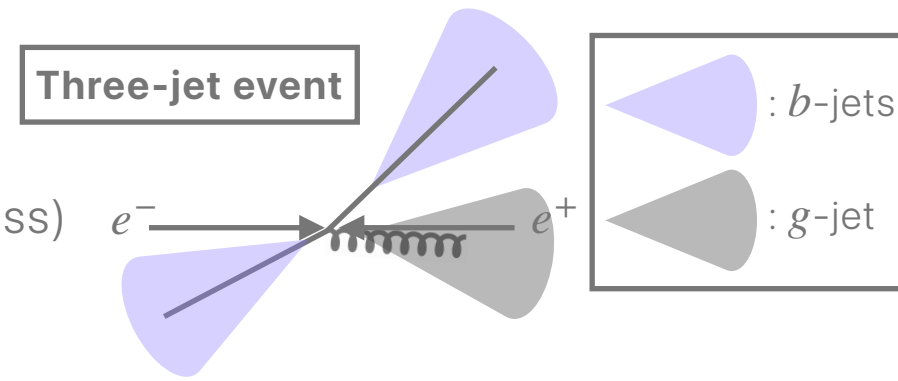


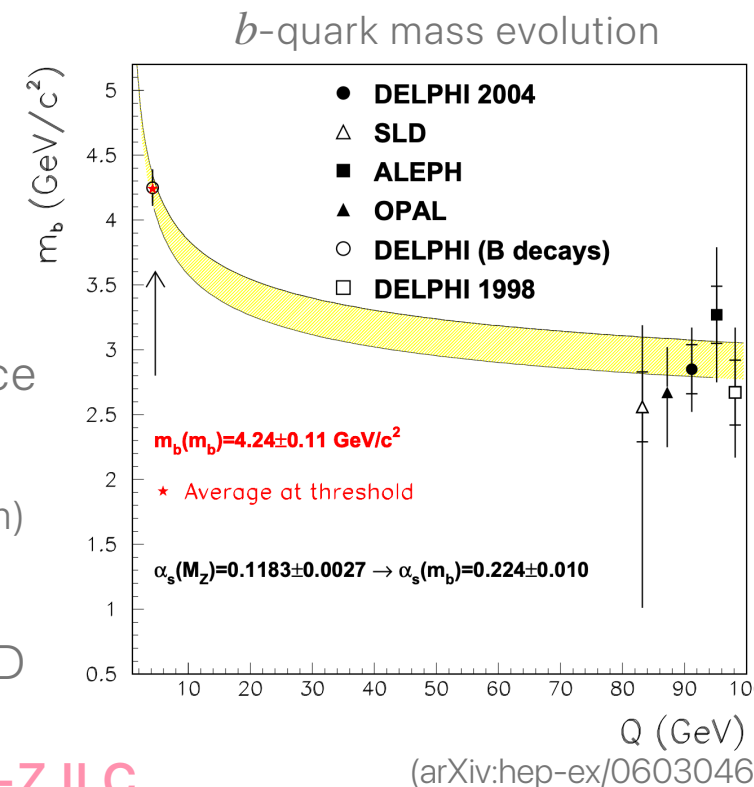
## 1. Running $b$ -quark mass

- Quarks can not be observed  
→ "Single" quark masses are not observables, and they are observed as running parameters (running mass)
- Running mass is described by RGE:  
$$\mu^2 \frac{\partial m_q(\mu)}{\partial \mu^2} = -\gamma(\alpha_s(\mu)) m_q(\mu)$$
  
 $\mu$ : renormalization scale  
 $\gamma(\alpha_s(\mu))$ : Perturbative function



## 2. Inferring of running $b$ -quark mass

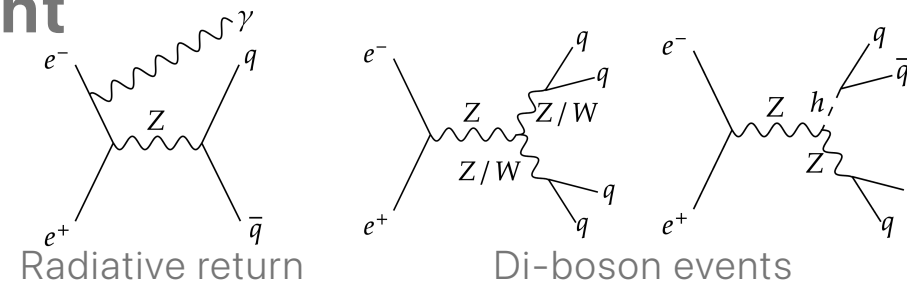
- Quarks and gluons appear as jets  
→ running quark masses are obtained from hadronic observables  
Exclusive observables (e.g. three jet rates) have better sensitivity (by a factor of 10 @Z-pole) of quark mass
  - Jet should be defined so that avoid infrared (soft/collinear) divergence  
→ Jet-Clustering algorithm (JADE, DURHAM, CAMBRIDGE...)
- $$R_3^f = \frac{\Gamma_{3j}^f(y_c)}{\Gamma^f} : \text{Total width for } e^+e^- \rightarrow f\bar{f}$$



- The  $b$ -quark mass at Z-pole has been measured precisely at LEP/SLD  
→  **$b$ -quark mass at higher energies at the ILC?**  
**Estimate dominant systematic errors at 250 GeV ILC and Giga-Z ILC**

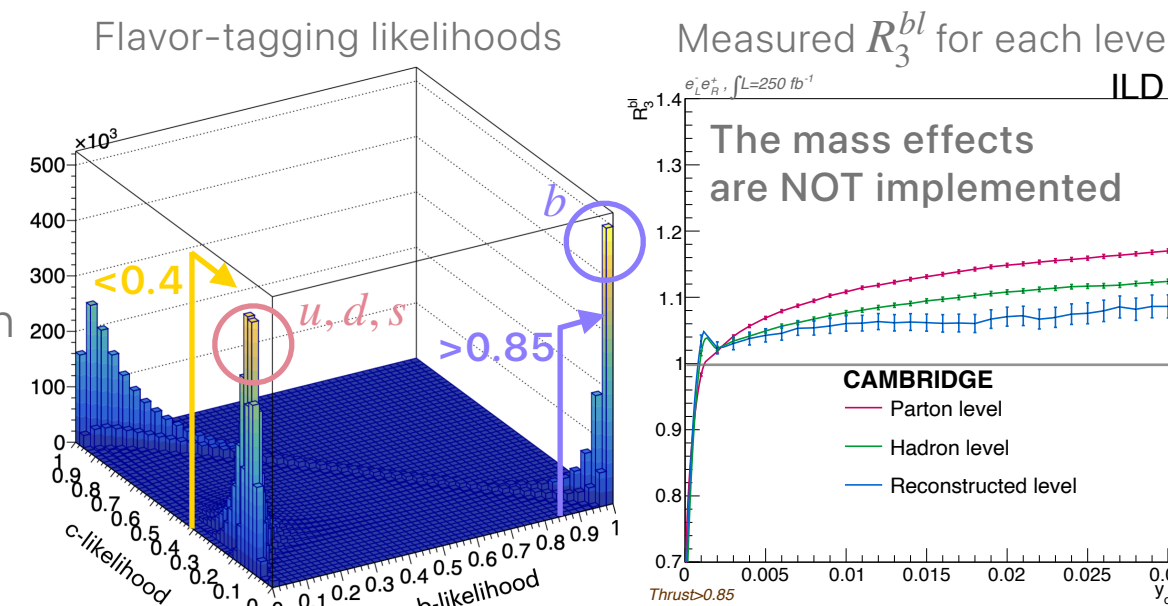
## 5. Environment of 250 GeV measurement

- Signal event:  $e^+e^- \rightarrow q\bar{q}$  ( $q = u, d, s, b$ )
- BKG events:
  - Radiative return (w/  $<50$  GeV ISR  $\gamma$ )
  - Di-boson events
- Luminosity:  $2ab^{-1}$  with two polarizations ( $P_{e^-}, P_{e^+}$ ) =  $(-0.8, +0.3)$  and  $(+0.8, -0.3)$
- Used old DBD sample → event generated by LO for massless quarks in WHIZARD  
Mass effects are only implemented in PYTHIA (PS+Hadronization)
- Situation is completely different from LEP's Z-pole measurement



## 6. Event selection

- Radiative return cut  
Construct ISR energy from 2-jets kinematically and remove invisible  $\gamma$ s  
$$K_{reco} = \frac{250 \text{ GeV} \sin \psi_{acol}}{\sin \psi_{acol} + \sin \theta_1 + \sin \theta_2}$$
  
 $\psi_{acol}$ : angle of btw 2 jets  
 $\theta_i$ : polar angle of each jet  
Visible  $\gamma$ s are removed by neutral PFO information
- Di-boson events cut: use Thrust  $> 0.85$
- Flavor-tagging  
Efficiency: 80% (for  $b$ ), 58% (for  $uds$ )  
Purity: 98.7% (for  $b$ ), 96.1% (for  $uds$ )
- Jet-reconstruction: CAMBRIDGE algorithm w/  $y_c = 0.01$



## 3. Definition of the Observable

- Consider double-ratio  $R_3^{bl}$  as the observable  
Cancel or reduce EW corrections and systematic uncertainties (hadronization effect)
- $$R_3^{bl} = \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^l(y_c)/\Gamma^l} = 1 + \frac{\alpha_s(\mu)}{\pi} a_0(y_c) + \bar{r}_b(\mu) \left( b_0(\bar{r}_b, y_c) + \frac{\alpha_s(\mu)}{\pi} b_1(\bar{r}_b, y_c, \mu) \right)$$
- ( $l = u, d, s$ )
- massless correction      massive LO correction      massive NLO correction
- $$\bar{r}_b(\mu) = m_b^2(\mu)/s$$
- $$\bar{b}_1 = b_1 + 2b_0 \left( 4/3 - \log \bar{r}_b + \log(\mu^2/s) \right)$$

## 4. Sensitivity of $b$ -quark mass at high energies

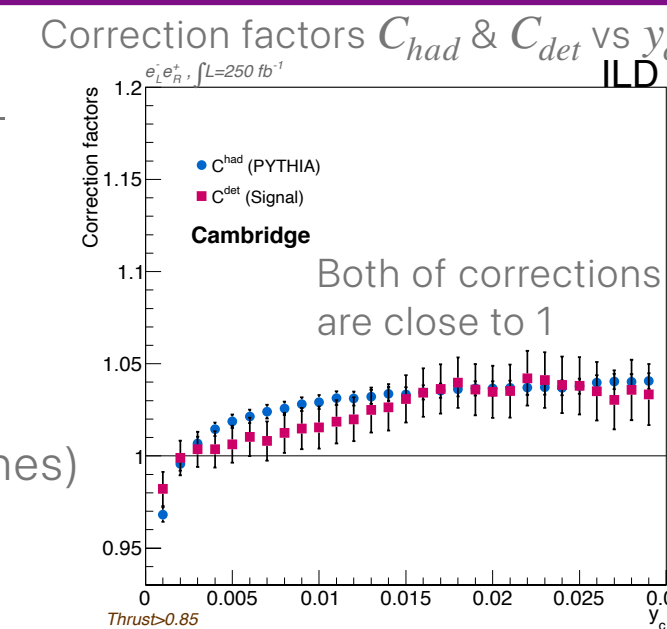
- Sensitivity of  $b$ -quark mass for  $R_3^{bl}$  is given by  $\Delta R_3^{bl} \sim \frac{2(1-R_3^{bl})}{m_b(\mu)} \Delta m_b(\mu)$  (e.g. CAMBRIDGE predictions are given below)
  - If we want  $\Delta m_b = 0.4$  GeV, we need to measure  $R_3^{bl}$  with a precision of 1% (for Z-pole) and 0.1% (for 250 GeV)  
→ **The sensitivity at 250 GeV is ~5 times deteriorated**
- 

## 7. Assessment of uncertainties

- The mass effects are not implemented in the current MC, but corrections between different levels are worthwhile:
- $$R_3^{bl} \Big|_{parton} = C_{had} \times C_{det} \times R_3^{bl} \Big|_{reco}$$
- Estimate systematic uncertainties from these corrections:
    - Hadronization model  
LEP's time: 0.2% uncertainty on  $C_{had}$  (Compare different hadronization models and tunes)  
→ assumed its half thanks for higher energy B-hadrons and more data
    - Detector  
Propagated flavor-tagging efficiency (0.1-0.5%) and BKG contaminations (O(1%)) to  $C_{det}$  through Toy-MC

$$C_{had} = \frac{R_3^{bl} \Big|_{parton}}{R_3^{bl} \Big|_{hadron}}$$

$$C_{det} = \frac{R_3^{bl} \Big|_{hadron}}{R_3^{bl} \Big|_{reco}}$$



- Statistical uncertainty is estimated at  $2ab^{-1}$  H2O scenario
- $b$ -quark mass precision for  $R_3^{bl} = 0.996$ ,  $m_b = 2.75$  GeV:  
$$\Delta m_b(250) = 0.76(stat.) \pm 0.59(exp.) \pm 0.34(had.) \pm 0.07(theo.) \text{ GeV}$$
- Giga-Z ILC gives better precision thanks for 100times larger statistics, superior flavor-tagging:
  - DELPHI:  $\Delta m_b(m_Z) = 0.18(stat.) \pm 0.13(exp.) \pm 0.19(had.) \pm 0.12(theo.) \text{ GeV}$
  - ILD:  $\Delta m_b(m_Z) = 0.02(stat.) \pm 0.02(exp.) \pm 0.09(had.) \pm 0.06(theo.) \text{ GeV}$

## 8. Conclusion and Prospects

- ILC 250 GeV measurement has limited  $b$ -quark mass sensitivity, but it will add a new point at never proved energies
- Giga-Z ILC will provide superior result at Z-pole than LEP and better QCD test

