CAIN

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Manual CAIN244bmanual.pdf and FORTRAN source file Cain244b.zip are uploaded to the indico site https://agenda.linearcollider.org/event/9396/

Versions

- Started in 1984, named ABEL, for beam-beam interaction in JLC
  - Pinch effect
  - Beamstrahlung
- Later, renamed to CAIN when beam-laser interaction was included for gamma-gamma colliders
- Latest version: not clear.
  - Perhaps, CAIN2.44b, but a beta version
  - CAIN2.43 is better
- Code
  - Written in FORTRAN90 since recursive calls were included
  - Windows version and UNIX version
  - I have almost stopped revising the code.
  - Ask Tauchi san for UNIX version and compilation
- Manual available. Uploaded to today's indico page
- Possible bugs
  - Sometimes a strange phenomena when too many macroparticles are used
  - Routines which are used only rarely

Structure of the Code

- All the particles (electron, positron, photon) are in one big array, containing the info of the space-time, energy-momentum, polarization, etc.
- Fields
  - ✓Beam-beam field
  - ✓External field (constant field, quadrupole field, laser field)

Math

- Simple arithmetic can be done in the input file
- Variables
  - Can use numerical variables, character strings, arrays
  - Pre-defined variables such as Pi, Emass…
- Expressions
  - Arithmetic, logical
- Pre-defined functions
  - Sin, Exp, Log, BesJ, etc.

Commands

- Math expression
  - SET, ARRAY
- Control :
  - DO, CYCLE, EXIT, ENDDO,
  - IF, ELSEIF, ELSE, ENDIF
- Output
  - WRITE, PRINT
  - PLOT (use very old software TOPDRAWER)
- Beam definition
  - BEAM
  - LASER
- Interaction control
  - BBFIELD beam-beam field
  - LASERQED
  - CFQED beamstrahlung, coherent pair creation,
  - LUMINOSITY
  - PPINT (incoherent pair)
- Beamline
  - MAGNET, BEAMLINE, BLOPTICS, MATCHING
  - TRANSPORT, ENDTRANSPORT
- Execution
  - PUSH, ENDPUSH
  - DRIFT

Polarization

- Spin of electron/positron and Stokes parameters of photons
- Treated as density matrix
- But not for all the interactions

		initial $e^{\pm}$	laser	final $e^{\pm}$	final γ
Beamstrahlung	$e^{\pm} \to e^{\pm} + \gamma$	LT	–	LT	LT
Linear laser-Compton	$e^{\pm} + \text{laser} \to e^{\pm} + \gamma$	LT	LT	LT	LT
Nonlinear laser-Compton	$e^{\pm} + n\cdot\text{laser} \to e^{\pm} + \gamma$	L	L*	L	L
	or	N	T*	N	T
		initial γ	laser	final $e^{\pm}$	
Coherent pair	$\gamma \to e^{+} + e^{-}$	LT	–	LT	
Linear laser-Breit-Wheeler	$\gamma + \text{laser} \to e^{+} + e^{-}$	LT	LT	LT	
Nonlinear laser-Breit-Wheeler	$\gamma + n\cdot\text{laser} \to e^{+} + e^{-}$	L	L*	L	
		initial	final pair		
Incoherent Breit-Wheeler	$\gamma + \gamma \to e^{+} + e^{-}$	L	N		
Incoherent Bethe-Heitler	$\gamma + e \to e + e^{+} + e^{-}$	N	N		
Incoherent Landau-Lifshitz	$e + e \to e + e + e^{+} + e^{-}$	N	N		
		initial	final		
Bremsstrahlung	$e + e \to e + e + \gamma$	N	N		

- L : Longitudinal spin of electron/positron (or circular polarization of photon).
- T : Transverse spin of electron/positron (or linear polarization of photon).
- * : 100% polarization only
- N : Not computed. (No change for existing particles, zero for created particles)
- - : Irrelevant

Beam Definition

- Courant-Snyder parameters: Following parameters can be used

$$N \text{ (bunch-population)},\ E,\ (t,x,y,s) \text{ (beam-center)},$$
$$\beta_{x,y},\ \alpha_{x,y},\ \eta_{x,y},\ \eta'_{x,y},\ \ \sigma_t,\ \sigma_\epsilon,$$
$$\psi_{x,y} \text{ (crab-angle)},\ \theta_{x,y} \text{ (crossing-angle)},\ \phi_{x,y} \text{ (}x\text{–}y\text{ role)},$$
$$(\zeta_x,\zeta_y,\zeta_s) \text{ (spin)},\ \text{etc.}$$

- Beam data can also be read from files
  - ✓ CAIN standard format, MATHEMATICA format, FORTRAN NAMELIST)
  - ✓ Or, user-defined format (see Sec.3.5.2 of the manual

Beam-Beam Field

- Longitudinal slices
  - ✓ No interaction between different slices (Lorentz contraction)
  - ✓ Longitudinal mesh size must be defined by the user
- Interaction within each slice (2D)
  - ✓ Main part
    - Equal-space, rectangular mesh
    - Kernel potential averaged over a mesh
    - Fast computation by FFT
  - ✓ Outside region
    - Ignore the contribution of the particles outside the mesh region
    - But the force from the main part to outside partcles is included by either direct Coulomb force or by harmonic expansion (see Sec.5.7 of the CAINmanual244b for more detail)
  - ✓ Size of the “main part” and the mesh size are decided by the input data, not automatically.

Beamstrahlung

- Use the formula in constant (within the slice) magnetic field

$$dW = \frac{\alpha m}{\sqrt{3}\pi\gamma}\left[Ki_{5/3}(z') + \frac{x^2}{1-x}K_{2/3}(z)\right]dx,$$

$$x \equiv \omega_\gamma/E_0, \qquad z \equiv \frac{2}{3\Upsilon}\frac{x}{1-x}, \qquad \Upsilon \equiv \gamma\frac{B}{B_{sch}}$$

$$B_{sch} = m^2/e = 4.4\times 10^9 \text{ Tesla}$$

$$Ki_\nu(z) = \int_z^\infty K_\nu(z')dz'$$

- Emission angle is not taken into account
- Polarization
  - ✓ Initial and final electron polarization (longitudinal and transverse) and the Stokes parameter of the final photon are included
  - ✓ So, the actual formula is much more complicated than above
- Special functions such as $Ki_{5/3}$ and $K_{2/3}$ are approximated by appropriated polynomials

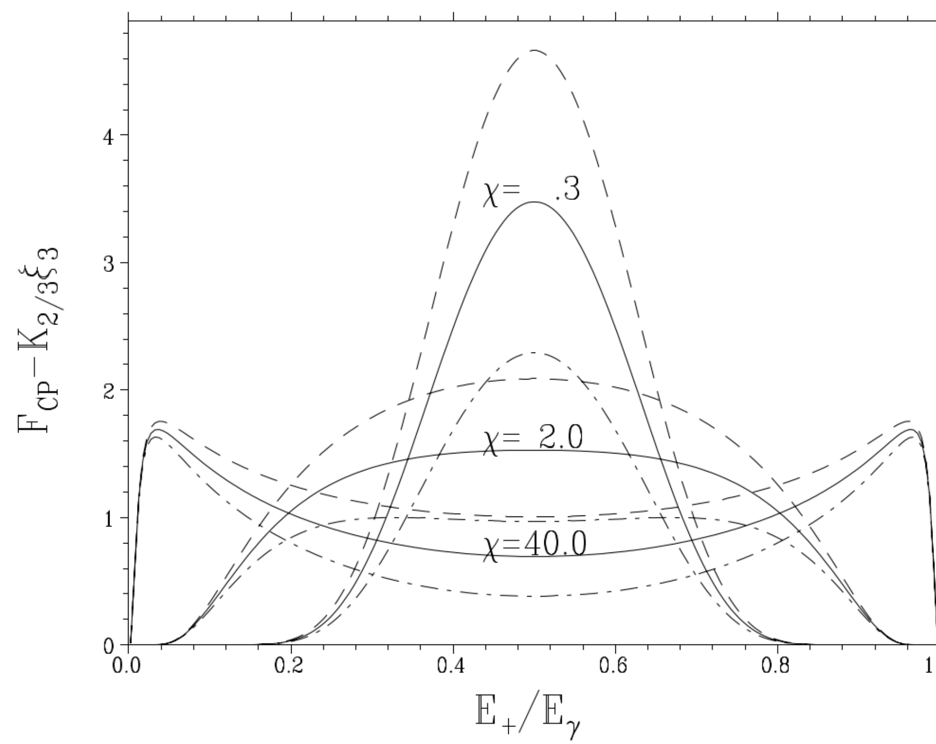
Coherent Pair Creation

- Constant field formula

$$dW = \frac{\alpha m^2 dE_+}{4\sqrt{3}\pi\omega_\gamma^2} \times \left[Ki_{1/3}(z) + \left(\frac{E_-}{E_+} + \frac{E_+}{E_-}\right)\right] K_{2/3}(z)$$

$$E_- = \omega_\gamma - E_+, \quad z = \frac{2}{3\chi}\frac{\omega_\gamma^2}{E_+ E_-}, \quad \chi \equiv \frac{\omega_\gamma}{mc^2}\frac{B}{B_{sch}}$$

- Polarization of (e^+, e^-, γ) is included
  - ✓ The formula is much more complex than above
- Creation angle ignored
- The algorithm is inefficient for large χ (>~ 1000)



Incoherent Processes

- Included processes

Breit-Wheeler	$\gamma + \gamma \rightarrow e^- + e^+$
Bethe-Heitler	$\gamma + e^{+-} \rightarrow e^{+-} + e^- + e^+$
Landau-Lifshitz	$e + e \rightarrow e + e + e^- + e^+$
Bremssstrahlung	$e + e \rightarrow e + e + \gamma$

- Breit-Wheeler process
  - ✓ Formula including circular polarization of initial photons is used
- Other processes are reduced to Breit-Wheeler by the virtual (almost real) photon approximation, i.e.,

Bethe-Heitler	γ + ‘γ‘ $\rightarrow e^- + e^+$
Landau-Lifshitz	‘γ‘ + ‘γ‘ $\rightarrow e^- + e^+$
Bremssstrahlung	e + ‘γ‘ $\rightarrow e + \gamma$

  where ‘γ‘ is the virtual photon
- These low energy pairs are somehow tracked
  - ✓ Use the exact formula of motion in a constant field (special mesh size is not introduced)
  - ✓ Time consuming
- See Sec.5.11 Incoherent processes of the manual

Beam-Laser Interaction

- Laser intensity parameter

$$\xi = \frac{e\sqrt{-A^\mu A_\mu}}{m} = \frac{\lambda_L}{2\pi m}\sqrt{\mu_0 c P}$$

$$A^\mu = \text{ vector potential}, \quad P = \text{power density}$$

  - ✓ Often denoted by a (plasma) or K (undulator)
- The laser is treated as an external field, but the created photons are treated as particles
- Laser field is defined by the parameters such wavelength, Rayleigh length, power density, Stokes parameters, profile (Gaussian, trapezoidal, cutoff, etc)

- Processes
  - ✓Laser-Compton
  - ✓Laser-gamma (Breit-Wheeler process between laser and gamma)

Laser-Compton

- Formulas expanded by Bessel functions are used
  - Valid for any ξ in principle but the convergence is poor for large ξ. Actually, $\xi \sim 3$ is the limit.
    - Laser-Compton was introduced to CAIN because of the gamma-gamma collider. ξ up to 1 was enough.
    - Sometimes QED people want very large ξ, but CAIN cannot treat such a case. I cannot find a good formula for large ξ.
  - Can also be used to simulate the radiation by electron/positron in undulators
- Polarization
  - Initial and final electron helicity
  - Final photon helicity
  - Laser polarization
    - Must be either 100% circular
    - or 100% linear
      - This case has not been checked well
      - Written for an experiment at BNL many years ago
    - “Unpolarized laser” impossible
    - TDL (times diffraction parameter) is adopted by physics is not clear
- Linear Compton formula can be used for very small ξ
  - Treated as particle-particle interaction
  - Almost all polarizations are included

Laser Breit-Wheeler

- Similar to laser-Compton (different channel)
  - Introduced also for gamma-gamma collider
    - Photons created by laser-Compton can disappear by pair creation in the same laser
  - Bessel function expansion
    - Poor convergence for large ξ
    - No linear polarization of the laser

Outputs

- Particle list
  - To a text file
  - At any time during the collision
    - Interaction proceeds as a loop of PUSH command as
      PUSH   Time=(t_{ini}, t_{fin}, n_{step})
      … … … any commands can be inserted
      ENDPUSH
- Graphic output
  - Only to for the very old software “topdrawer” developed > 40 years ago
  - You have to create graphic data unless you have “topdrawer”
- Luminosity
  - Any combination of particle species (γ, e^+, e^-) for right/left going
  - For Topdrawer or numerical table
  - Helicity can be separated

What else is needed?

- A few years ago, Daniel Jeans asked me if physics events can be generated during the collision simulation. This makes it possible to generate events with the vertex position recorded.
- CAIN2.44b introduced an operand "PPDATA=n_f" of the LUMINOSITY command. Then, CAIN writes particle info (energy-momentum, space-time, etc.) **at any close encounter** of 2 particles on the file #n_f.
- However, this will create a huge file (> several hundred Giga bytes). Not very practical.
- For that purpose, perhaps, you have to edit the subroutine PPINT in the source file.

What else is needed? (continued)

- Big fix for large number of macroparticles
  - This is hard
- Very large Upsilon or χ ?
- Angle of beamstrahlung?
- Compton scattering (not as laser-Compton)?
- Laser with very large ξ.