Investigating Center-of-Mass Energy Determination with Dimuons

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Introduction

- ILC will make precision measurements of the **masses** of known fundamental particles (M_{H}, M_{t}, M_{W}, M_{Z}), and Γ_{Z}. Measure new ones, M_{X}.
- A primary issue is the measurement of the **absolute center-of-mass energy scale** for most determinations. The proposed $\sqrt{s}_p$ method uses only the momenta of muons in dimuon events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K}^0_{\mathrm{S}}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K}^0_{\mathrm{S}}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

Today,

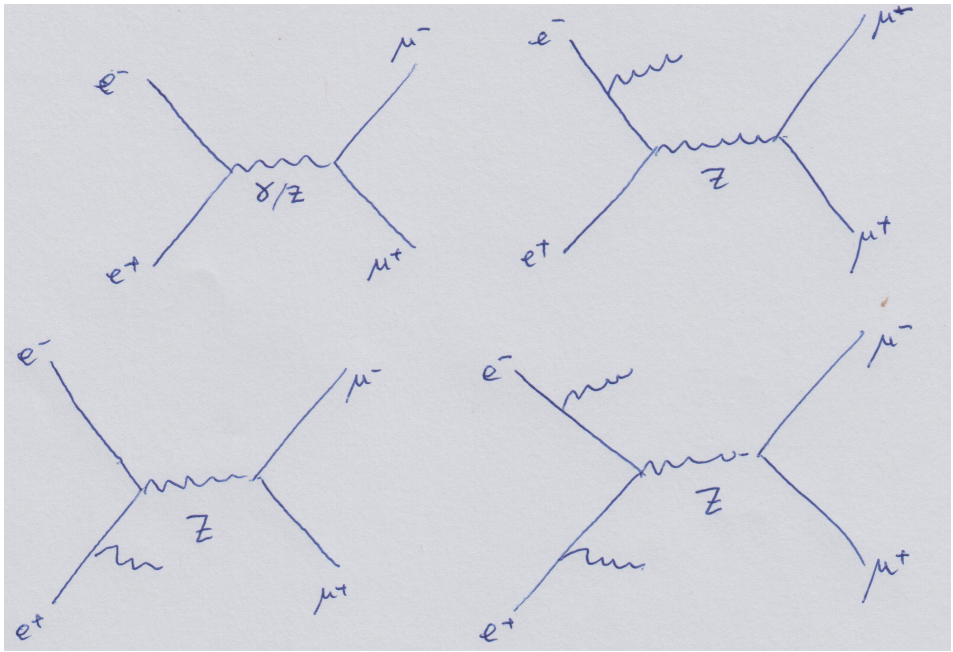
- Look more carefully at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$.
- Include crossing angle, full simulation and reconstruction with ILD, track error matrices, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum.
- In progress, treatment of detected ISR/FSR photons and vertex fitting.
- Bonus. Physics: M_{Z}. Beam knowledge: **luminosity spectrum**, $\mathrm{d}L/\mathrm{d}\sqrt{s}$.

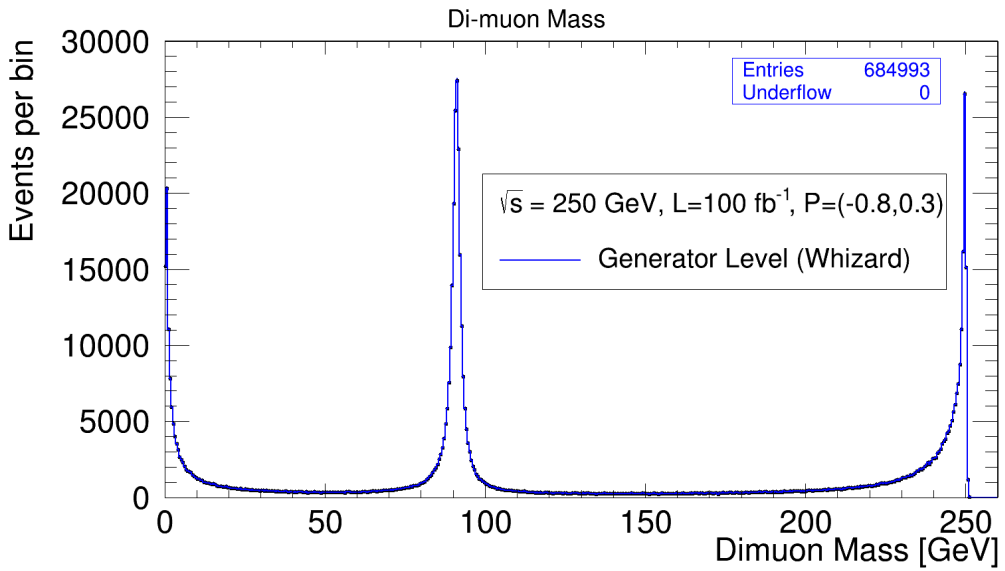
Dimuons

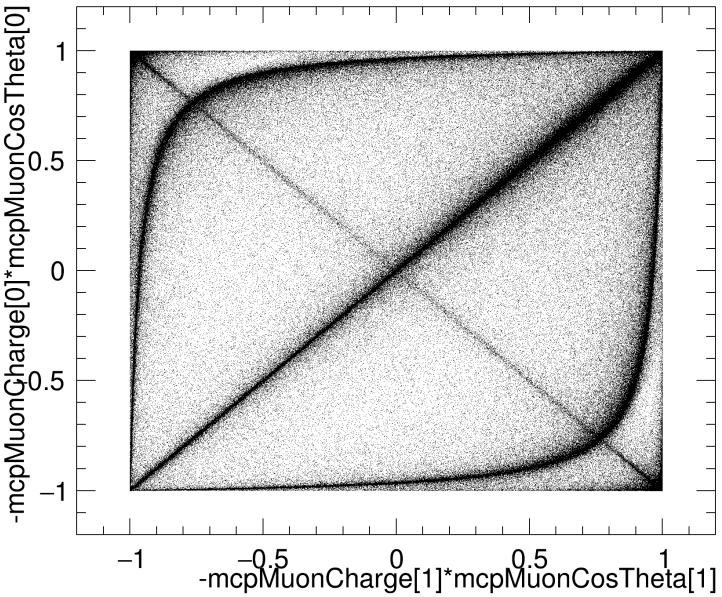
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass, $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

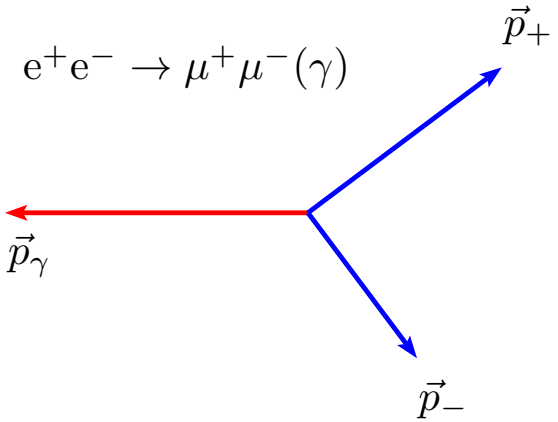
- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.







$\sqrt{s_p}$ Method in a Nutshell



Measure $\sqrt{s_p}$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

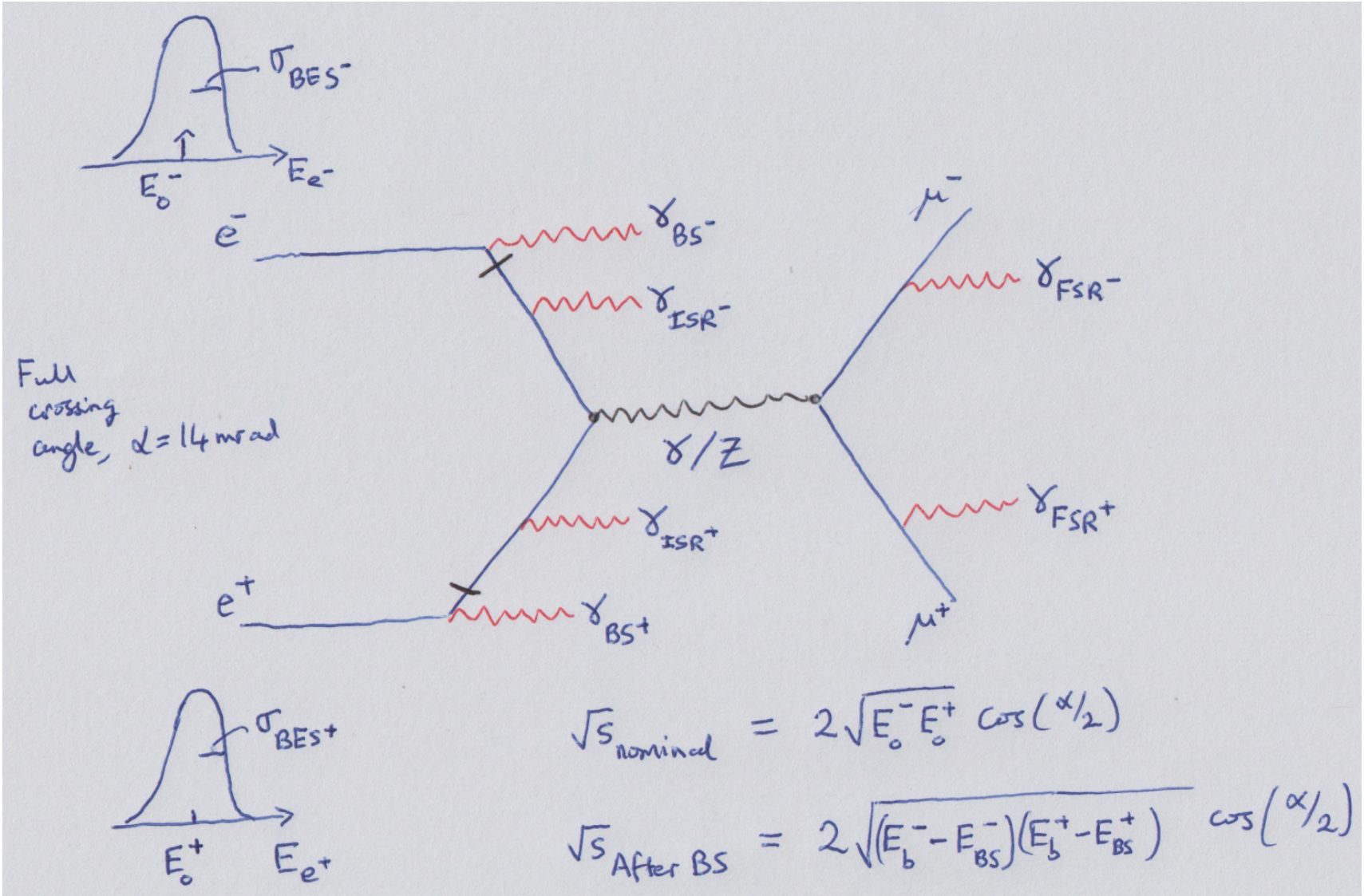
- Equal beam energies, E_b
- The lab is the CM frame, $(\sqrt{s} = 2\,E_\mathrm{b},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is massless

$$\sqrt{s} = \sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

$$\sqrt{s_p} = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

Getting More Realistic



See more verbose explanations on next slide.

Beam Effects

The main idea is to use the kinematics of $e^+e^- \to \mu^+\mu^-(\gamma)$ events and measurements of the final-state particles to measure the distribution of the center-of-mass energy of collisions.
We identify 3 effects needed to make a more realistic model of the collision:

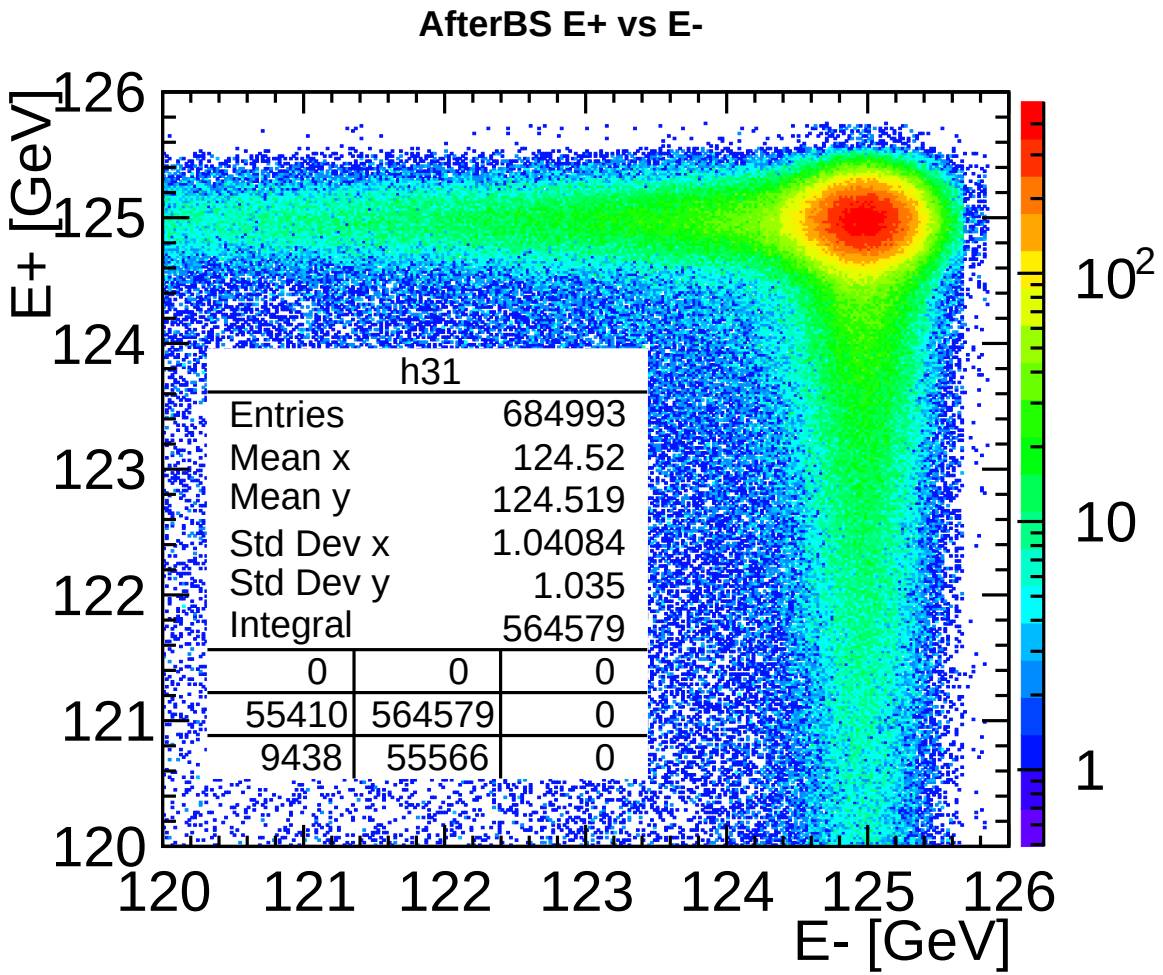
0. Nominal. Each beam is a δ-function centered at a particular beam energy.
1. **Beam energy spread.** Each beam has a Gaussian distribution with rms width, σ_E, centered at a particular beam energy.
2. **Beamstrahlung.** The collective interaction of the two beams leads to radiation of collinear photons from the beams, resulting in the colliding e^+ and e^- having a *beamstrahlung-reduced center-of-mass energy*.
3. **Initial-state-radiation (ISR).** All e^+e^- physics processes may have ISR, where the invariant mass of the annihilating e^+ and e^- and the resulting particle system is further reduced cf 2 due to the emitted ISR photon(s).

We are primarily concerned with evaluating the **beamstrahlung-reduced center-of-mass energy**. This is *after* beam energy spread and beamstrahlung radiation, but *before* emission of any ISR photons. We should allow for differences in the energy of each beam and for a **beam crossing angle**, α, defined as the horizontal plane angle between the two beam lines. For ILC, α, is 14 mrad.

What do we really want to measure?

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Now let's look at the related 1-d distributions ($E_+, E_-, \sqrt{s}, p_z$) with empirical fits.



Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

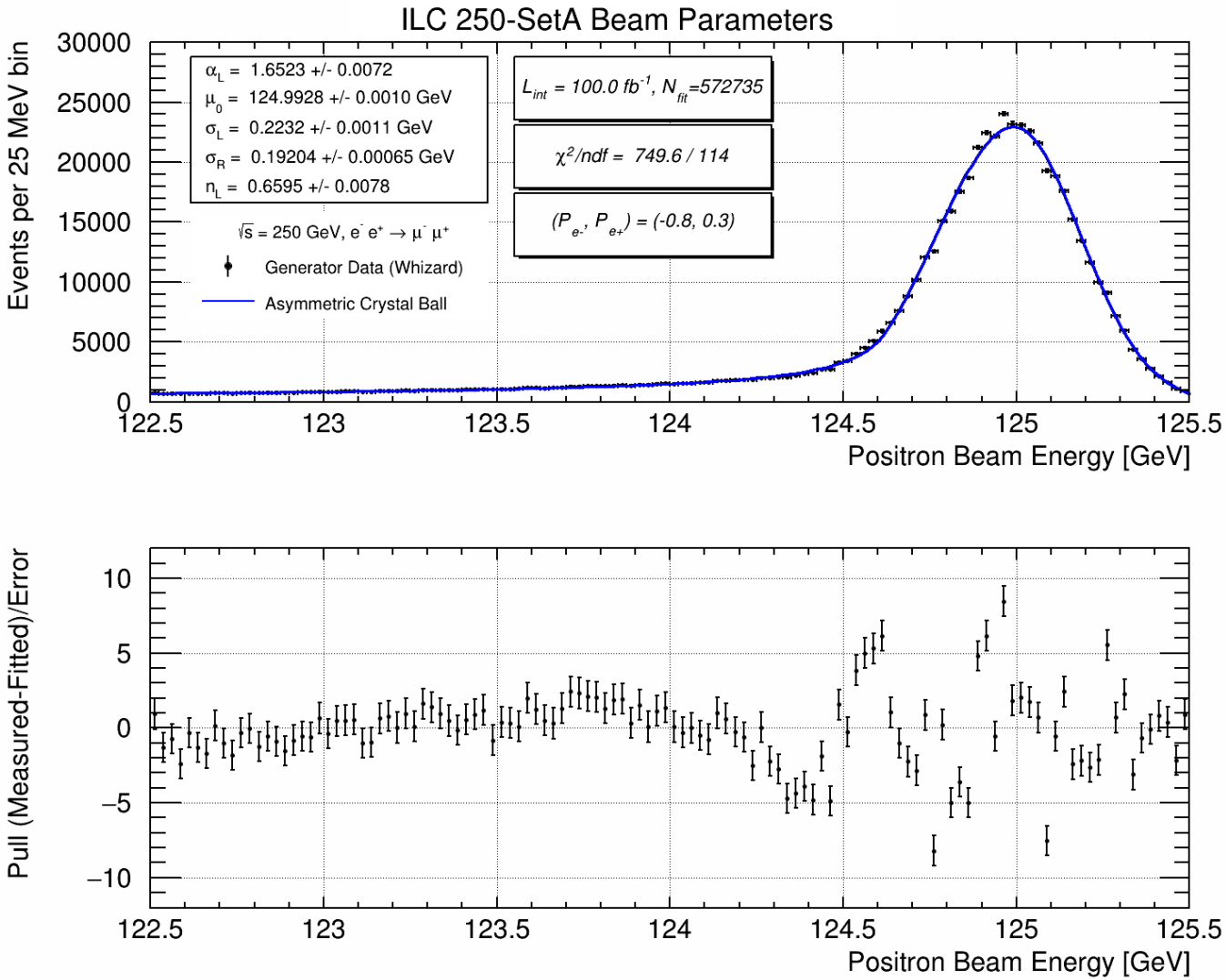
Aside on Crystal Ball Empirical Fit Functions

- The 1-d distributions generally feature a Gaussian **peak** associated with beam energy spread and a long **tail** with harder beamstrahlung
- These can be fit qualitatively well - although not well enough - with a Crystal Ball function. This piece-wise function has a Gaussian core and a power-law tail with a continuous first-derivative at the transition points.
- The generalized asymmetric double-sided Crystal Ball is

$$f(E; \mu_0, \sigma_L, \alpha_L, n_L, \sigma_R, \alpha_R, n_R)$$

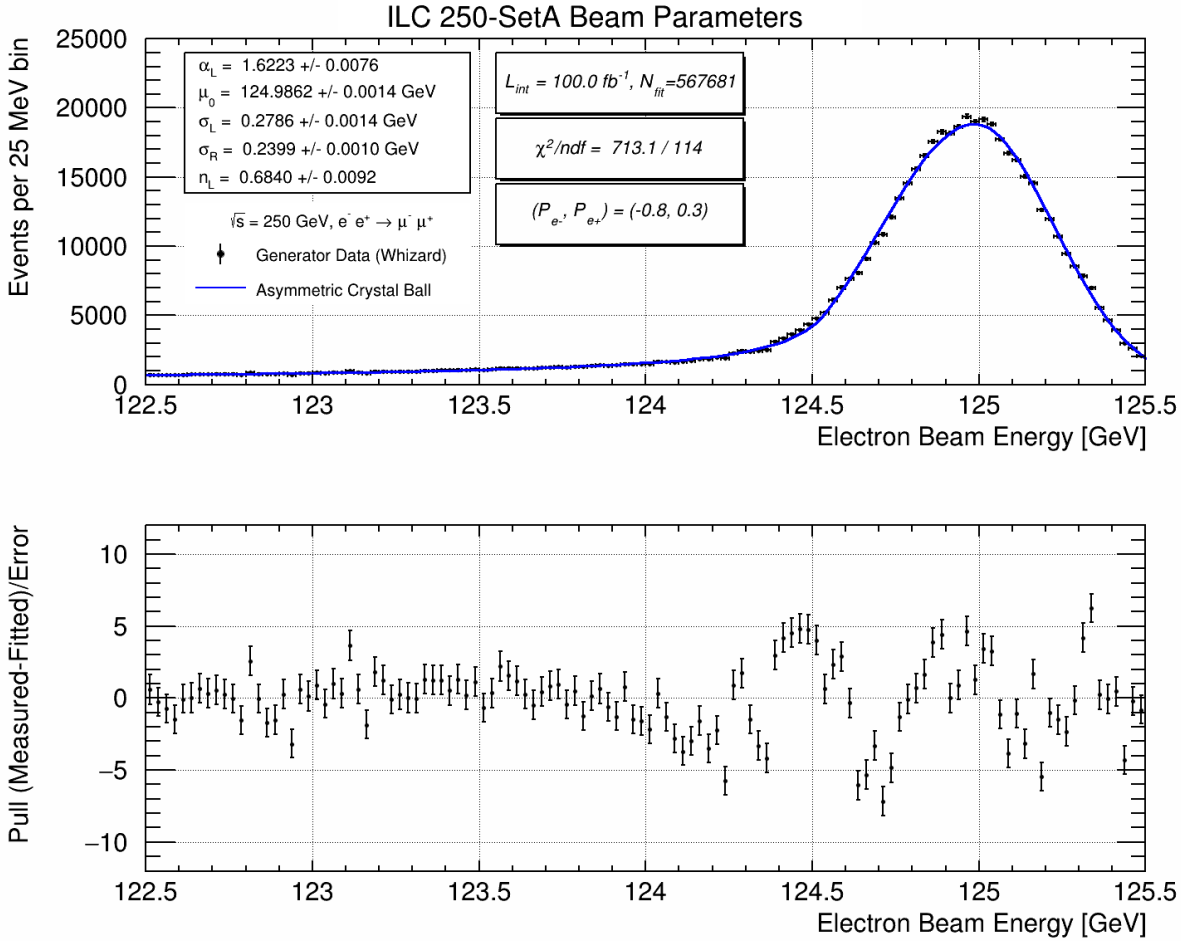
  where μ_0 is the Gaussian peak **mode**, σ_i are the Gaussian **widths** (on L&R), α_i are the Gaussian/power-law **transition points** in units of σ_i (on L&R), and n_i are the power law **exponents** (on L&R)
- With the beam energy related distributions, only a 5-parameter version is applicable with parameters, $\mu_0, \sigma_L, \alpha_L, n_L, \sigma_R$ with the right-hand power-law tail disabled. The classic 1-sided Crystal Ball (4-parameters) $\mu_0, \sigma_L, \alpha_L, n_L$ fits are included for reference in the backup slides.
- See RooCrystalBall for implementation details

Positron Beam Energy (After Beamstrahlung)



$$\sigma_{\mathrm{R}}/E = 0.1536 \pm 0.0005\% \text{ (cf 0.152\% in TDR)}$$

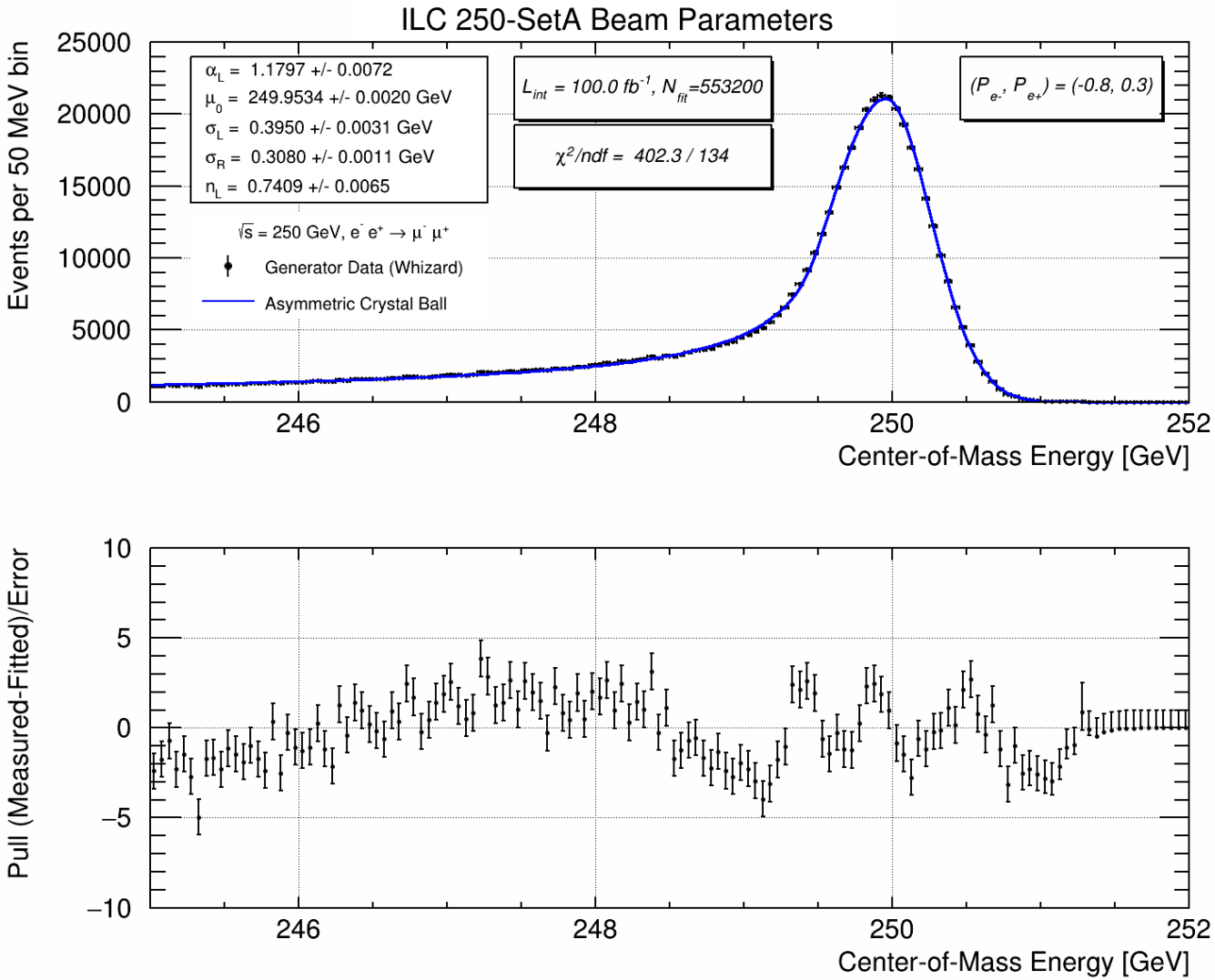
Electron Beam Energy (After Beamstrahlung)



$$\sigma_{\mathrm{R}}/E = 0.1919 \pm 0.0008\% \text{ (cf 0.190\% in TDR)}$$

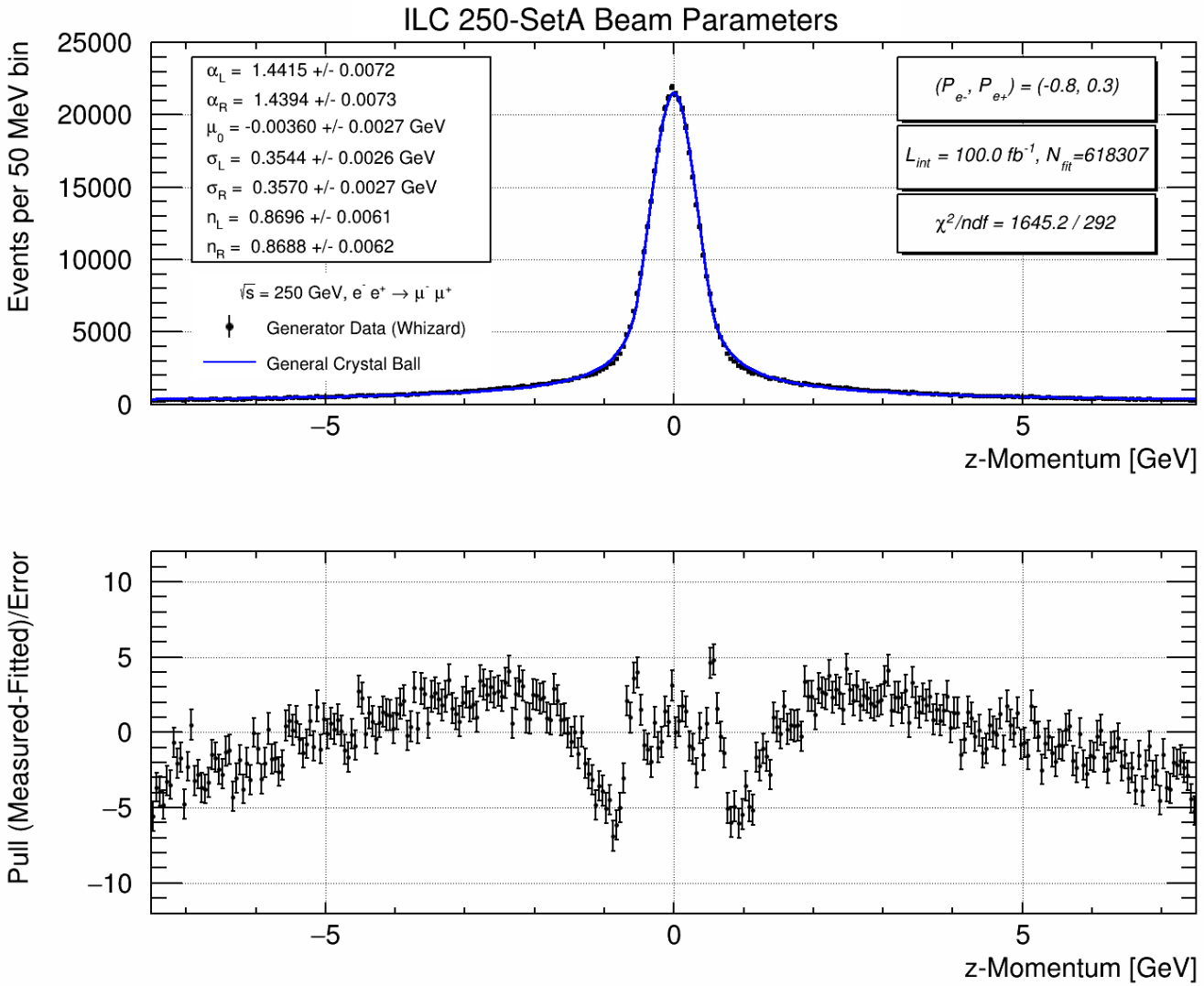
Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.

Center-of-Mass Energy (After Beamstrahlung)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1232 \pm 0.0004\%$ (cf 0.122% in TDR (0.190% $\oplus$ 0.152%)/2)

z-Momentum (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the electron beam and positron beam respectively.
Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$

$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$

$$p_y = 0$$

$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α is known, evaluation of the center-of-mass energy of this collision amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \ , \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^2 - (\overline{\Delta E_{\mathrm{b}}})^2} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ process. Denote the μ^+ as particle 1, the μ^- as particle 2, and the rest-of-the event (RoE) as system 3.
We can write this final-state system 4-vector as

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Then applying $(E, \vec{p})$ conservation and assuming $m_3 = 0$ we obtain,

$$(E_1 + E_2 + E_3) = E_1 + E_2 + p_3 = 2\ E_{\mathrm{ave}} \tag{1}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{2}$$

In general the RoE may not be fully detected and needs to be inferred using $(E, \vec{p})$ conservation. Here we have these 4 equations and 5 unknowns, namely the 3 components of the RoE momentum $(\vec{p_3})$ and E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$.
One approach is to solve for E_{ave} for various assumptions on $\overline{\Delta E_{\mathrm{b}}}$. Specifically we then focus on using the simplifying assumption that $\overline{\Delta E_{\mathrm{b}}} = 0$. Note this is often a poor assumption event-by-event for the p_z conservation component.

The Averaged Beam Energy Quadratic

Using the outlined approach results in a quadratic equation in E_{ave}, $(AE_{\mathrm{ave}}^2 + BE_{\mathrm{ave}} + C = 0)$, with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

$$C = (M_{12}^2)/4 + p_{12}^z \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2) - \overline{\Delta E_{\mathrm{b}}}^2 \cos^2(\alpha/2)$$

Based on this, there are three particular cases of interest to solve for E_{ave}.

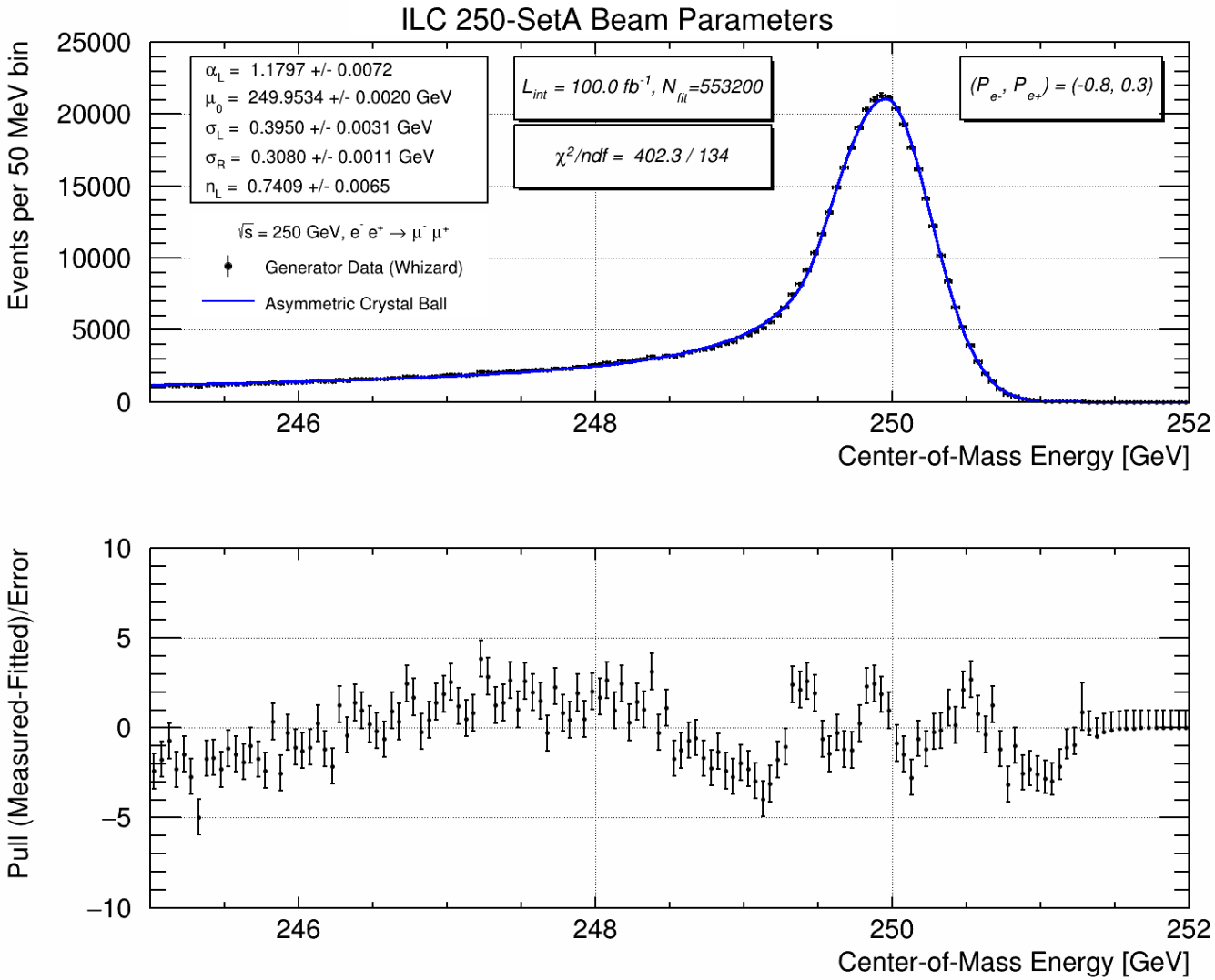
1. Zero crossing angle, $\alpha = 0$, and zero beam energy difference.
2. Crossing angle and zero beam energy difference.
3. Crossing angle and non-zero beam energy difference.

The original formula,

$$\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$$

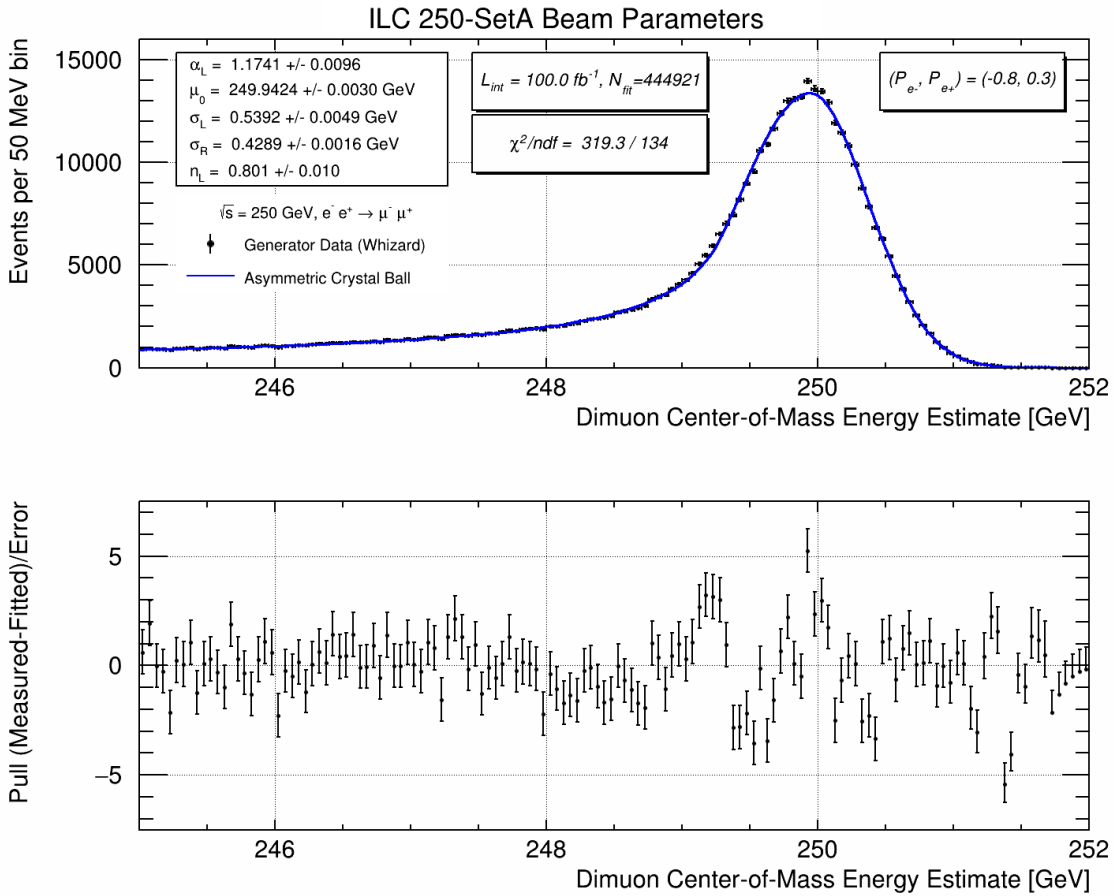
arises trivially in the first case. In the rest of this talk I will use the $\sqrt{s}$ estimate from the largest positive solution of the second case as what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

Center-of-Mass Energy (After Beamstrahlung) (Repeated)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1232 \pm 0.0004\%$ (cf 0.122% in TDR (0.190% $\oplus$ 0.152%)/2)

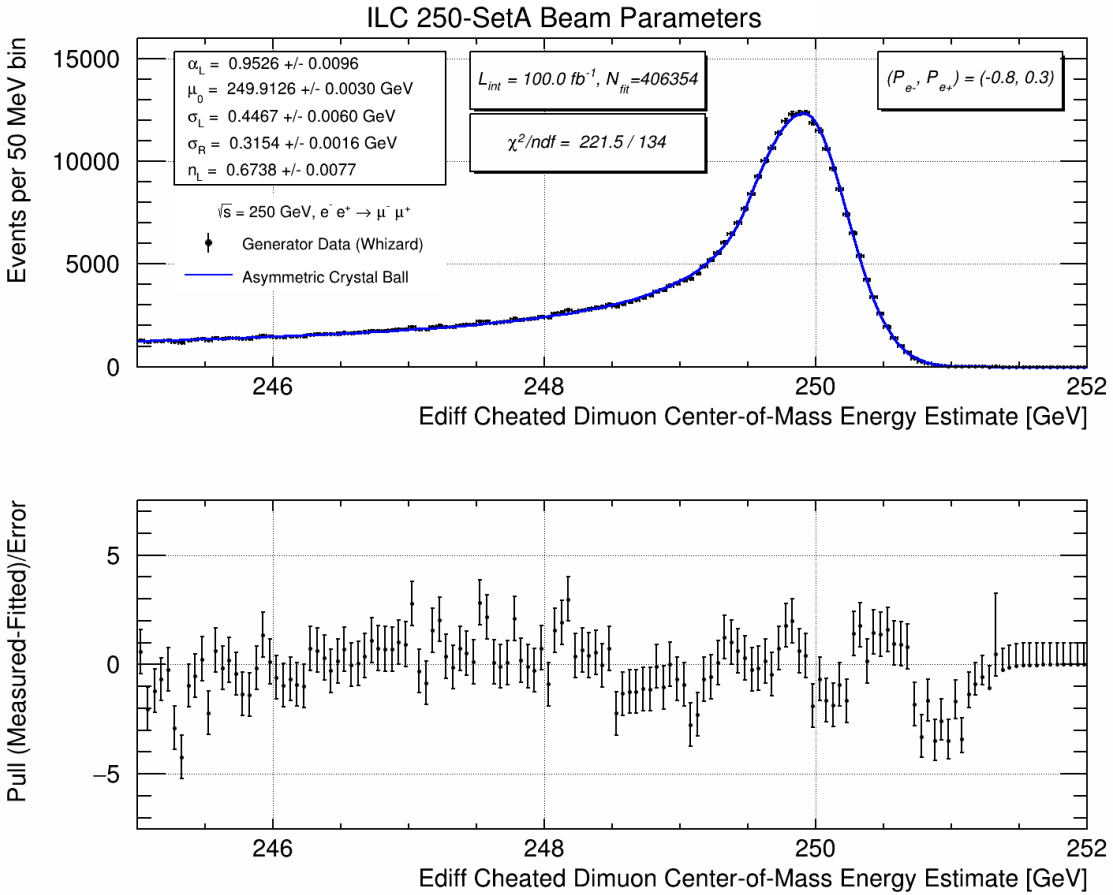
Dimuon Estimate of Center-of-Mass Energy (After BS)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1716 \pm 0.0006\%$ (cf 0.1232% with true $\sqrt{s}$)

- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Likely because some events violate the assumptions that $\overline{\Delta E_{\mathrm{b}}} = 0$ and $m_3 = 0$
- The former is no surprise given the p_z distribution
- The latter can be associated with events with 2 or more non-collinear ISR/FSR photons

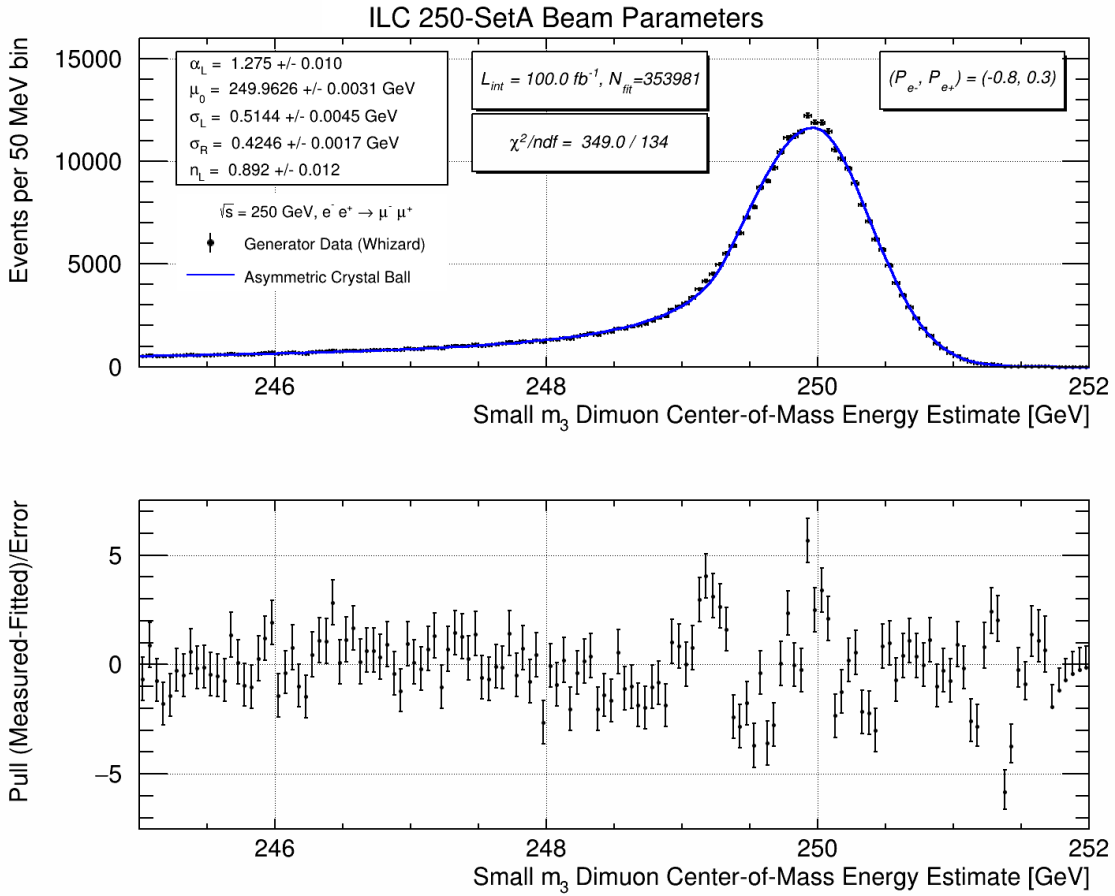
Cheated Dimuon Estimate of $\sqrt{s}$ (After BS)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1259 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

- This is the generator-level $\sqrt{s_p}$ calculated from the 2 muons
- But using the true $\overline{\Delta E_{\mathrm{b}}}$ in the equations
- Why so few events in range?

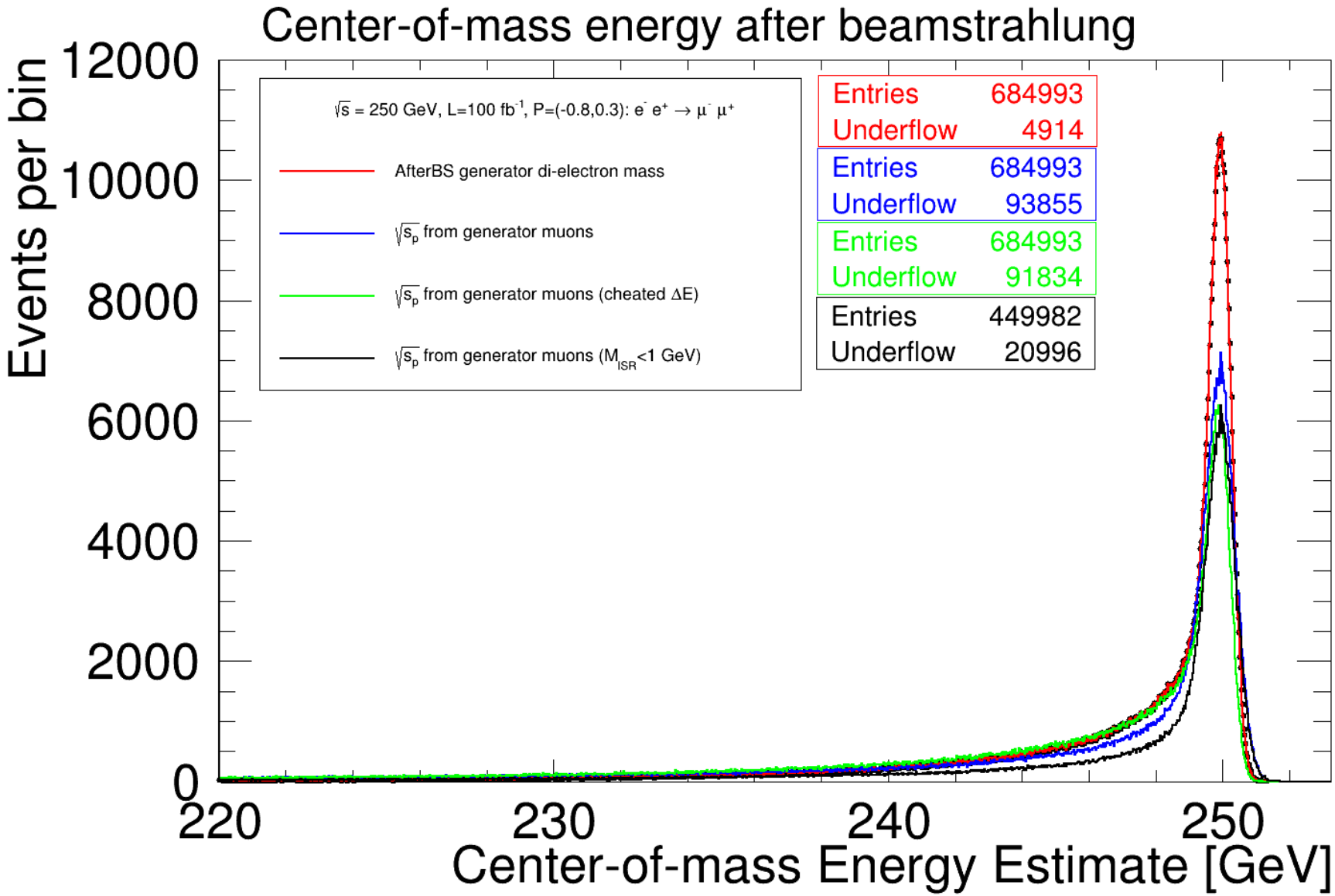
Dimuon Estimate of $\sqrt{s}$ (Low m_3) (After BS)



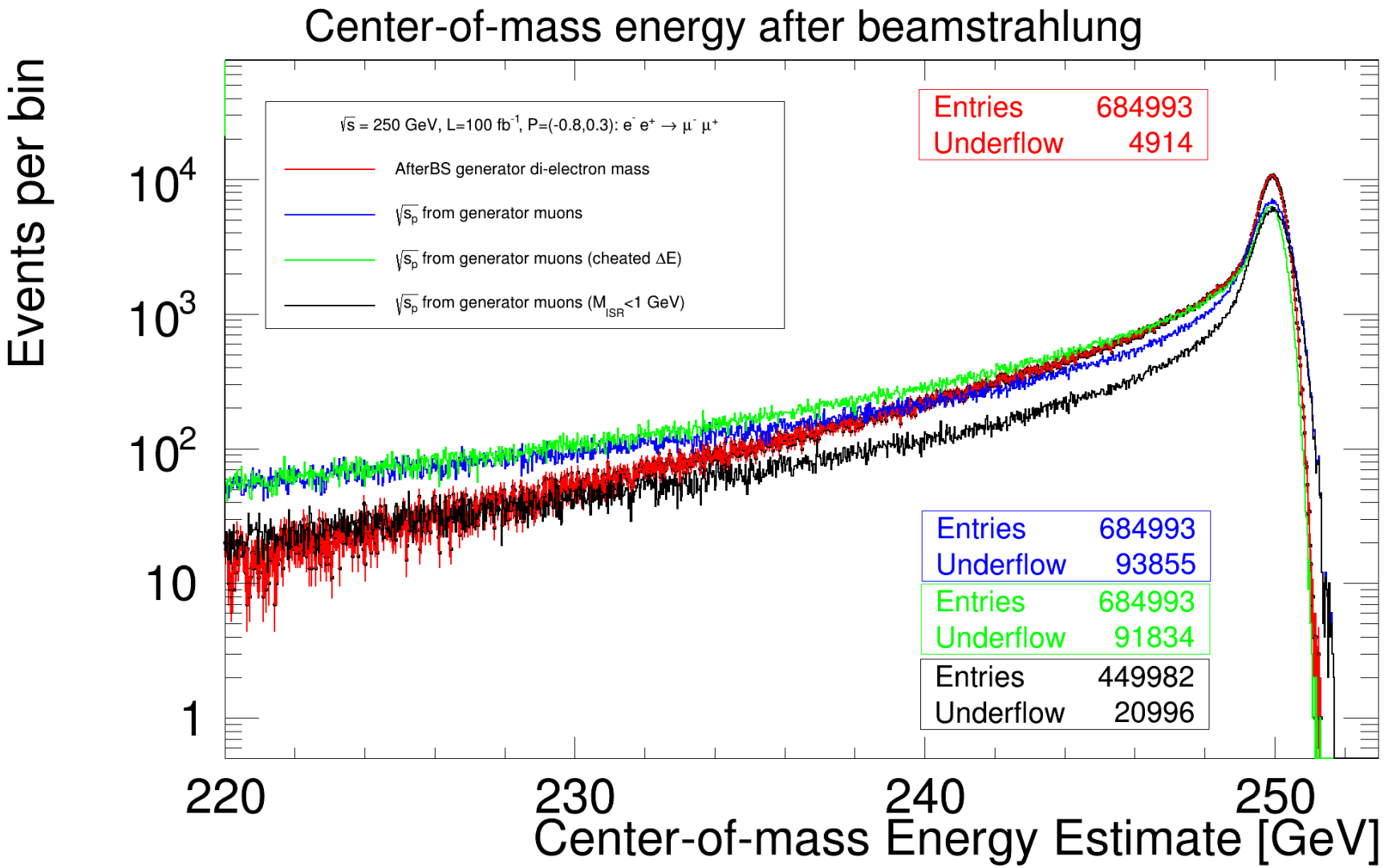
- This is the generator-level $\sqrt{s_p}$ calculated from the 2 muons
- For events with ISR photon system mass < 1 GeV
- Looks like the p_z issue dominates

$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1698 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

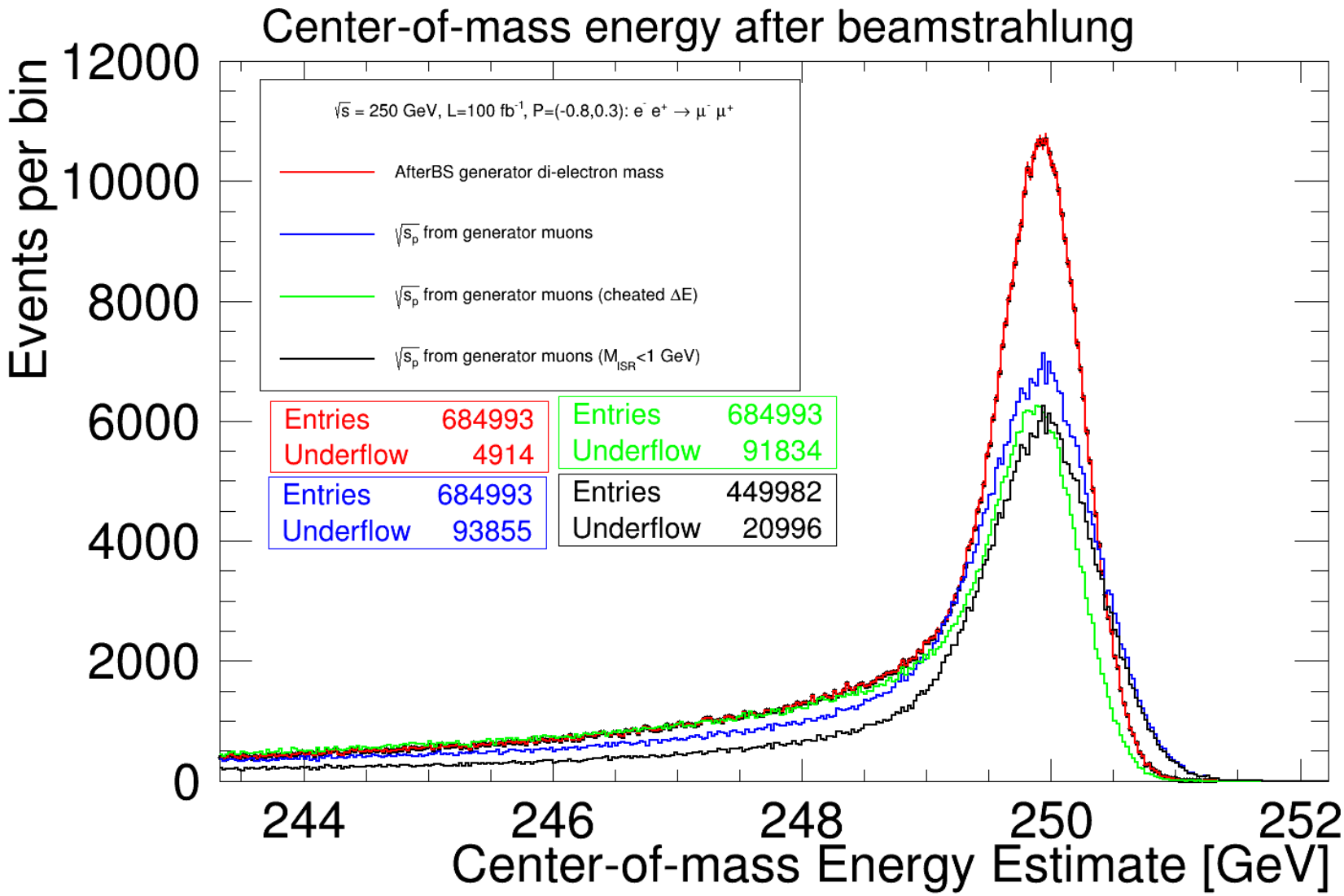
Comparisons I (After BS) Linear



Comparisons II (After BS) Log

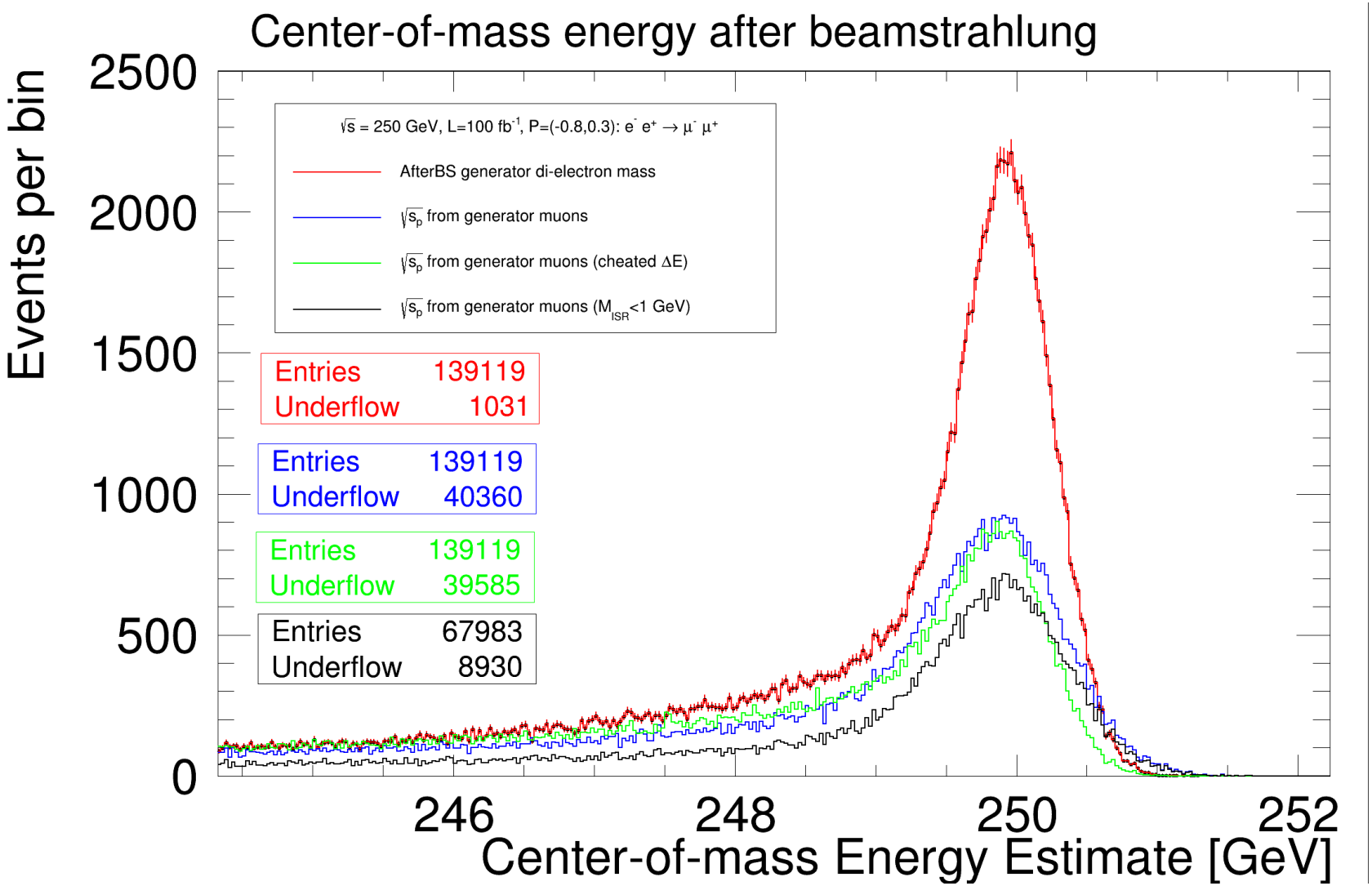


Comparisons III (After BS) Linear Zoomed



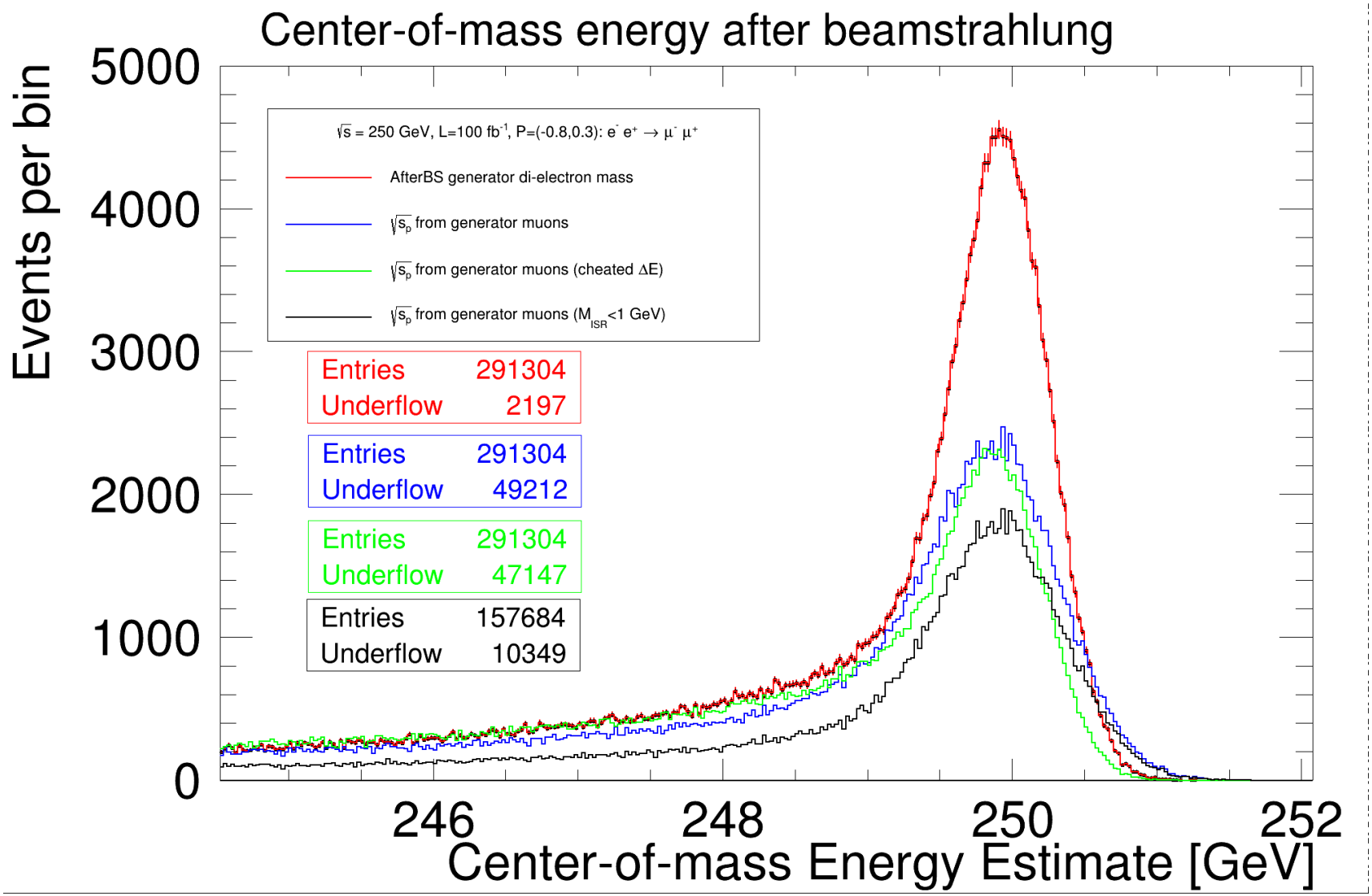
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III Low Dimuon Mass (After BS) Zoomed



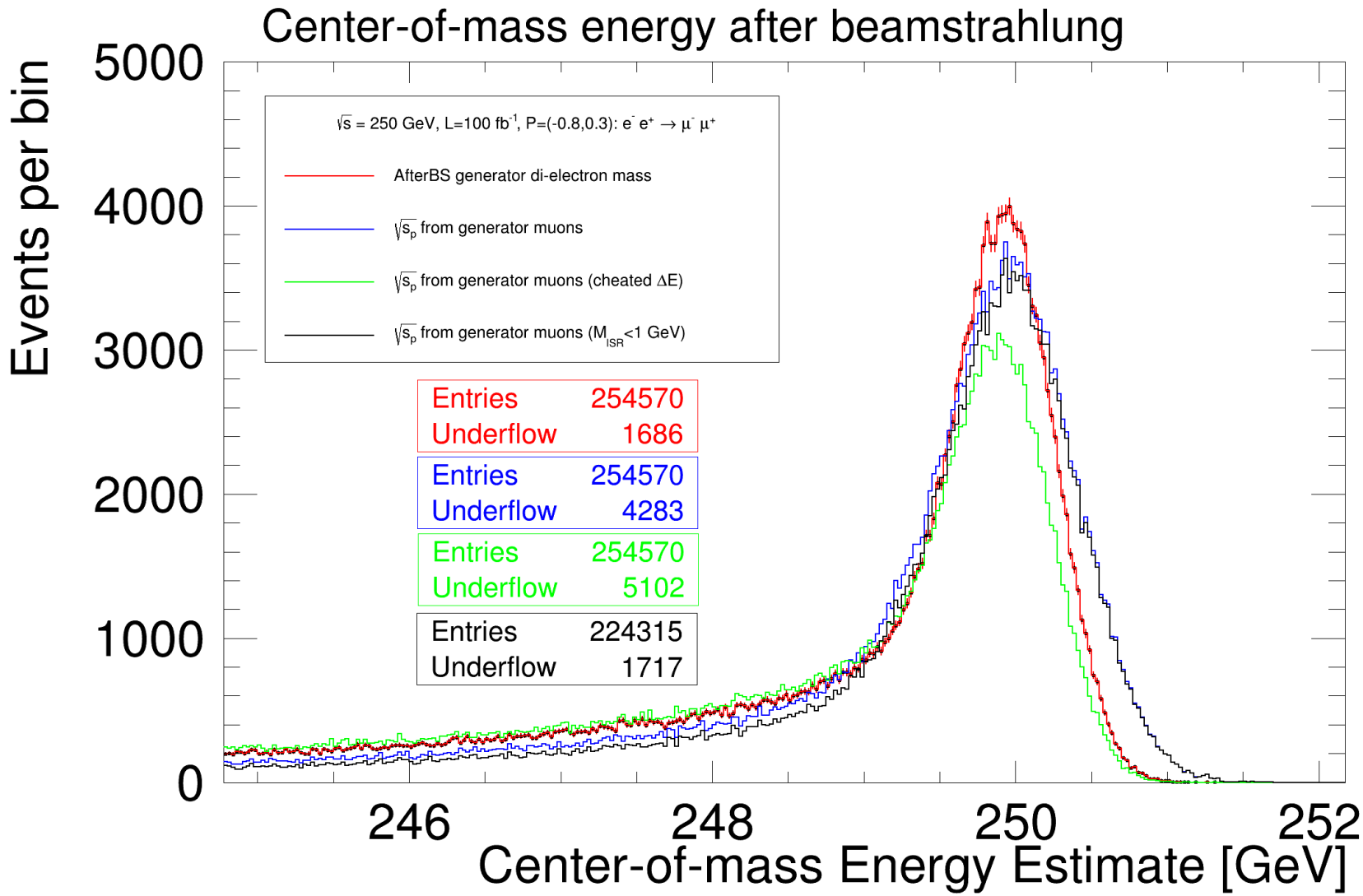
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III Medium Dimuon Mass (After BS) Zoomed



Note: Underflow statistics still refer to < 220 GeV.

Comparisons III High Dimuon Mass (After BS) Zoomed

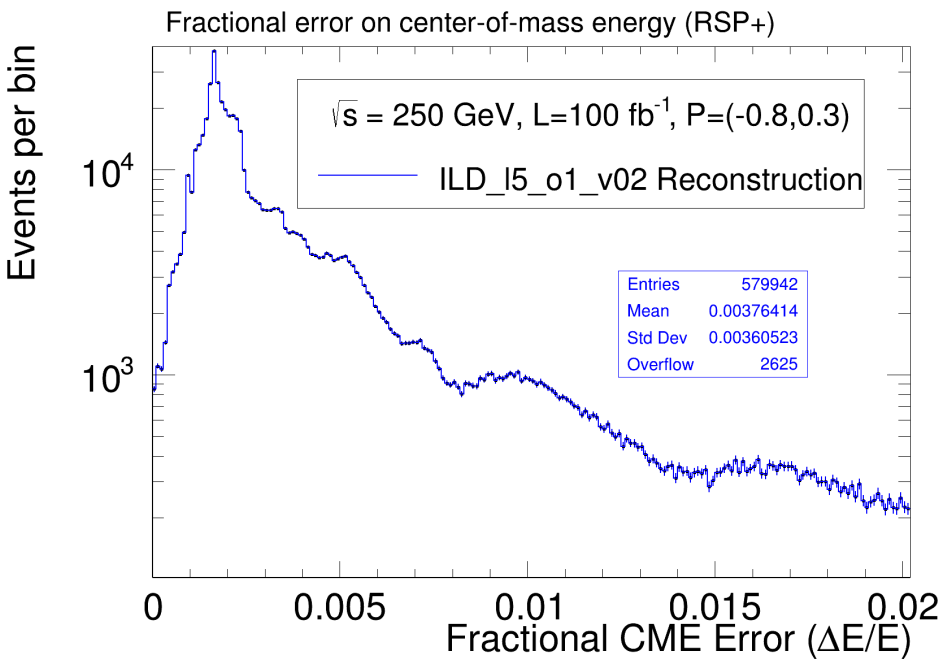


Note: Underflow statistics still refer to < 220 GeV.

Event Selection Requirements

Currently rather simple.
Use latest ILD simulation/reconstruction at 250 GeV.

- Require at least two PFOs identified as muons in the PandoraPFO collection.
- At least one opposite sign pair of muons (dimuon).
- Select the highest mass OS dimuon as the dimuon candidate of the event.
- Require uncertainty on estimated $\sqrt{s_p}$ of the event of less than 0.8% based on propagating PFO track-based error matrices.
- Categorize reconstruction quality as gold (<0.15%), silver ([0.15, 0.30]%), bronze ([0.30, 0.80]%)



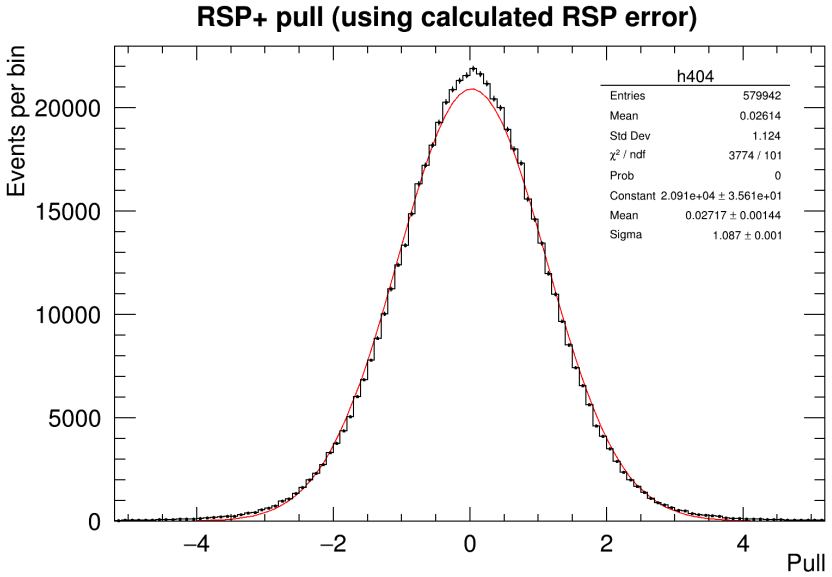
Selection efficiencies for (80%/30%) beam polarizations:

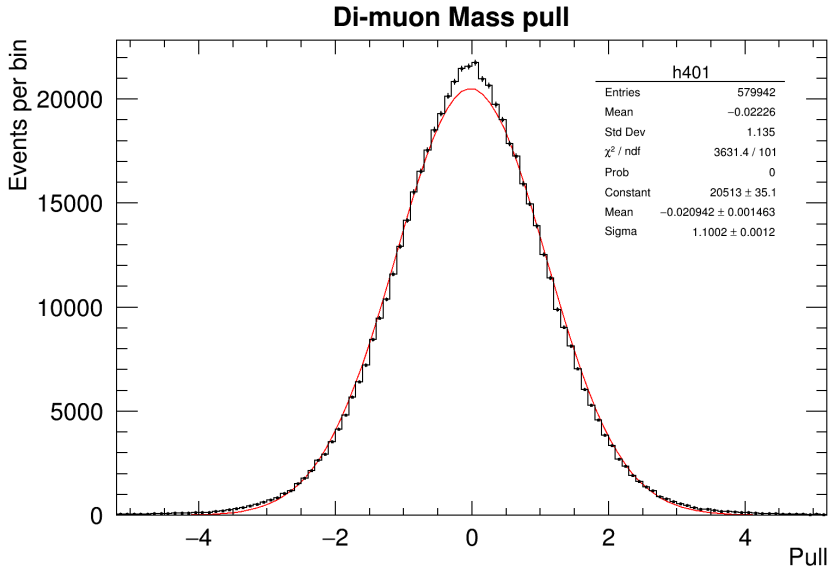
- $\varepsilon_{-+} = 72.73 \pm 0.05$ %
- $\varepsilon_{+-} = 70.21 \pm 0.06$ %
- $\varepsilon_{--} = 72.42 \pm 0.05$ %
- $\varepsilon_{++} = 70.60 \pm 0.06$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is rather small and of no import for the $\sqrt{s}$ peak region).

Dimuon Pull Distributions

- Pull $\equiv$ (meas - true)/error.
- Track-based estimates of the errors on both the $\sqrt{s}_p$ quantity (left) and the di-muon mass (right) agree well with the modeled uncertainties for reconstructed dimuon events.





- In both cases the fitted rms over this range is about 10% larger than ideal. Central range well described. Suspect tails should be non-Gaussian given the non-Gaussian tails of multiple scattering.
- In practice this is pretty good!

Vertex Fit (W.I.P. - not included in following plots)

Given that the track errors are well modeled and the 2 muons should originate from a common vertex consistent with the interaction point, we can perform:

- Vertex Fit: Constrain the two tracks to a common point in 3-d.
- Beam-spot Constrained Vertex Fit.

The ILC beam-spot size is $(\sigma_x, \sigma_y) = (516, 7.7)$ nm, $\sigma_z = 0.202$ mm

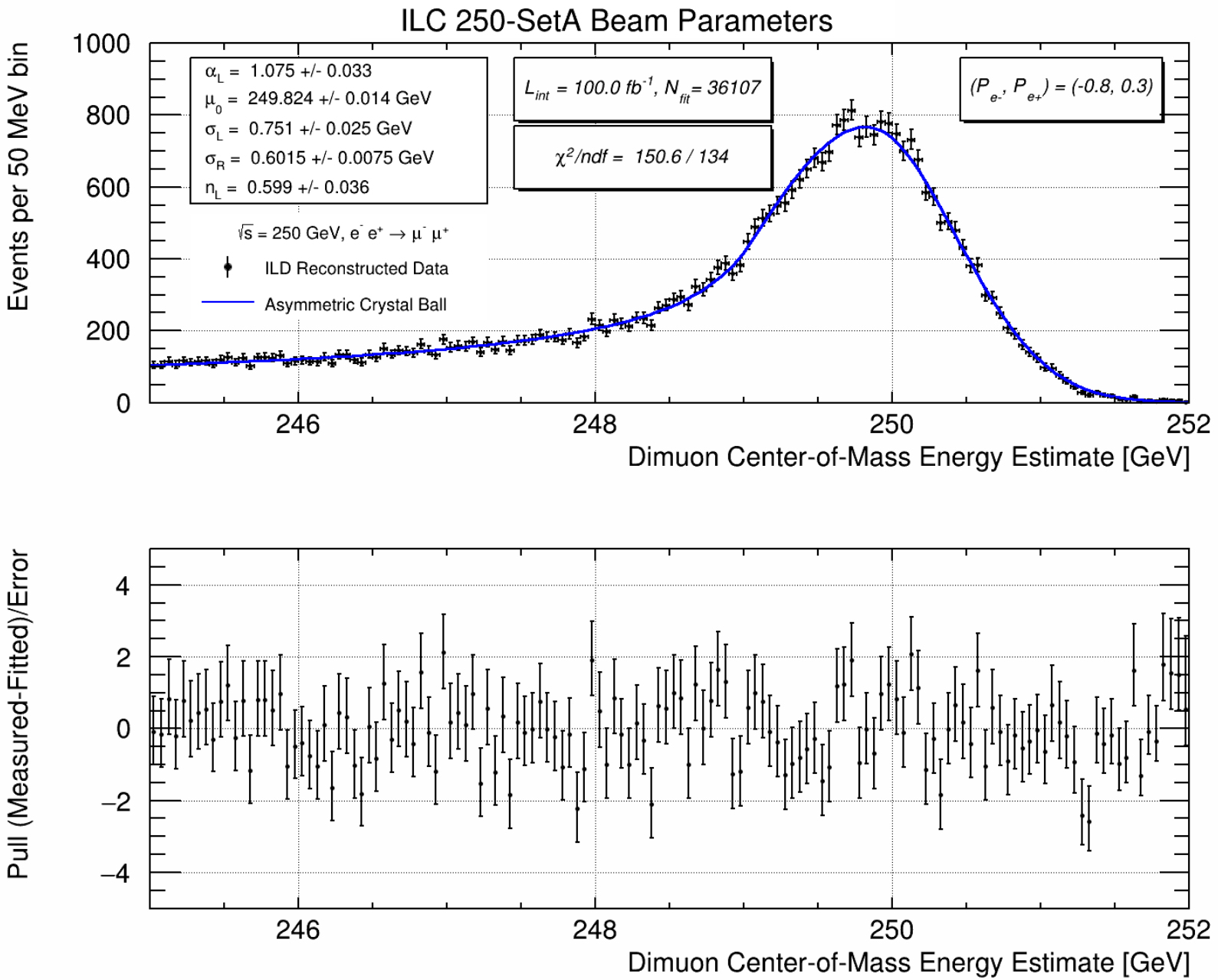
- Vertex fit along same lines as AWLC2014 talk has been re-implemented using the fully simulated data for technical checks. χ^2 distributions look good.
- Also have imposed beam-spot constraints.
- Need to port this directly to ilcsoft and/or use existing processors.

What good is this?

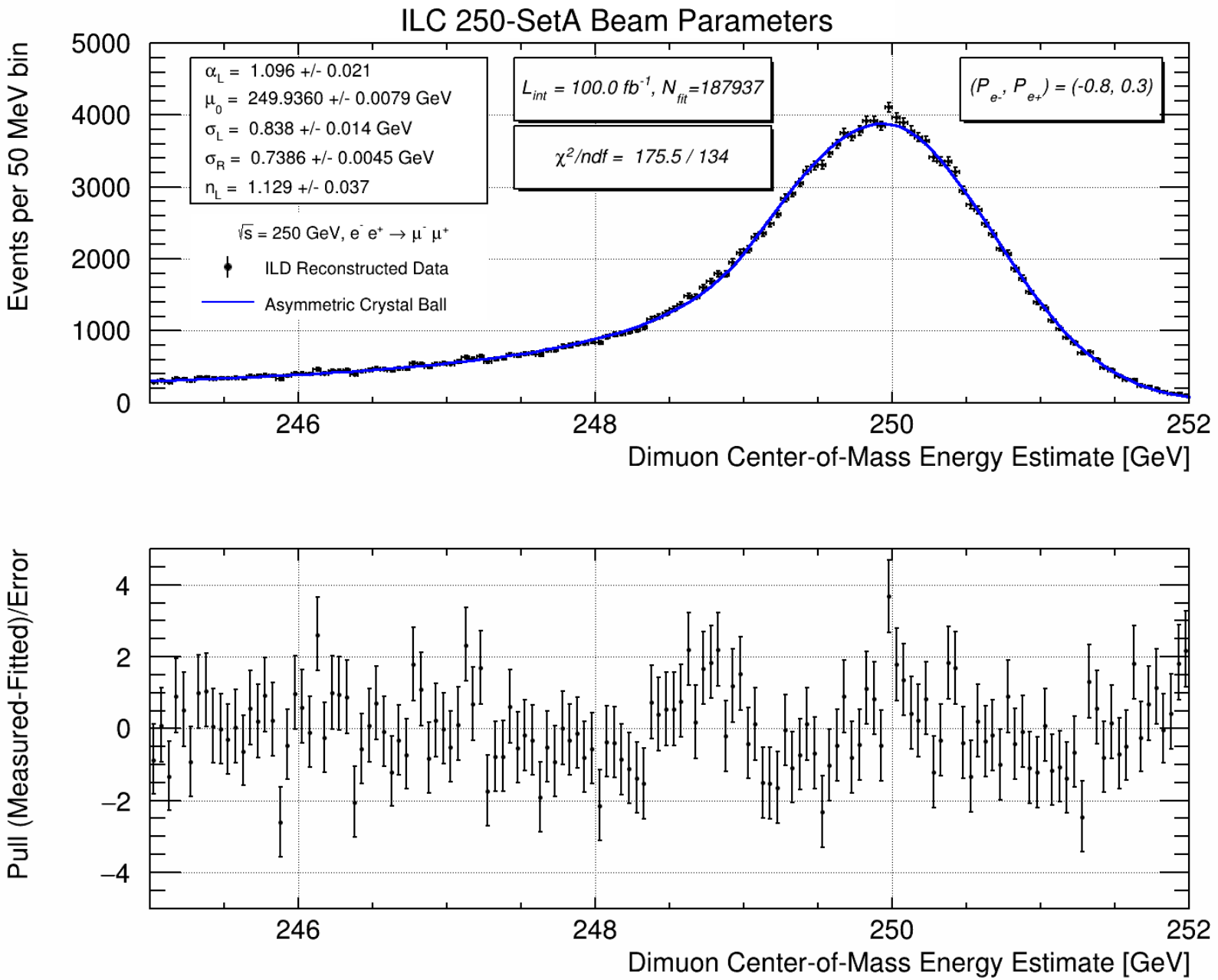
- Residual background rejection (examples $\tau^+\tau^-$ and $\mu^\pm\tau^\mp\nu\bar{\nu}$)
- Additional handle for rejecting or deweighting mis-measured events
- Expect some improvement in di-muon kinematic quantities
- Should also be used for $\mathrm{H} \to \mu^+\mu^-$ and for ZH recoil.

Note: simulated data does not simulate the transverse beam-spot ellipse.

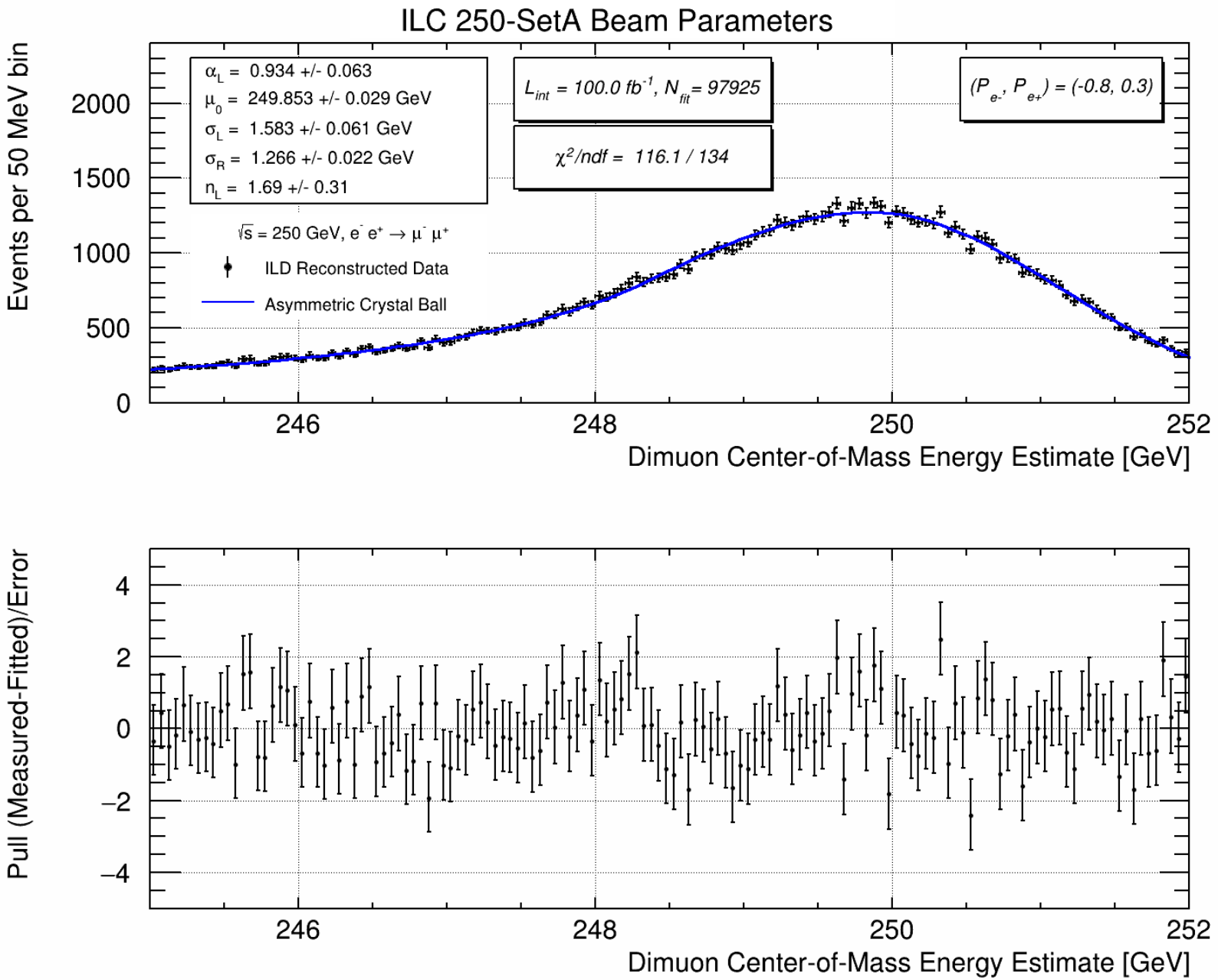
Gold Quality Dimuon PFOs (After BS)



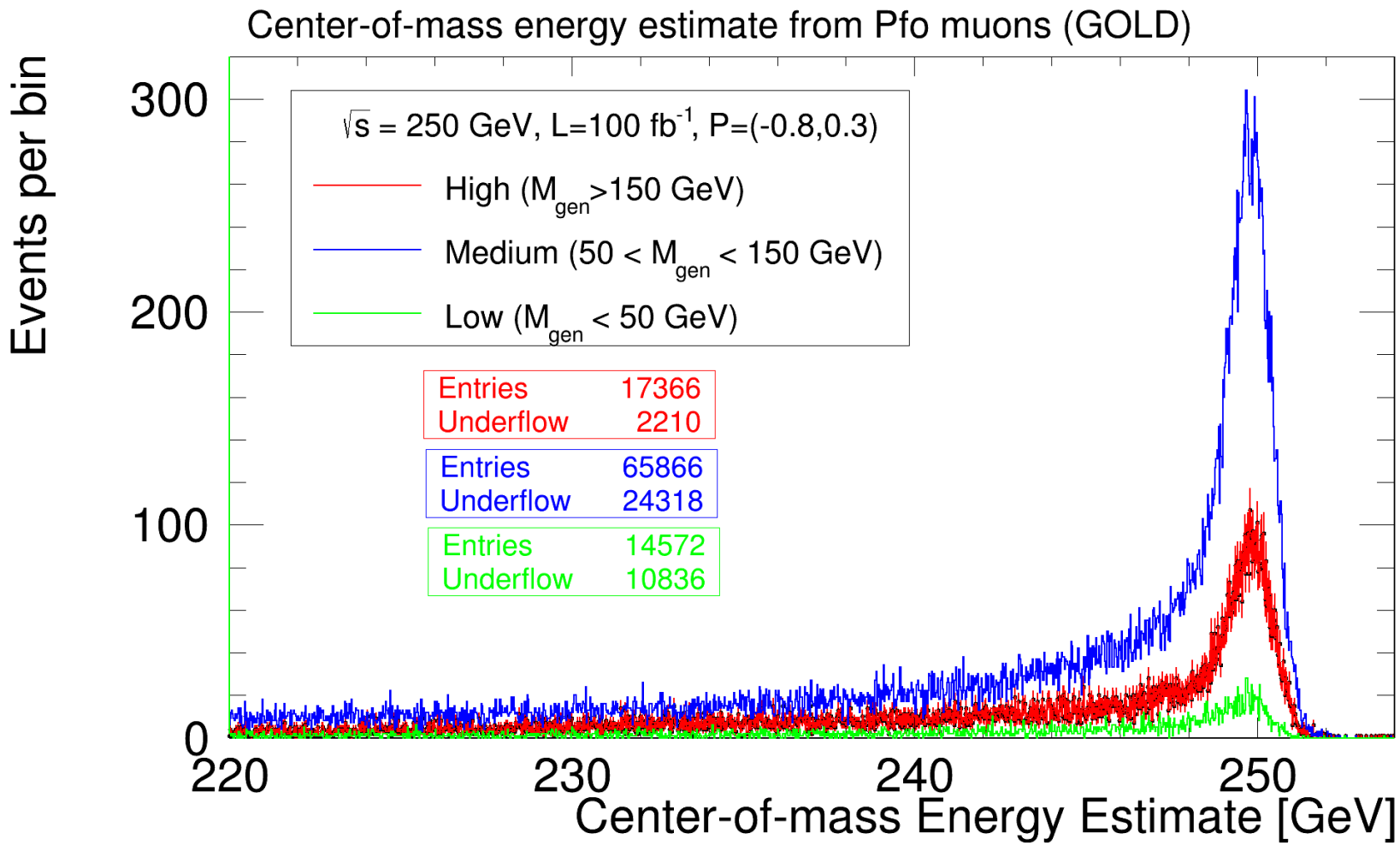
Silver Quality Dimuon PFOs (After BS)



Bronze Quality Dimuon PFOs (After BS)

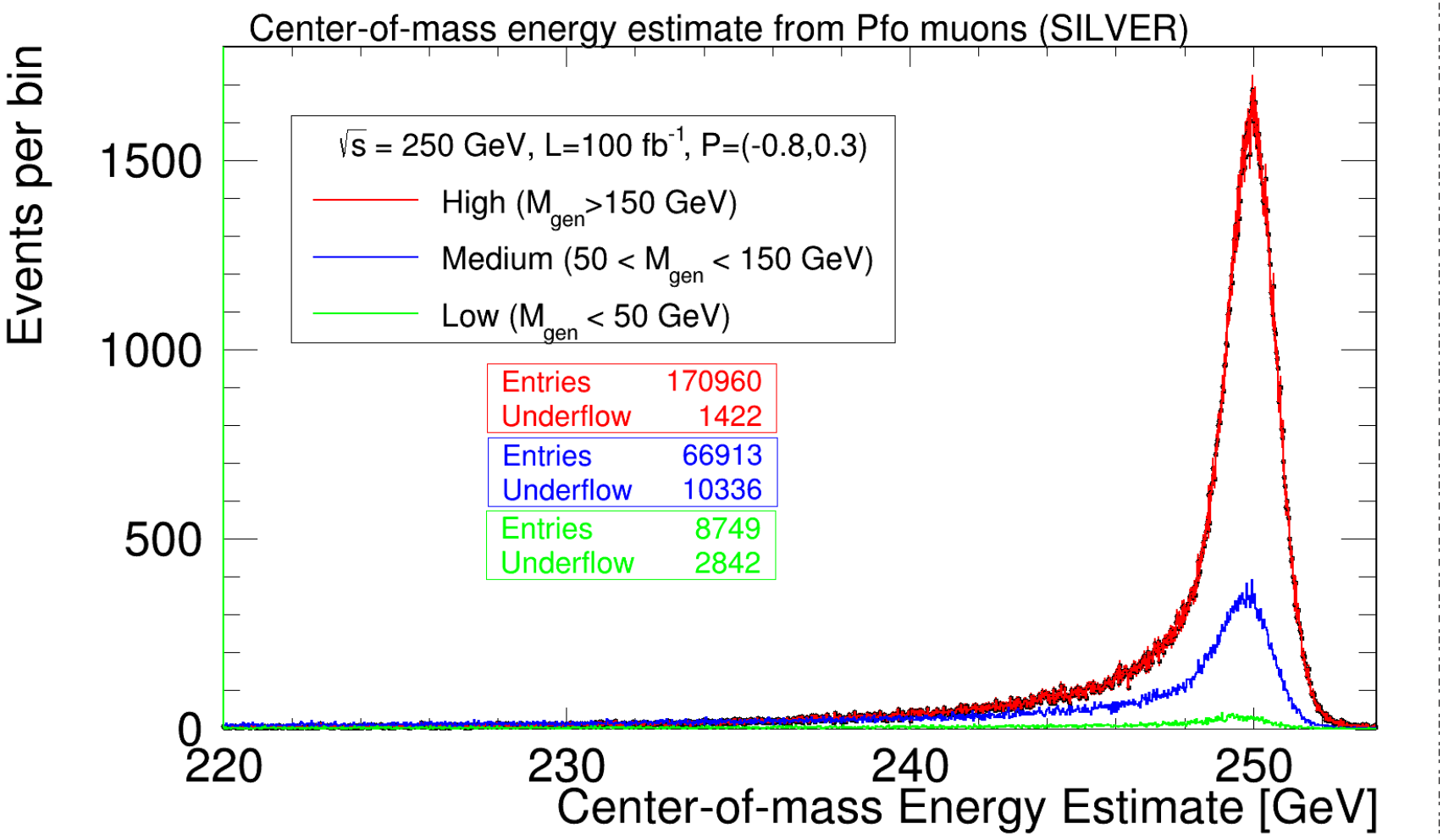


Gold Quality Dimuon PFOs (After BS)



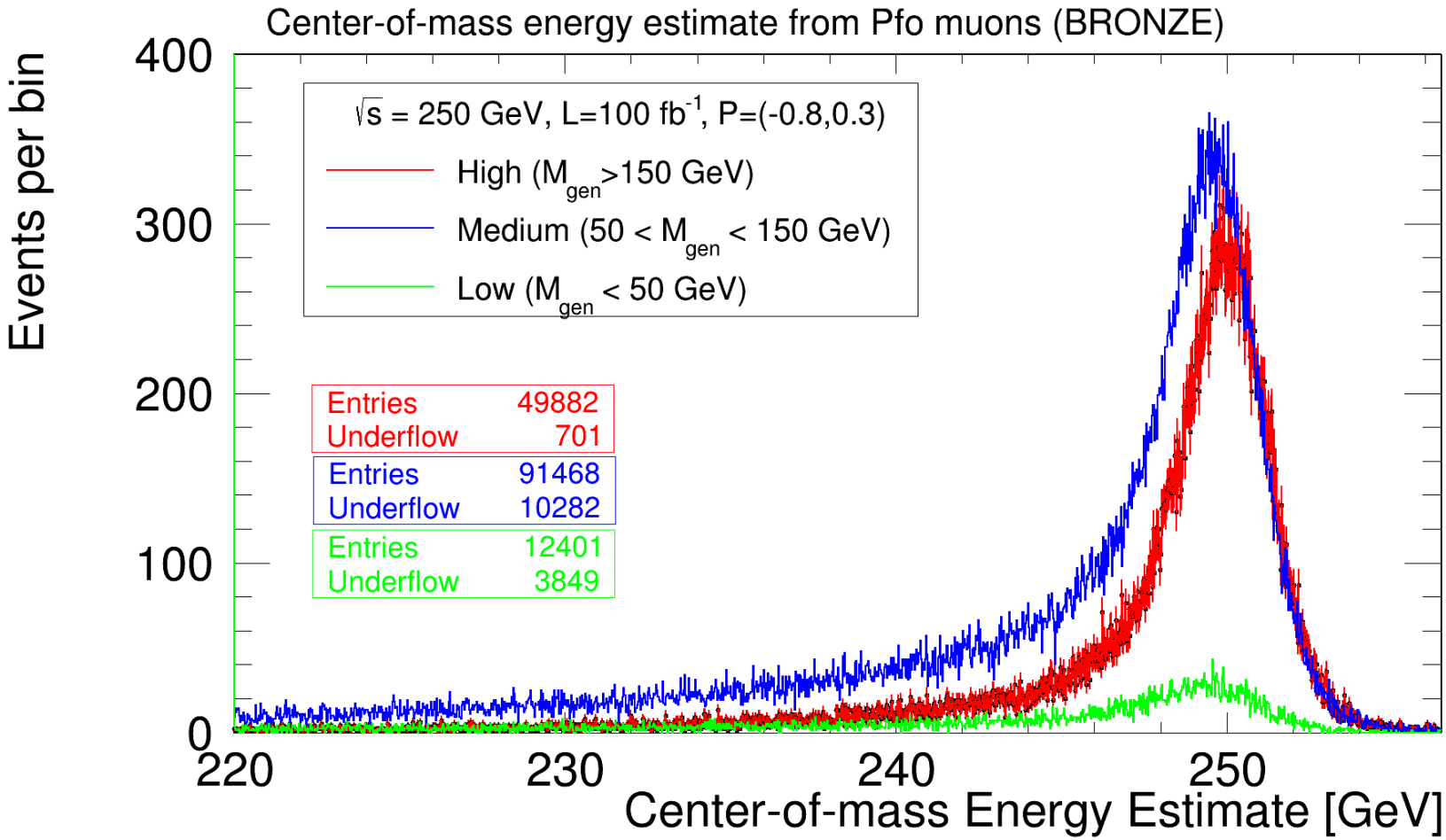
Mostly Z-like

Silver Quality Dimuon PFOs (After BS)



Mostly high mass

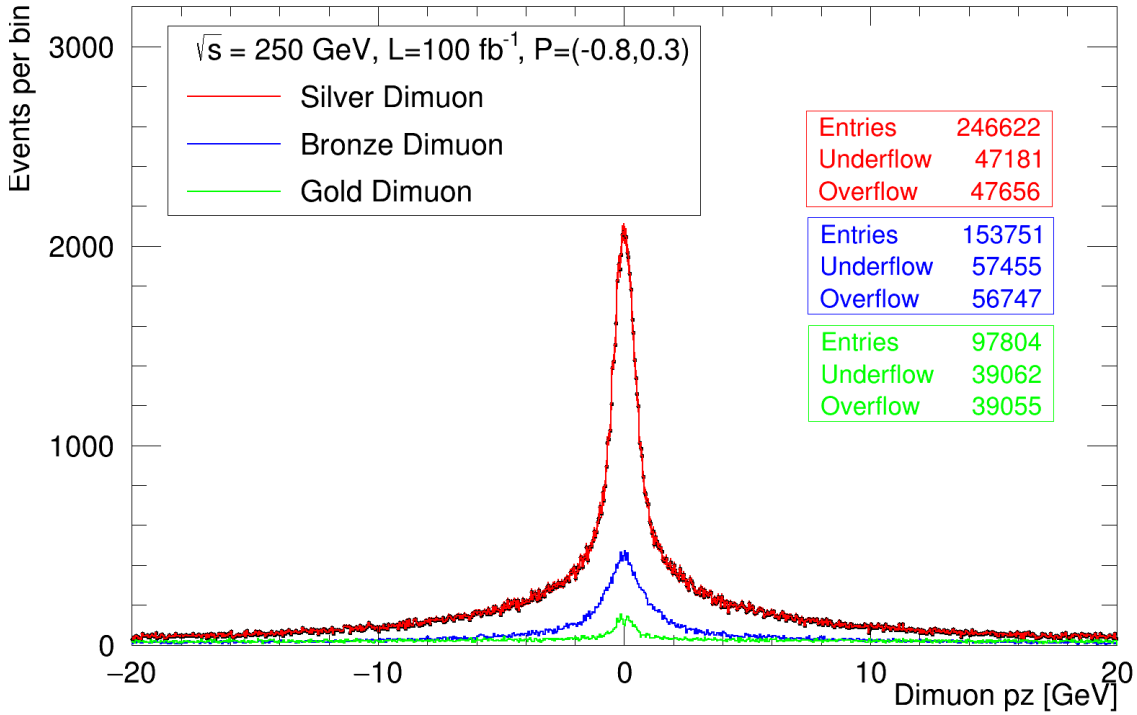
Bronze Quality Dimuon PFOs (After BS)



Mix of high mass and Z-like. Z-like with one forward muon?

Measuring the z-imbalance

Likely can use both p_z and acolinearity (for high mass events).



Will be sensitive to energy asymmetries. The suggestion by Tim Barklow in 2005 (which I now understand) is to measure

$$E_{\mu^+\mu^-} + p_z(\mu^+\mu^-) = (E_+ + E_-) + (E_- - E_+) = 2E_-$$

$$E_{\mu^+\mu^-} - p_z(\mu^+\mu^-) = (E_+ + E_-) - (E_- - E_+) = 2E_+$$

Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Current Status

- To be frank - still in flux.
- Prior approach. Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error.
- Following a similar approach, but using estimates of the statistical error on μ_0 for 4-parameter Crystal Ball fits to fully simulated data in the various resolution categories (example gold, silver, bronze fits in backup slides).
- See table on next slide with these estimates.

Current Thinking

- The luminosity spectrum and absolute center-of-mass energy are the same problem or at least very related.
- Likely need either a convolution fit or a reweighting fit.
- Currently trying to proceed with a convolution fit where we parametrize the underlying (E_-, E_+) distribution, and model quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR and FSR) and cross-section effects.

$\sqrt{s}$ Sensitivity Estimate

$\mathcal{L}$ (P_{e^-}, P_{e^+})	Gold	Silver	Bronze	G+S+B
$900\,fb^{-1}$ $(-80\%, +30\%)$	11.1	4.8	16.2	4.3
$900\,fb^{-1}$ $(+80\%, -30\%)$	12.0	5.5	18.1	4.8
$100\,fb^{-1}$ $(-80\%, -30\%)$	42.7	19	64	16.5
$100\,fb^{-1}$ $(+80\%, +30\%)$	45.6	21	68	18.3
$2\,ab^{-1}$	7.9	3.5	11.7	3.1

4-parameter Crystal Ball fit. Note this fit is pessimistic (assumes all shape parameters needs to be adjusted) but is optimistic in the sense that it does not have as many parameters as are needed.

Fractional errors on μ_0 parameter (mode of peak) in parts per million (ppm).

Measuring M_Z Using Old Study Methods (ECFA 2013)

M_Z from $m_{\mu\mu}$

Revisited old study of $\sqrt{s_p}$ at $\sqrt{s} = 250, 350, 500, 1000$ GeV. Used smeared MC.

Fitted $m_{\mu\mu} \in [75, 105]$ GeV with sum of two Voigtians.

Scaled statistical uncertainties on the peak parameter, m_z, to full ILC program.

$\sqrt{s}$ [GeV]	$\int \mathcal{L}$ [fb^{-1}]	Polarization	Sharing	ΔM_Z [MeV]
250	2000	80/30	0.45, 0.45, 0.05, 0.05	1.20
350	200	80/30	0.675, 0.225, 0.05, 0.05	5.99
500	4000	80/30	0.40, 0.4, 0.1, 0.1	2.55
1000	8000	80/20	0.4, 0.4, 0.1, 0.1	5.75
				1.05 MeV

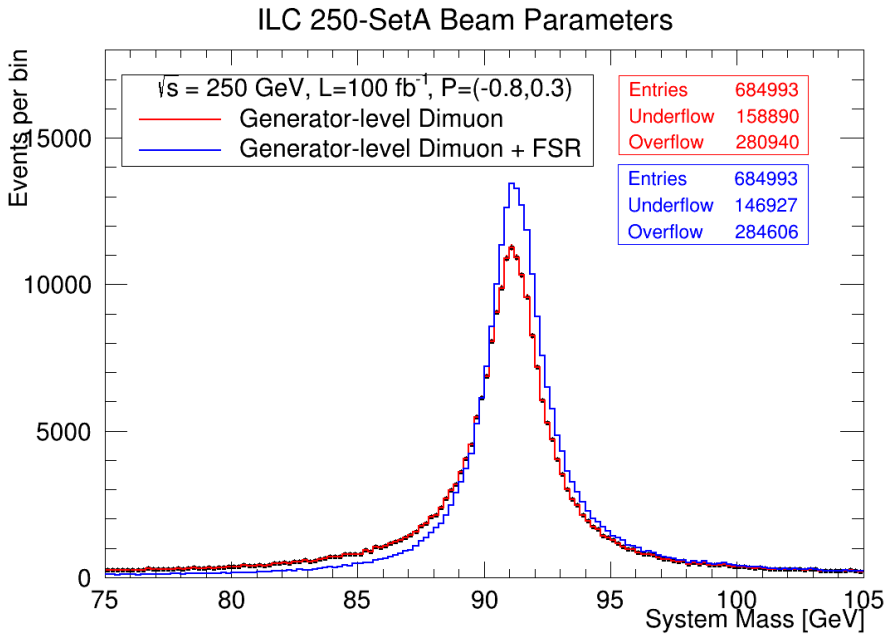
Current PDG uncertainty on m_z = 2.1 MeV

Use TDR beam parameters

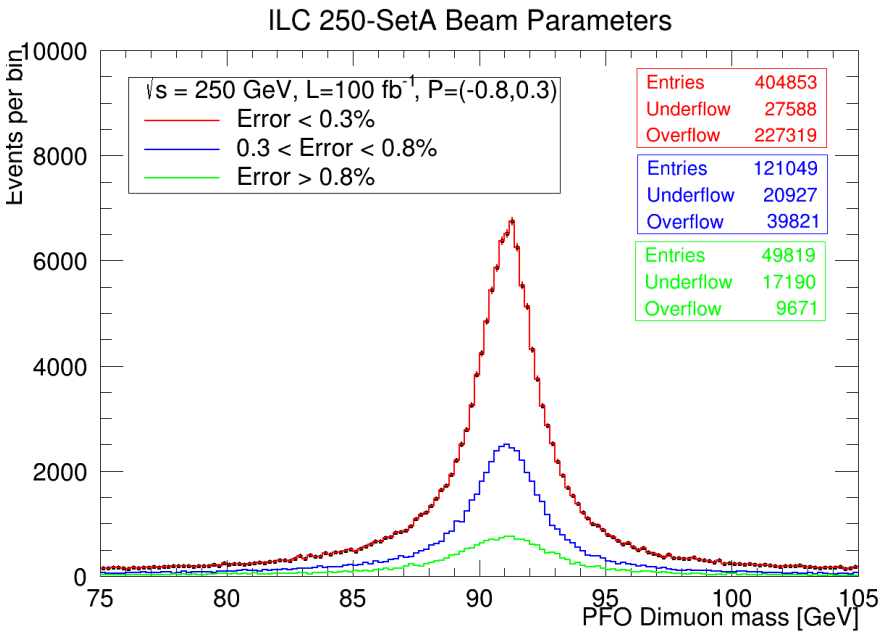
Statistical only. FSR makes effective BW width larger and shifts the peak.

Measuring M_Z Today

More statistics - fully simulated.



Adding in FSR photon(s) reduces the peak width to be consistent with Γ_Z. Improves statistical sensitivity by 10–20%.



Di-muon mass resolution is much less than Γ_Z/M_Z. Prior sensitivity estimates will be reasonable.

Main systematics: momentum-scale, FSR modeling.

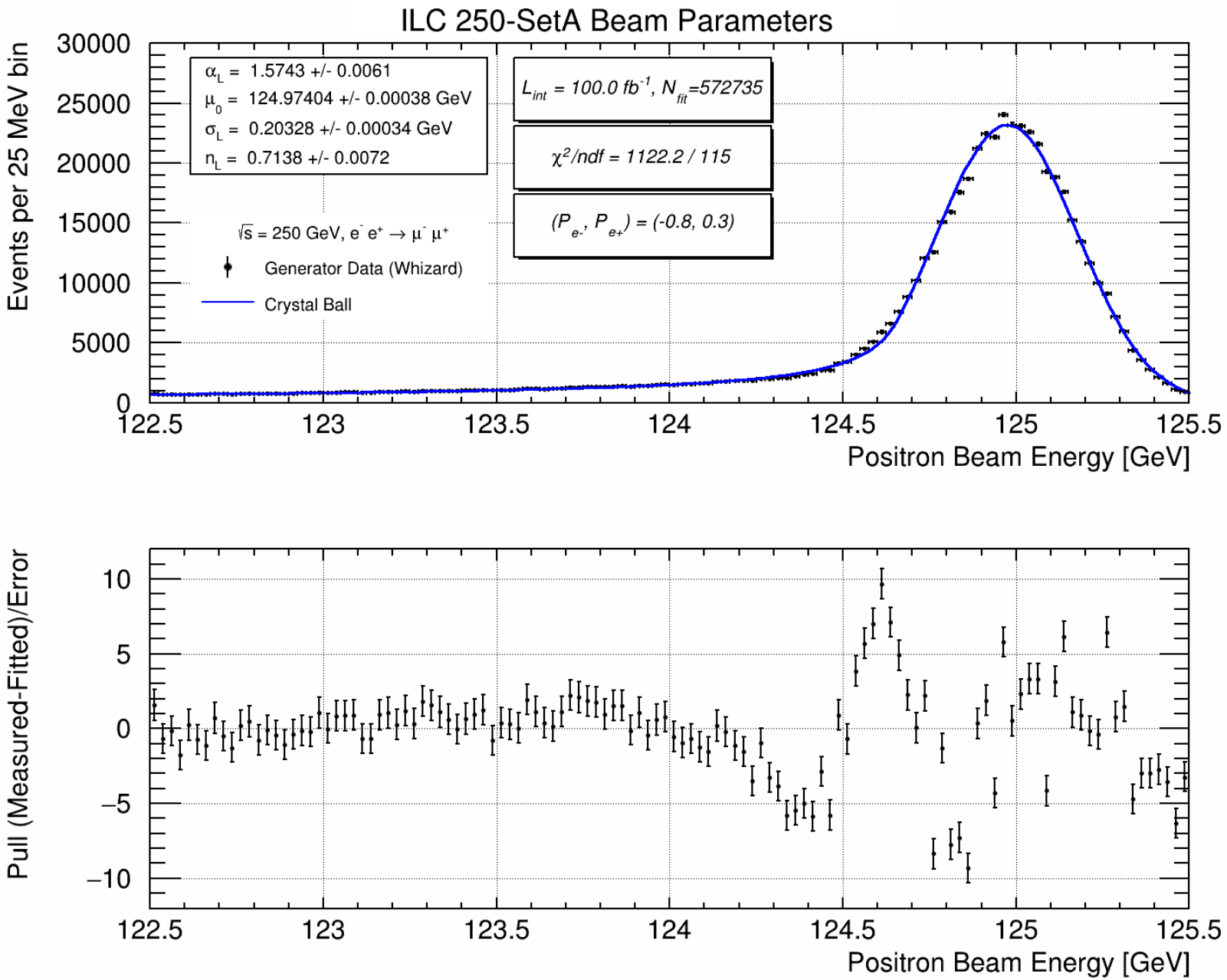
Summary and Outlook

- ILC tracking detectors have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events).
- At $\sqrt{s} = 250$ GeV, sufficient precision on $\sqrt{s}$ (sub 10 ppm) that this should not be a limiting factor for measurements such as M_{W}.
- Potential to improve M_{Z} by a factor of two using 250 GeV $\mu^+\mu^-$ data alone.
- Applying the same techniques to running at the Z-pole can enable a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge.

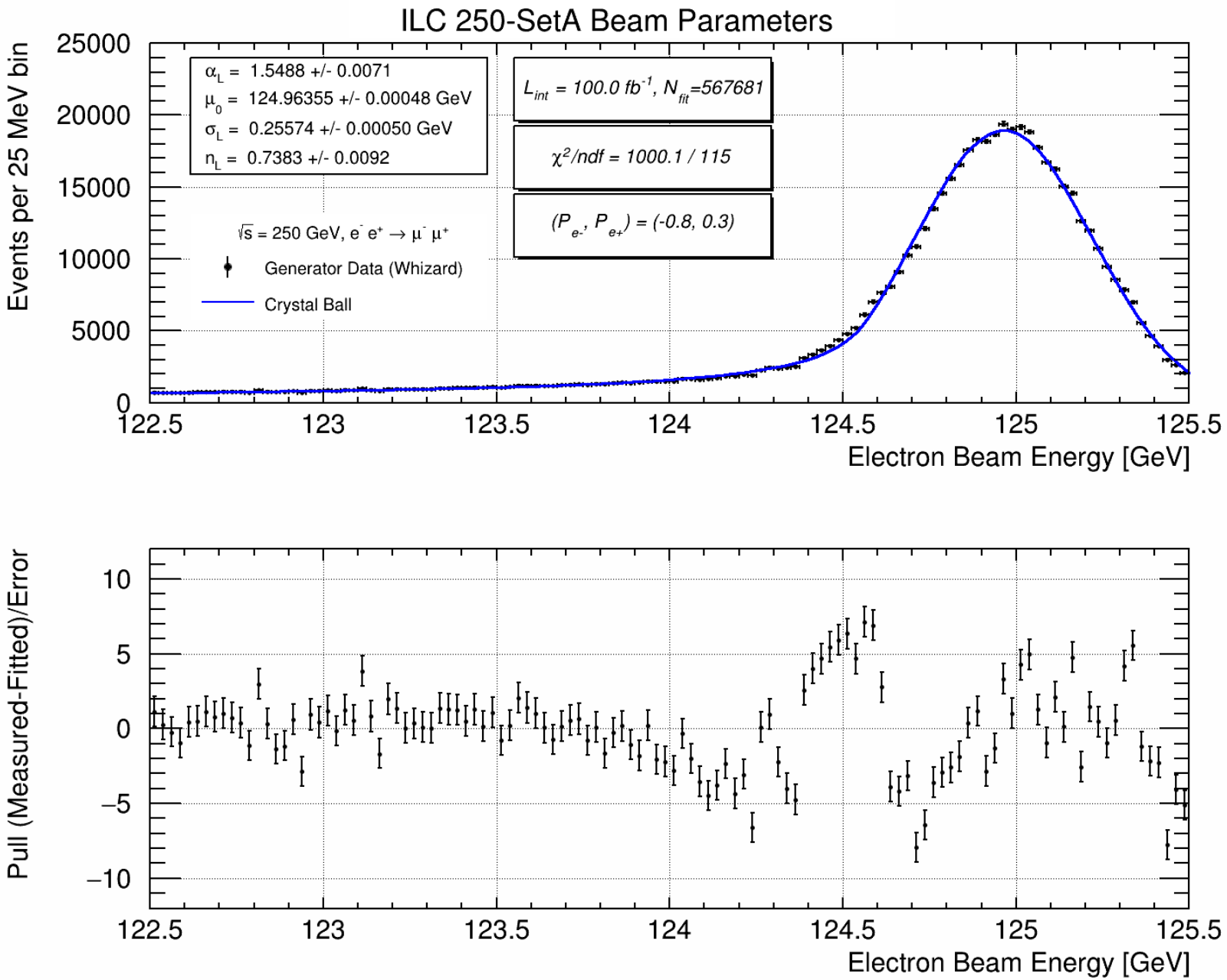
Backup Slides

- Most of these are 4-parameter Crystal Ball fits. Particularly for those with more sharply resolved features, the χ^2 is substantially worse than the 5-parameter asymmetric fits shown earlier.
- The fits generally need the additional σ_{R} parameter to describe the beam energy spread feature while σ_{L} accommodates the convolution of beam energy spread with soft beamstrahlung.
- On the other hand these 4-parameter fits may better represent the statistical error on the mode parameter when able to better constrain the shape of the distributions such as with external knowledge of the beam energy spread.

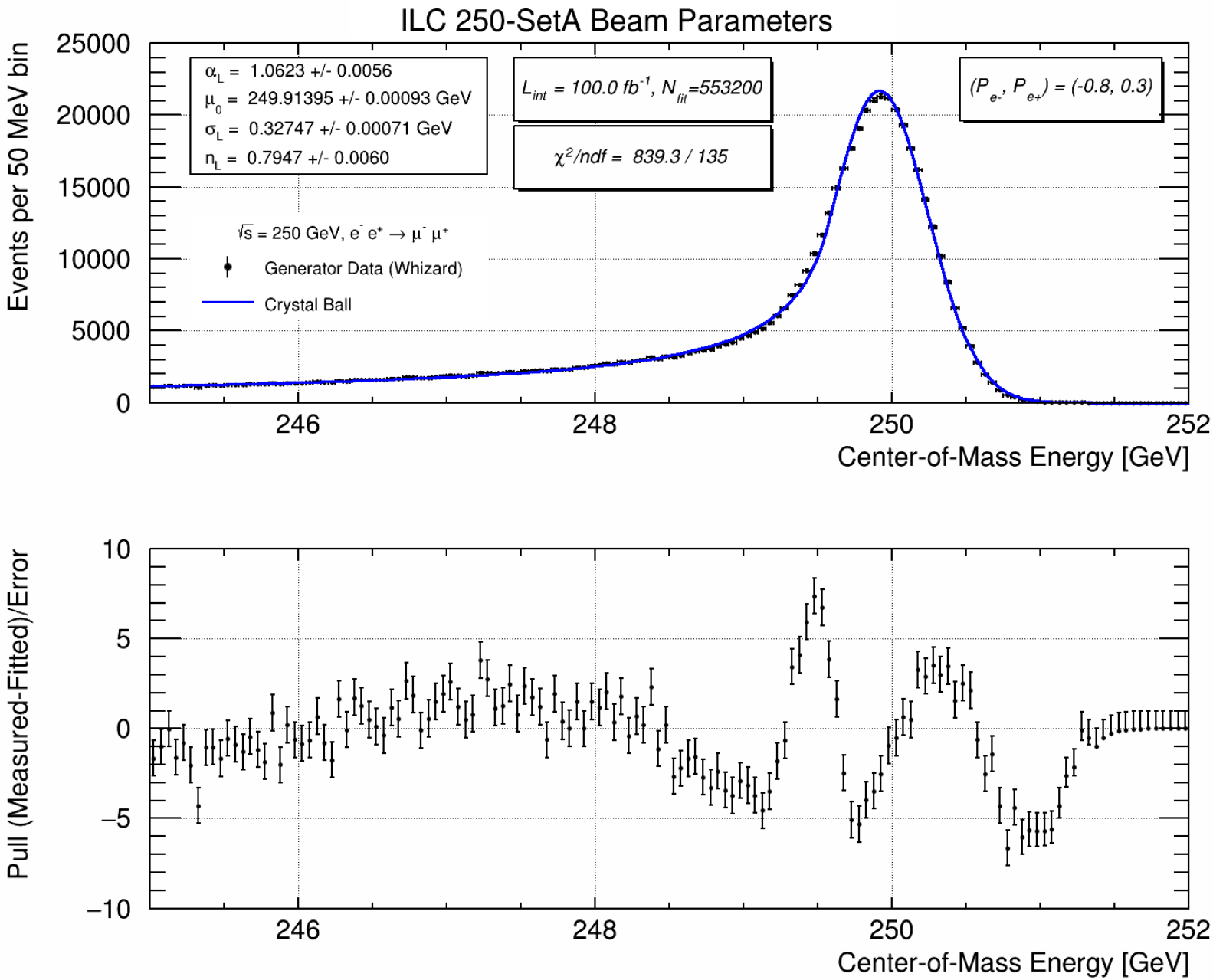
Positron Beam Energy (After Beamstrahlung)



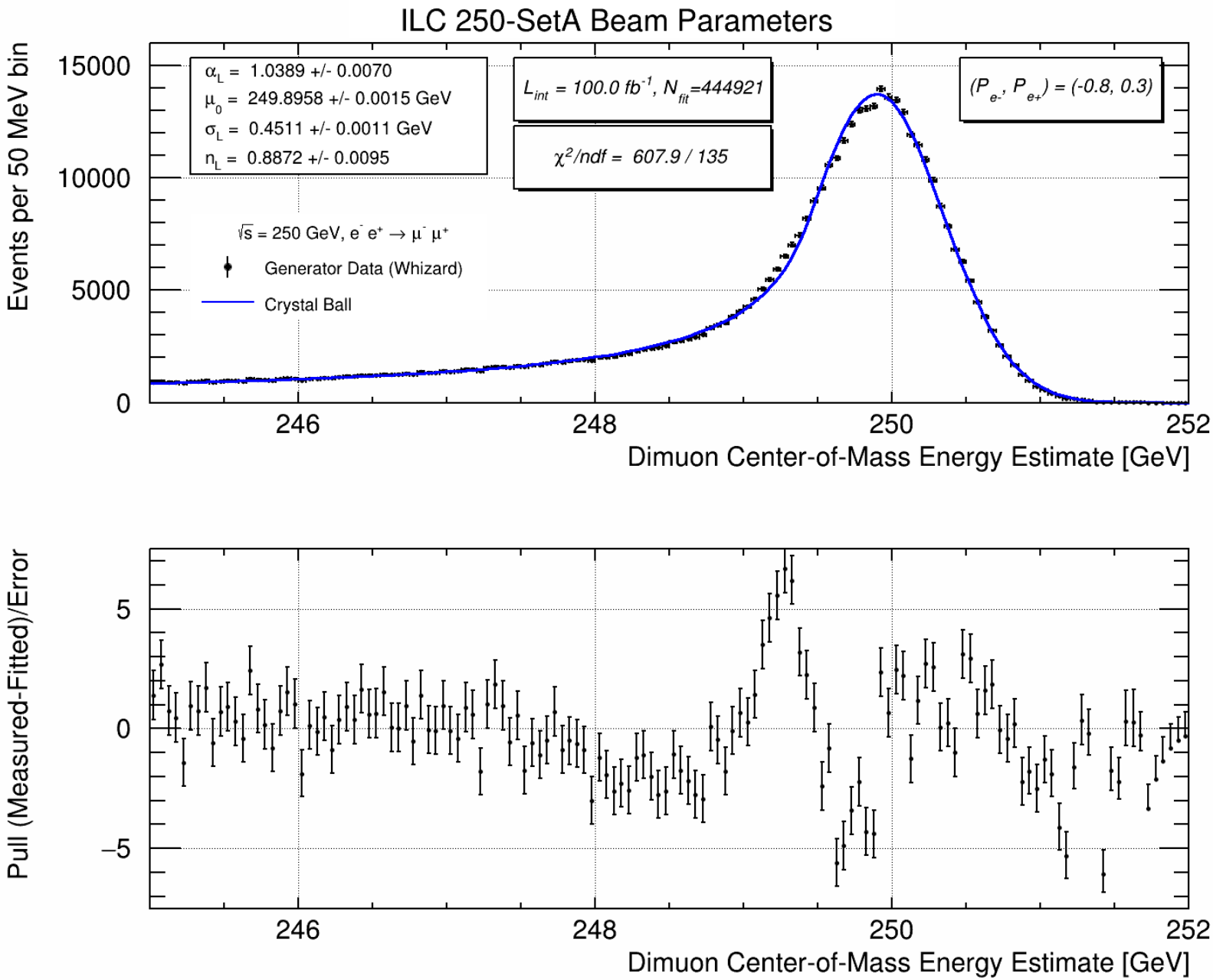
Electron Beam Energy (After Beamstrahlung)



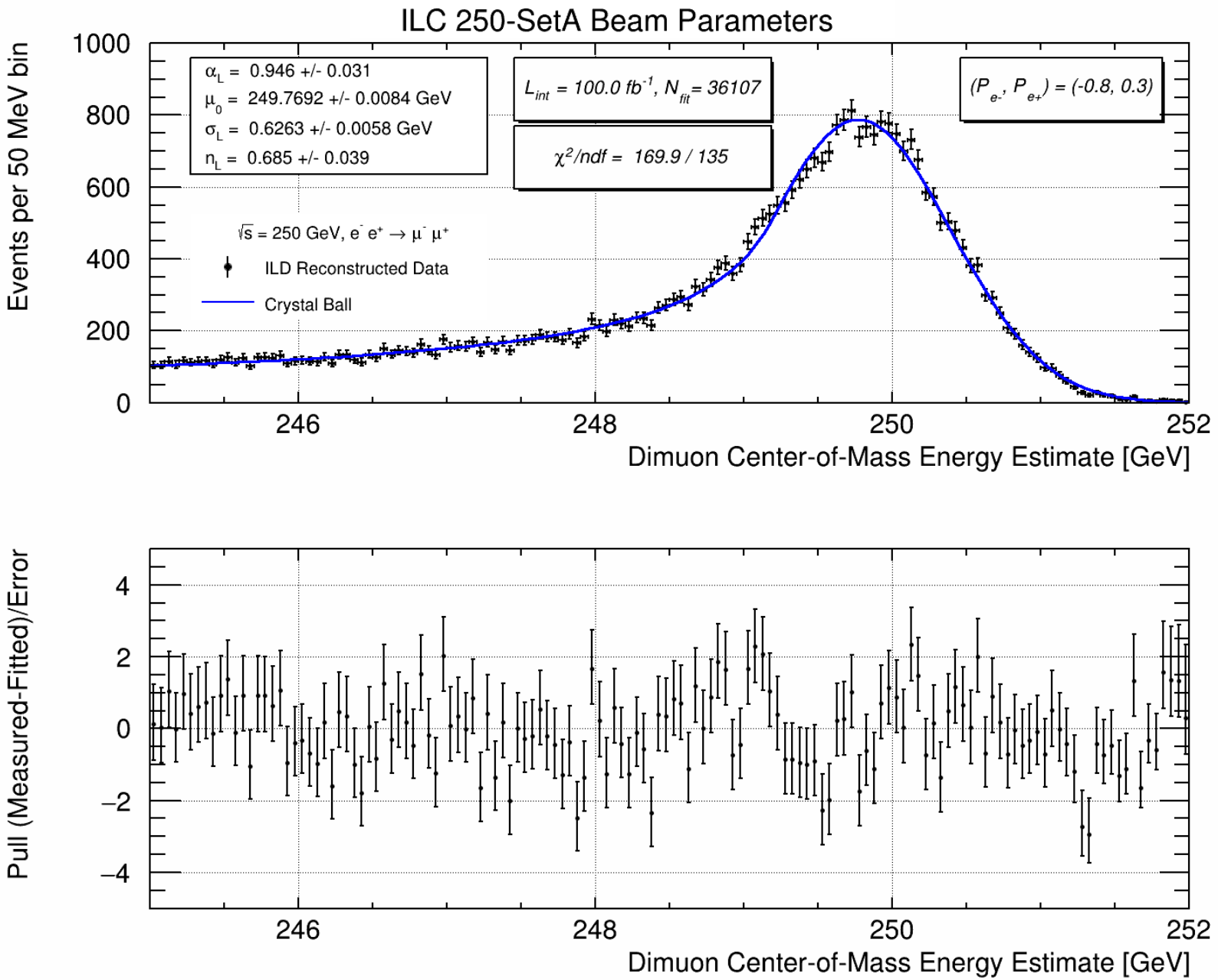
Center-of-Mass Energy (After Beamstrahlung)



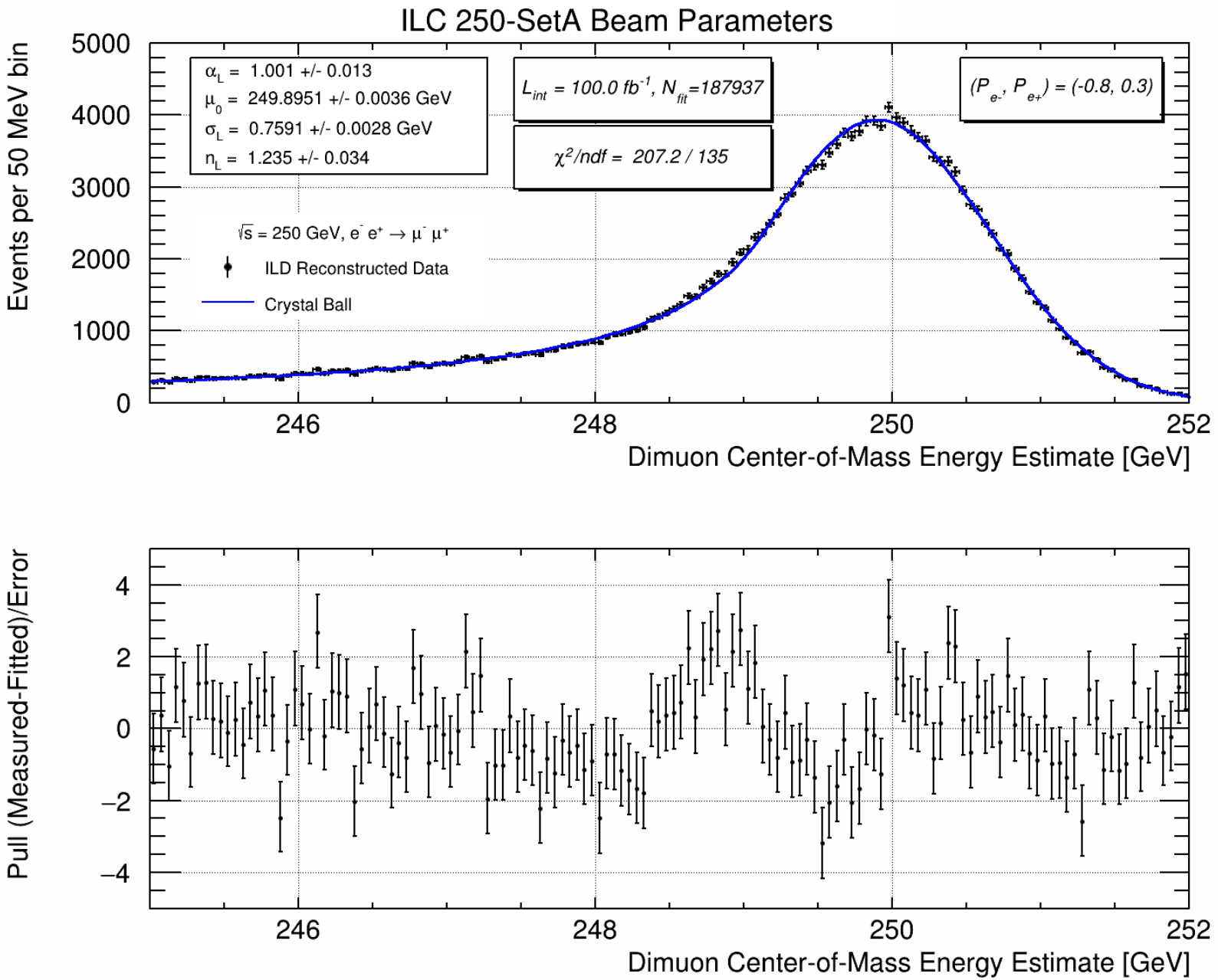
Dimuon Estimate of Center-of-Mass Energy (After BS)



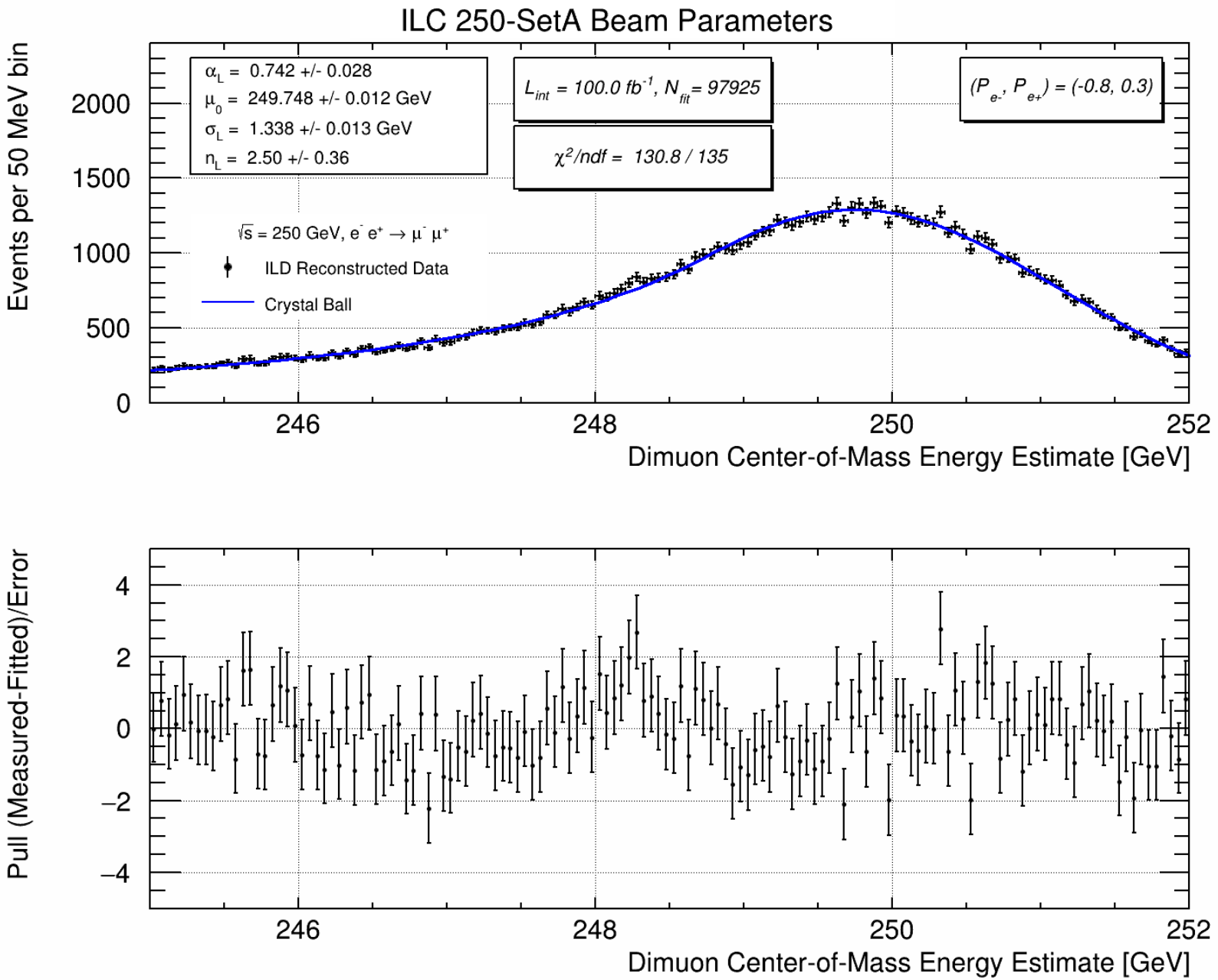
Gold Quality Dimuon PFOs (After BS)



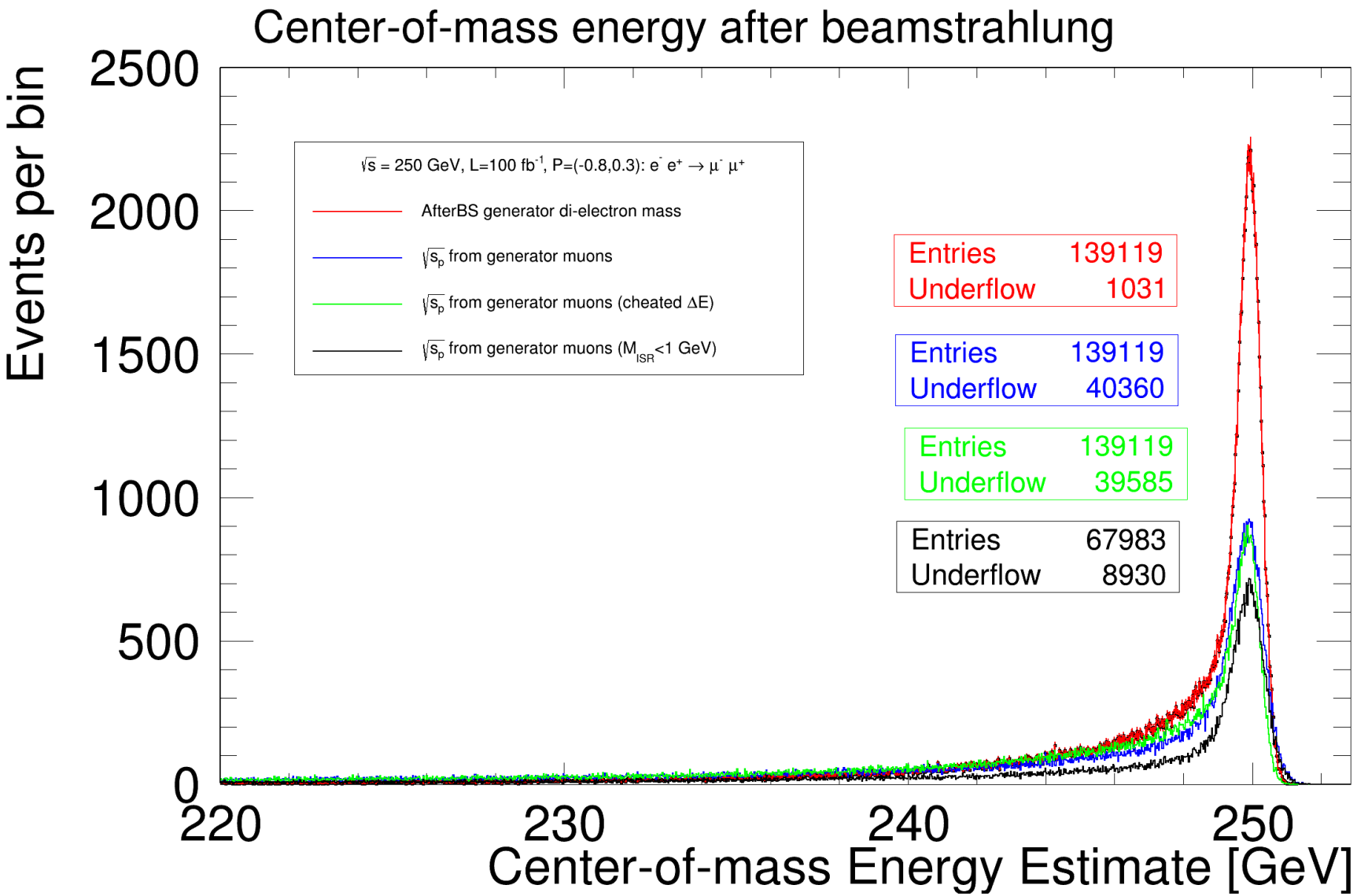
Silver Quality Dimuon PFOs (After BS)



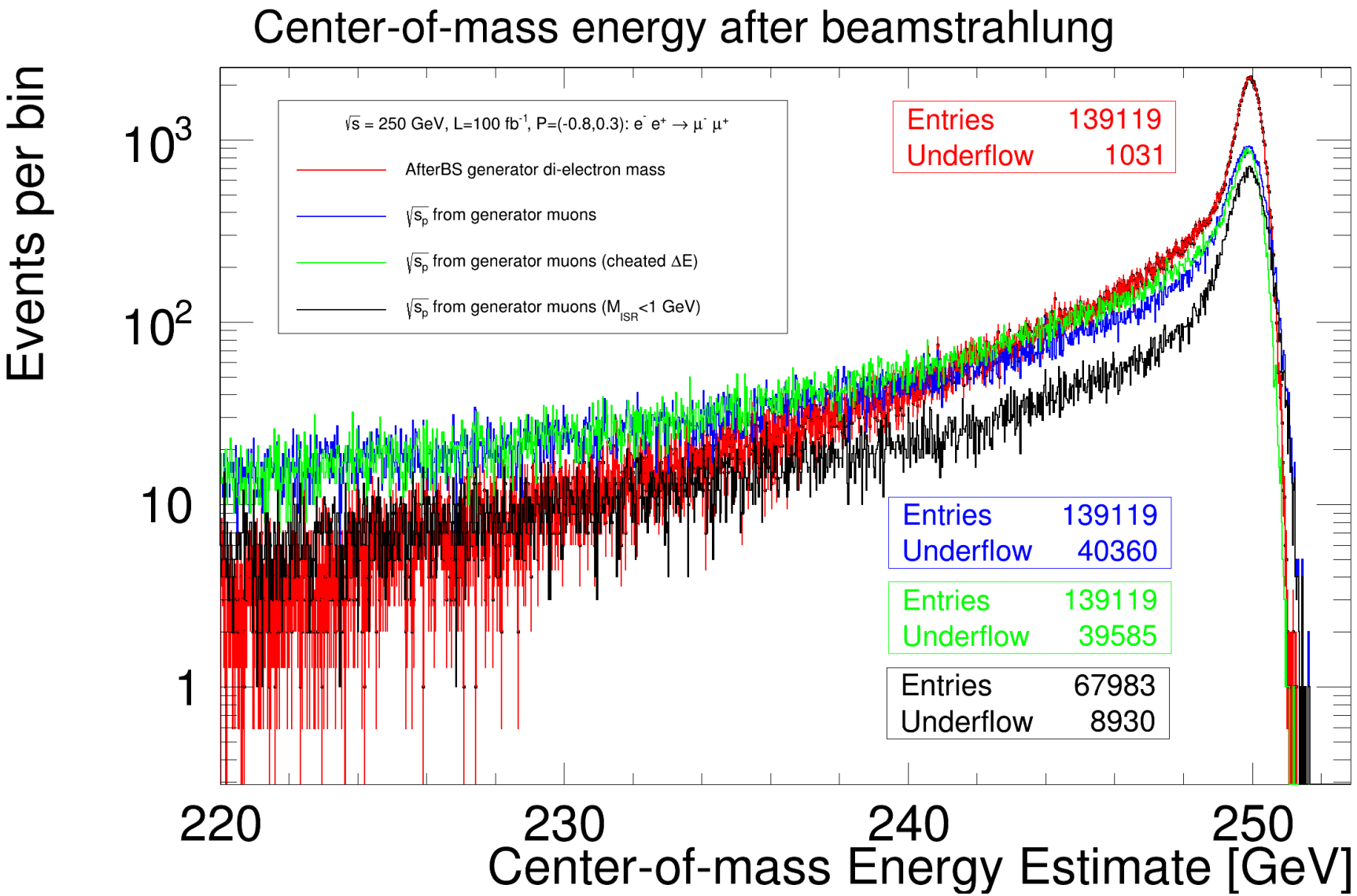
Bronze Quality Dimuon PFOs (After BS)



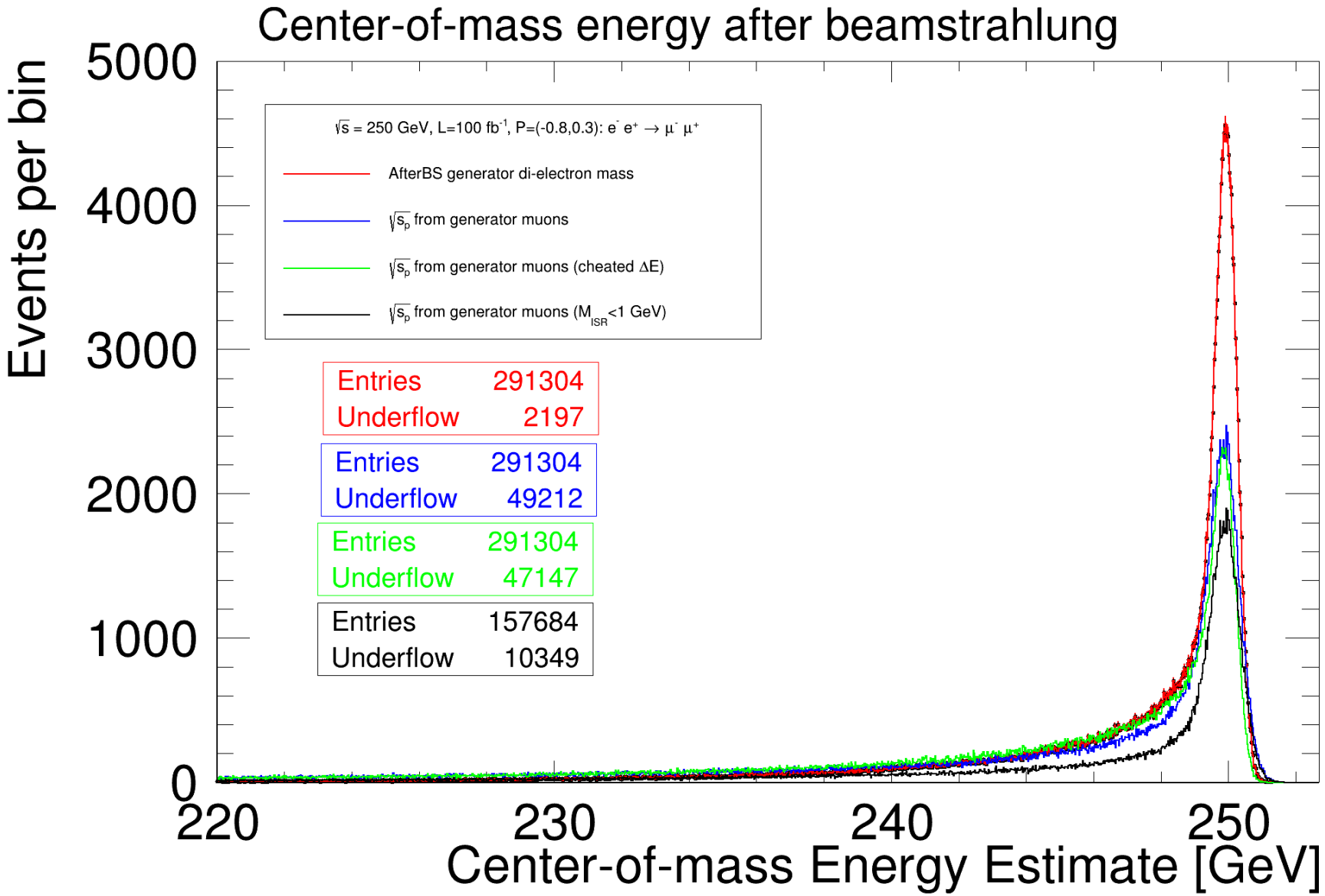
Comparisons I Low Dimuon Mass (After BS)



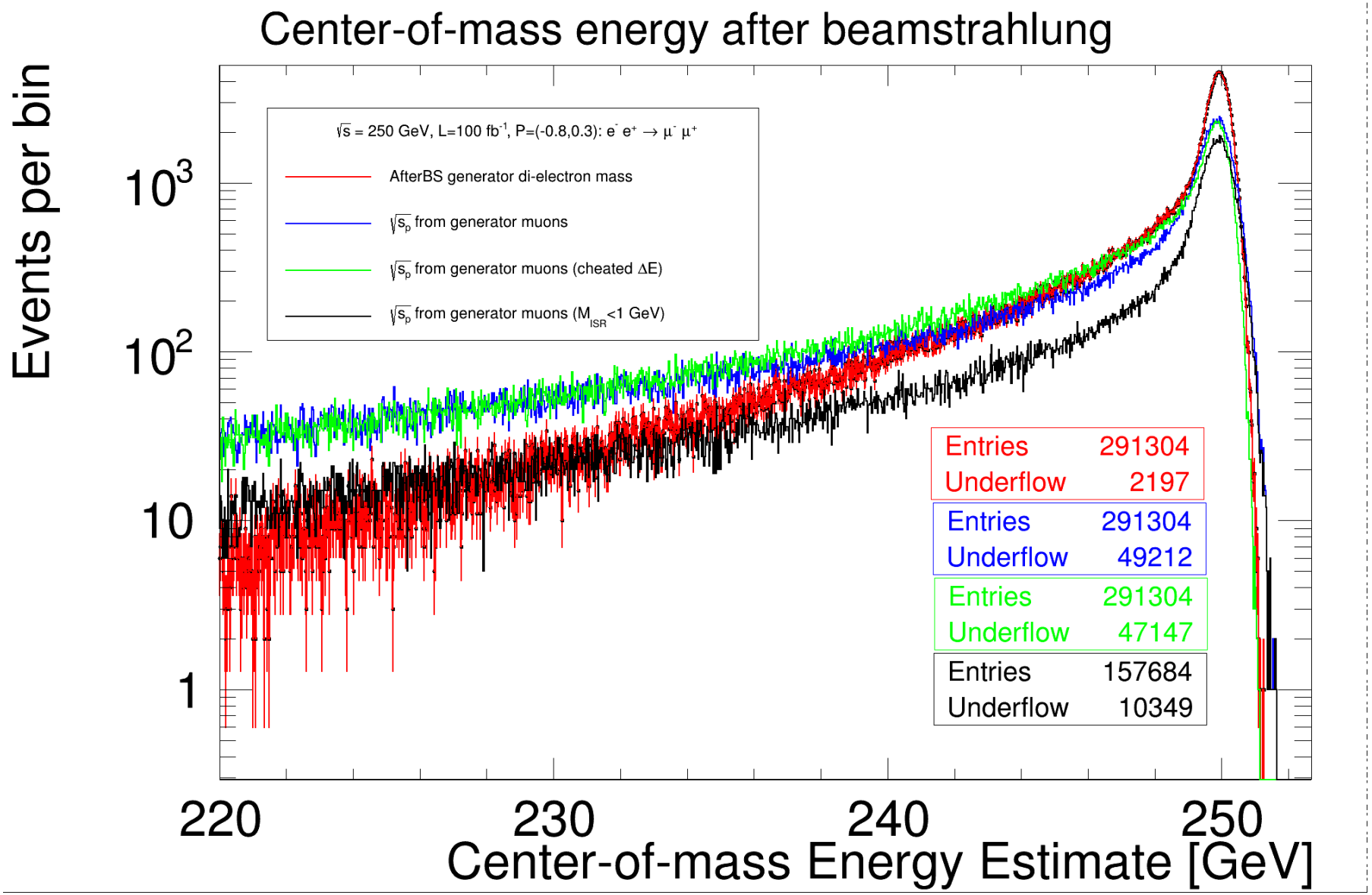
Comparisons II Low Dimuon Mass (After BS)



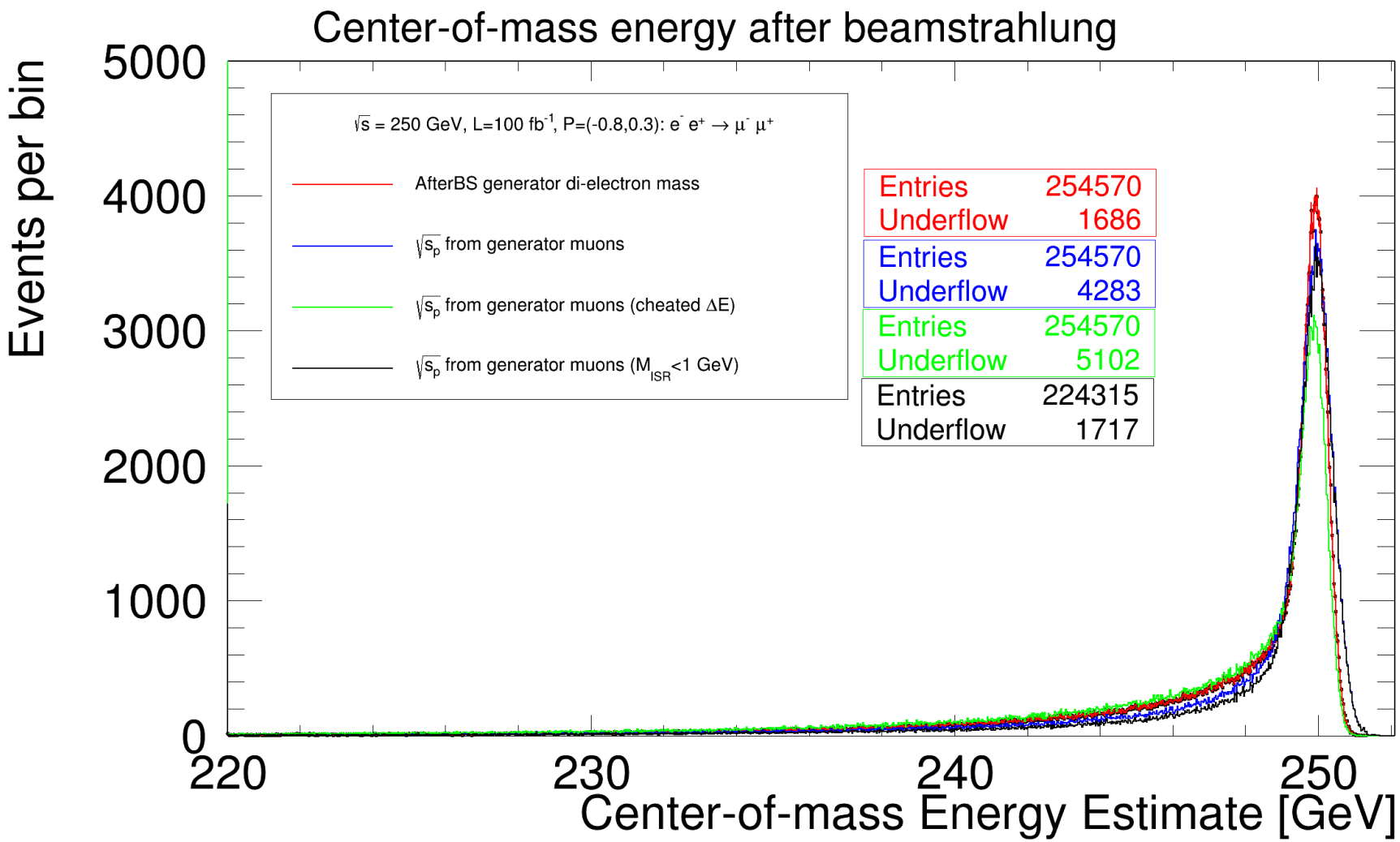
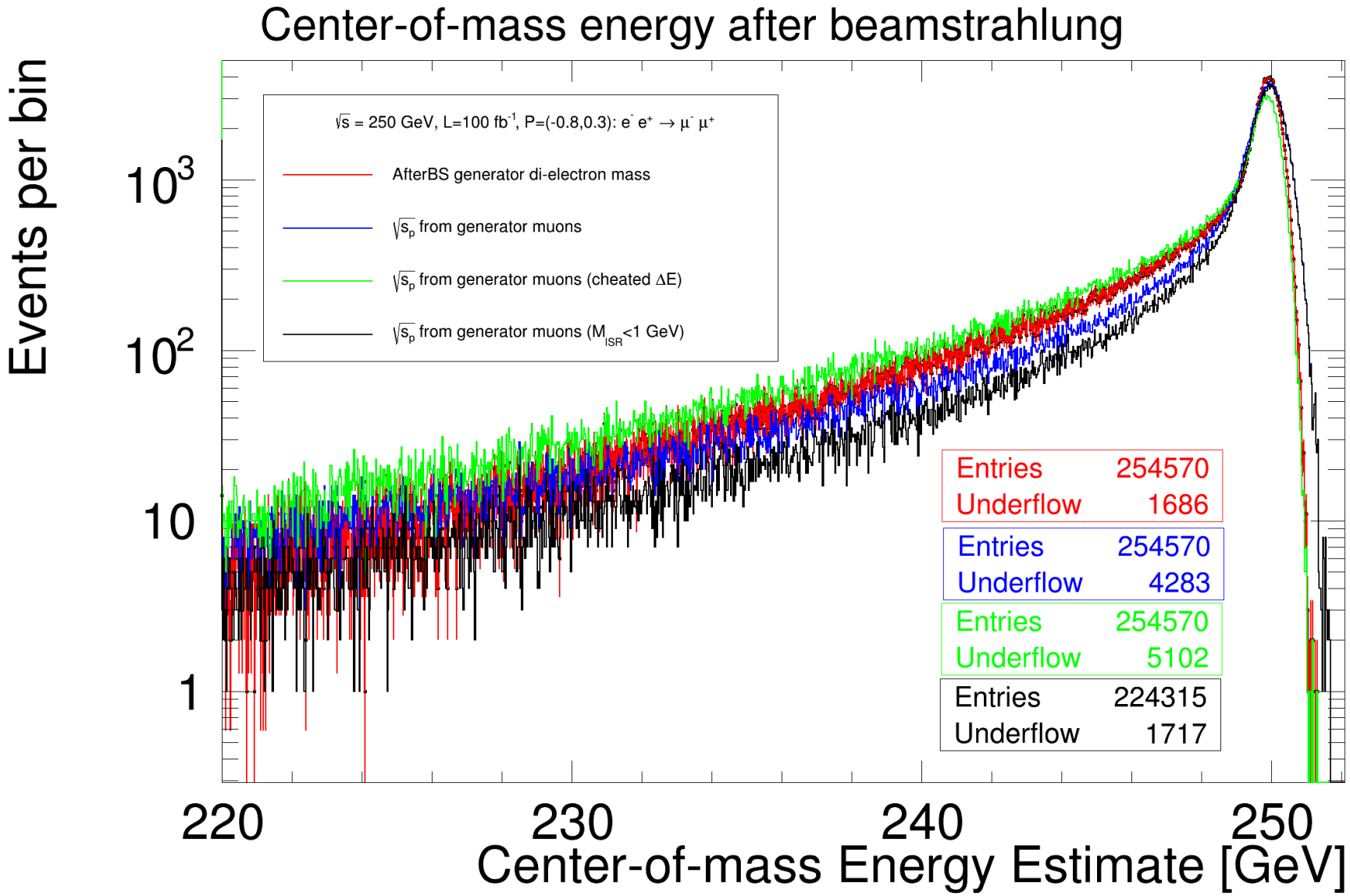
Comparisons I Medium Dimuon Mass (After BS)



Comparisons II Medium Dimuon Mass (After BS)



Comparisons I High Dimuon Mass(After BS)



Comparisons II High Dimuon Mass (After BS)