Jet Energy Scale Calibration using $e^+e^- \to \gamma Z$ process

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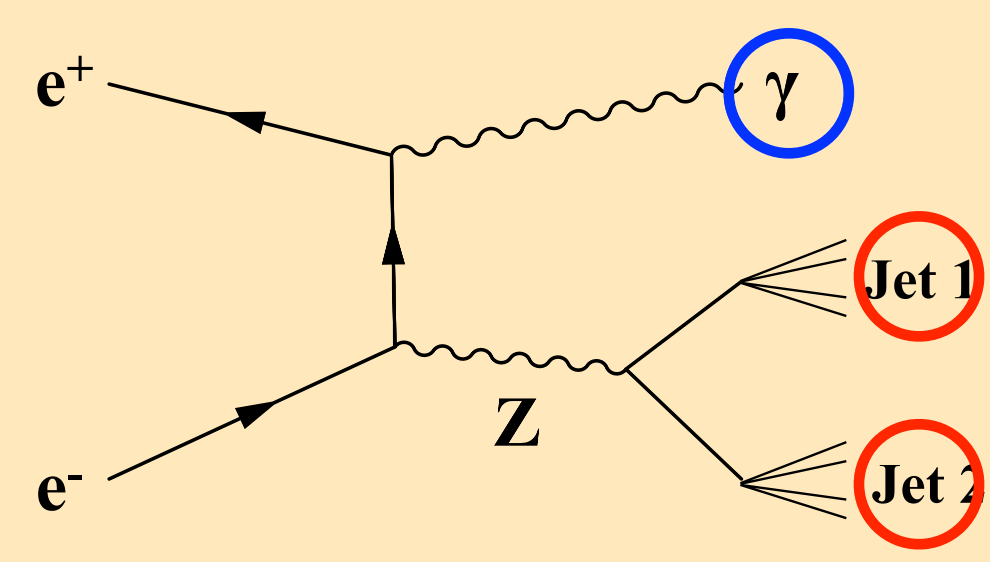
Introduction

- Jet energies can be reconstructed using measured direction of 2 jets and γ and mass of 2 jets in the $e^+e^- \to \gamma Z$, $Z \to$ 2 Jets process.

- The results using **DBD samples** were reported at the ILD group meeting held on 13/10/2020. Please look at https://agenda.linearcollider.org/event/8657/ for the detail.

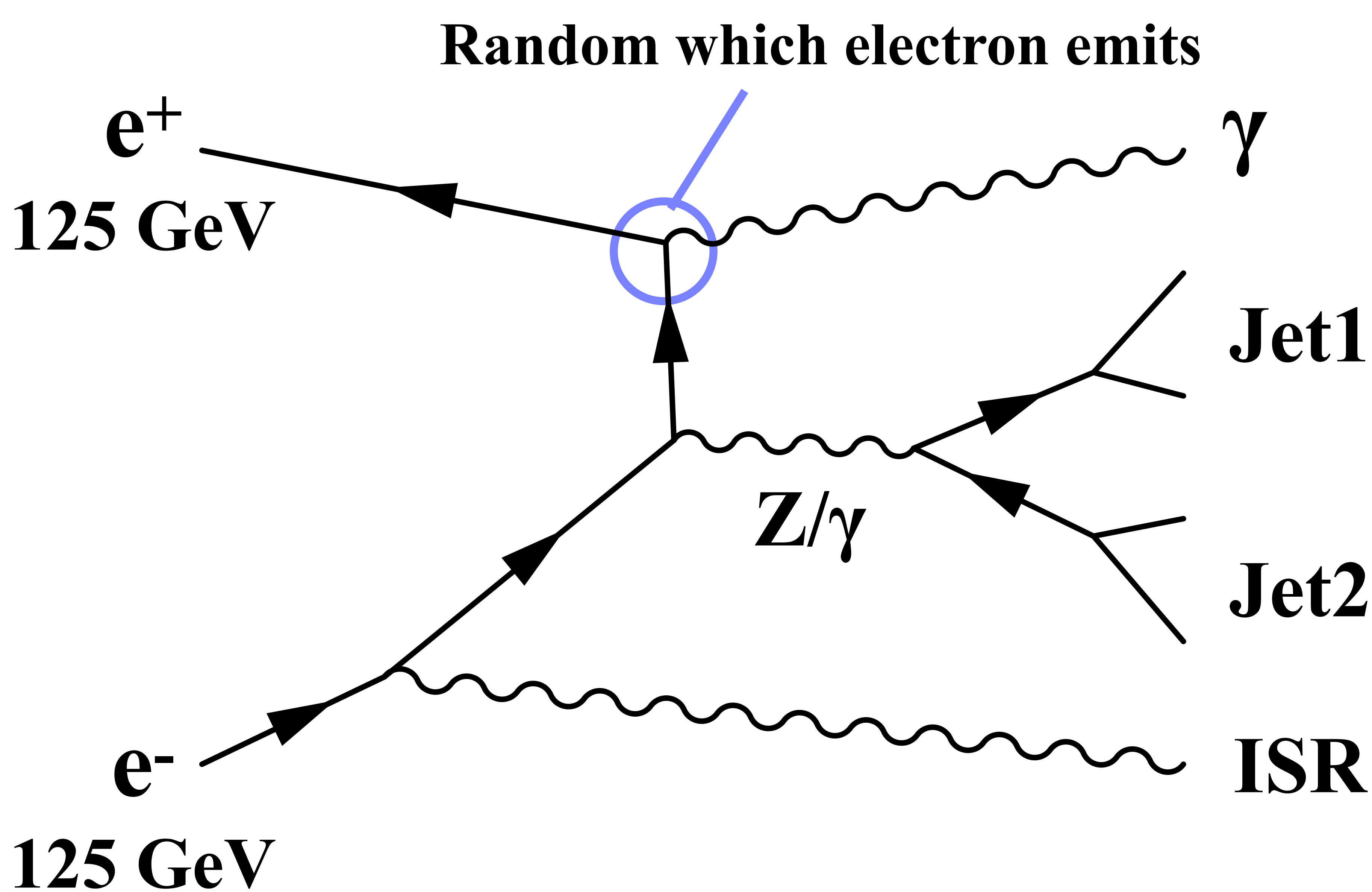
- In this talk, I will focus on the new results using **mc-2020 samples** and show the difference.

Jet Energy Scale Calibration



Full simulation

- **Geant4-based full detector simulation** is performed for the $e^+e^- \rightarrow \gamma Z$, $Z \rightarrow$ 2 Jets process using a **realistic ILD detector model**, at **E_{CM}= 250 GeV** with $\int Ldt$=900fb^{-1} each for 2 beam polarizations: (P_{e^-}, P_{e^+}) = (-0.8, +0.3) and (+0.8, -0.3).



Event selection

Signal Photon Selection

Events signature = **1 isolated energetic photon + 2 jets**

Signal photon is selected as follows:
1. choose neutral particles with particle ID = 22 (Pandora PFA ID)
2. require energy > 50 GeV
3. choose the photon candidate with energy closest to 108.4 GeV

Other photons inside the cone (with the angle $\cos\theta > 0.998$ from the signal photon) are merged with the signal photon.

#Signal Photon

- #Photon = 0 : 82.2% of the generated eLpR samples
- #Photon = 1 : 17.8% of the generated eLpR samples

Event selection

Jet Clustering

- All Particle Flow Objects (PFOs) other than the selected photon are clustered into 2 jets with Durham algorithm (done by LCFIPlus)
- The jet with higher reconstructed energy is defined as “jet 1” and the other as “jet 2”

2 Definitions for MCTruth

- All-MC : contains all MC particles
- Detected-MC : contains only particles linked to the detected PFOs

Both MC were used

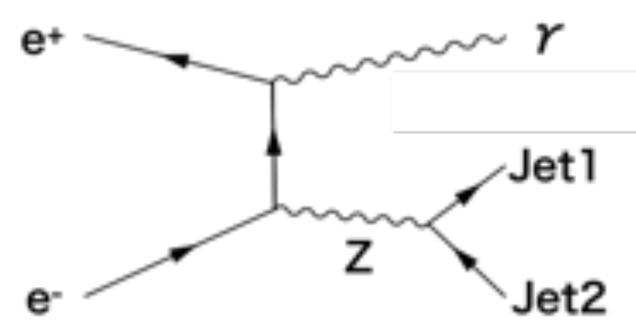
Reconstruction Method

Main idea: Reconstructing jet energies based on **jet, photon angles and jet masses** using 4-momentum conservation

Inputs and outputs

Using $(\theta_{J1},\theta_{J2},\theta_{\gamma},\varphi_{J1},\varphi_{J2},\varphi_{\gamma},m_{J1},m_{J2})$

-> Determine $(P_{J1},P_{J2},P_{\gamma},P_{ISR})$



Direction Angle

θ: polar angle

ϕ: azimuthal angle

$$\left\{\begin{array}{l} \sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}| = E_{CM} \quad ① \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_{\gamma}cos\phi_{\gamma} \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_{\gamma}sin\phi_{\gamma} \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_{\gamma} \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_{\gamma} \end{pmatrix} = \begin{pmatrix} (E_{CM}-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{array}\right.$$

Matrix A — Inverse

Beam Crossing Angle ≡2α = 14.0 mrad

ISR photon = additional unseen photon

Irrational equation for each sign of the ISR -> 8 possible solutions

Choose the solution with

(i) **Real and positive value with $<E_{CM}/2$**

(ii) $\sqrt{P_{J1}^2+m_{J1}^2}$ **>0 and** $\sqrt{P_{J2}^2+m_{J2}^2}$ **>0**

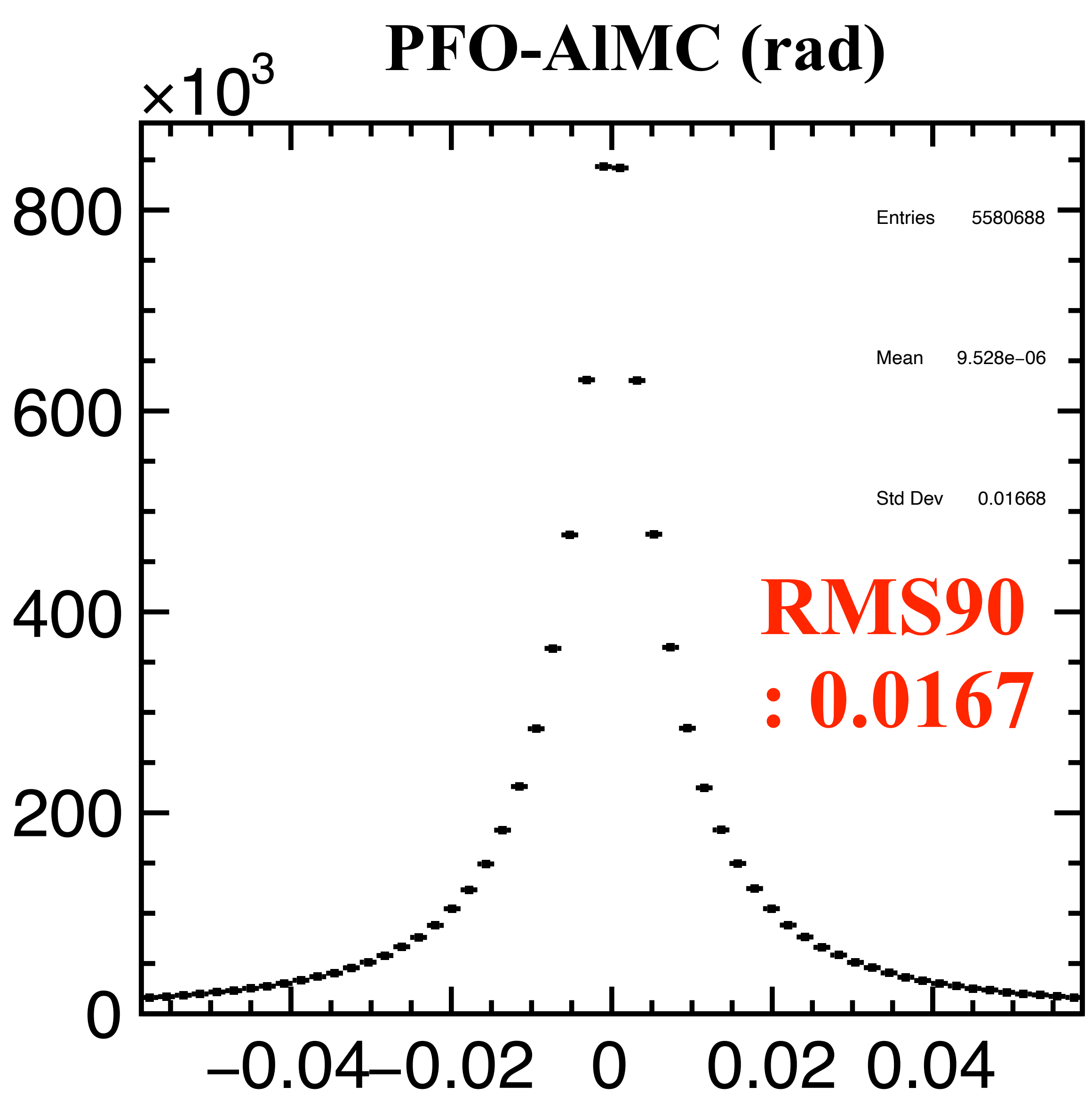
(iii) **P_{J1}, P_{J2}, P_{γ} >0**

(iv) **solved P_{γ} closest to the measured P_{γ}**

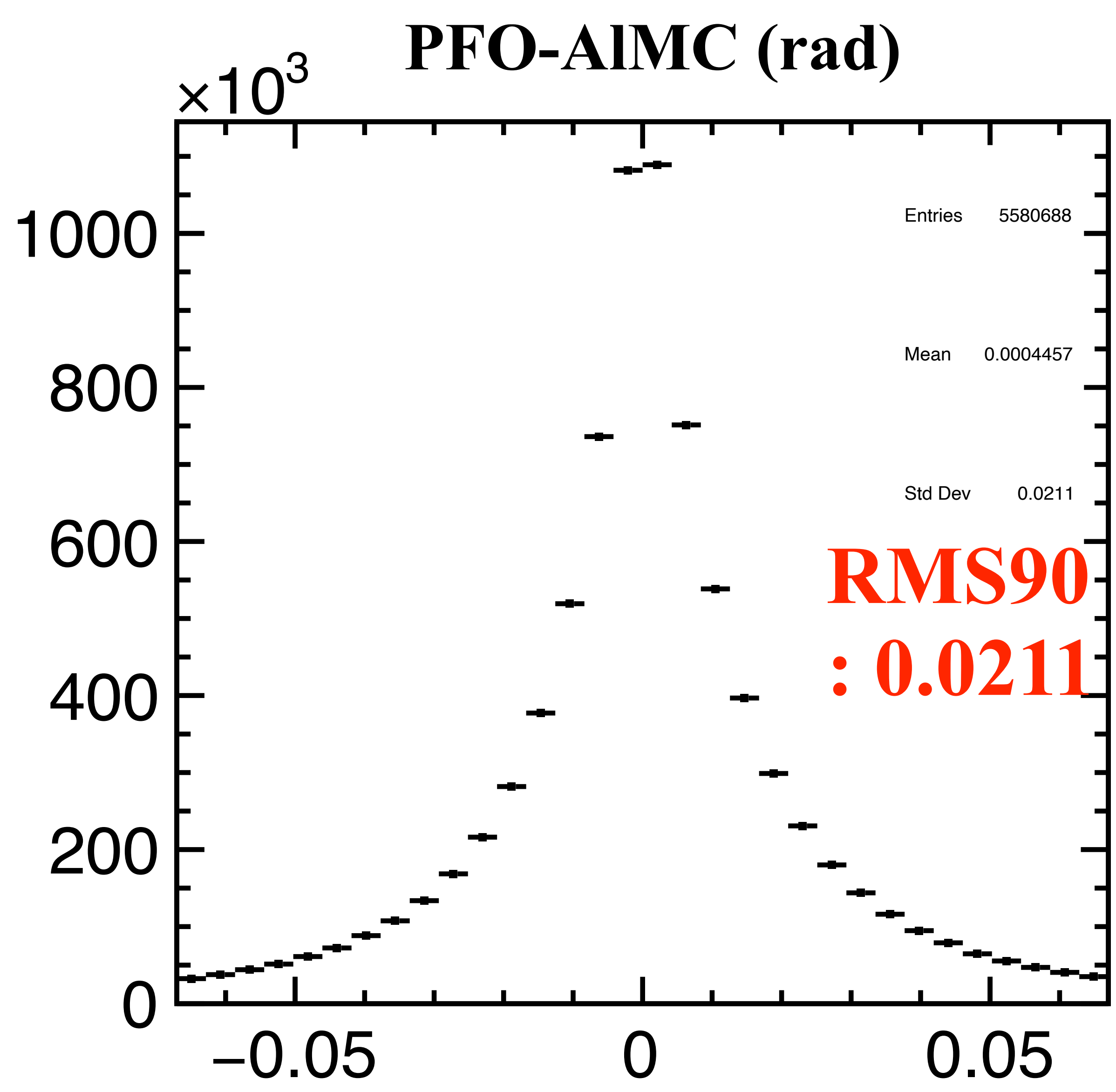
Input Variables Correctness

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

Theta



Phi



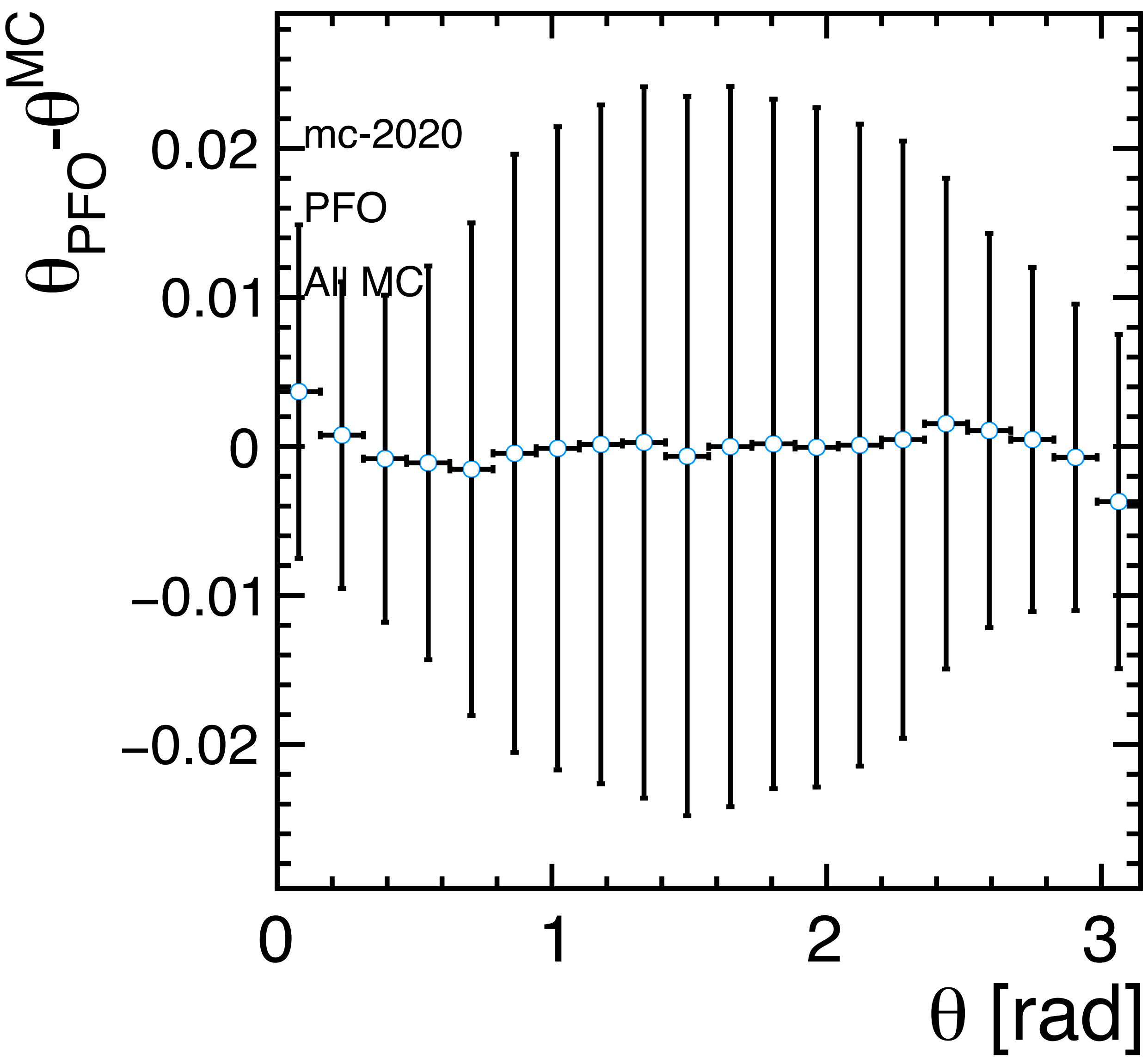
Both have ~0.02 rad RMS90.

Abs. Differences

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

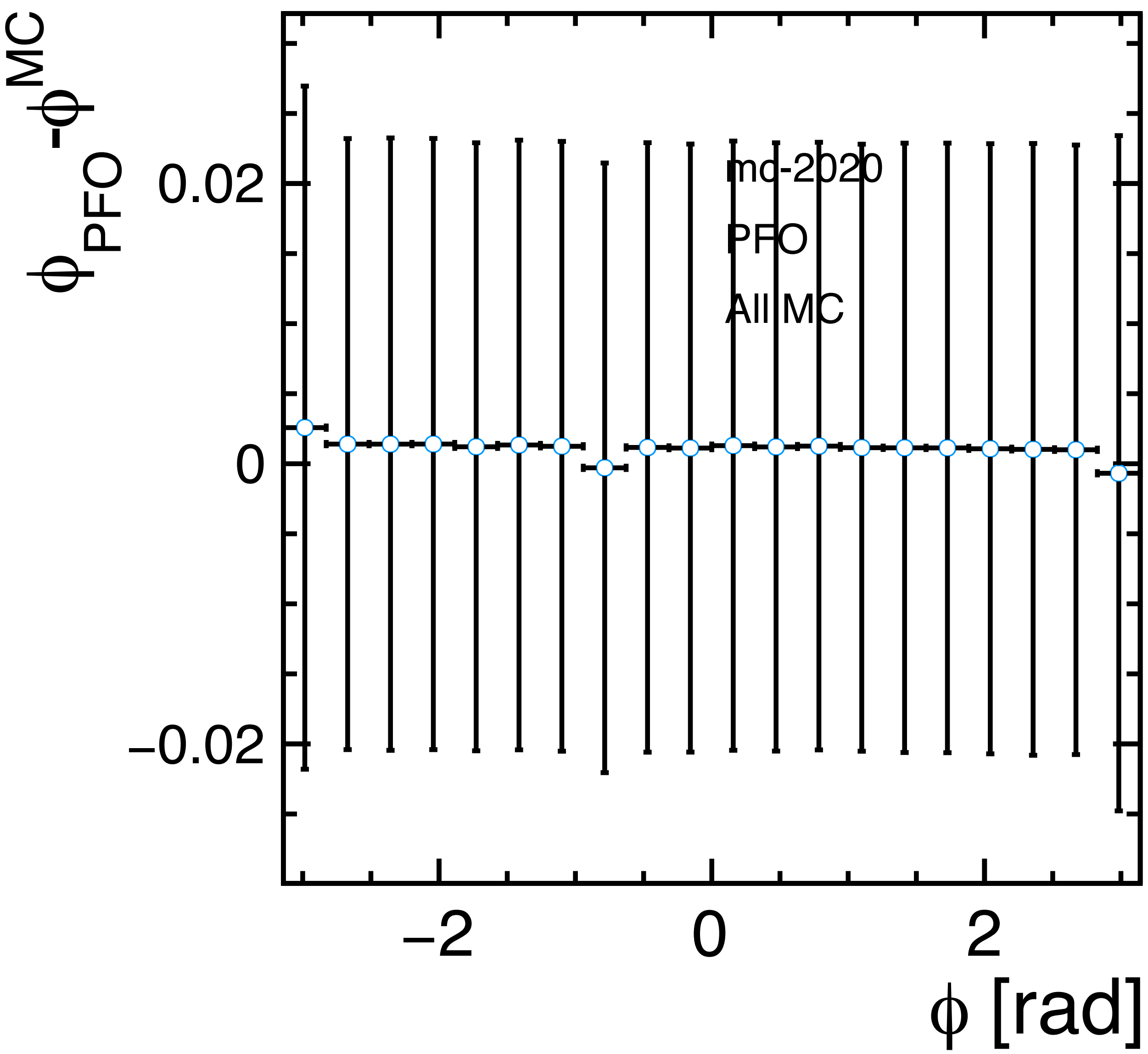
Circle points are mean90 and bars are RMS90.

Theta Difference



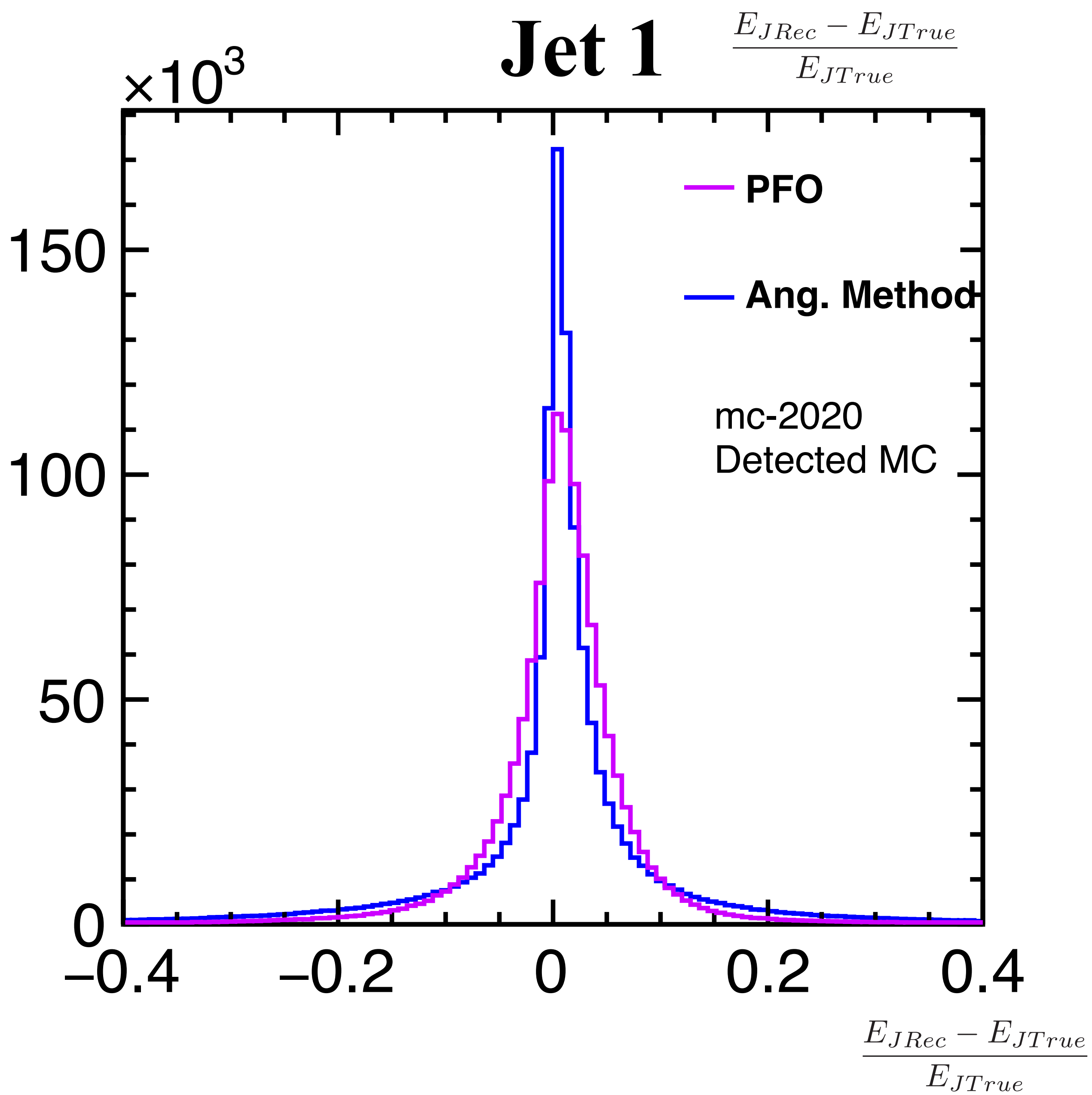
Bias with structure

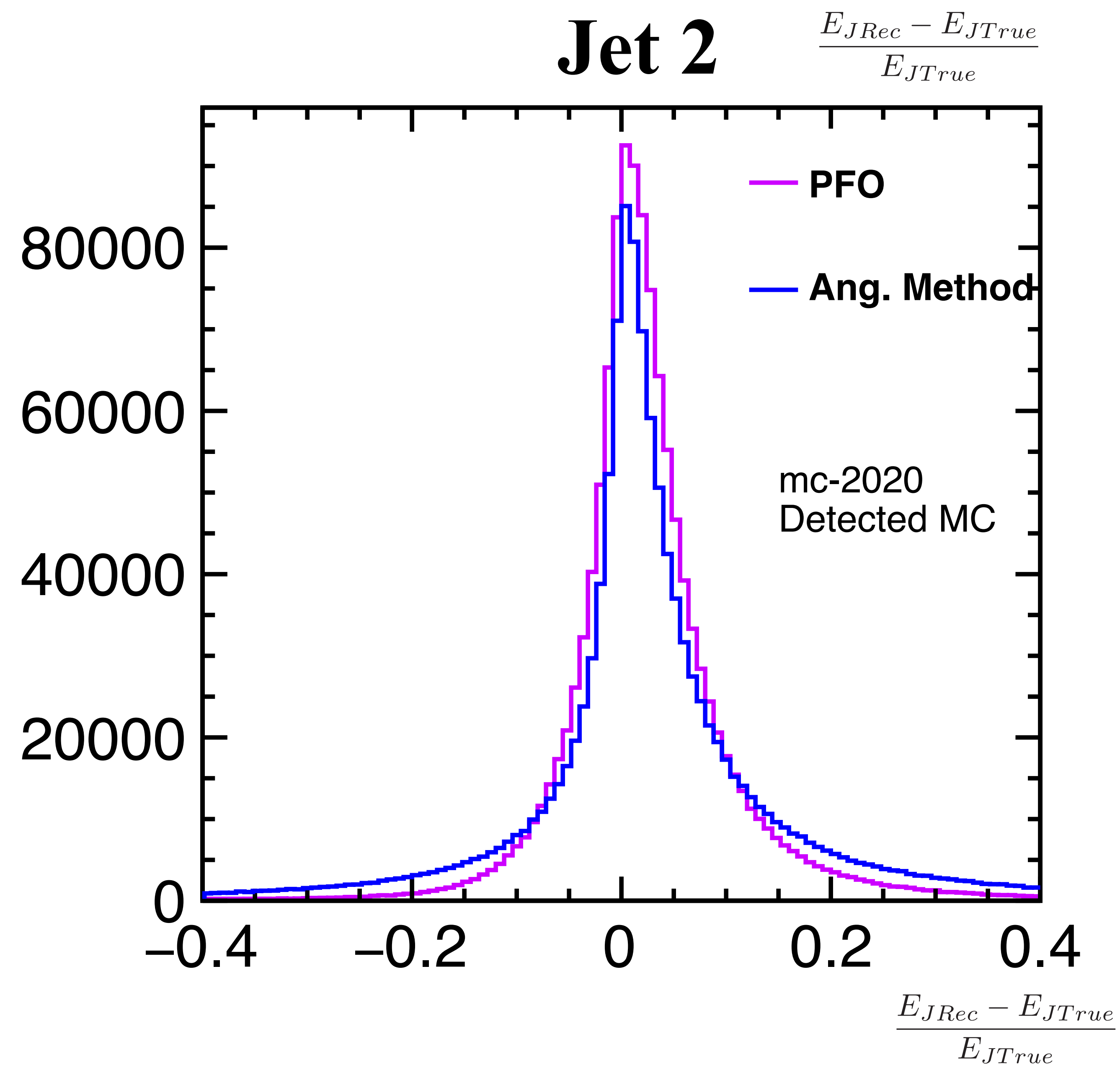
Phi Difference



Uniformly positive bias

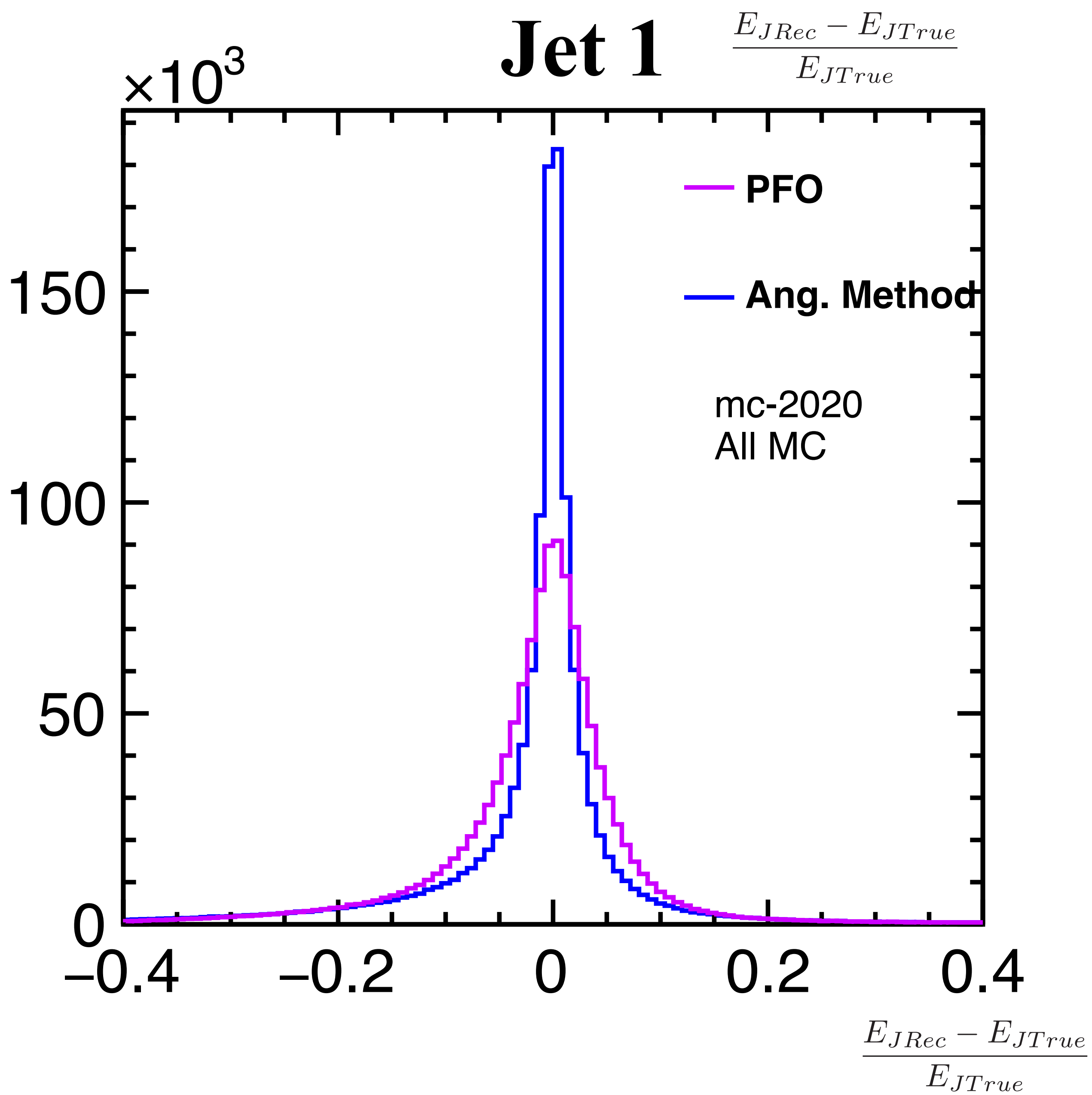
Method Comparison (De-MC)

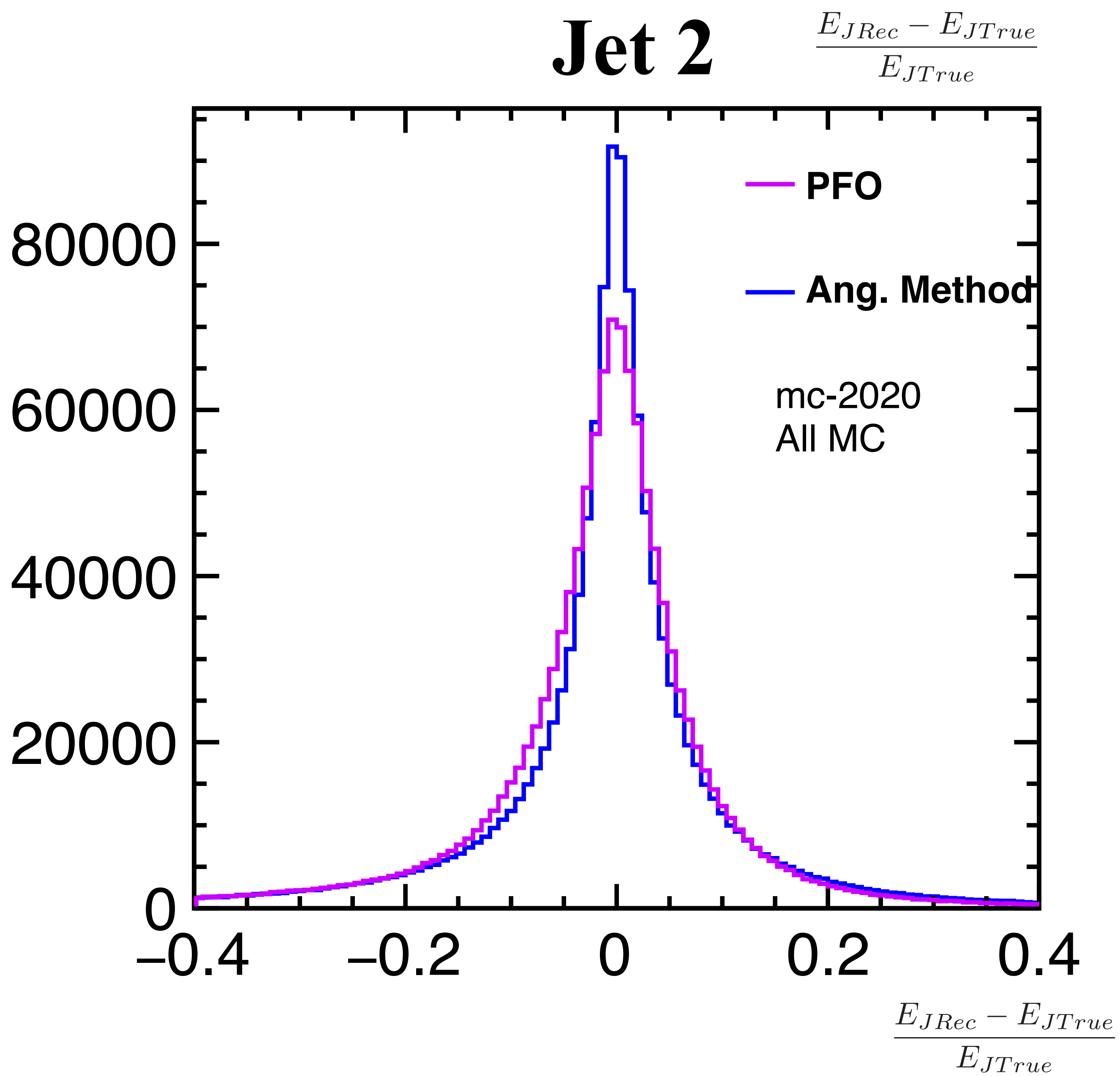




- **Ang. Method has good resolution and peak position is between 0 and 0.008.**

Method Comparison (Al-MC)

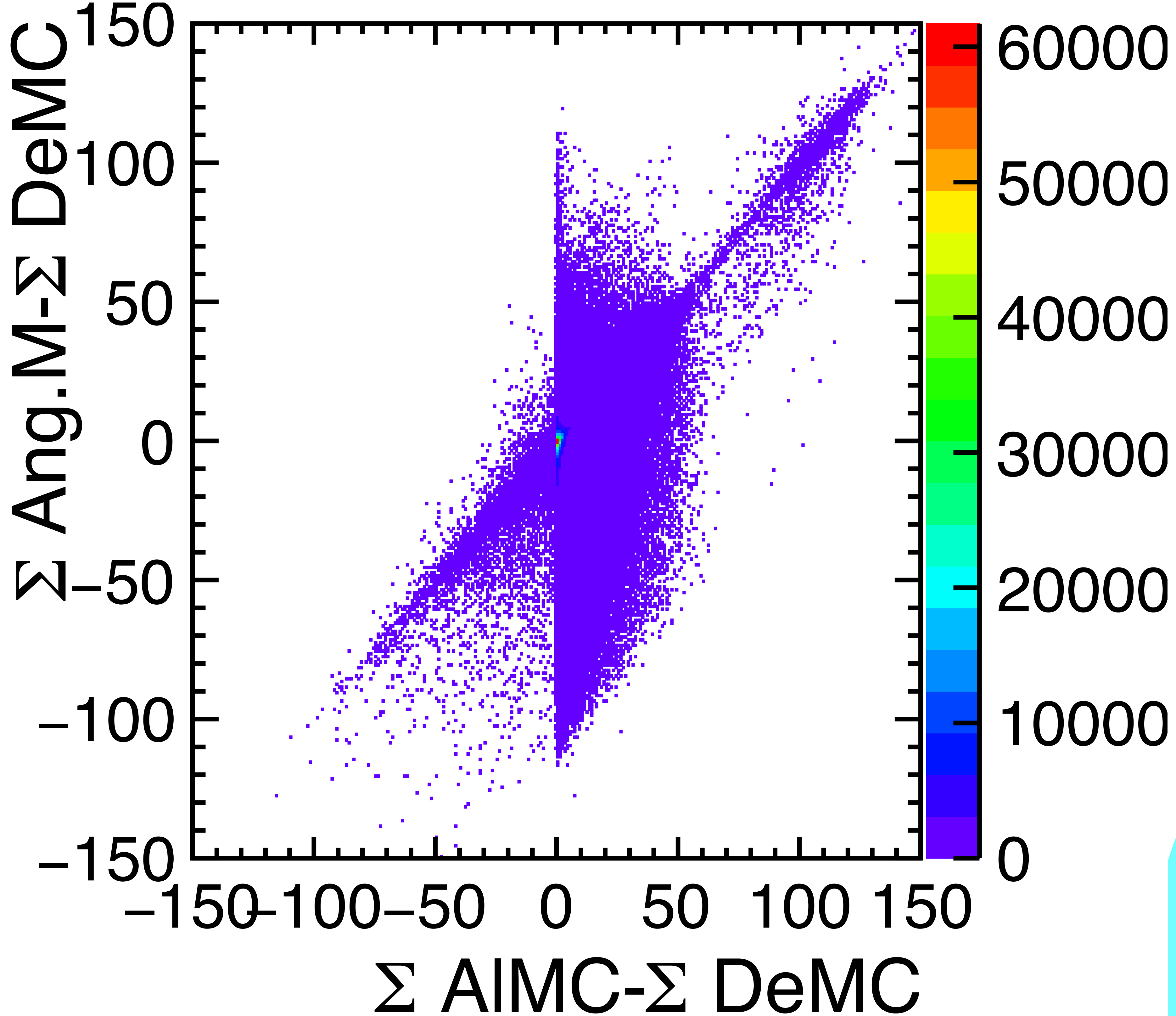




- **Ang. Method is much better. This means Ang. Method is rather closer to the all MC than the detected MC. It can recover non-detected particles.**

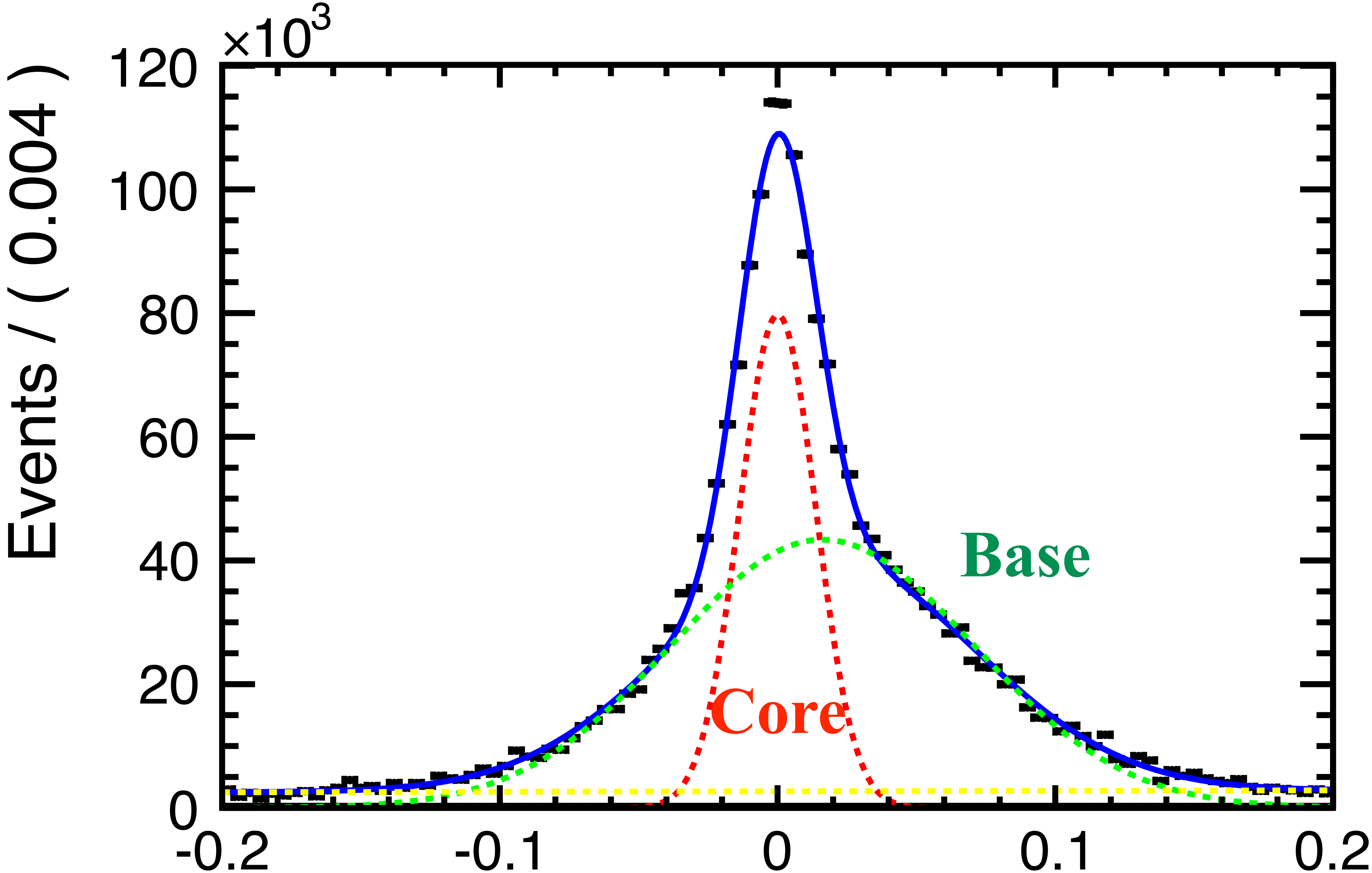
Missing particles recovery in Method 3

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer



This will be more clear in the flavor dependence plot

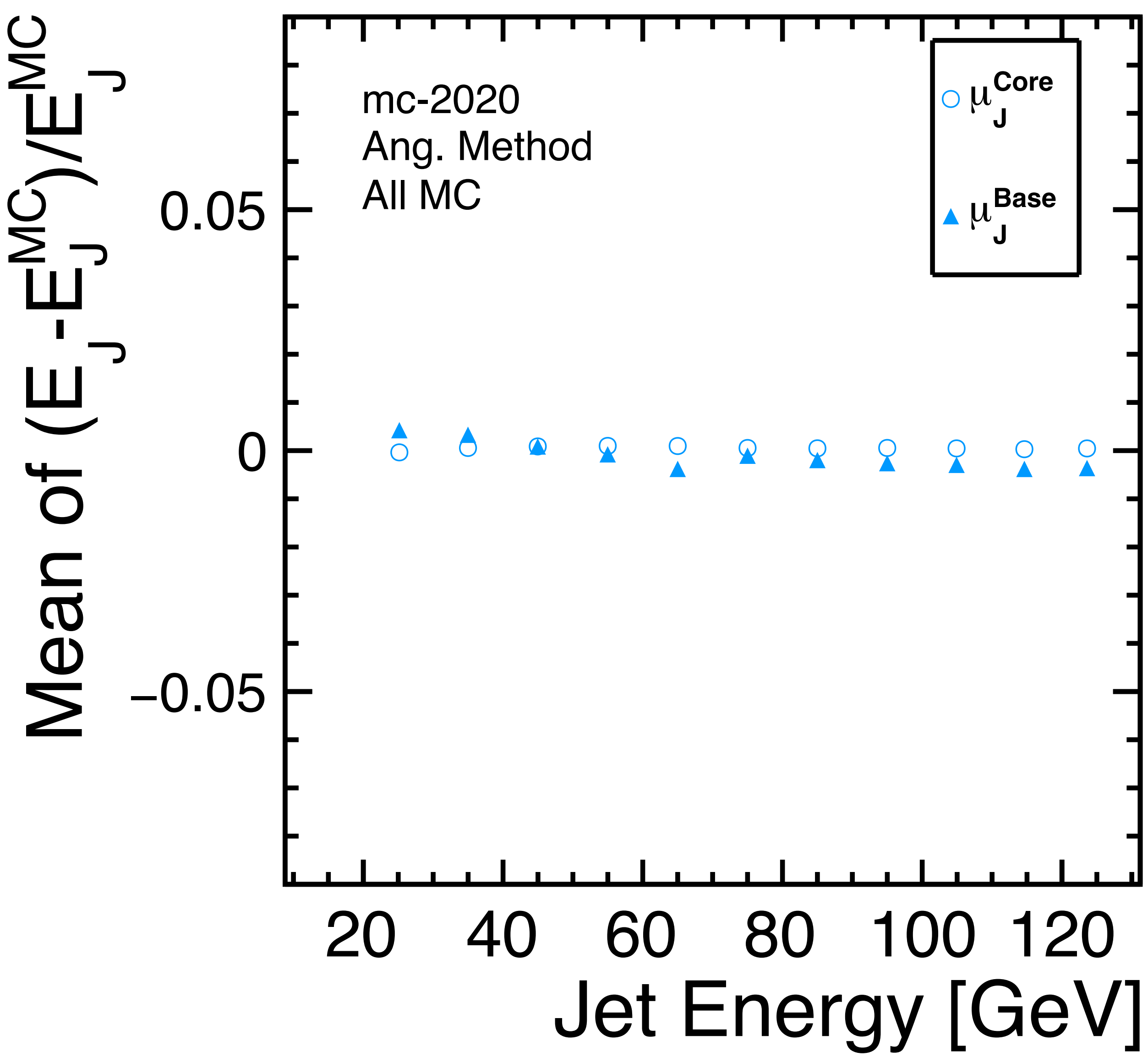
Fit the relative difference of reconstructed jet energy with
Gaus (Core)+Gaus (Base)+exponential
Calibration is based on the mean value of the Gaus (Core).



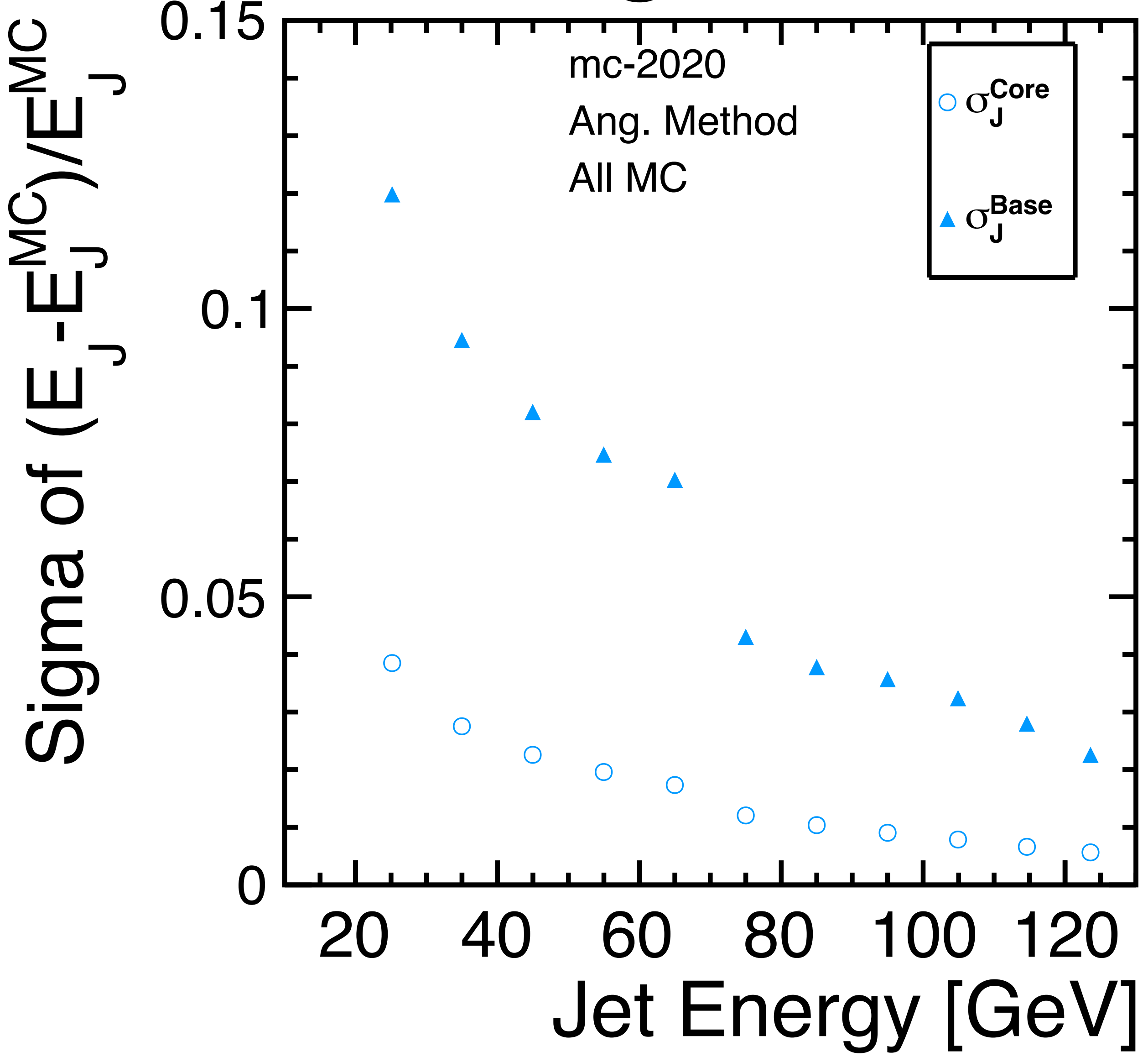
-> Check the theta, energy, and flavor dependence.

Ang. Method E-Dep (Al-MC)

Mean

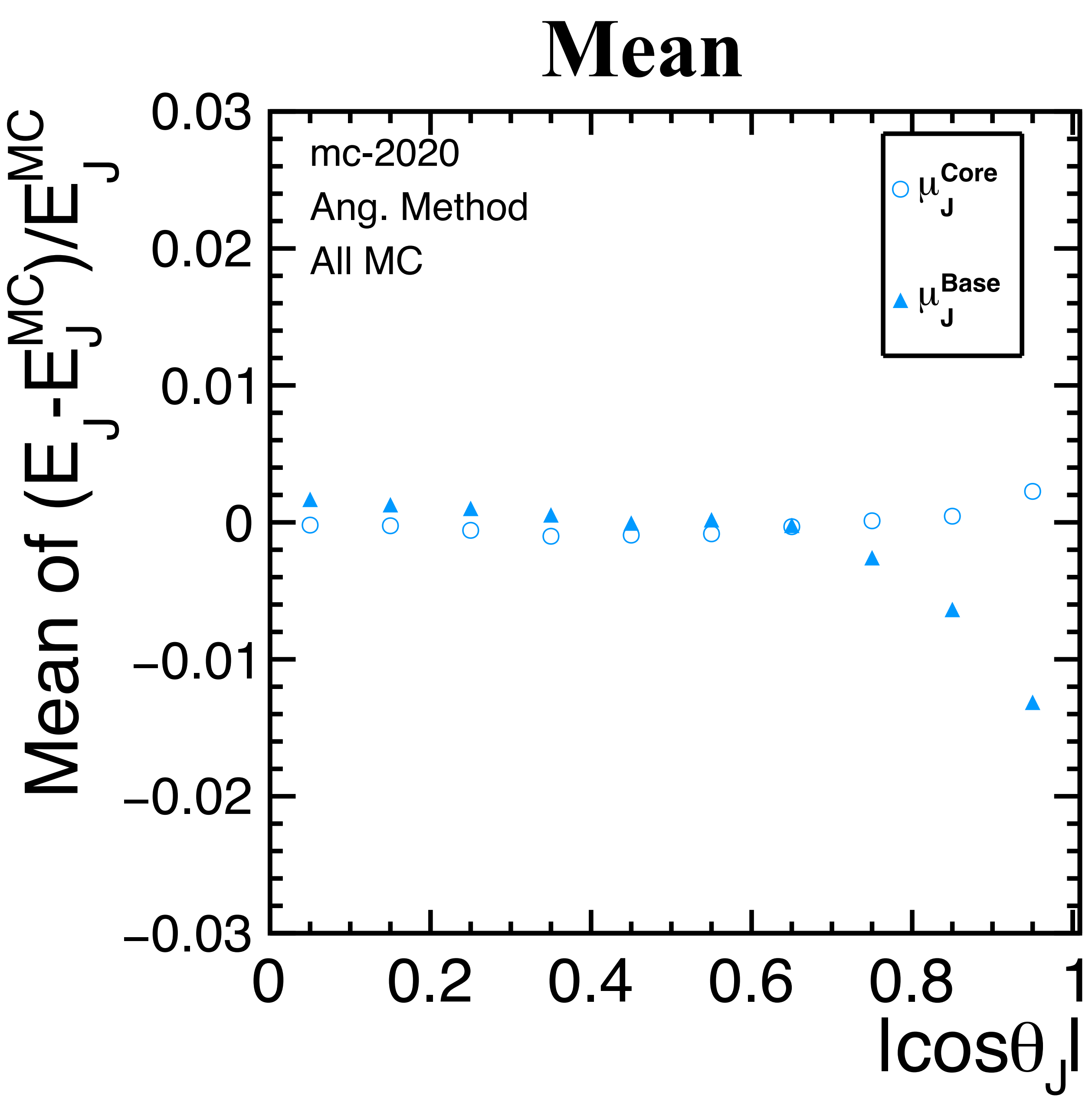


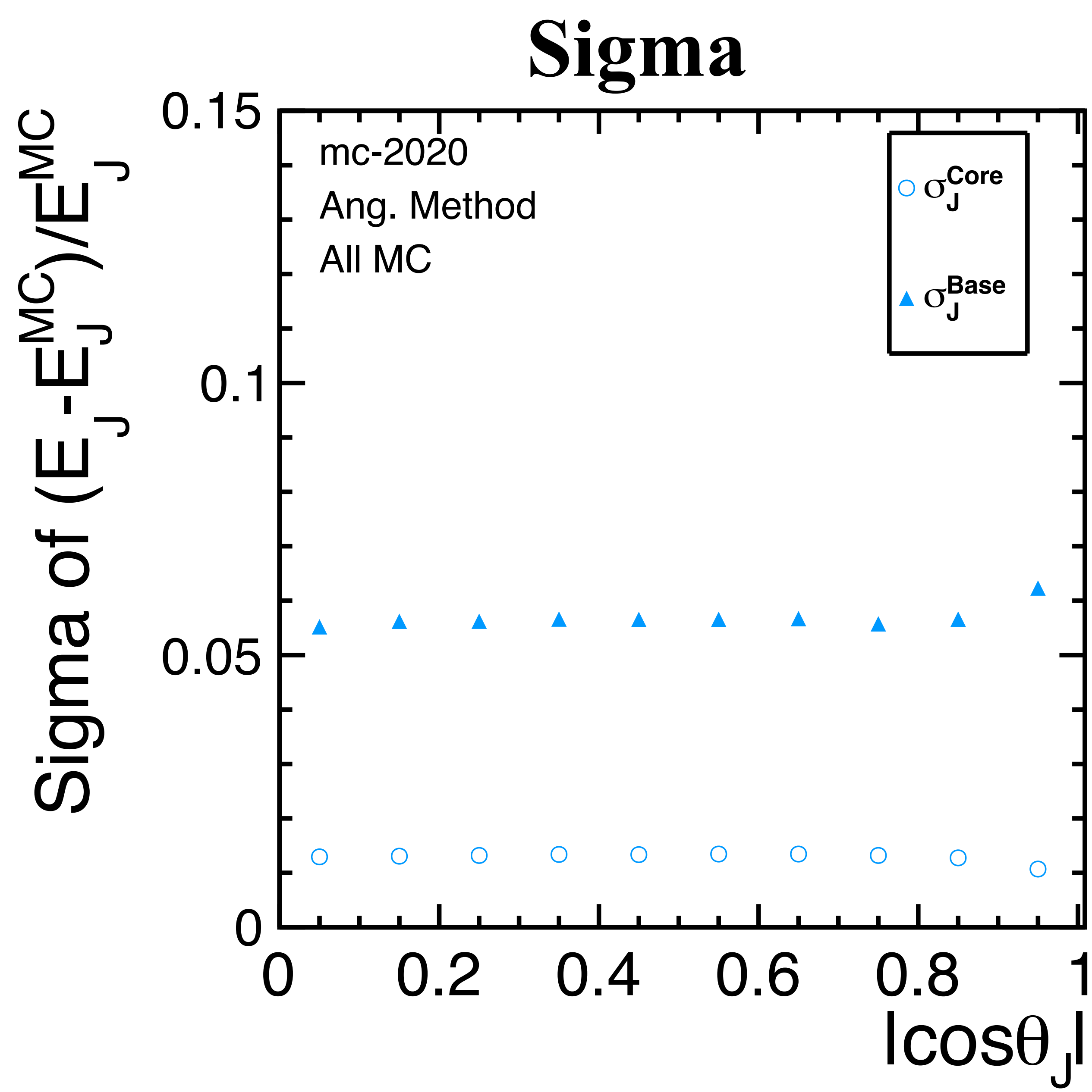
Sigma



Mean value of the core gaussian is order of 10^{-4} independent on the jet energy.
Higher energy jet has negative bias and lower one has positive bias.

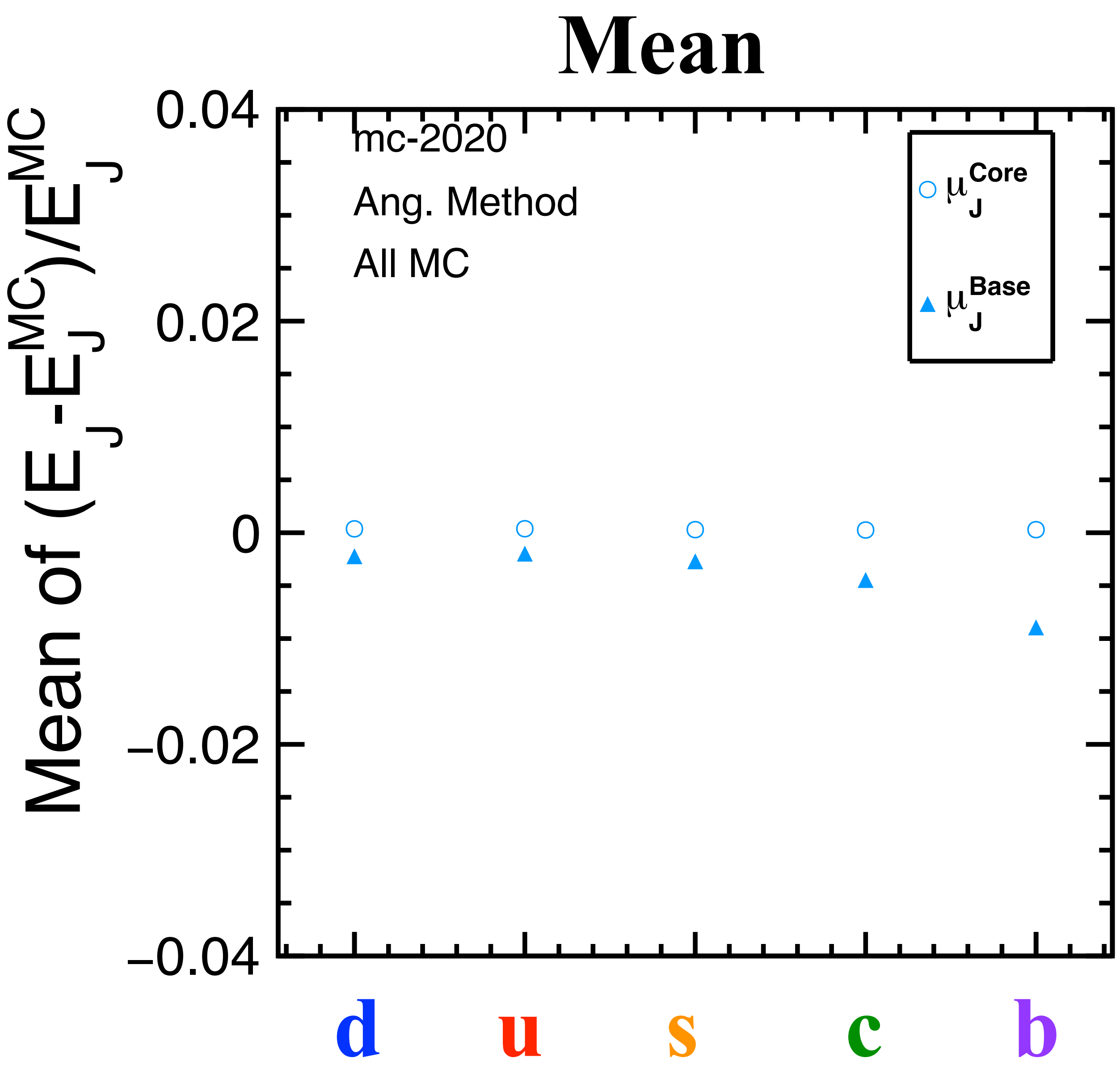
Ang. Method T-Dep (Al-MC)

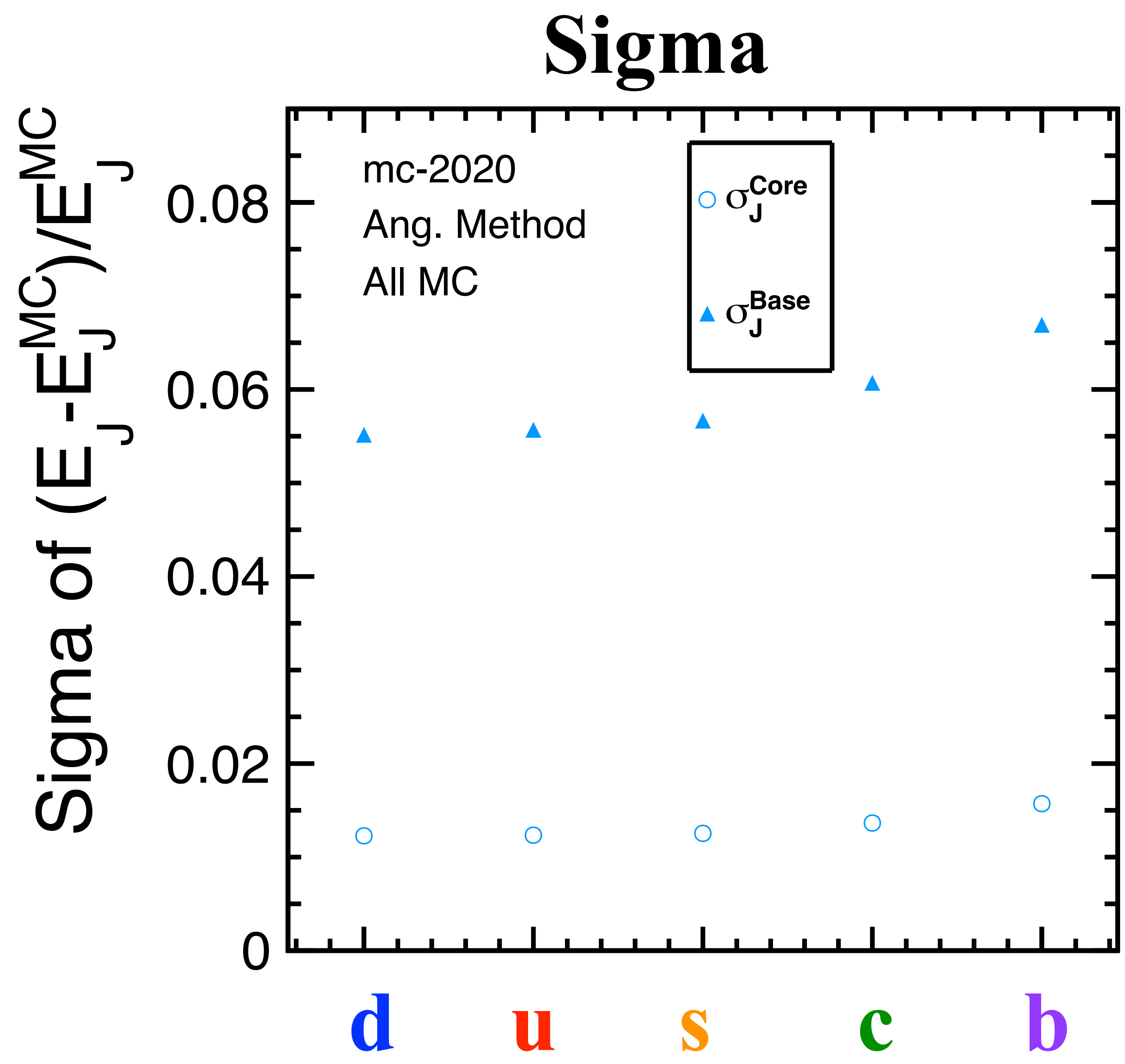




Forward jet makes slight positive bias on the core gaussian and barrel region jet makes slight negative bias on the core gaussian.

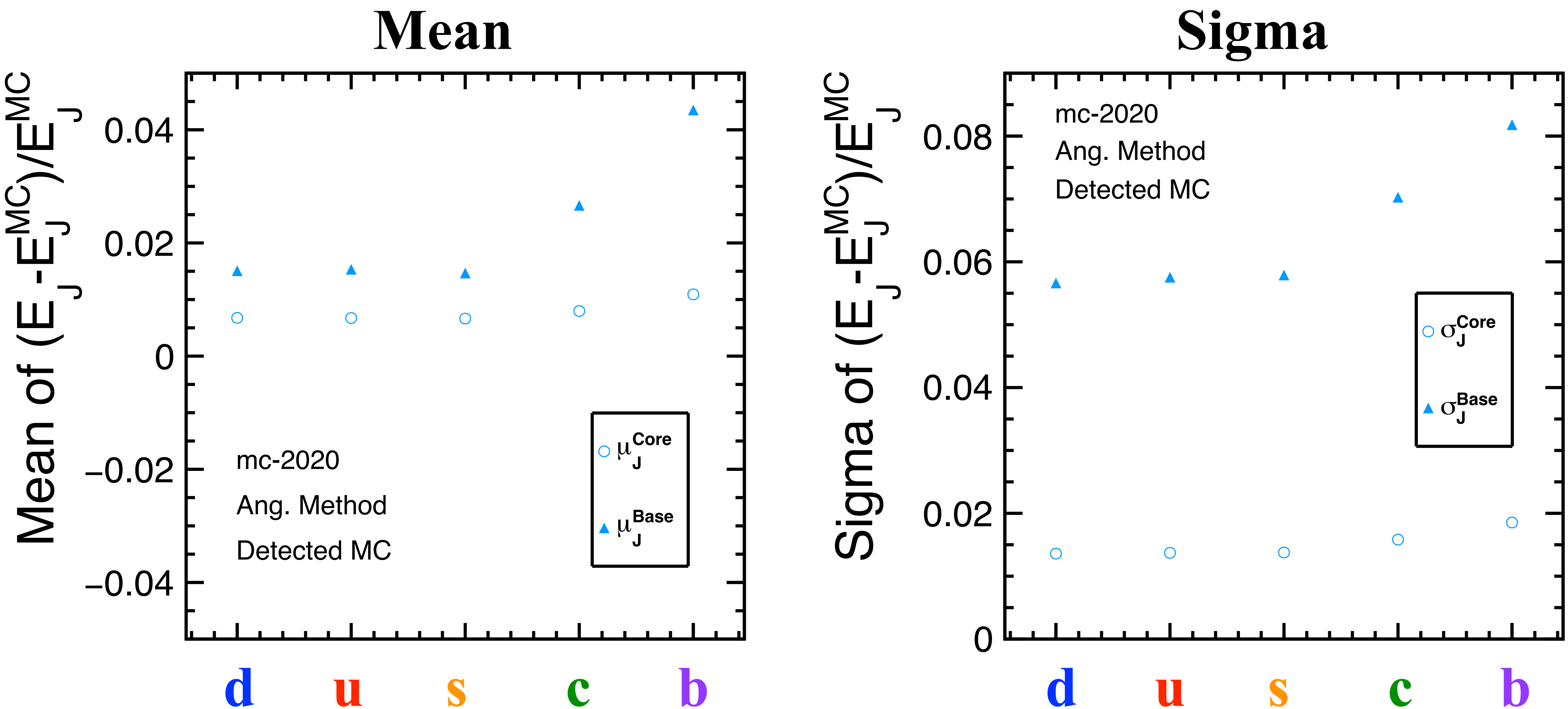
Ang. Method F-Dep (Al-MC)





Mean value of the core gaussian is order of 10^{-4} independent on the flavor.

Ang. Method F-Dep (De-MC)



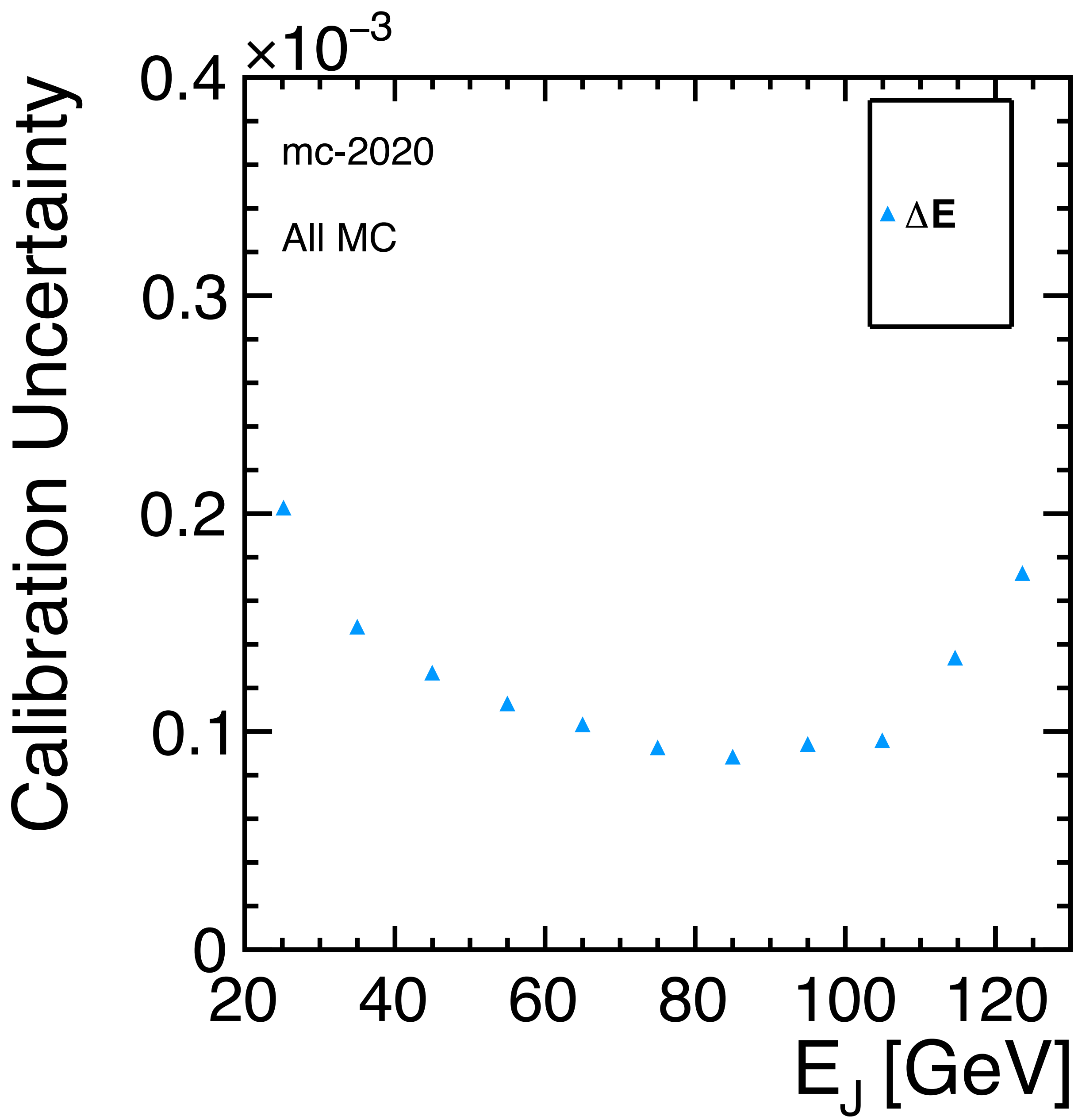
Mean value of the core gaussian is always positive and larger in the heavy flavor. This is because heavy flavor jet emits more neutrinos and Ang. Method recovers the missing energy.

Calibration Uncertainty (AI-MC)

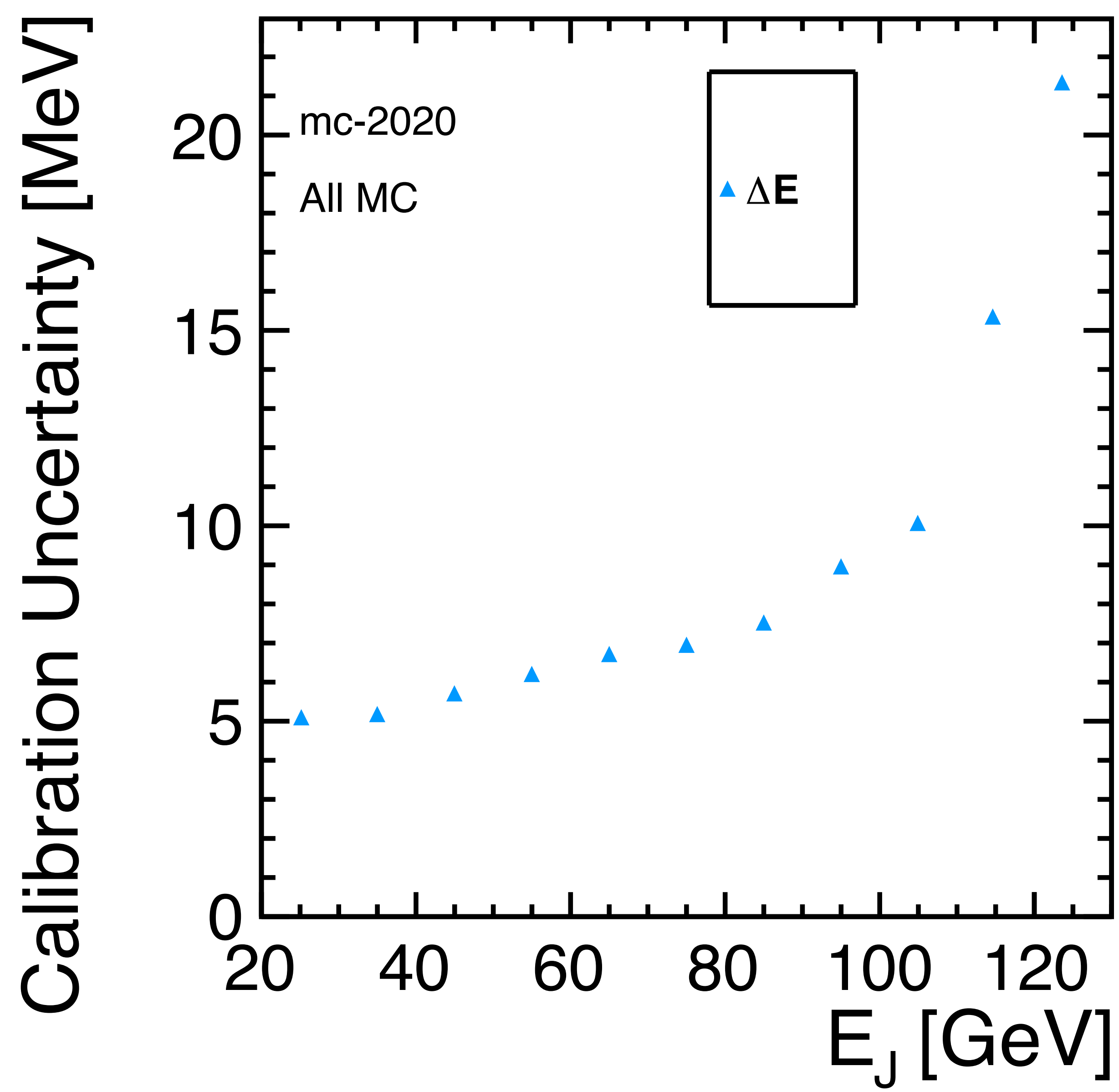
Calibration uncertainty := $\sqrt{(\Delta\mu_{Detector})^2 + (\Delta\mu_{Reconstructed})^2}$

Square root of the squared sum of the error of the mean

Relative uncertainty



Absolute uncertainty



We can calibrate the jet energy scale with about 10^{-4} accuracy, which corresponds to ~10 MeV.

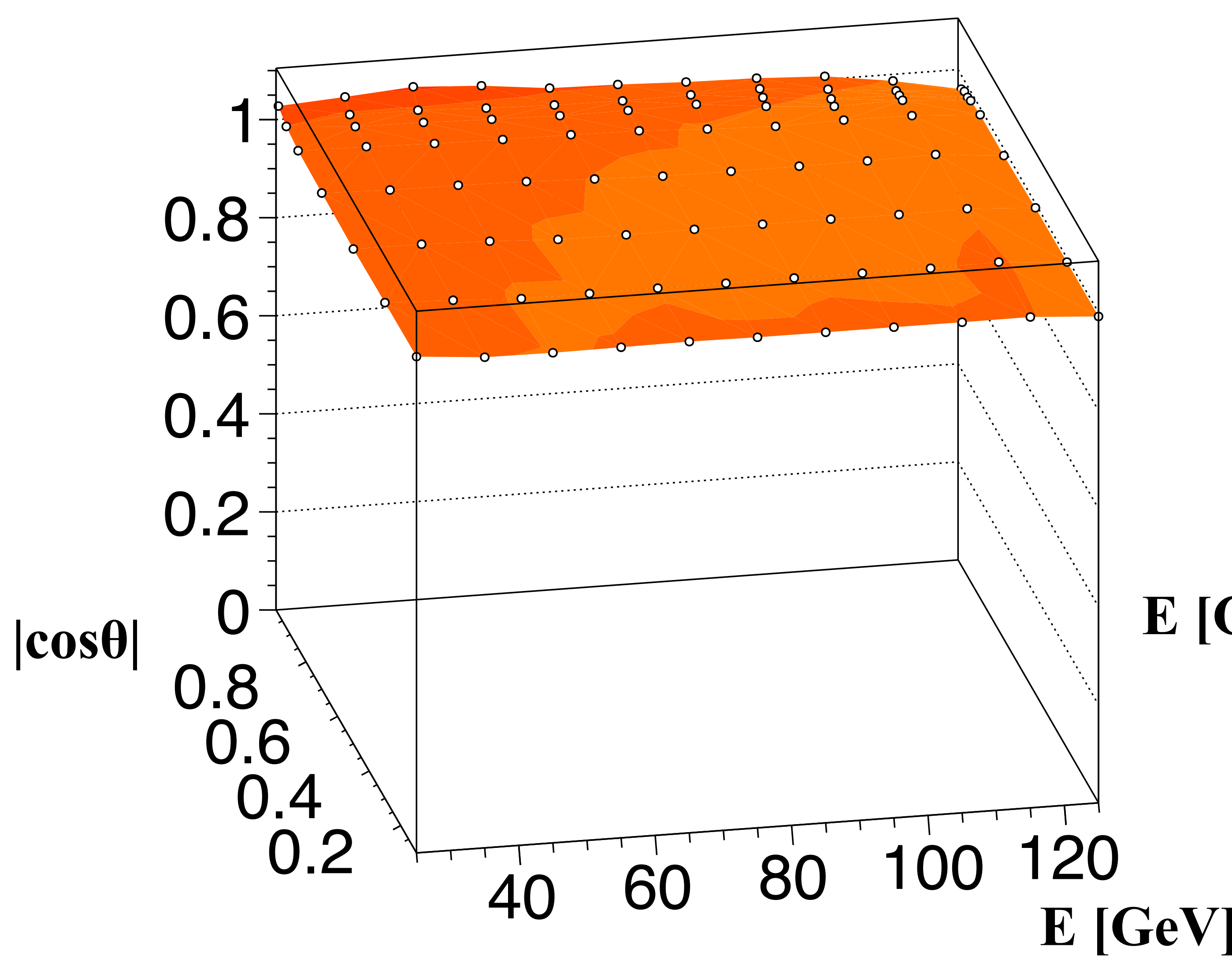
Calibration Factors

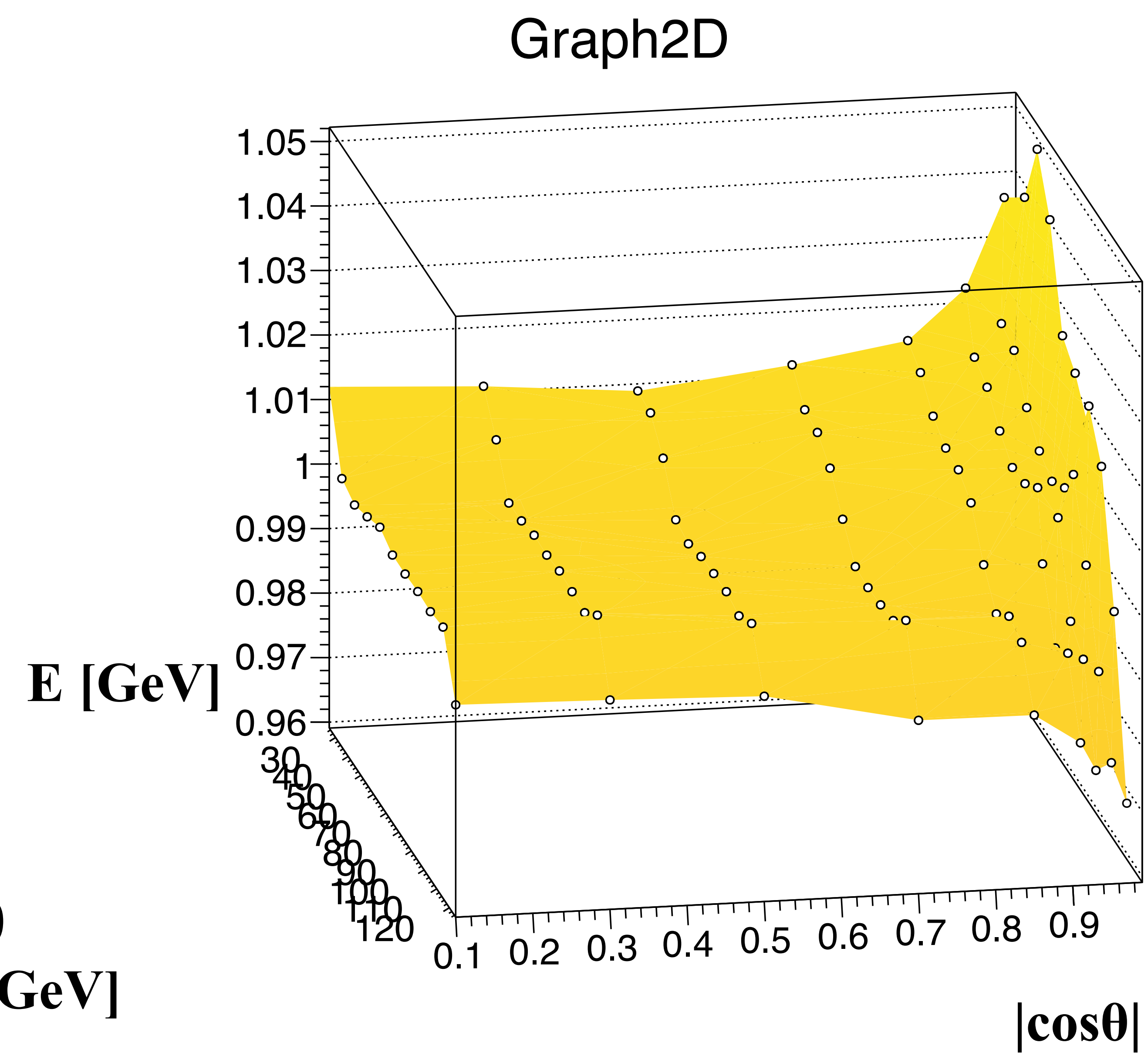
Fit the (E_PFO-E_Ang.Method)/E_Ang.Method and derive the mean values of Core-Gaussian "μ" as a function of energy and |cosθ|
Calibration Factor := E_Ang.Method/E_PFO = 1/(μ+1)

Energy	Upperbound of |cosθ|
20-30	0.2,0.4,0.6,0.8,0.9,0.95,1.0
30-40	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
40-50	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
50-60	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
60-70	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
70-80	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
80-90	0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0
90-100	0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0
100-110	0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0
110-120	0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0
120-130	0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0

Calibration Factor

Fit the (E_PFO-E_Ang.Method)/E_Ang.Method and derive the mean values of Core-Gaussian "μ" as a function of energy and |cosθ|
Calibration Factor := E_Ang.Method/E_PFO = 1/(μ+1)





Except |cosθ| = 0.95 to1.0 && E = 20 to 30 GeV bin (Now fitting failed)

Conclusion

- Full simulation is performed using mc-2020 samples in order to access jet energy calibration uncertainty.
- Jet energy can be reconstructed using the $e^+e^- \to \gamma Z$, $Z \to 2$ Jets process. Reconstructed jet energy resolution is better than the measured one.
- Calibration uncertainty is calculated as a function of energy and angle. It is $\sim 10^{-4}$ accuracy which corresponds to ~ 10 MeV.
- Calibration factor for the jet energy calibration is estimated.

Back up

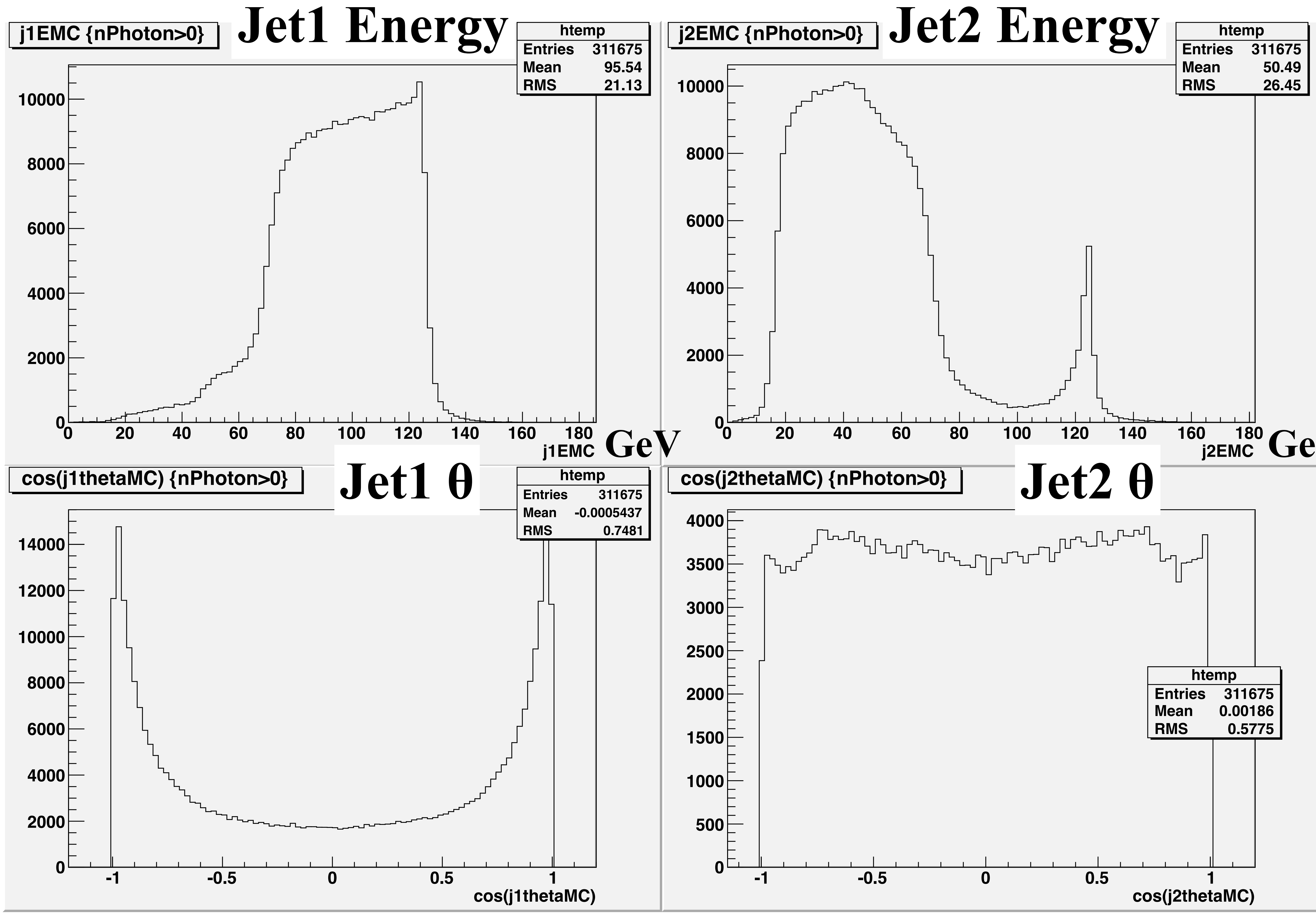
Calibration Constant

Fit the (E_PFO-E_Ang.Method)/E_Ang.Method and derive the mean values of core-Gaussian "μ" as a function of energy and |cosθ|
Calibration constant := 1/(μ+1)

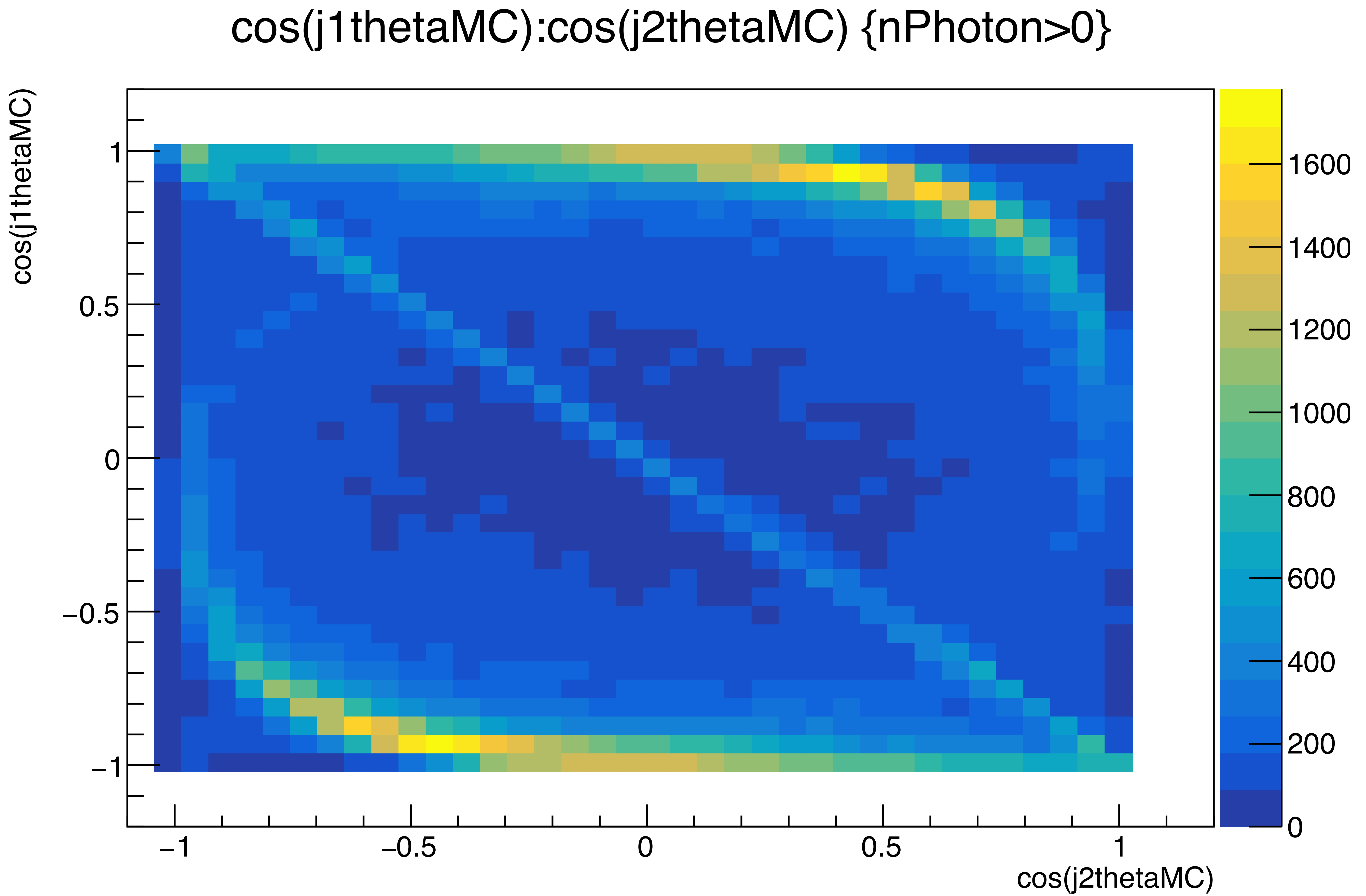
Upper bound of |cosθ|

```
0.2,0.4,0.6,0.8,0.9,0.95,1.0,                    //for 020-030 GeV
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //030-040
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //040-050
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //050-060
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //060-070
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //070-080
0.2,0.4,0.6,0.8,0.9,0.94,0.97,1.0,               //080-090
0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0,     //090-100
0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0,     //100-110
0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0,     //110-120
0.2,0.4,0.6,0.8,0.9,0.92,0.94,0.96,0.98,1.0      //120-130
```

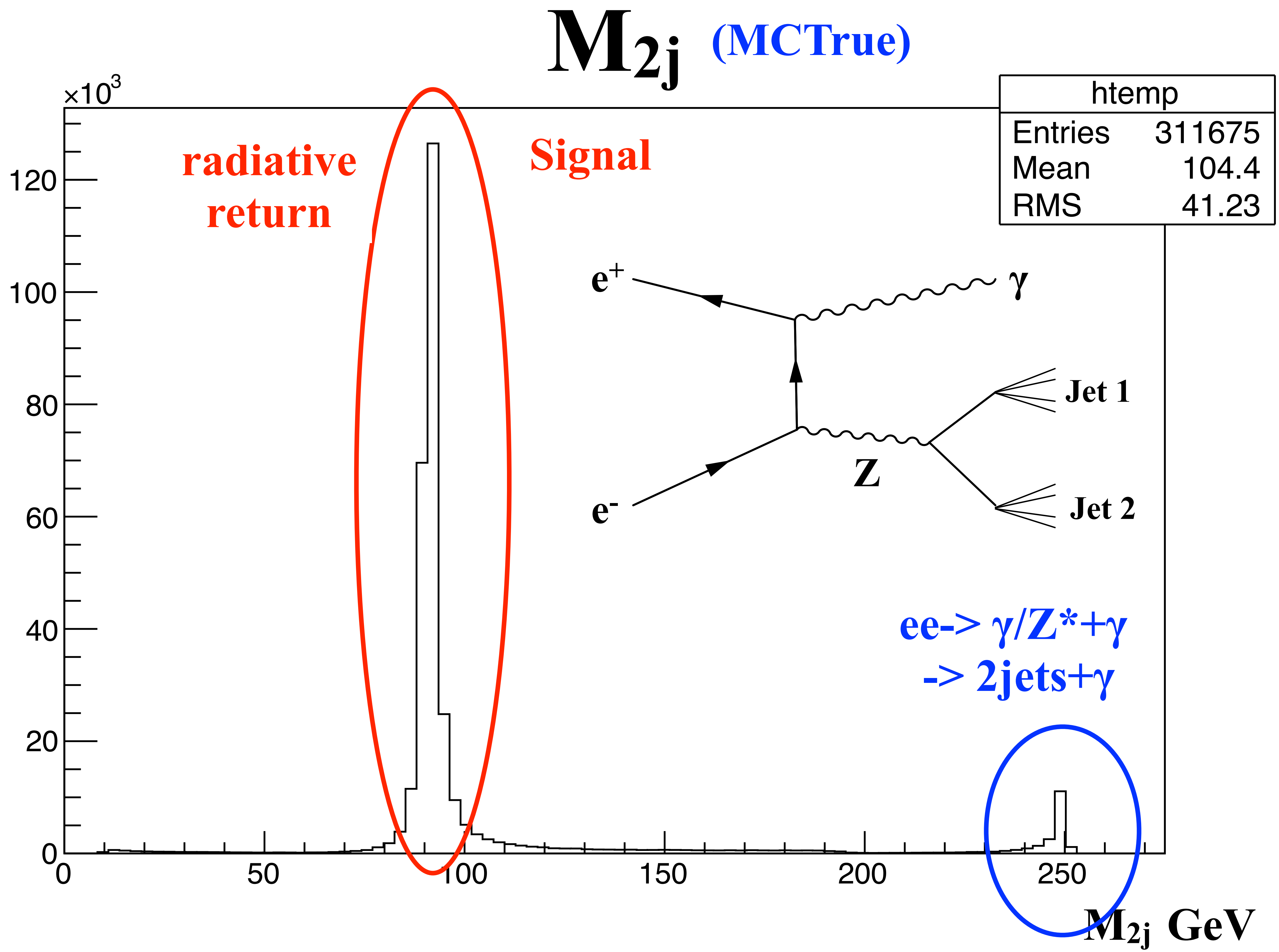
Energy and theta of jets (#photon>0)



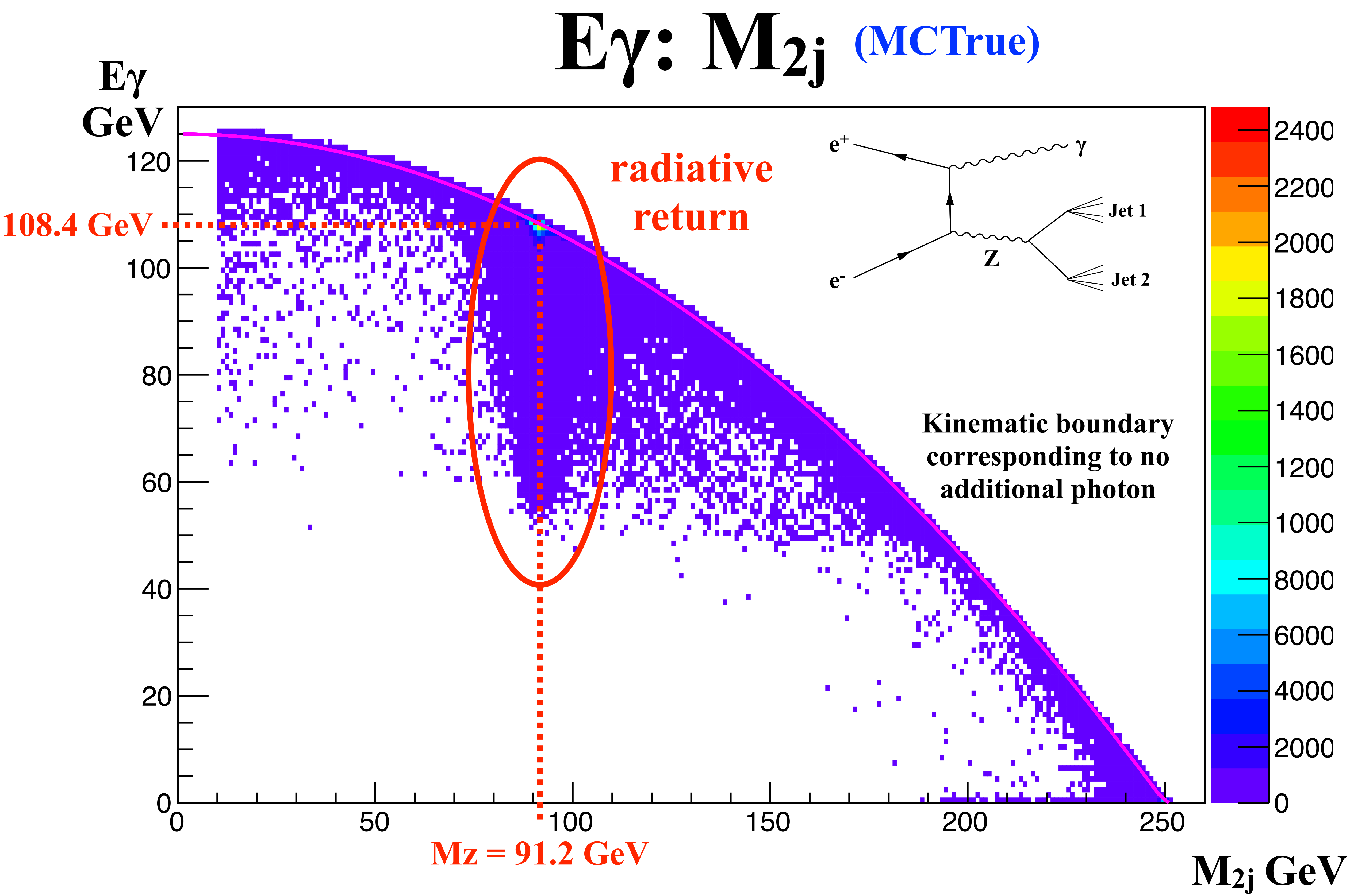
Energy and theta of jets (#photon>0)



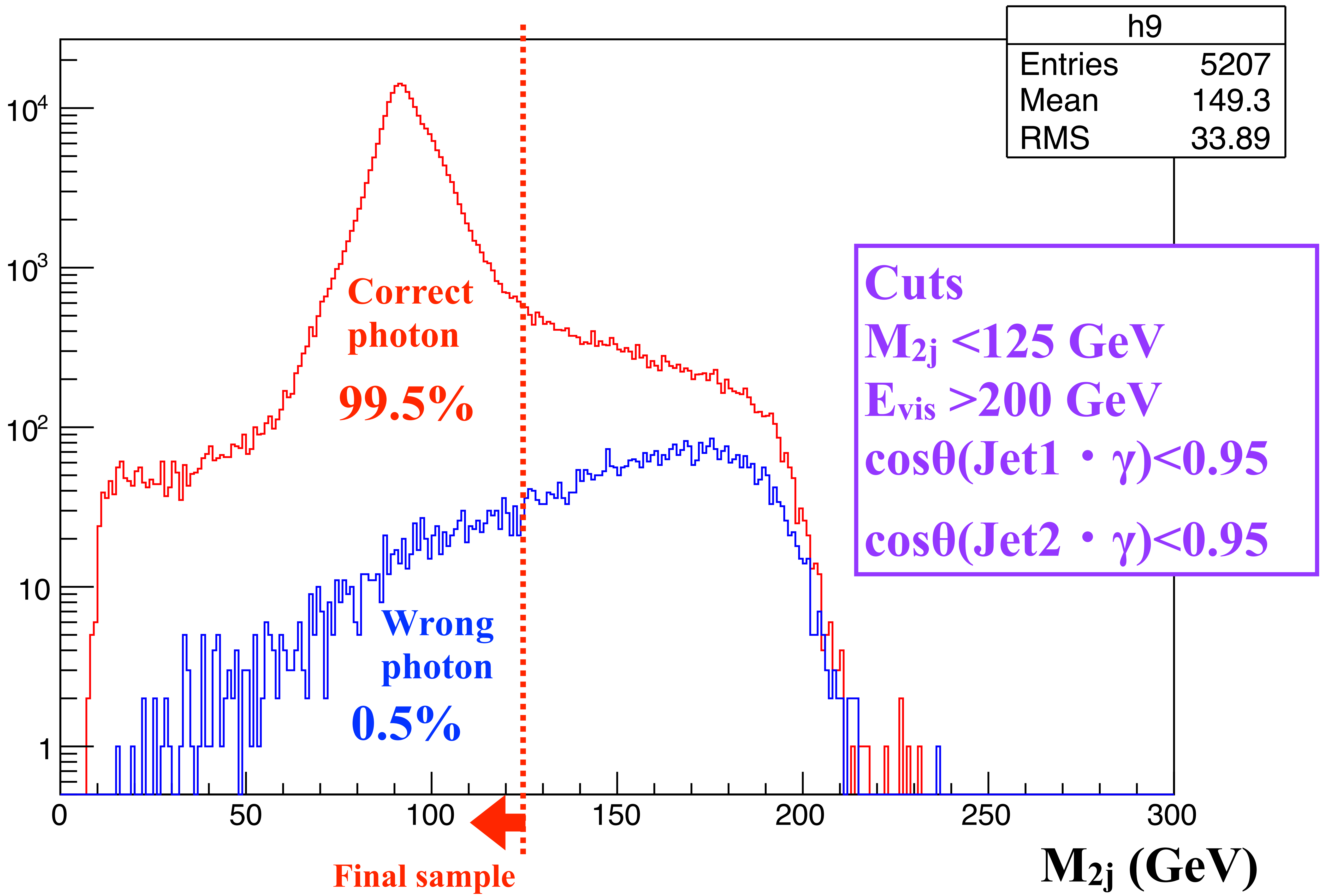
M_{2j} distribution



Photon energy & M_{2j} distribution



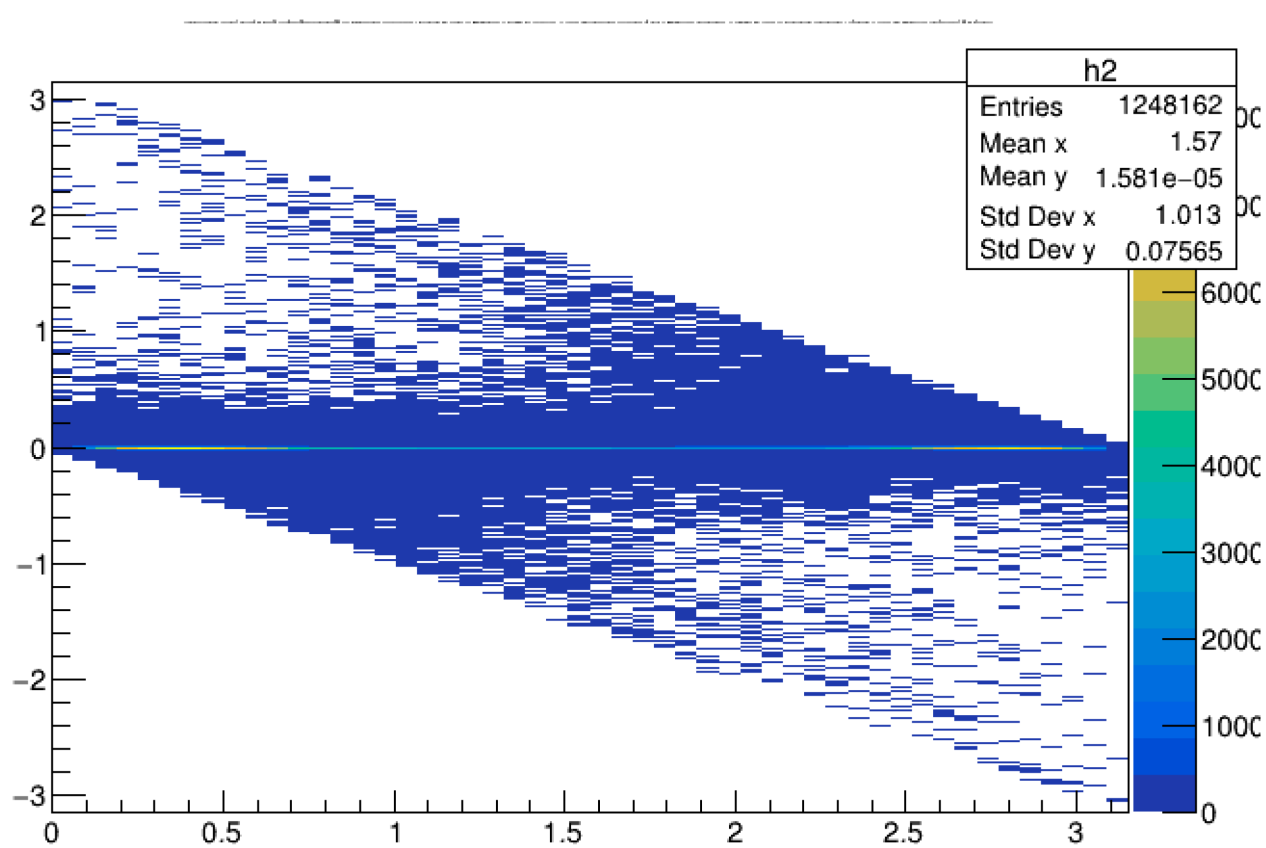
Correct photon selection cuts



Theta Abs. Difference

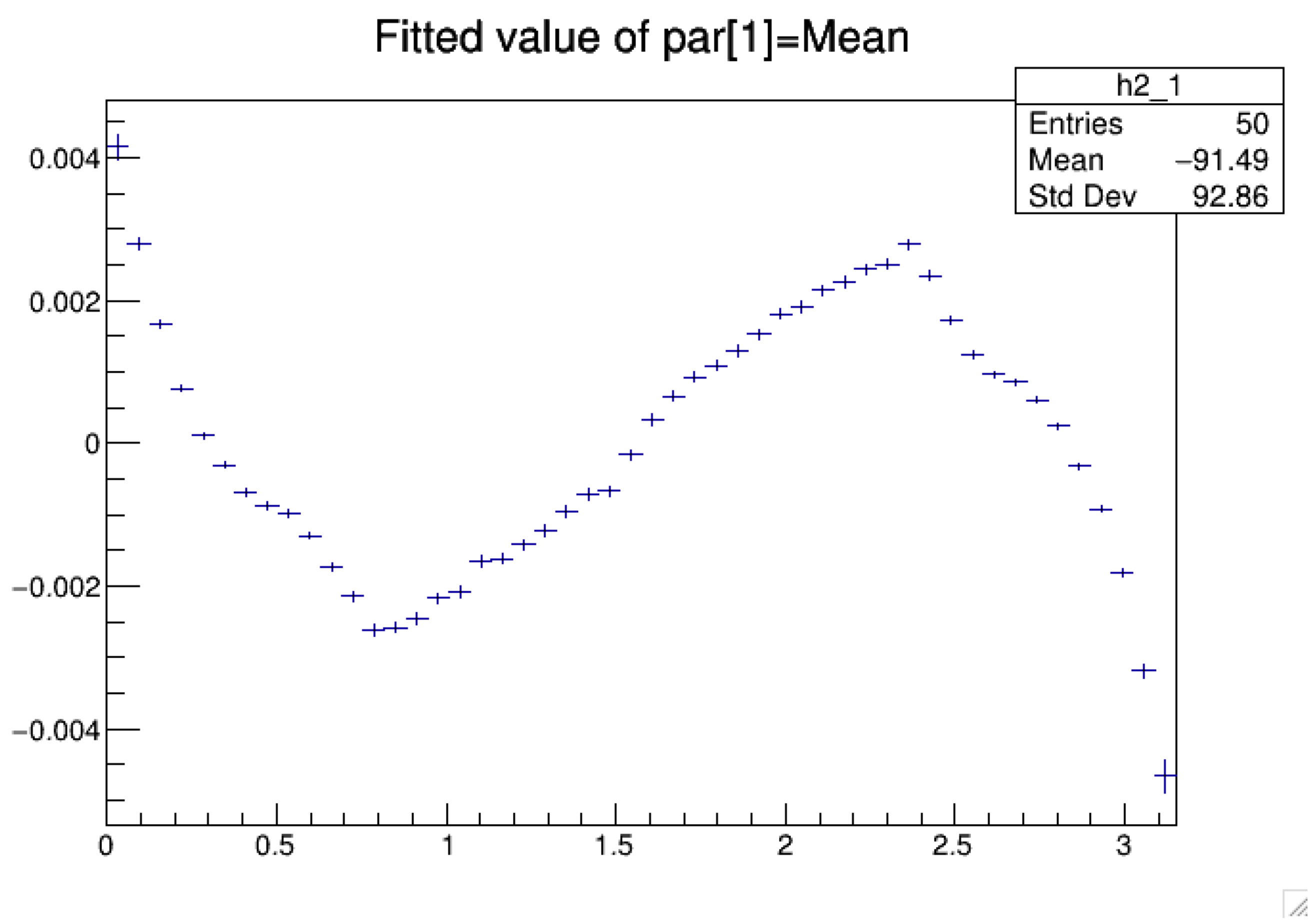
eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

Theta-dependence



Theta

FitSlicesY

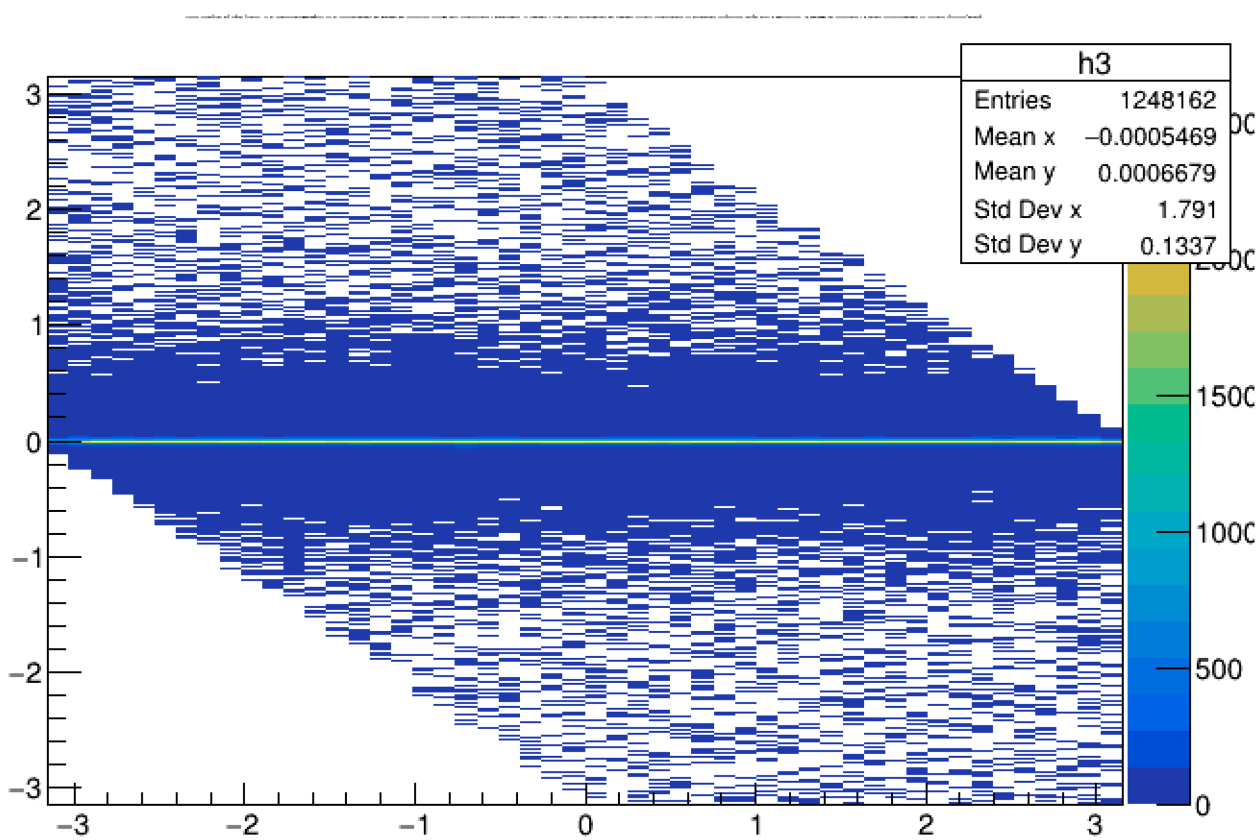


Theta

Phi Abs. Difference

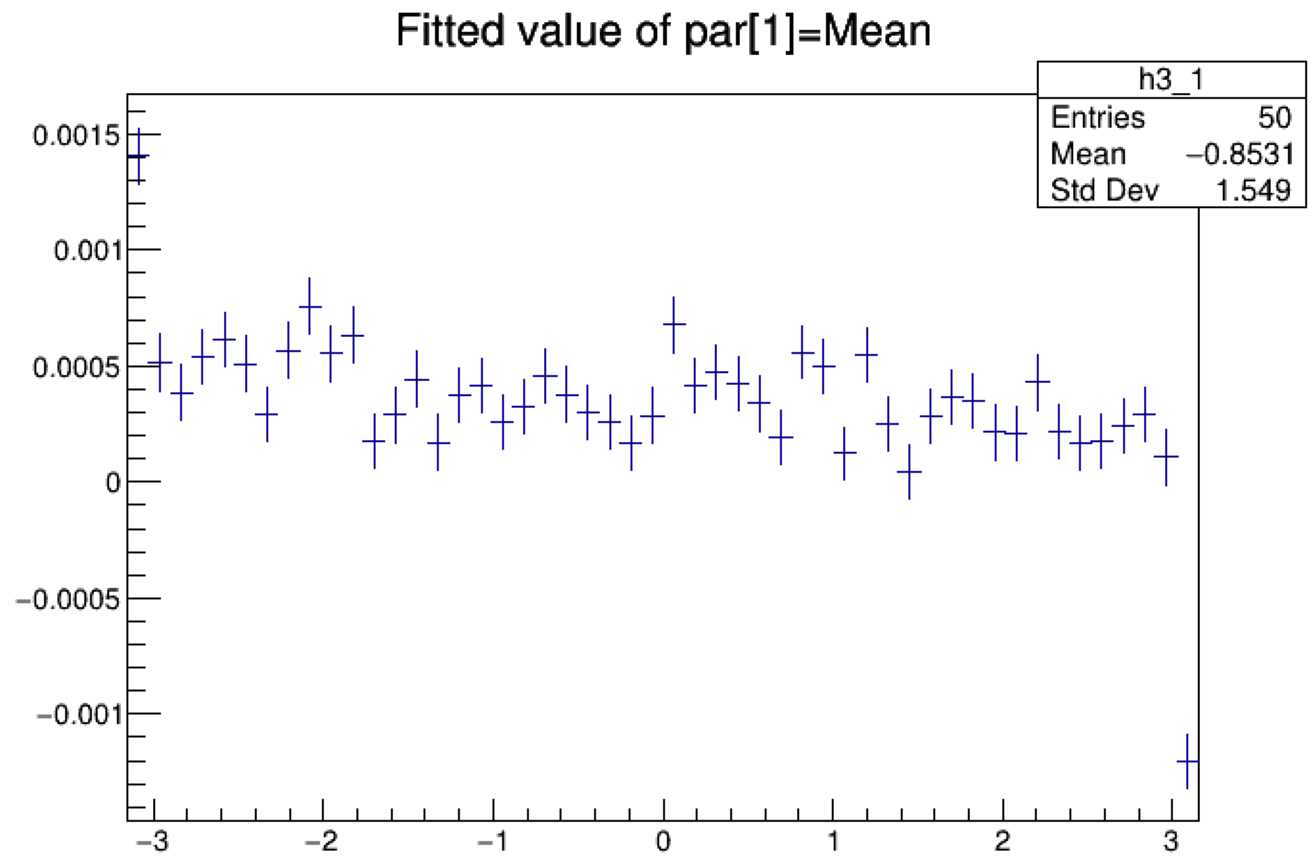
eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

Phi-dependence



Phi

FitSlicesY

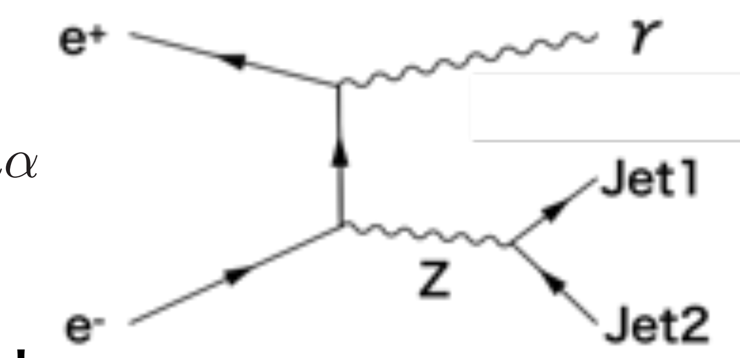


Phi

Reconstruction Method

Based on 4-momentum conservation

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| + |P_{ISR}| = 500 \\ P_{J1} sin\theta_{J1} cos\phi_{J1} + P_{J2} sin\theta_{J2} cos\phi_{J2} + P_\gamma sin\theta_\gamma cos\phi_\gamma + |P_{ISR}| sin\alpha = 500 sin\alpha \\ P_{J1} sin\theta_{J1} sin\phi_{J1} + P_{J2} sin\theta_{J2} sin\phi_{J2} + P_\gamma sin\theta_\gamma sin\phi_\gamma = 0 \\ P_{J1} cos\theta_{J1} + P_{J2} cos\theta_{J2} + P_\gamma cos\theta_\gamma \pm |P_{ISR}| cos\alpha = 0 \end{cases}$$



Beam Crossing Angle ≡2α : α = 7.0 mrad

Direction Angle
θ: polar angle
ϕ: azimuthal angle

- ISR photon = **additional** unseen photon
- Several reconstruction methods (Method **1**, **2'**, **2**, **3**, **and** **4**) are considered.

Method 1: Ignore ISR
Using ($\theta_{J1}, \theta_{J2}, \theta_\gamma, \varphi_{J1}, \varphi_{J2}, \varphi_\gamma, m_{J1}, m_{J2}$) -> Determine ($P_{J1}, P_{J2}, P_\gamma$)

$$\begin{cases} \sqrt{P_{J1}^2 + m_{J1}^2} + \sqrt{P_{J2}^2 + m_{J2}^2} + |P_\gamma| = 500 \\ \begin{pmatrix} sin\theta_{J1} cos\phi_{J1} & sin\theta_{J2} cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1} sin\phi_{J1} & sin\theta_{J2} sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} 500 sin\alpha \\ 0 \\ 0 \end{pmatrix} \end{cases}$$

Matrix A — Inverse

Reconstruction Method

Method 2: Use measured P_γ as input and Consider ISR
Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2},P_\gamma)$ -> Determine (P_{J1},P_{J2},P_{ISR})

$$
\begin{cases}
\sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}|=500 \quad ① \\
\begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\alpha \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & 0 \\ cos\theta_{J1} & cos\theta_{J2} & \pm cos\alpha \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ |P_{ISR}| \end{pmatrix} = \begin{pmatrix} 500sin\alpha - sin\theta_\gamma cos\phi_\gamma P_\gamma \\ -sin\theta_\gamma sin\phi_\gamma P_\gamma \\ -cos\theta_\gamma P_\gamma \end{pmatrix}
\end{cases}
$$

Matrix A

Inverse

2 solutions for each sign of P_{ISR}
-> choose the best answer which satisfies ① better

Method 2': Use measured P_γ as input and Ignore ISR
Using $(\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2},P_\gamma)$ -> Determine (P_{J1},P_{J2})

$$
\begin{cases}
\begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \end{pmatrix} = \begin{pmatrix} 500sin\alpha - sin\theta_\gamma cos\phi_\gamma P_\gamma \\ -sin\theta_\gamma sin\phi_\gamma P_\gamma \end{pmatrix}
\end{cases}
$$

Reconstruction Method

Method 3: Consider ISR and solve the full equation
Using ($\theta_{J1},\theta_{J2},\theta_\gamma,\varphi_{J1},\varphi_{J2},\varphi_\gamma,m_{J1},m_{J2}$) -> Determine ($P_{J1},P_{J2},P_\gamma,P_{ISR}$)

$$\begin{cases} \sqrt{P_{J1}^2+m_{J1}^2}+\sqrt{P_{J2}^2+m_{J2}^2}+|P_\gamma|+|P_{ISR}| = \mathbf{E_{CM}} \quad ① \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_\gamma cos\phi_\gamma \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_\gamma sin\phi_\gamma \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_\gamma \end{pmatrix} \begin{pmatrix} P_{J1} \\ P_{J2} \\ P_\gamma \end{pmatrix} = \begin{pmatrix} (\mathbf{E_{CM}}-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{cases}$$

Matrix A —— Inverse

Irrational equation for each sign of the ISR -> 8 possible solutions

Choose the solution with
- **(i) Real and positive value with <E_{CM}/2**
- **(ii)** $\sqrt{P_{J1}^2+m_{J1}^2}$ **>0 and** $\sqrt{P_{J2}^2+m_{J2}^2}$ **>0**
- **(iii)** P_{J1}, P_{J2}, P_γ **>0**
- **(iv) solved** P_γ **closest to the measured** P_γ

Reconstruction Method

Jet mass "m" can be expressed as "P/γβ" (P: momentum of the jet)
-> Irrational equation ① is reduced to be a linear equation!

Method 4: Represent the equation with P_{ISR}
Using ($\theta_{J1},\theta_{J2},\theta_{\gamma},\varphi_{J1},\varphi_{J2},\varphi_{\gamma},$**$\gamma\beta_{J1},\gamma\beta_{J2}$**) -> Determine ($P_{J1},P_{J2},P_{\gamma},P_{ISR}$)

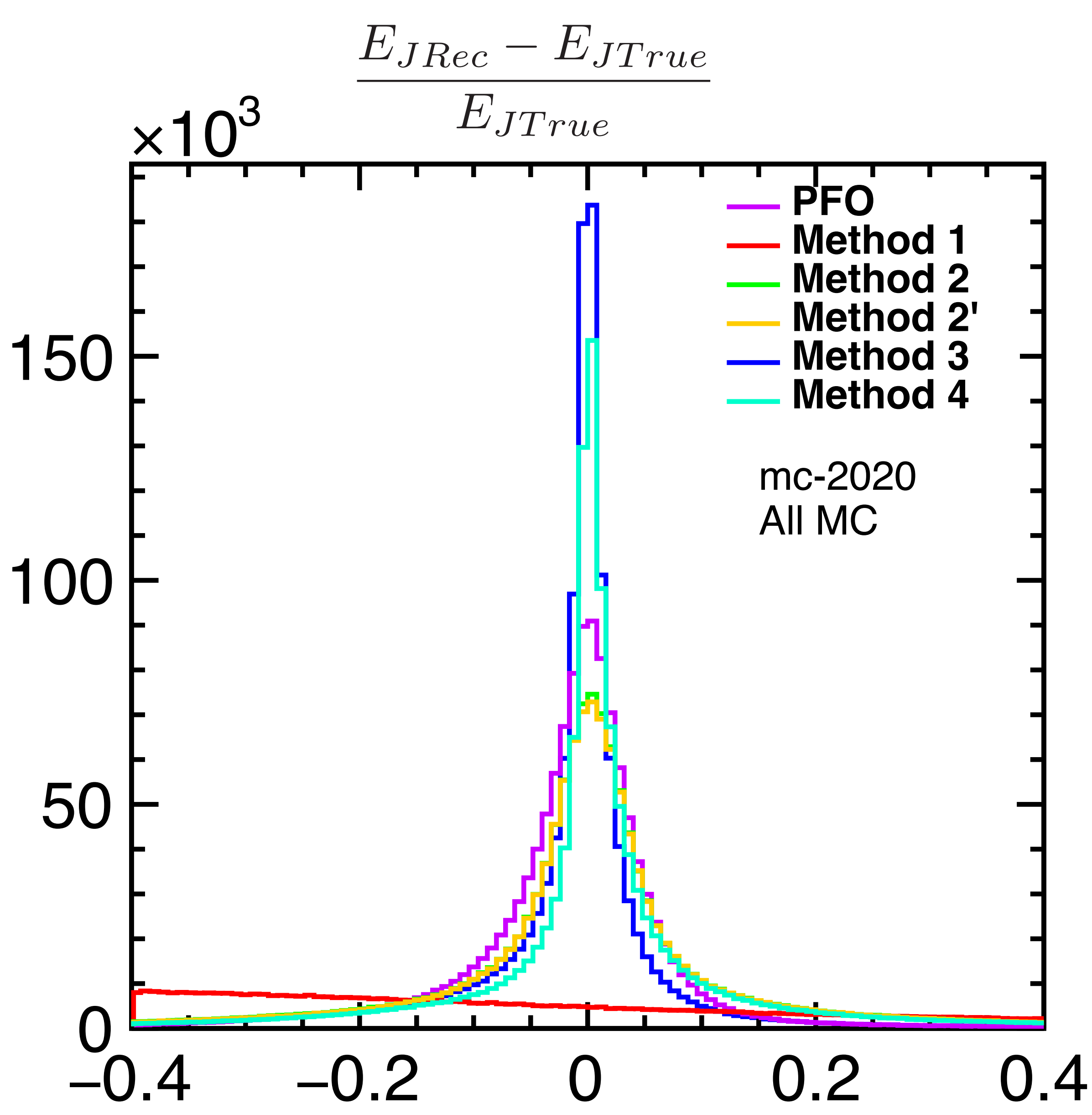
$$\begin{cases} |P_{J1}|\sqrt{1+\frac{1}{(\gamma\beta)^2_{J1}}}+|P_{J2}|\sqrt{1+\frac{1}{(\gamma\beta)^2_{J2}}}+P_{\gamma}+|P_{ISR}|=E_{CM} \quad ① \\ \begin{pmatrix} sin\theta_{J1}cos\phi_{J1} & sin\theta_{J2}cos\phi_{J2} & sin\theta_{\gamma}cos\phi_{\gamma} \\ sin\theta_{J1}sin\phi_{J1} & sin\theta_{J2}sin\phi_{J2} & sin\theta_{\gamma}sin\phi_{\gamma} \\ cos\theta_{J1} & cos\theta_{J2} & cos\theta_{\gamma} \end{pmatrix}\begin{pmatrix} P_{J1} \\ P_{J2} \\ P_{\gamma} \end{pmatrix}=\begin{pmatrix} (E_{CM}-|P_{ISR}|)sin\alpha \\ 0 \\ \pm|P_{ISR}|cos\alpha \end{pmatrix} \end{cases}$$

Choose the solution with solved P_{γ} closest to the measured P_{γ}

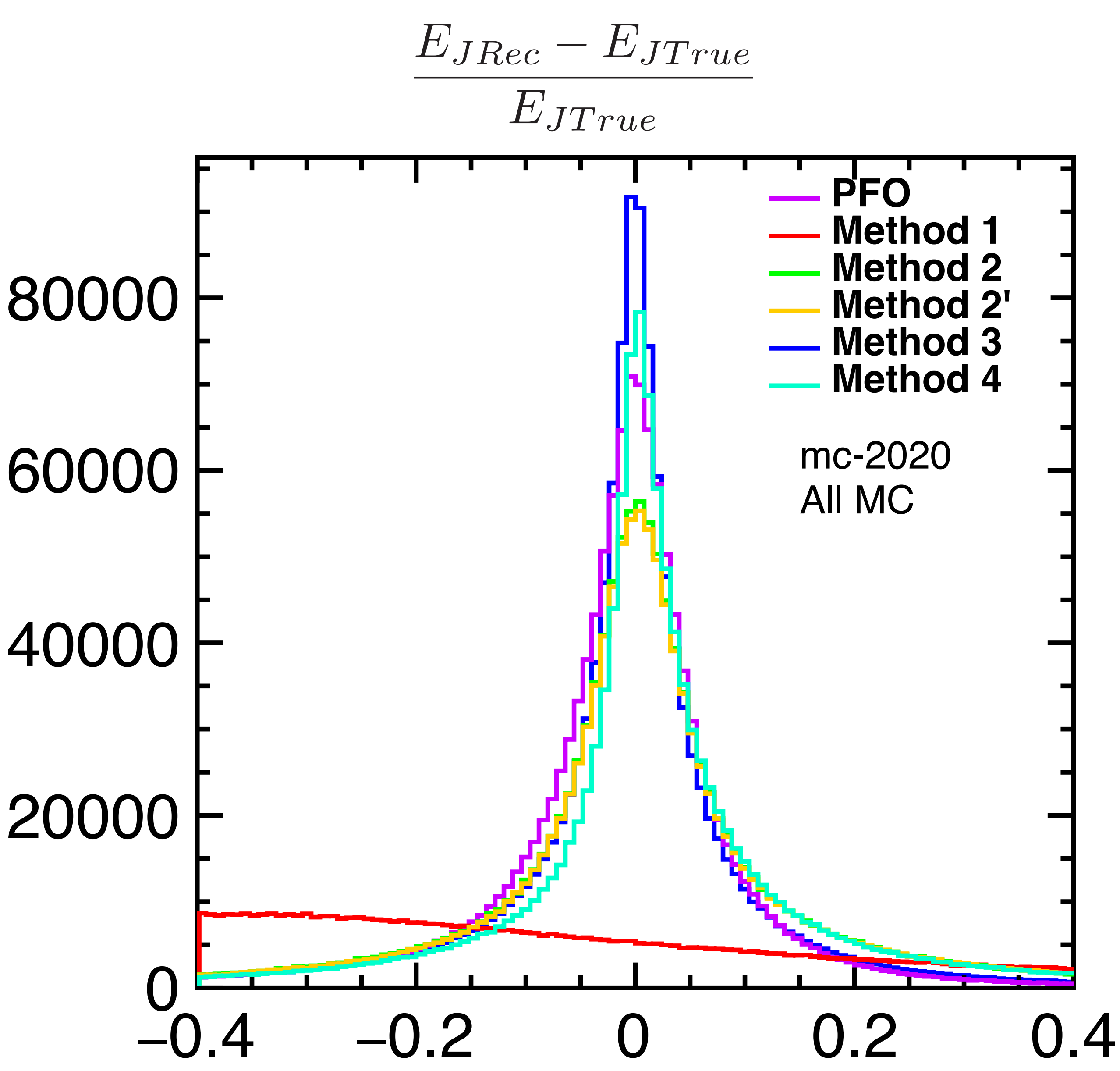
Jet Energy Reconstruction Result (All-MC)

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

Jet 1



Jet 2



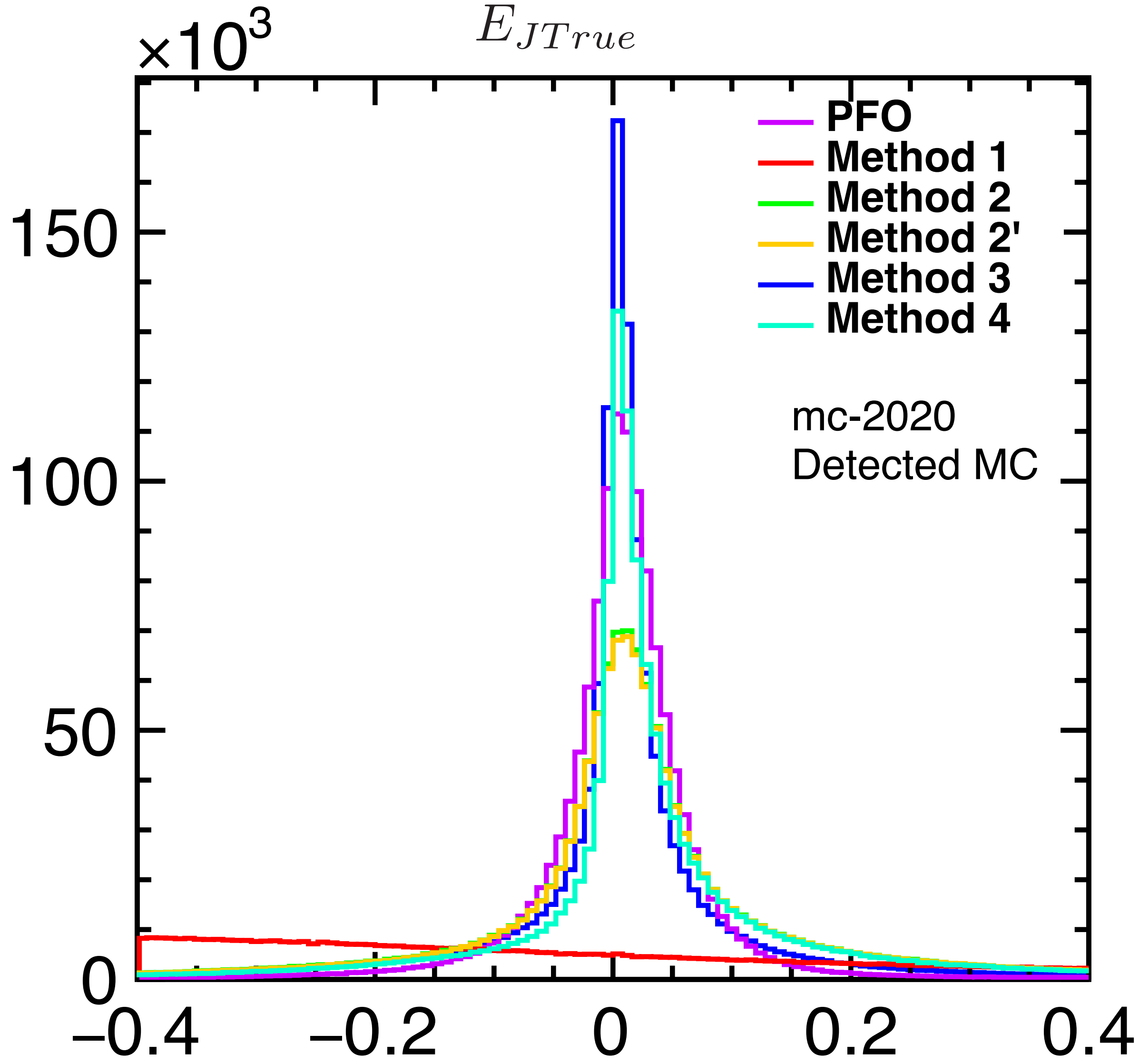
Reconstructed energy not only can calibrate the measured energy, but also has better resolution.

Jet Energy Reconstruction Result (Detected-MC)

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

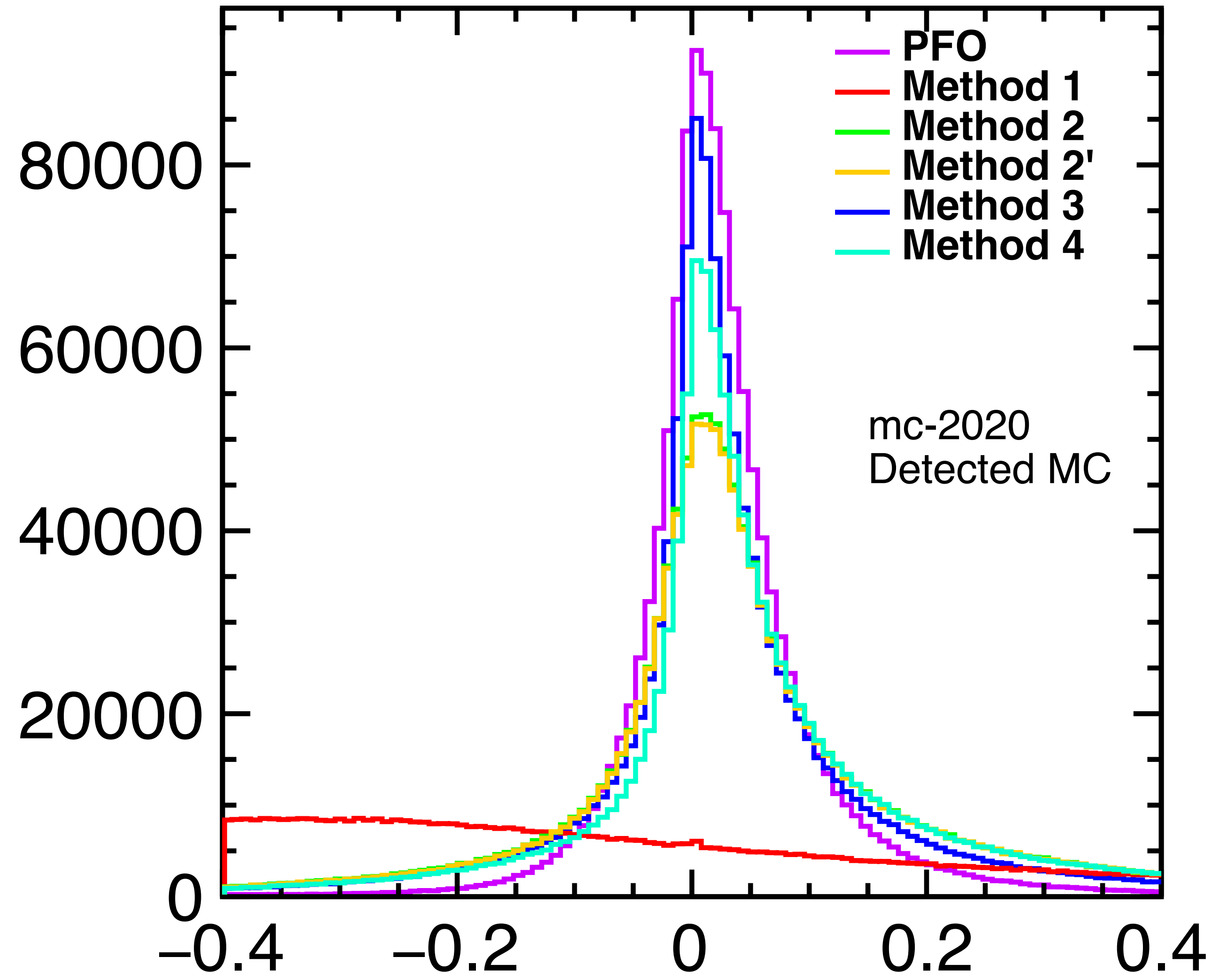
Jet 1

$$\frac{E_{JRec} - E_{JTrue}}{E_{JTrue}}$$



Jet 2

$$\frac{E_{JRec} - E_{JTrue}}{E_{JTrue}}$$

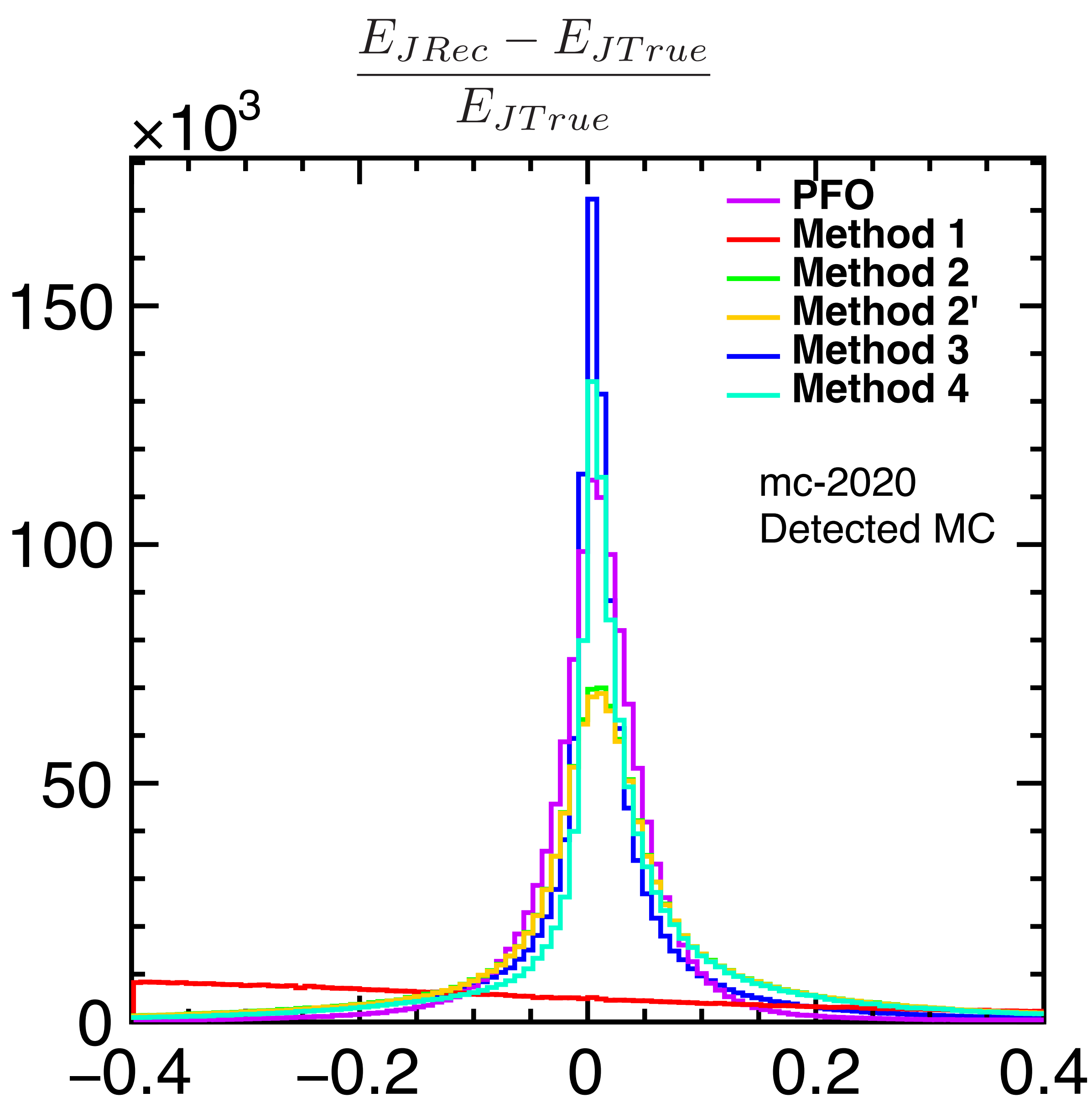


Reconstructed energy not only can calibrate the measured energy, but also has better resolution.

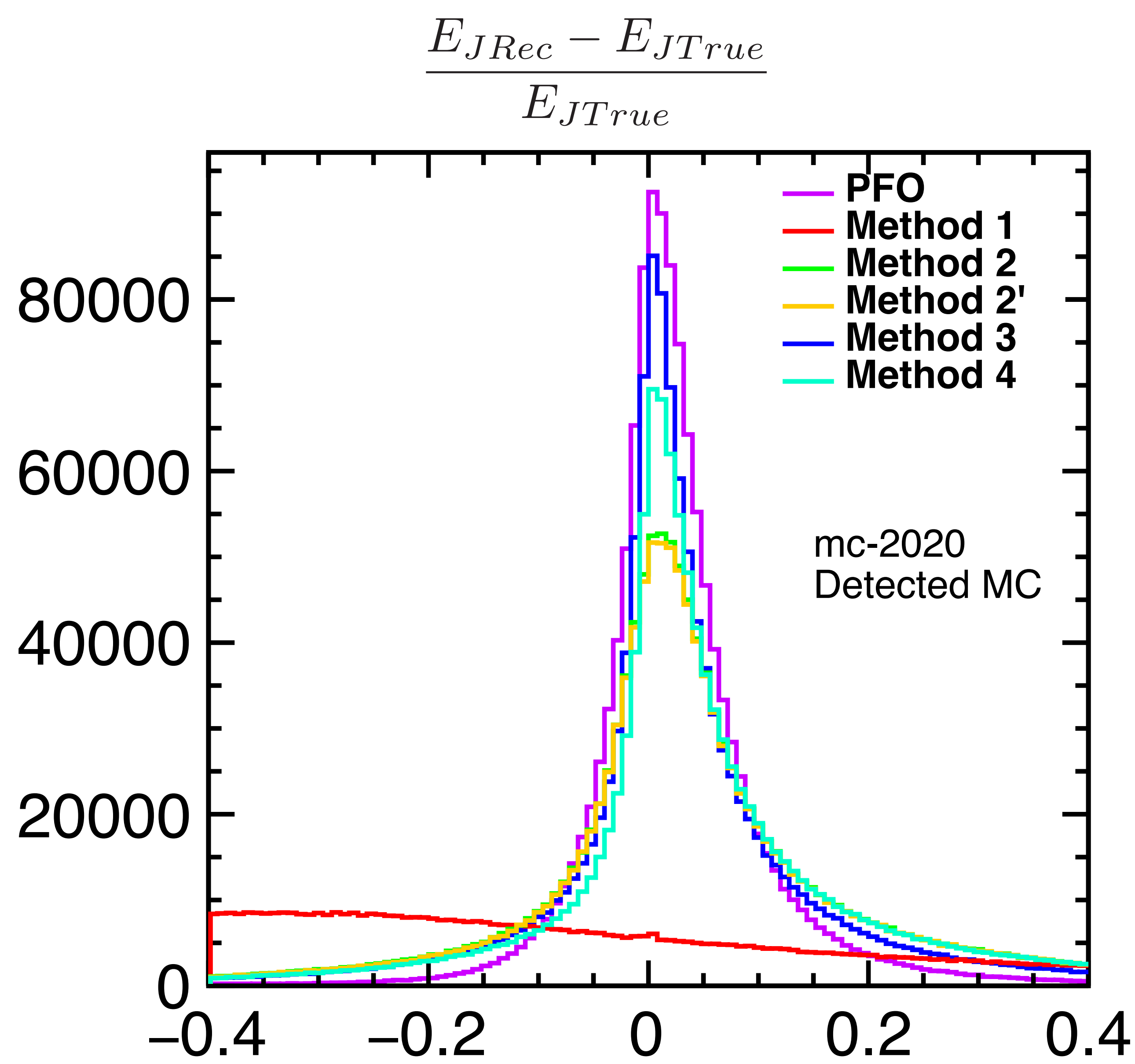
Jet Energy Reconstruction Result (Detected-MC)

eLpR Samples
MC Cut:
Correct photon selection
Method 3 has answer

Jet 1



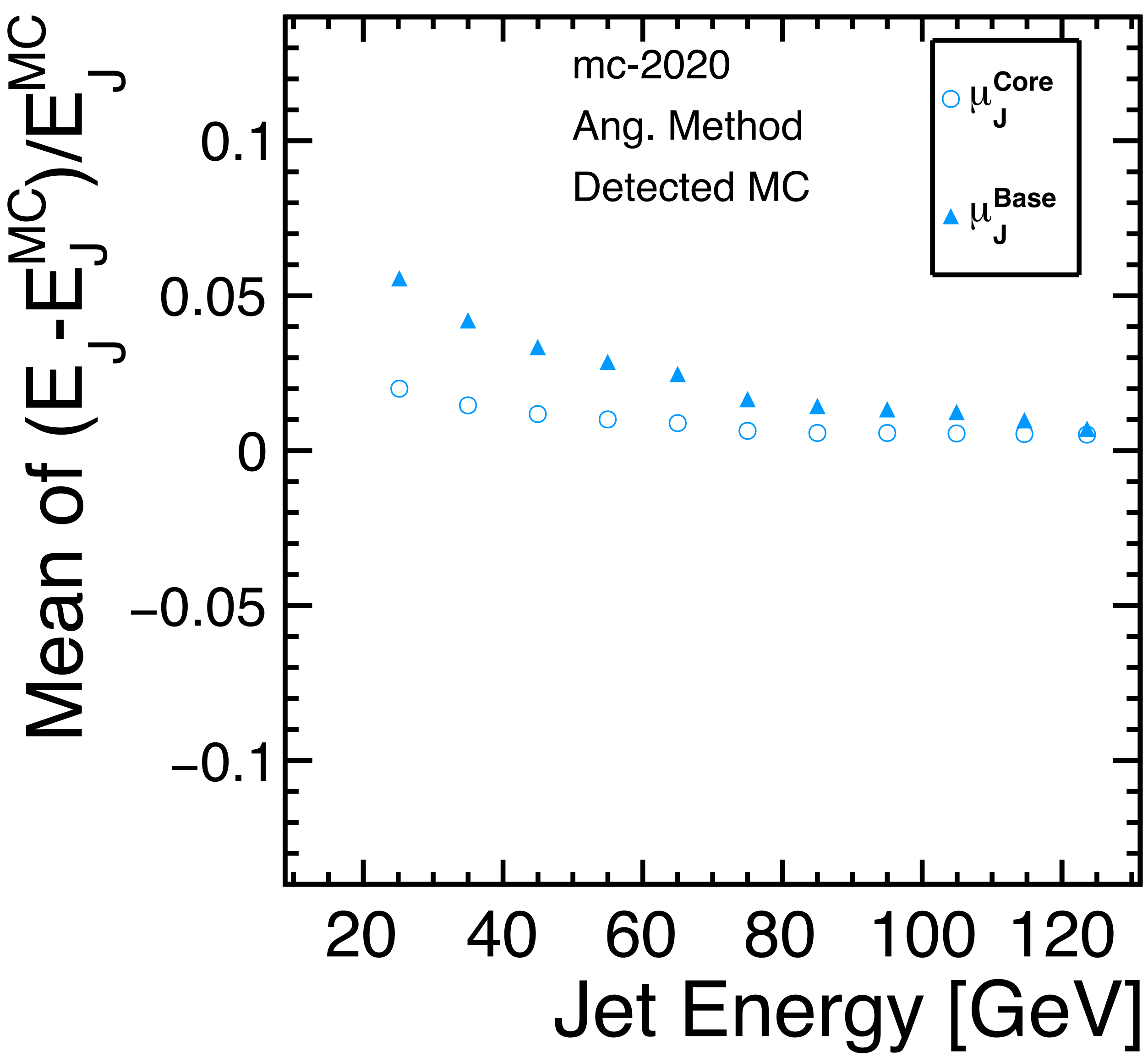
Jet 2



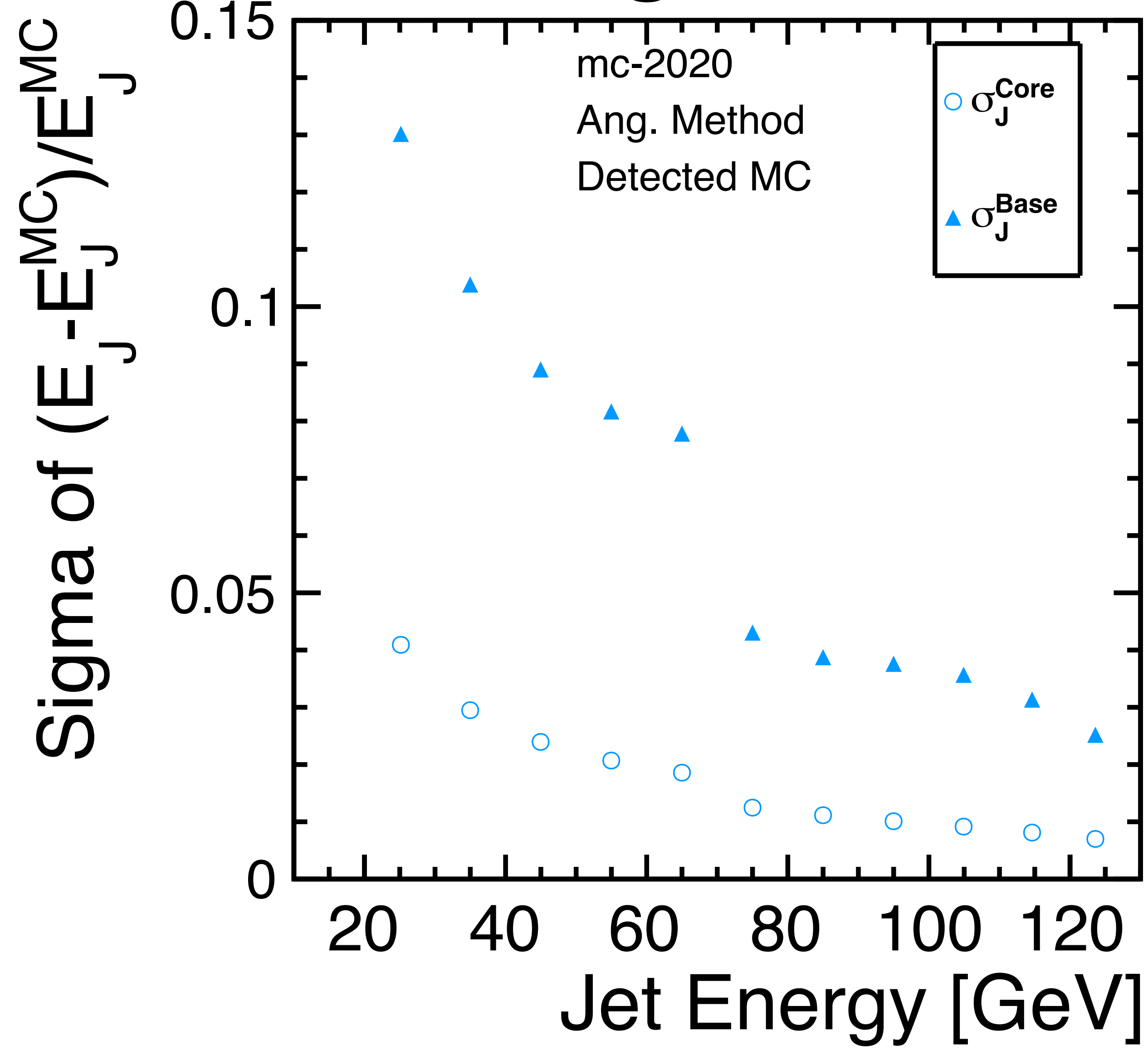
Decided to use Method 3 and rename this as "Ang. Method"

Ang. Method E-Dep (De-MC)

Mean

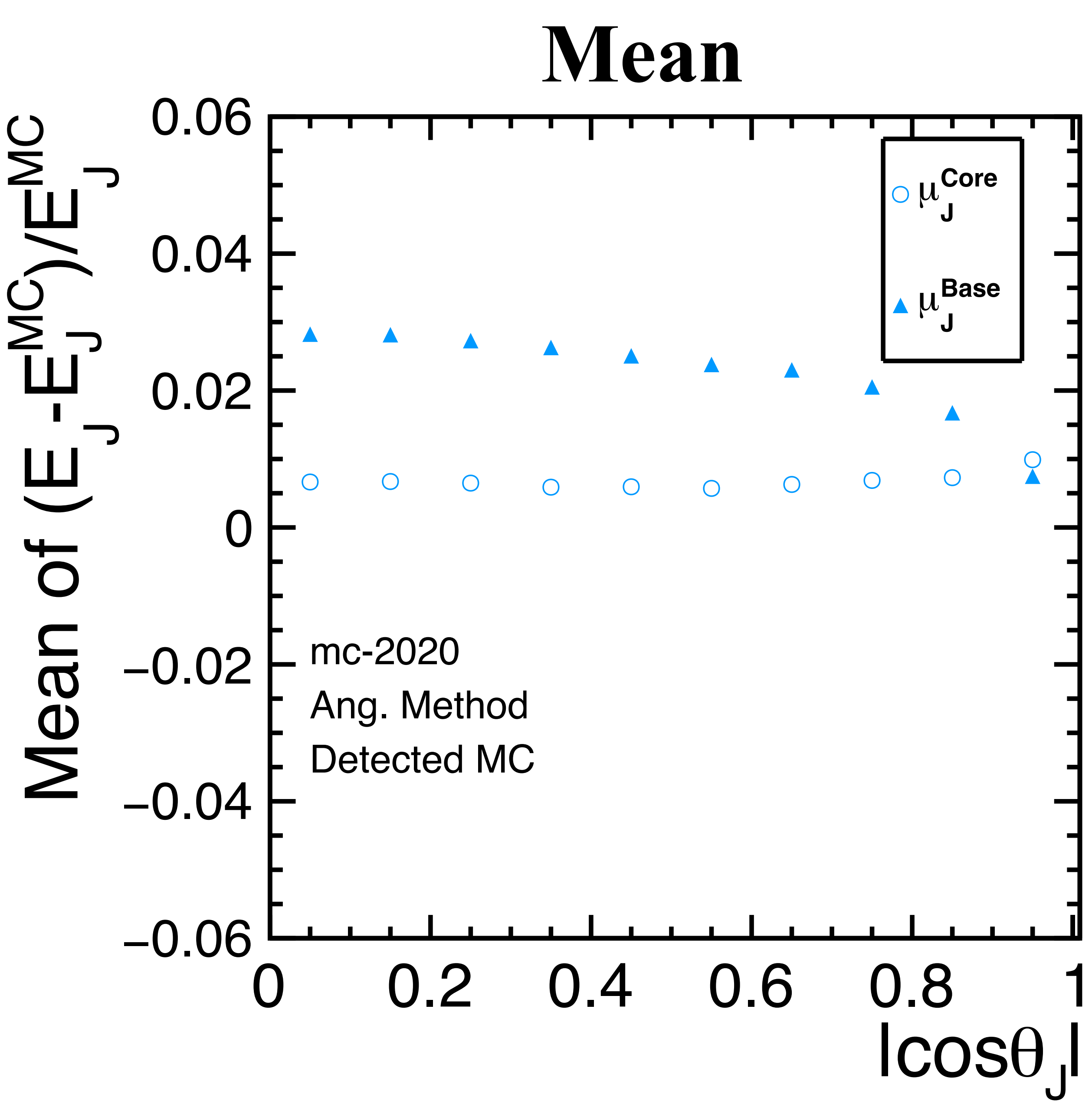


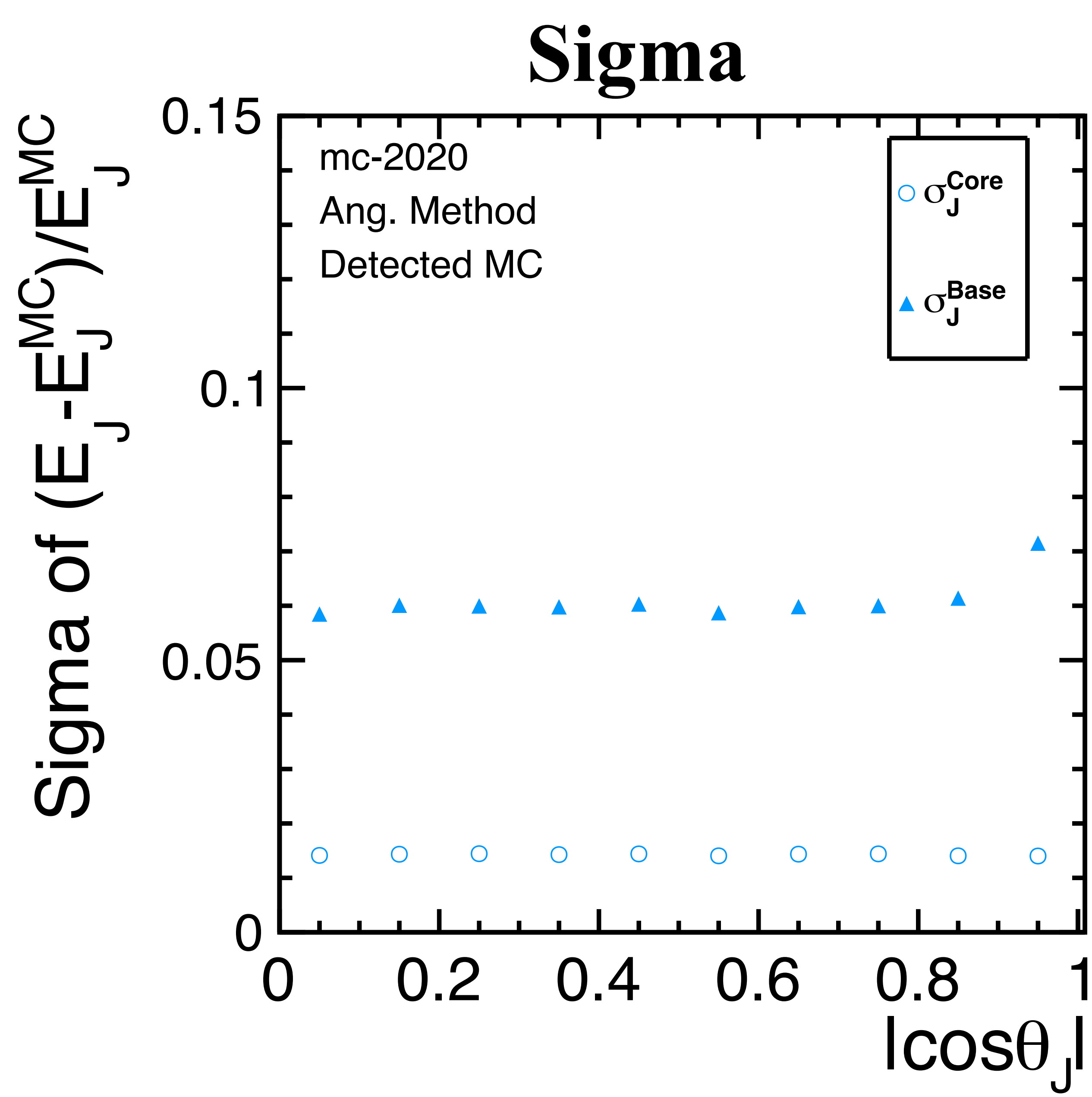
Sigma



Values are positive as Ang. Method recovers missing particles.

Ang. Method T-Dep (De-MC)



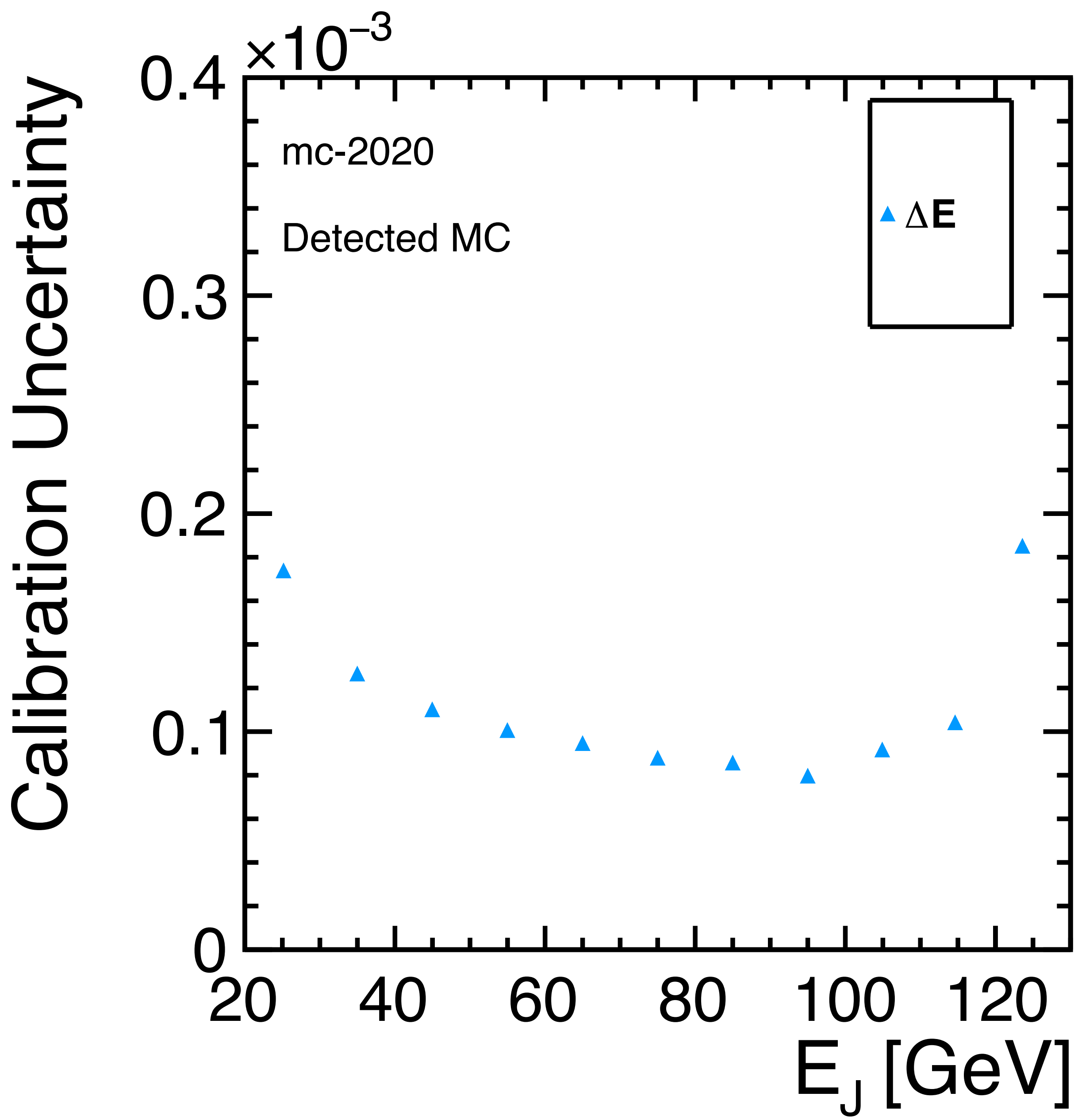


Calibration Uncertainty (De-MC)

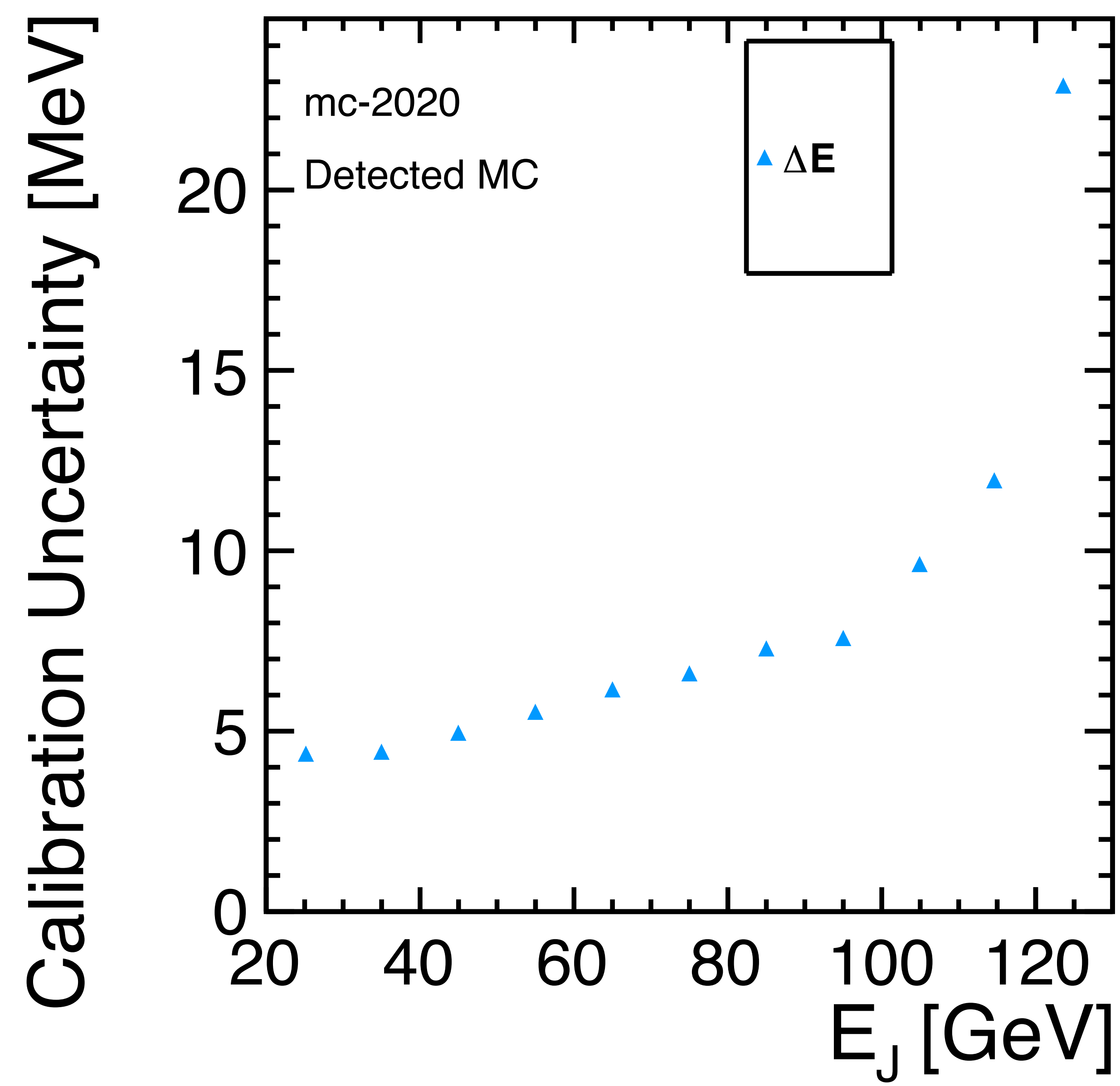
Calibration uncertainty := $\sqrt{(\Delta\mu_{Detector})^2 + (\Delta\mu_{Reconstructed})^2}$

Square root of the squared sum of the error of the mean

Relative uncertainty



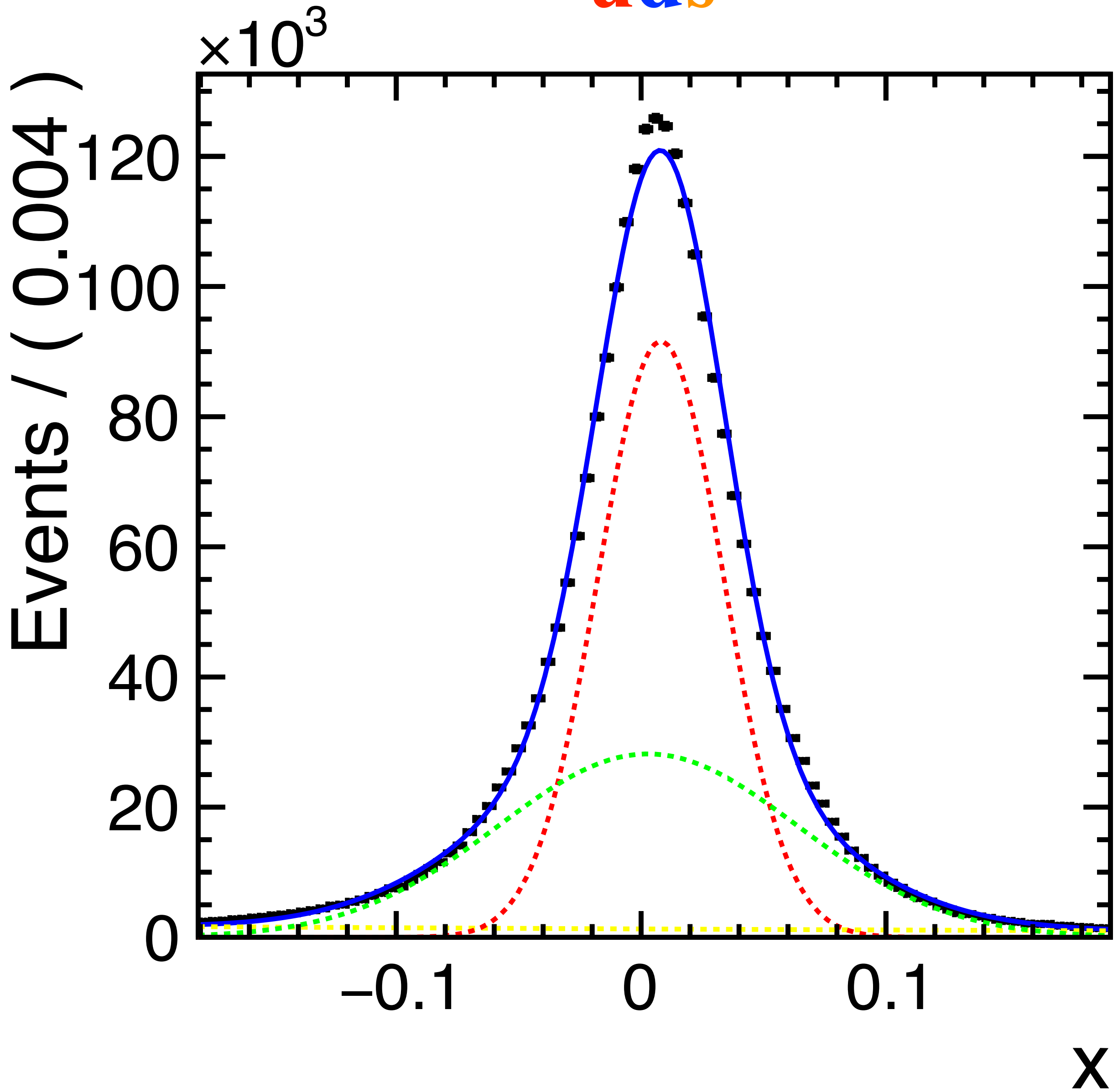
Absolute uncertainty



We can calibrate the jet energy scale with about 10^{-4} accuracy, which corresponds to ~10 MeV.

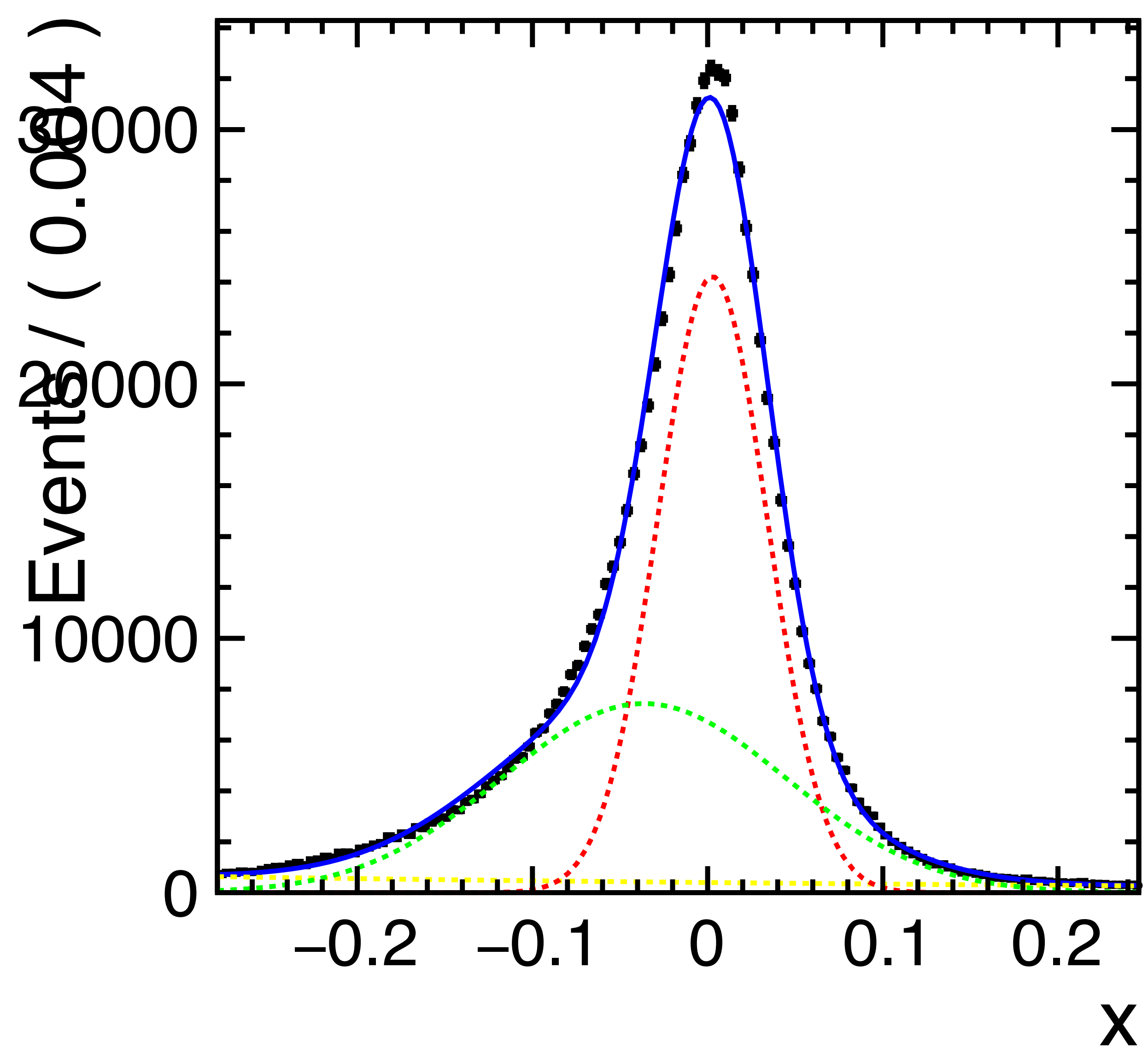
PFO total jet energy “PFO-DeMC”

uds




```
mean = 0.0080543 +/- 3.35e-05  L(-0.1 - 0.1)
mean2 = 0.0028019 +/- 9.26231e-05  L(-0.1 - 0.1)
sigma1 = 0.0256053 +/- 6.1407e-05  L(0.005 - 0.05)
sigma2 = 0.0612324 +/- 0.000176353  L(0.05 - 0.2)
sig1frac = 0.57608 +/- 0.00224723  L(0 - 1)
bkgfrac = 0.0610457 +/- 0.000354142  L(0 - 1)
```

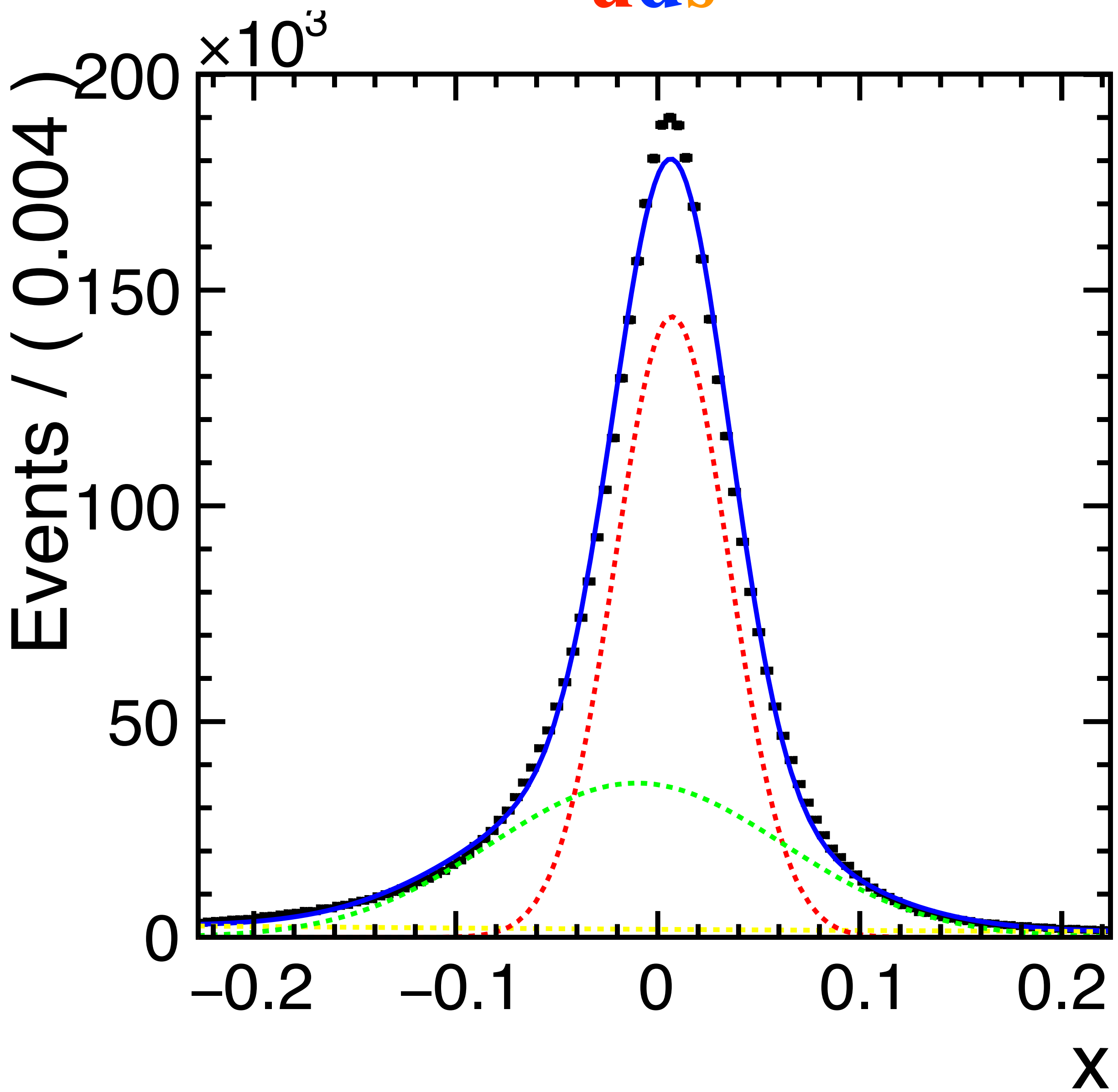
b




```
mean = 0.00275763 +/- 8.00636e-05
mean2 = -0.0364699 +/- 0.000303912
sigma1 = 0.0313553 +/- 0.000100174
sigma2 = 0.0811313 +/- 0.000272633
sig1frac = 0.557376 +/- 0.00234477
bkgfrac = 0.0699624 +/- 0.00071931
```

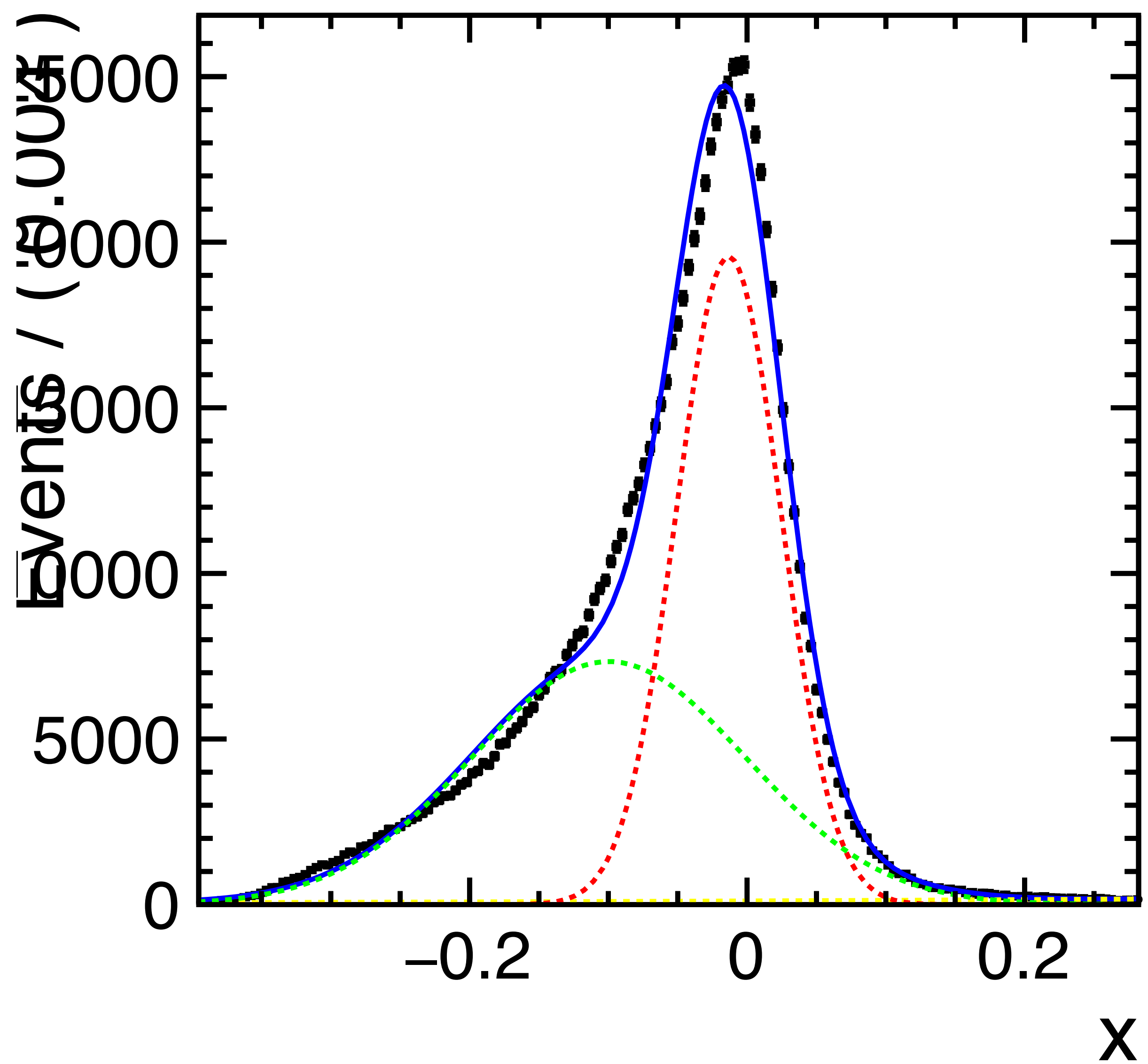
PFO total jet energy "PFO-MC(quarks)"

uds




```
mean = 0.00235774 +/- 3.43218e-05  L(-0.1 - 0.1)
mean2 = -0.00626636 +/- 9.58478e-05  L(-0.1 - 0.1)
sigma1 = 0.0256816 +/- 5.90396e-05  L(0.005 - 0.05)
sigma2 = 0.059851 +/- 0.000161193  L(0.05 - 0.2)
sig1frac = 0.578097 +/- 0.00220472  L(0 - 1)
bkgfrac = 0.0550327 +/- 0.000332775  L(0 - 1)
```

b




```
mean = -0.0133071 +/- 0.000103814
mean2 = -0.0997099 +/- 0.000341045
sigma1 = 0.037782 +/- 0.000108181
sigma2 = 0.0984826 +/- 0.000240112
sig1frac = 0.505844 +/- 0.00174362
bkgfrac = 0.0191361 +/- 0.00086861
```