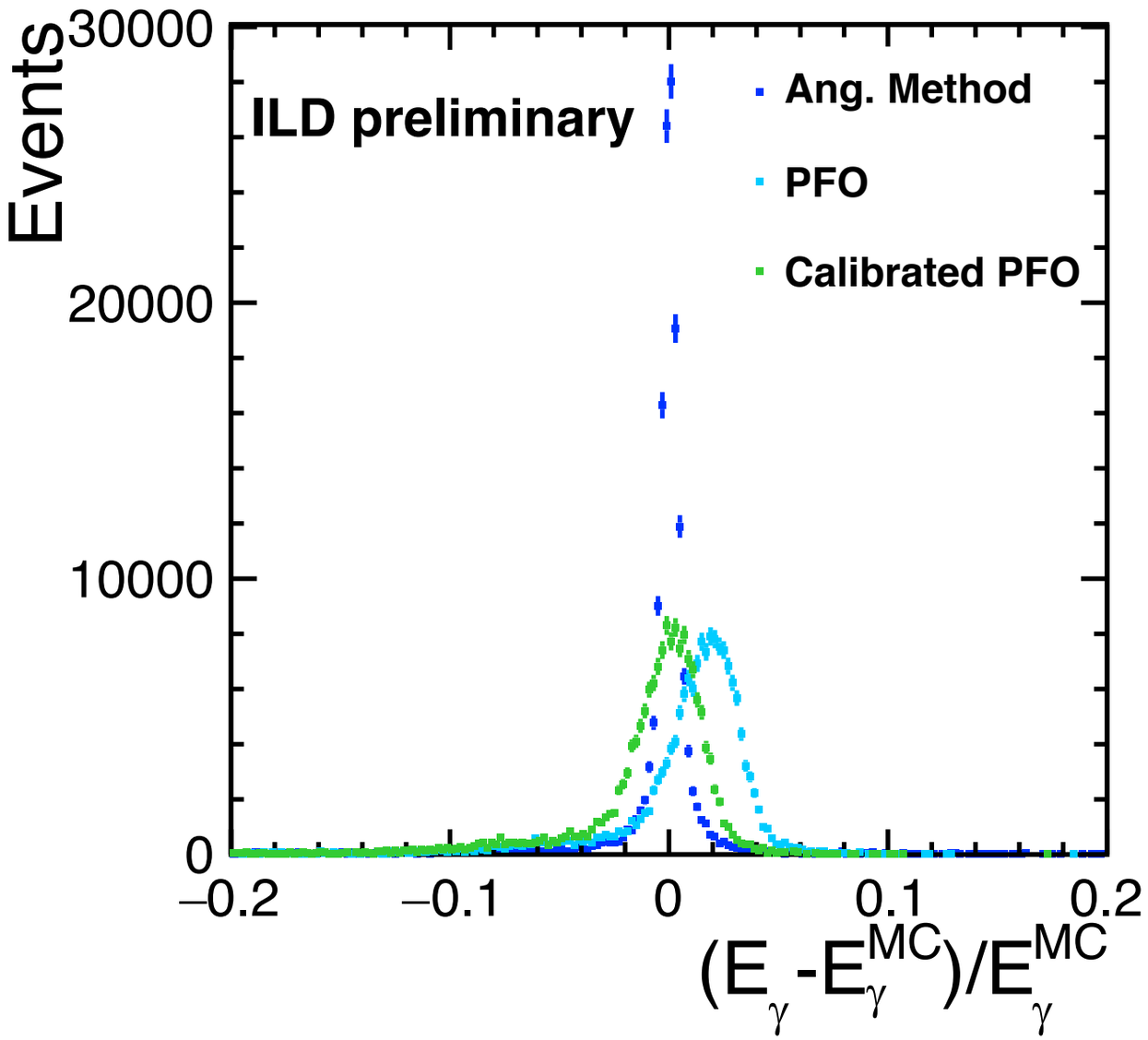
$$f(|\cos\theta_\gamma|) = \frac{\langle E_{\gamma,\text{Angular Method}}(|\cos\theta_\gamma|)\rangle}{\langle E_{\gamma,\text{PFO}}(|\cos\theta_\gamma|)\rangle}$$

-> Calibrate photon energy using f as

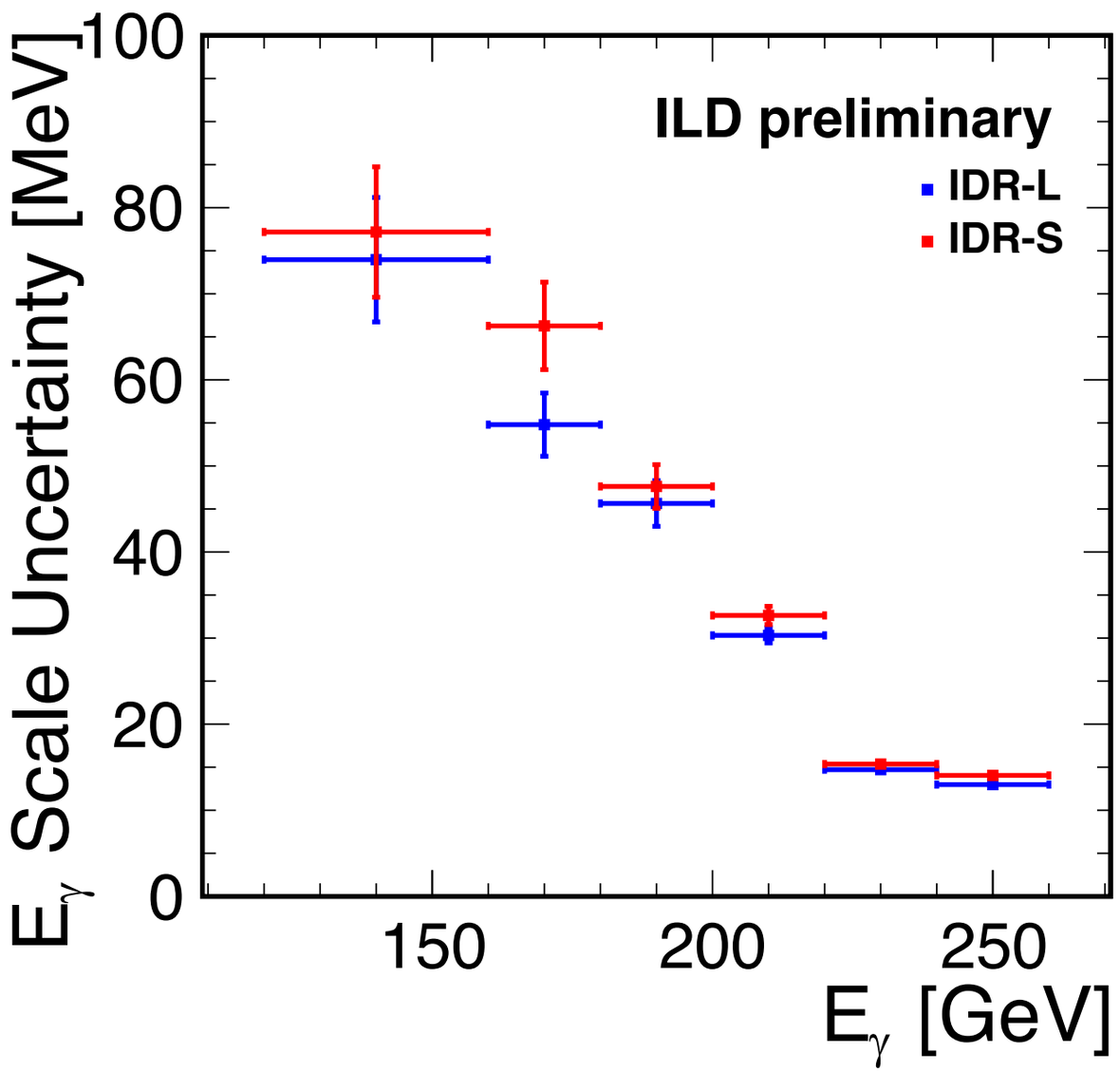
$$E_{\gamma,\text{PFO,Corr.}}(|\cos\theta_\gamma|) = f(|\cos\theta_\gamma|) \times E_{\gamma,\text{PFO}}(|\cos\theta_\gamma|)$$

Calibration Result

Comparison of $(E\text{-}E_{MC})/E_{MC}$ among PFO, calibrated PFO, and Ang. Method



E_γ Scale Uncertainty



The calibration procedure removes the overall bias in the raw PFO photon energy.

$$E_\gamma \text{ Scale Uncertainty} = \sqrt{(\text{PFO Energy Uncertainty})^2 + (\text{Ang. Method Uncertainty})^2}$$

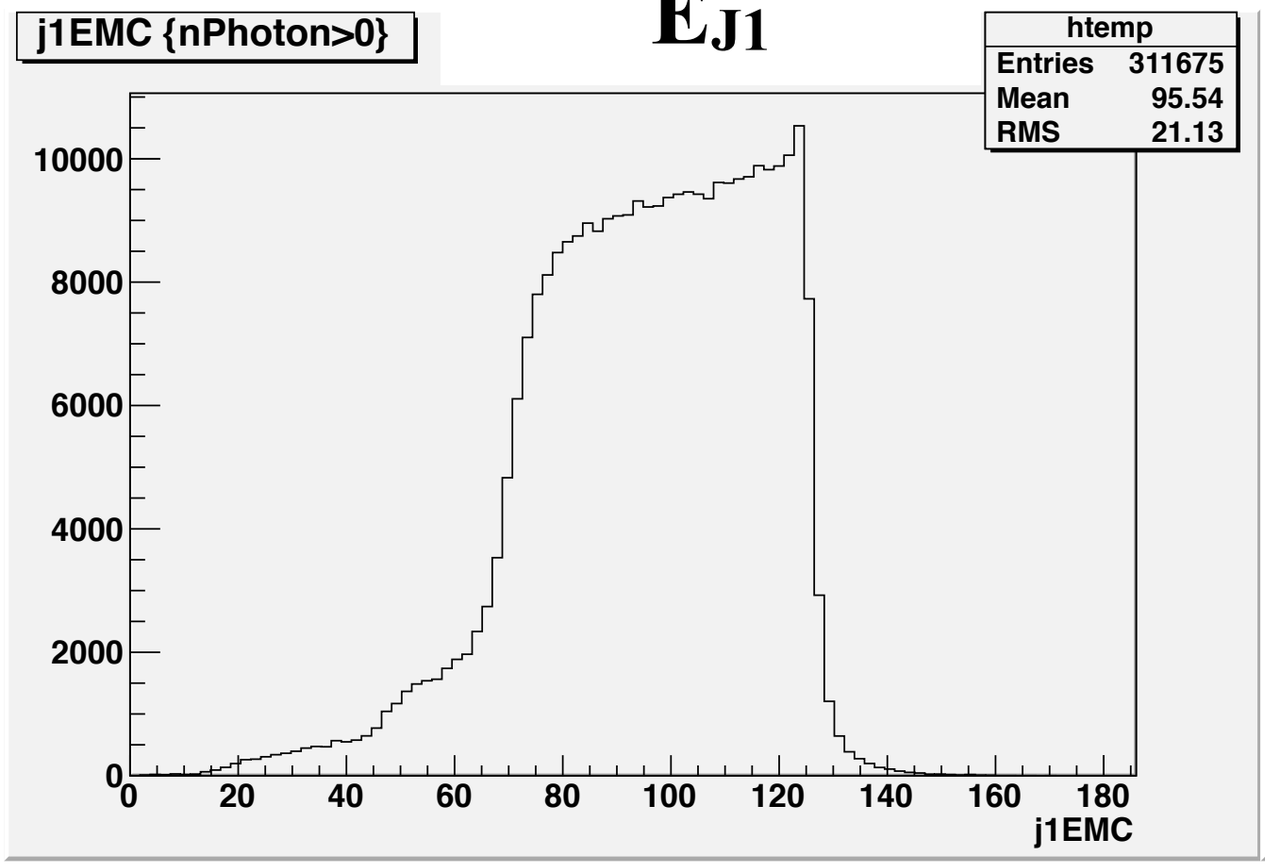
We can calibrate photon energy to better than 100 MeV for Eγ > 120 GeV.

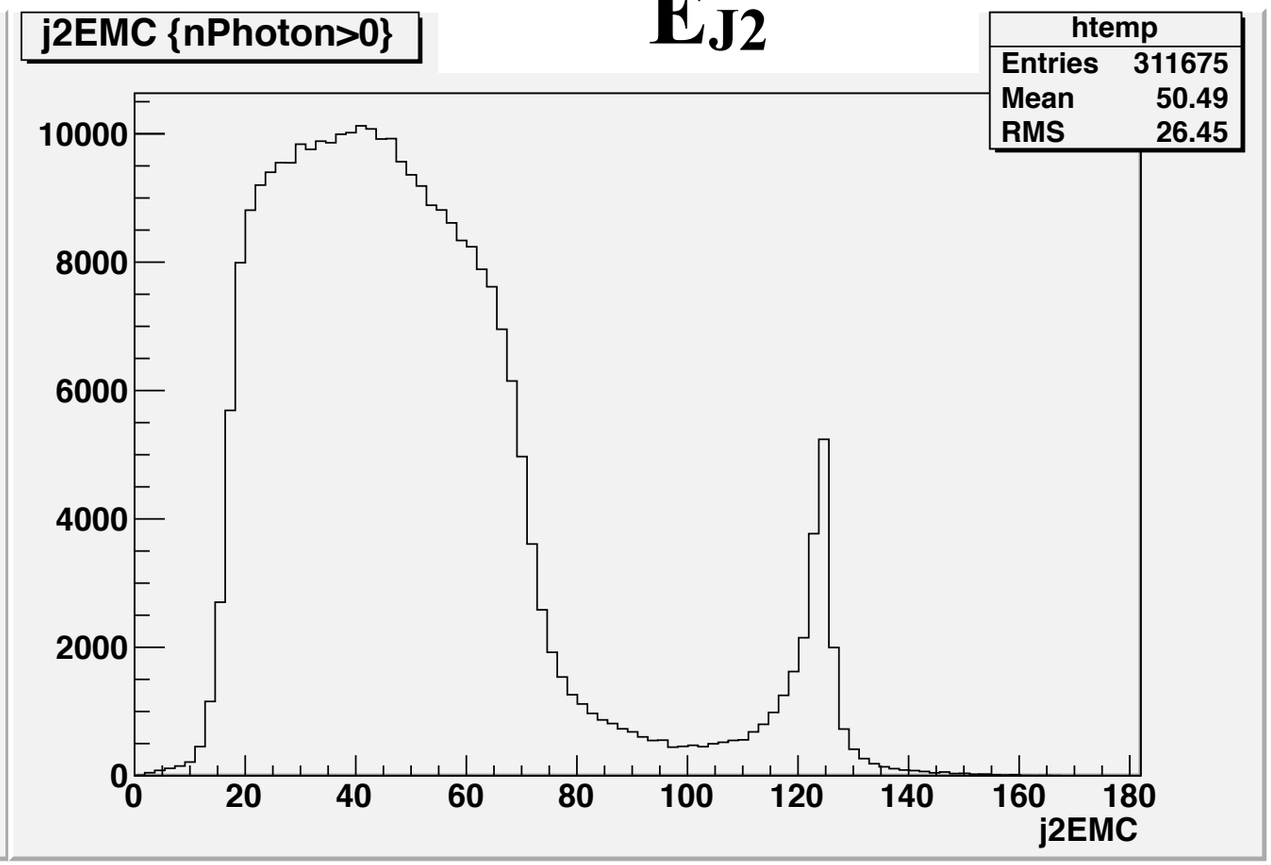
Jet energy scale calibration

Jet1 Energy: E_{J1}

$E_{J1} > E_{J2}$

Jet2 Energy: E_{J2}





GeV **GeV**

E_{CM} = 250 GeV

$\int \mathscr{L} dt$ =900 fb^{-1} for each of the 2 polarization combinations

The selected jets cover energy range from ~20 to 130 GeV.

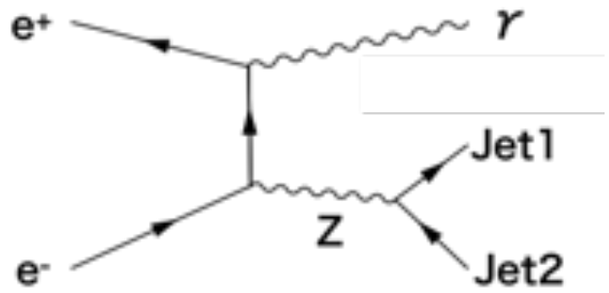
Jet Energy Reconstruction Method

Angular Method: Kinematically reconstruct jet energies based on **4-momentum conservation** using **jet, photon angles and jet masses**

Inputs and outputs

Using (θ_{J1},θ_{J2},θ_γ,φ_{J1},φ_{J2},φ_γ,m_{J1},m_{J2})

-> Determine (P_{J1},P_{J2},P_γ,P_{ISR})



Direction Angle

θ : polar angle

ϕ : azimuthal angle

$$\begin{cases} \sqrt{|P_{J1}|^2+m_{J1}^2}+\sqrt{|P_{J2}|^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}|=E_{CM} & ① \\ \begin{pmatrix} \sin\theta_{J1}\cos\phi_{J1} & \sin\theta_{J2}\cos\phi_{J2} & \sin\theta_\gamma\cos\phi_\gamma \\ \sin\theta_{J1}\sin\phi_{J1} & \sin\theta_{J2}\sin\phi_{J2} & \sin\theta_\gamma\sin\phi_\gamma \\ \cos\theta_{J1} & \cos\theta_{J2} & \cos\theta_\gamma \end{pmatrix} \begin{pmatrix} |P_{J1}| \\ |P_{J2}| \\ |P_\gamma| \end{pmatrix} = \begin{pmatrix} (E_{CM}-|P_{ISR}|)\sin\alpha \\ 0 \\ \pm|P_{ISR}|\cos\alpha \end{pmatrix} \end{cases}$$

Matrix A — Inverse

Beam Crossing Angle ≡2α = 14.0 mrad

ISR photon = additional unseen photon

Irrational equation for each sign of the ISR → 8 possible solutions

Choose the best solution that satisfies

(i) Real and positive value with $<E_{CM}/2$

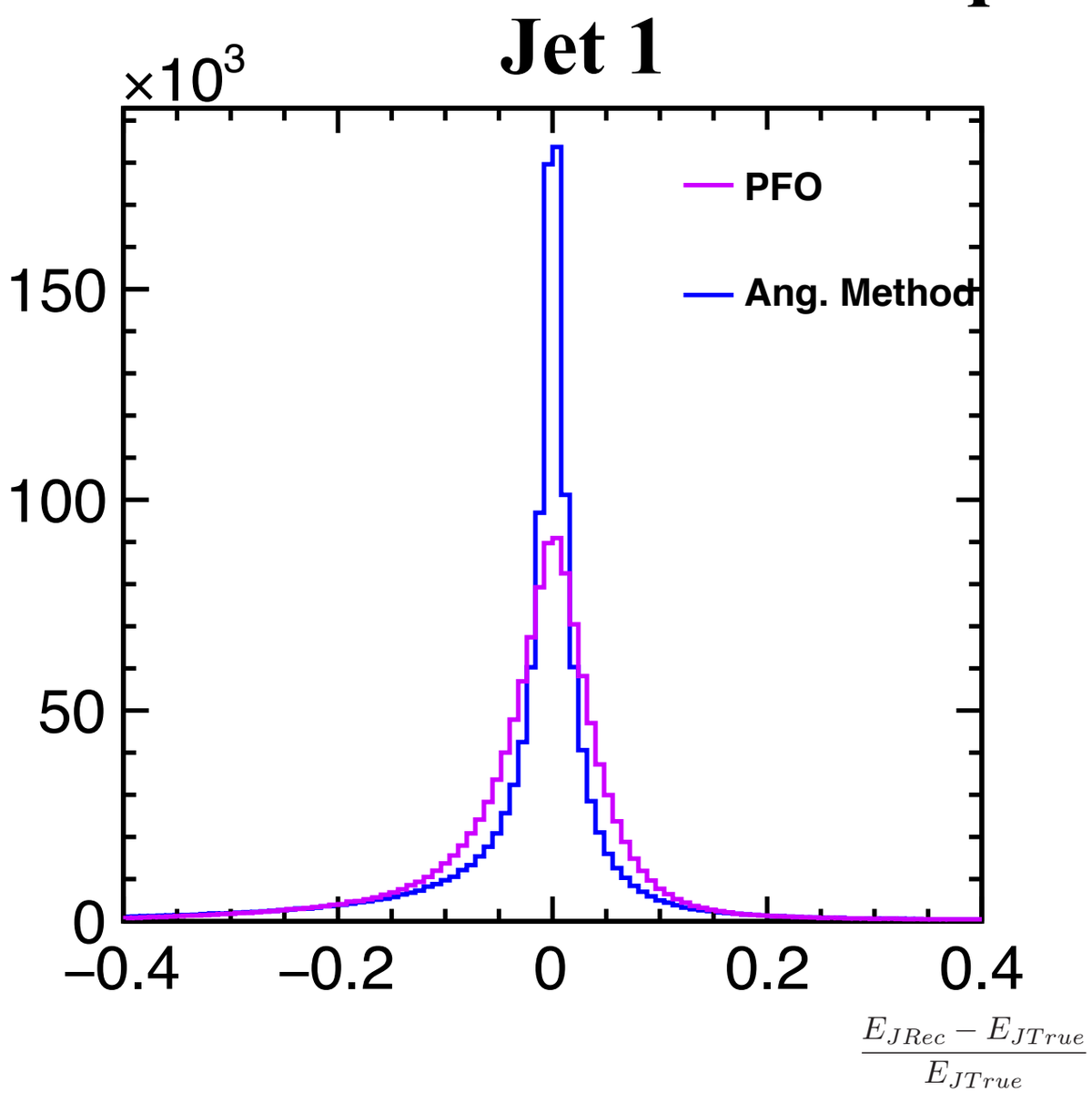
(ii) $\sqrt{P_{J1}^2+m_{J1}^2}>0$ and $\sqrt{P_{J2}^2+m_{J2}^2}>0$

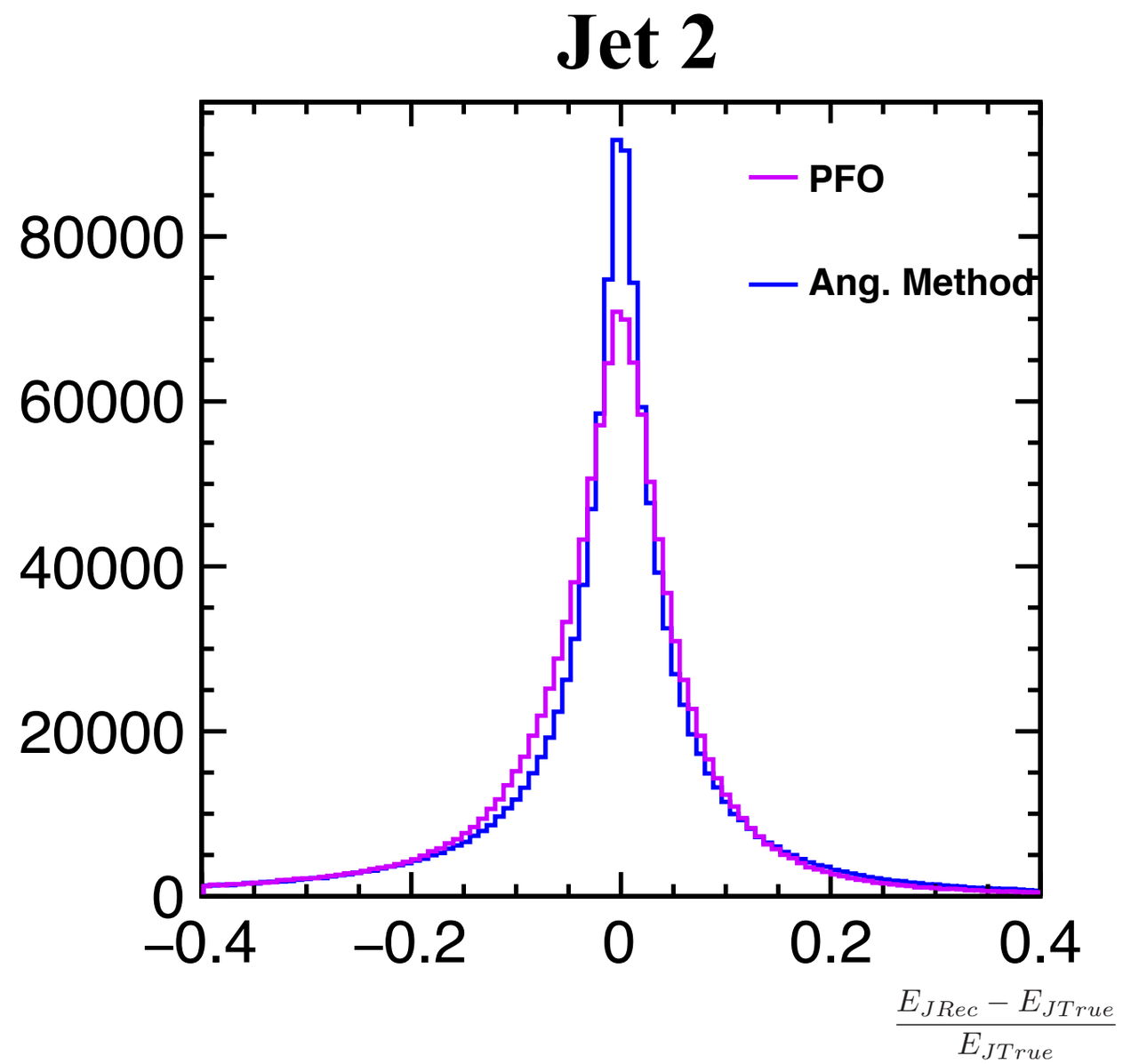
(iii) P_{J1}, P_{J2}, P_γ >0

(iv) solved P_γ closest to the measured P_γ

Relative bias in jet energy (All-MC)

All quark flavors



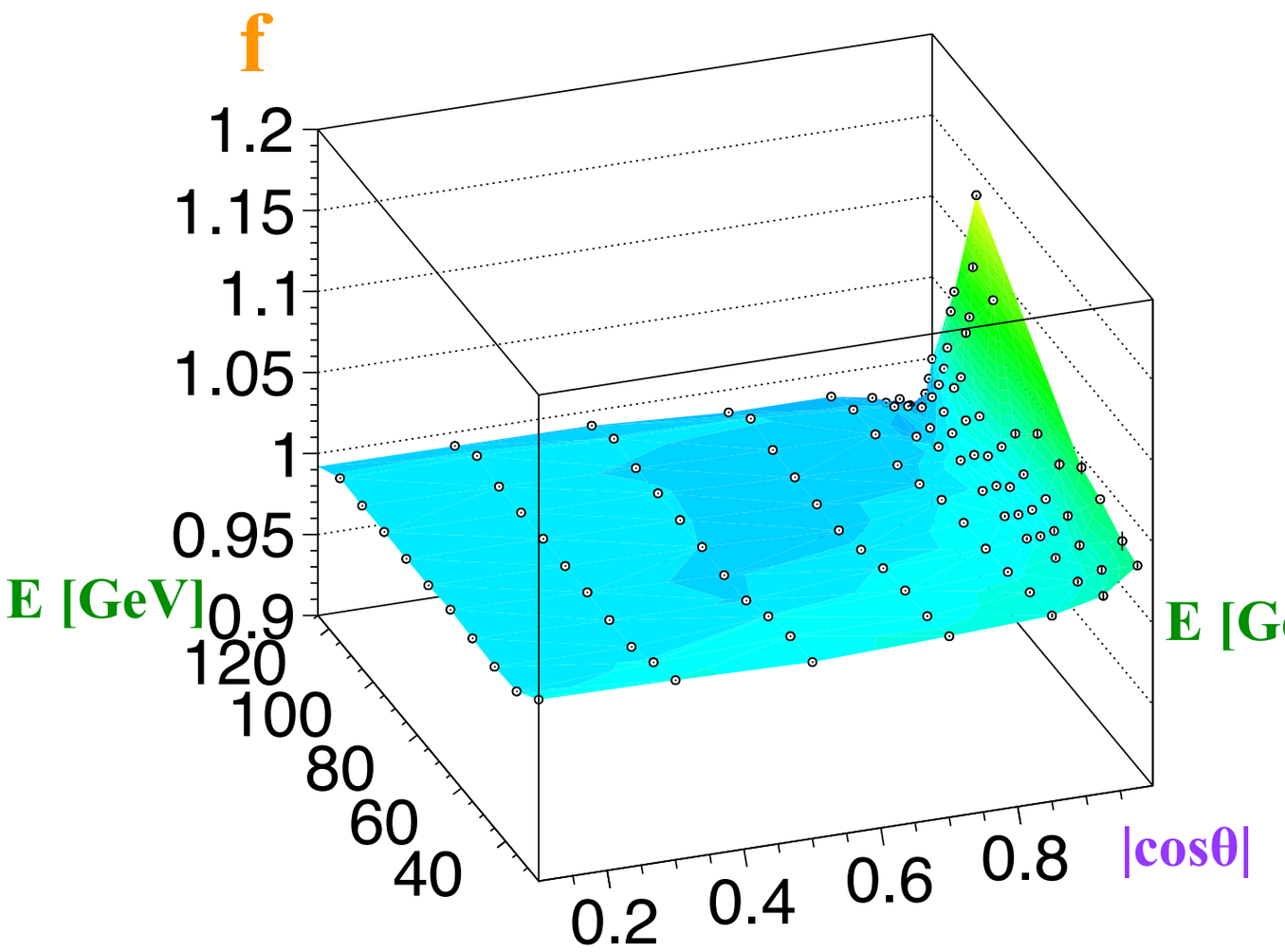


- **Ang. Method gives sharper peak with the bias much smaller than 0.8% implying that Ang. Method can recover energies of undetected or invisible particles.**

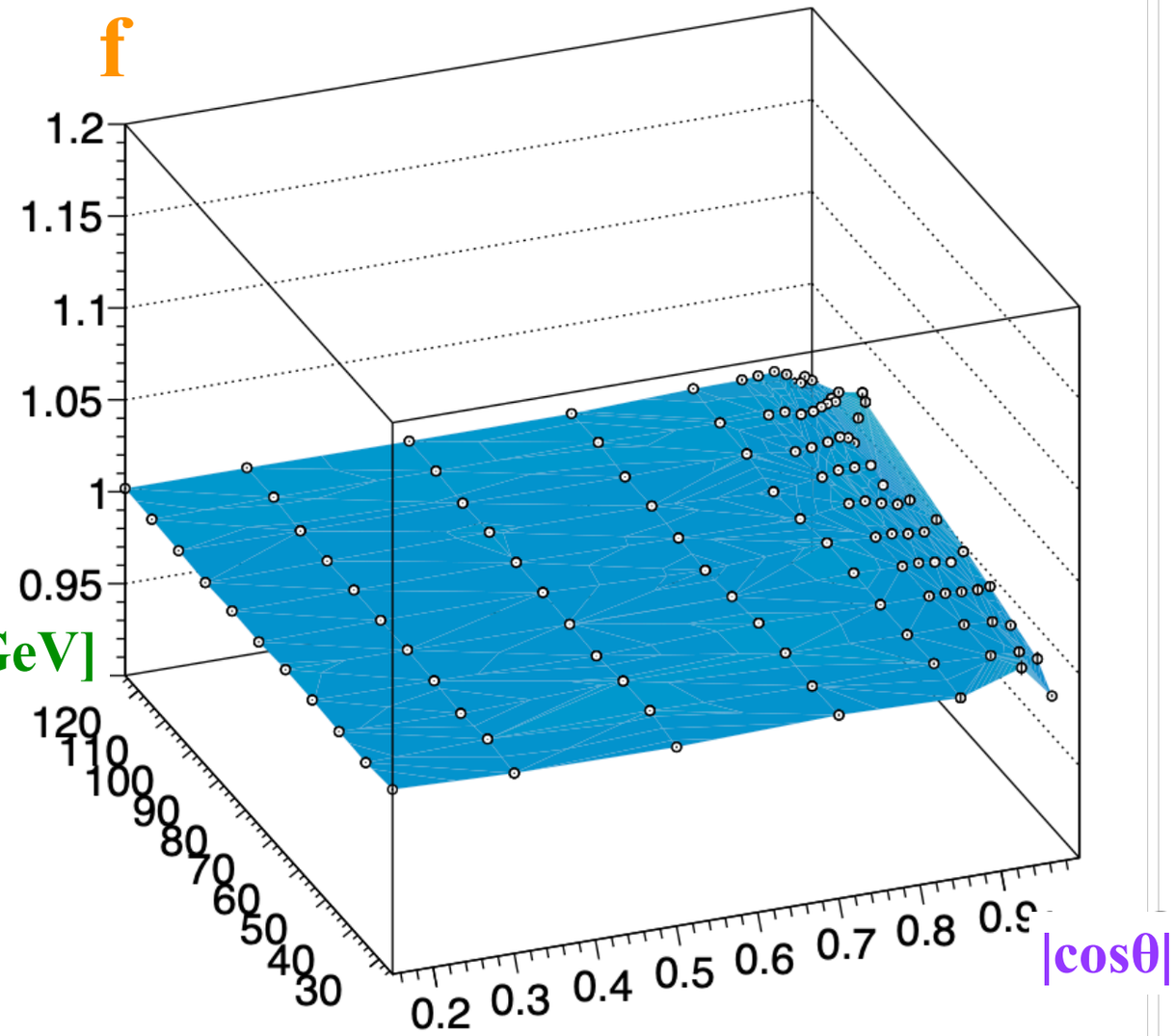
Determination of calibration factor

Calibration Factor f:= E_Ang.Method/E_PFO

Look at the mean value of Core-Gaussian "μ" for the uds jets as a function of E and |cosθ| and define calibration factor as f:=1/(μ+1)



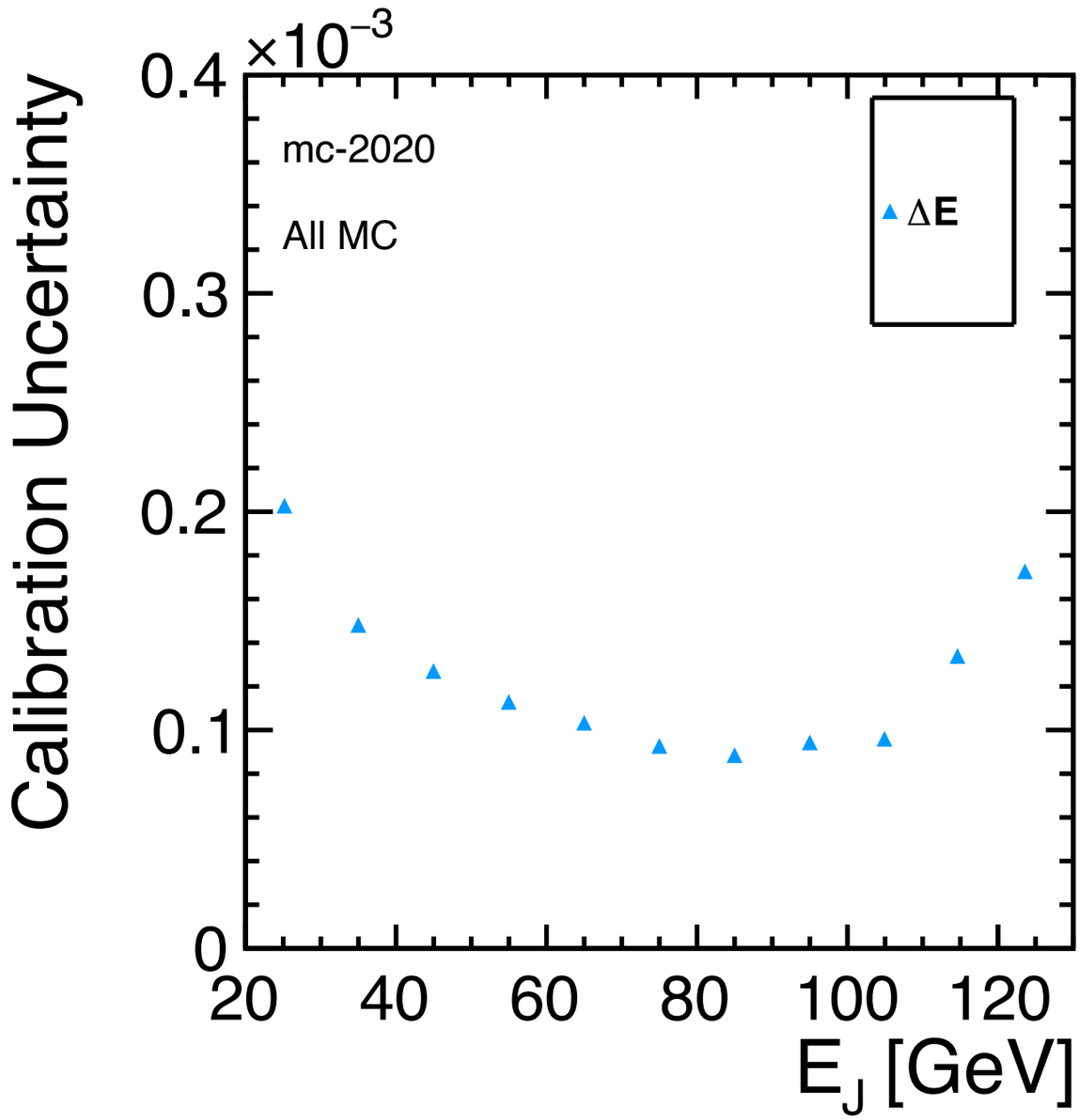
After the calibration



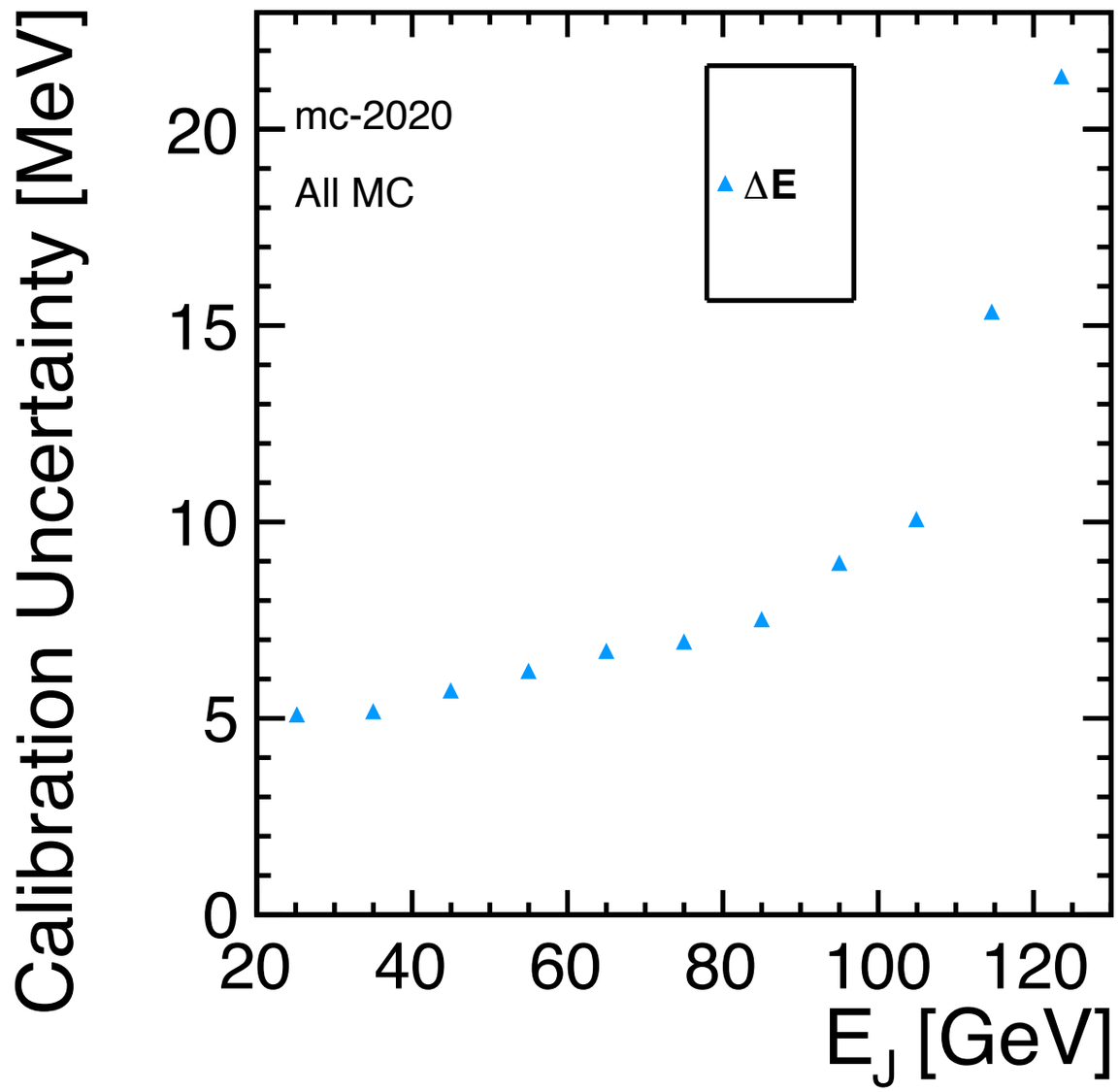
Calibration Uncertainty

Calibration Uncertainty := $\sqrt{(\Delta\mu_{PFO})^2 + (\Delta\mu_{Ang.Method})^2}$

Relative uncertainty



Absolute uncertainty



We can calibrate the jet energy scale with about 10^{-4} accuracy, which corresponds to absolute energy scale uncertainty of about 5 to 20 MeV.

Conclusion

- Radiative return process $e^+e^- \rightarrow \gamma Z$ provides a large set of Z-pole data at initial stage of ILC at 250 GeV.
- We performed the first full simulation study for precise measurement of A_{LR} using this process based on ILD, including realistic SM background events.
- The **statistical error on the A_{LR}** is estimated to be **1.8×10^{-4}.** Most of the systematic error will be canceled out and the dominant systematic error would be polarization uncertainty, we think. In this case, total error on A_{LR} is **2.5×10^{-4}** i.e. **8.8 times better precision than that from the SLC (0.00219)**.
- We can use radiative return process in order to reduce the uncertainties of the photon and jet energy scales.