In situ Measurements of ILC Beam/Center-of-Mass Energy

Graham W. Wilson

University of Kansas

January 13, 2022

Key issue: systematic control for the absolute scale of center-of-mass energy (in collision...) at **all** center-of-mass energies.

Apologies for inadequate references to prior work and especially the diagnostics side which I'm not so familiar with.

ILC Physics Targets — Energy Requirements *

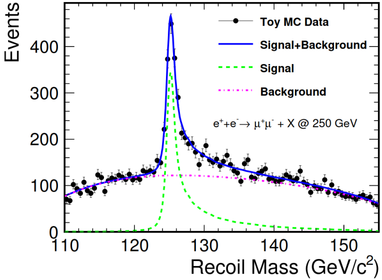
Core Program

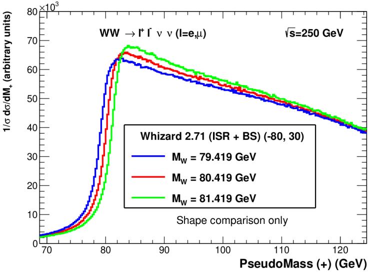
Observable	M_{H}	M_{t}	M_{W}	M_{X}
Method	Recoil mass	Scan	Reconstruction	Scan?
Best $\sqrt{s}$ [GeV]	250	350	250	Highest?
Current precision [MeV]	170	300	12	–
Target precision [MeV]	10	20	2	?
$\sqrt{s}$ contribution [MeV]	3	6	0.5	?
$\sqrt{s}$ uncertainty goal [ppm]	100	200	10	100?

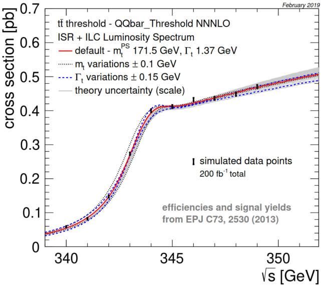
Ultimate Impact/Reach

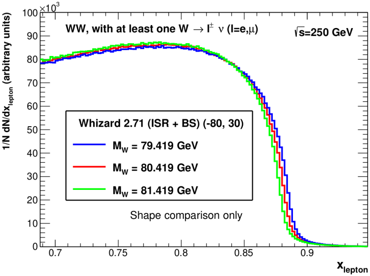
Observable	M_{W}	M_{Z}	Γ_{Z}	A_{LR}
Method	Scan	Scan	Scan	Count/Scan
Best $\sqrt{s}$ [GeV]	161	91	91	91
Current precision	12	2.1	2.3	1.9×10^{-3}
Target precision	2 MeV	0.2 MeV	0.11 MeV	3.5×10^{-5}
$\sqrt{s}$ contribution	0.8 MeV	0.2 MeV	small	1.8×10^{-5}
$\sqrt{s}$ uncertainty goal [ppm]	10	2	5*	10

Example Physics Plots









Beam Energy and Beam Energy Spread

- Upstream diagnostics. Chicane BPM spectrometer. Energy target: $\mathcal{O}(10^{-4})$.
- Downstream diagnostics. Targets $\mathcal{O}(10^{-4})$. SLC-style synchrotron radiation stripes spectrometer - sees beams after beam-beam effects.
- Beam energy spread?, and distribution?
- Energy-z correlations?
- Also pass-through non-collision mode (to inter-calibrate upstream/downstream)?

While these may not provide the ultimate absolute beam energy uncertainty, they should be extremely useful for tracking **relative** beam energies especially for scans and for short-term variations.

So expect: $< E_-^U >$, $< E_+^U >$, $< E_-^D >$, $< E_+^D >$ on a bunch-by-bunch basis?

Beam/Center-of-Mass Energy, Luminosity Spectrum

Center-of-Mass Energy

- Naively, $\sqrt{s} = 2E_{\mathrm{b}}$
- Less naively, $\sqrt{s} = 2\sqrt{E_{-}^{C} E_{+}^{C}} \cos(\alpha/2)$ ($\alpha = 14$ mrad crossing-angle)
- E_{-}^{C}, E_{+}^{C} are the collision energies (after probable beamstrahlung)

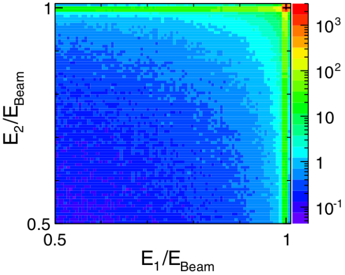
Collision Momentum Imbalance

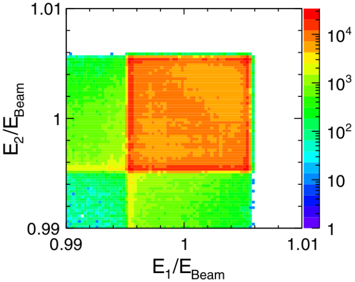
- Mostly in z, but also in x
- $p_x = (E_{-}^{C} + E_{+}^{C}) \sin(\alpha/2)$
- $p_z = (E_{-}^{C} - E_{+}^{C}) \cos(\alpha/2)$

What is most important is the **distribution** of the collision initial-state 4-vector weighted by luminosity.
This is usually called the **luminosity spectrum**, and is either 1-d ($\sqrt{s}$) or 2-d (E_{-}^{C}, E_{+}^{C}). Potentially even 3-d or more, eg. in (E_{-}^{C}, E_{+}^{C}) for slices in z_{int}.
Needs to be unfolded from collision physics events gathered over long time periods. Necessarily averages over all the variations in conditions.

Luminosity Spectrum

There are a number of studies of the luminosity spectrum, incl. (Frary, Miller), Moenig, (Boogert, Miller), Sailer, and (Poss, Sailer). Use **Bhabhas** with $\theta > 7^\circ$. State of the published art is Poss and Sailer study for CLIC 3 TeV.





$$\begin{aligned}
\mathscr{L}(x_1, x_2) = {} & p_{\text{Peak}}\,\delta(1-x_1) \otimes \text{BES}(x_1; [p]^1_{\text{Peak}}) \\
& \delta(1-x_2) \otimes \text{BES}(x_2; [p]^2_{\text{Peak}}) \\
& + p_{\text{Arm1}}\,\delta(1-x_1) \otimes \text{BES}(x_1; [p]^1_{\text{Arm1}}) \\
& \text{BB}(x_2; [p]^2_{\text{Arm1}}, \beta^{\text{Arm}}_{\text{Limit}}) \\
& + p_{\text{Arm2}}\,\text{BB}(x_1; [p]^1_{\text{Arm2}}, \beta^{\text{Arm}}_{\text{Limit}}) \\
& \delta(1-x_2) \otimes \text{BES}(x_2; [p]^2_{\text{Arm2}}) \\
& + p_{\text{Body}}\,\text{BG}(x_1; [p]^1_{\text{Body}}, \beta^{\text{Body}}_{\text{Limit}}) \\
& \text{BG}(x_2; [p]^2_{\text{Body}}, \beta^{\text{Body}}_{\text{Limit}}),
\end{aligned}$$

Parametrize the lumi spectrum resulting from beam-beam simulations (Guinea-PIG) and incorporate in measurement using (E_1, E_2, θ_{acol}). [Currently working on related parametrization approach for ILC using reweighting fits.]

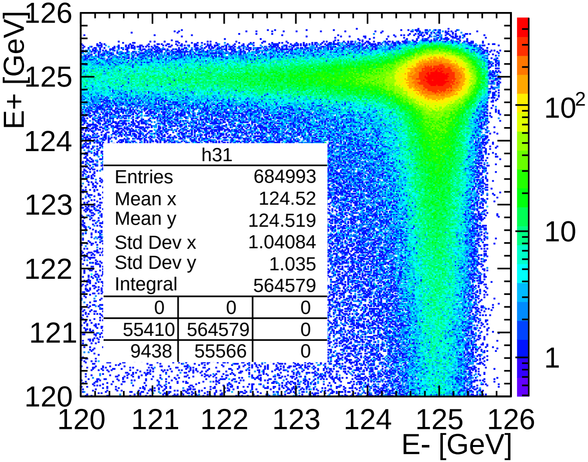
What do we really want to measure?

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Shortly, we'll look at the related 1-d distributions $(E_+, E_-, \sqrt{s}, p_z)$ with empirical fits.

Absolute energies of peak position (E) and shape (LS)

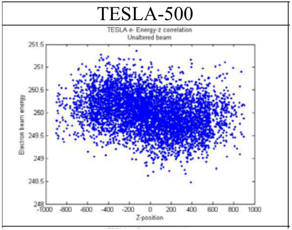
[dL/d$\sqrt{s}$: see work by Boogert, Frary, Miller, Moenig, Sailer, Poss]

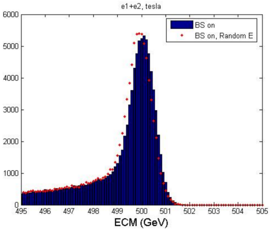


Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

Upstream Issues/Diagnostics/Correlations

- One very important issue is understanding the E-z distribution of the beams presented to the interaction point.
- Wakefield effects can distort the E-z distribution. Also RF phasing/kink instability avoidance? (BNS damping??)
- Plot shows modeled ECM distribution with correlation and without (red) from Woods/Florimonte study of 2005.
- Are there more recent studies?





Current centralized Whizard simulations assume uncorrelated Gaussian beams as do my initial Guinea-PIG forays.

Request: Would really appreciate validated ILC beam input files

In situ Methods Related to Beam Energy

There are three main techniques currently envisaged using collision physics events. They are inter-related and should be carried out in a global analysis.

Methods

1. $\sqrt{s}_A$: The radiative return to the Z method. (Wilson - Munich96, LEP2, Moenig, Hinze)
2. $\sqrt{s}_p$: The dilepton momenta method. (Barklow - LCWS05, Wilson)
3. θ_{acol}: Bhabha acollinearity angle. (Frary-Miller 91)

Comments

All three use particle direction measurements and a ≤ 3 particle final-state approximation

1: Relies on M_{Z} for energy scale

2: Relies on tracker momentum scale for energy scale

3: More focused on lumi. spectrum to date than energy

1+2: focus of existing studies has been $\mu^+\mu^-$

2: Includes radiative return and full energy events.

Use radiative return events to the Z with precision angular measurements.

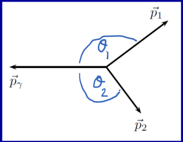
Assume one photon recoiling from $\mu^+\mu^-$

$$x_\gamma \equiv \frac{E_\gamma}{E_b} = 1 - \frac{m_{12}^2}{s}$$

At $\sqrt{s} = 250$ GeV, $x_\gamma = 0.867$, $E_\gamma = 108$ GeV, for $m_{12} = M_Z$.

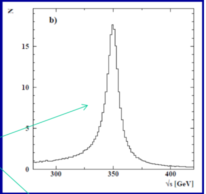
Write $m_{12}^2/s = f(\theta_1, \theta_2)$. Then assume, $m_{12} = M_Z$.

$e^+ e^- \to Z(\gamma) \to \mu^+\mu^-(\gamma)$



GWW – MPI 96
LEP Collabs.

Hinze & Moenig



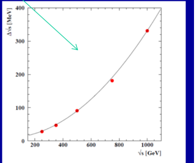


Figure 3: Energy dependence of $\Delta\sqrt{s}$ for $\mathcal{L} = 100$ fb^{-1}.

Photon often not detected.
Use muon angles to (photon/beam-axis).
Requires precision polar angle.

$$\sqrt{s} = m_Z \sqrt{\frac{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1+\theta_2)}{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1+\theta_2)}}$$

Statistical error per event of order Γ/M = 2.7%

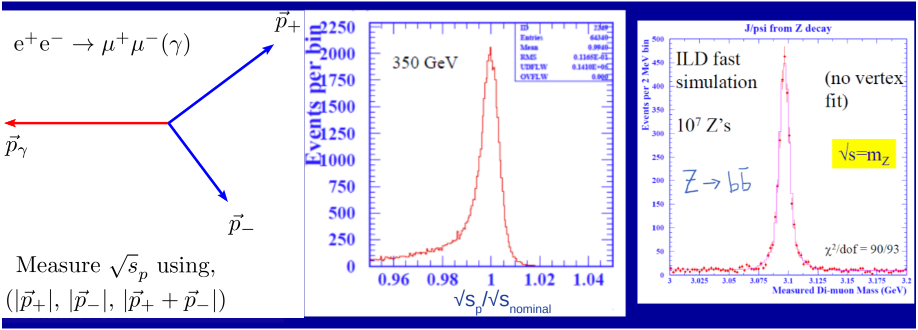
Acceptance degrades quickly at high √s

- uses M_Z and is limited in ultimate precision by its knowledge (23 ppm).
- can also use e^+e^-, and even $\tau^+\tau^-$ decays of the Z (maybe also $Z \to q\bar{q}$)
- per event uncertainty poor given Γ_Z

Most recent study in K. Moenig talk and proceedings from LCWS05.

$\sqrt{s}_p$ Method for Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.



Tie detector p-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc).

Bhabhas and acollinearity

Forward Bhabhas ($\mathrm{e^+e^- \rightarrow e^+e^-}$) with scattering angles above 7° are widely discussed mainly for luminosity spectrum measurements.

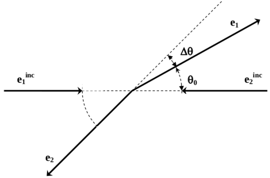
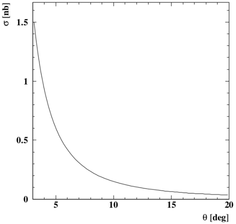


Figure 22 - Bhabha scattering after radiation loss by the incoming particle e_2^{inc} due either to ISR or beamsstrahlung. The acolinearity angle ΔΘ arises from the energy difference in the incoming particles e_1^{inc} and e_2^{inc}.



$\sqrt{s} = 500$ GeV

The original literature focused on the acollinearity angle, that measures the **momentum imbalance** of the two beams, (rewritten here using E given $E \approx p$),

$$\Delta p = (E_- - E_+) = \frac{E_b \theta_{\mathrm{acol}}}{\sin\theta_0}$$

One can also use x_γ or s'/s notation as before (with the photon along the direction of lost momentum). No reference energy scale like M_{Z}. Need to rely on spectrometer info or on direct energy measurements. Foreseen endcap E,p resolution not great.
Large statistics. Δp uncertainty gets amplified by $1/\sin\theta_0$ term at very forward angle - so not so much to gain with wider acceptance. Can explore $\sqrt{s_p}$ too.

Recent studies related to $\sqrt{s_p}$ method

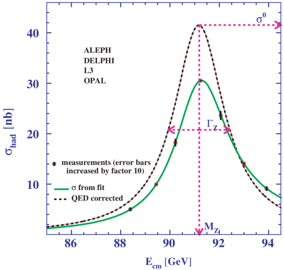
- Critical issue for $\sqrt{s_p}$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s_p}$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

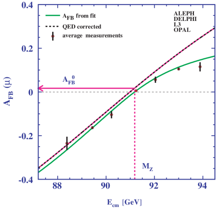
Recently,

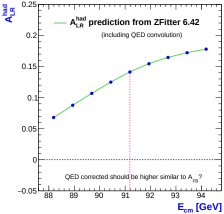
- Several talks on $\sqrt{s_p}$. Latest ones at ILCX and in December.
- Includes a more careful look at the $\sqrt{s_p}$ method prospects with $\mu^+\mu^-$. Include crossing angle, full simulation and reconstruction with ILD, track error matrices, vertex fitting, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- Also a look at colliding beam-energy/interaction-vertex correlations and more of a focus on $\mathrm{d}L/\mathrm{d}\sqrt{s}$ issues.
- Prospects for Z lineshape with a polarized scan including energy systematics.

Essentially, perform LEP/SLC-style measurements in all channels but also with $\sqrt{s}$ dependence of the polarized asymmetries, A_{LR} and $A^{f}_{FB,LR}$, in addition to A_{FB}. (Also polarized $\nu\bar{\nu}\gamma$ scan.) Not constrained to LEP-style scan points.



LEP: $\Delta M_{\mathrm{Z}} = 2100$ keV, $\Delta\Gamma_{\mathrm{Z}} = 2300$ keV





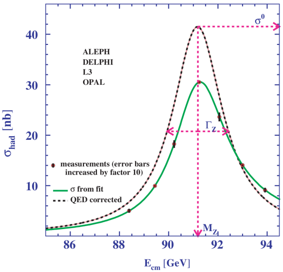
With 0.1 ab^{-1} polarized scan around M_{Z}, find **statistical** uncertainties of 35 keV on M_{Z}, and 80 keV on Γ_{Z}, from LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER for QED convolution.

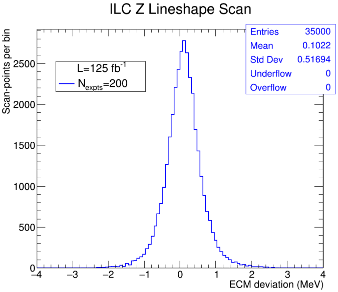
Exploiting this fully needs in-depth study of $\sqrt{s}$ **calibration systematics**

ILC $\mathcal{L}$ is sufficient for M_{Z}

Γ_{Z} systematic uncertainty depends on $\Delta(\sqrt{s}_+ - \sqrt{s}_-)$, so expect $\Delta\Gamma_{\mathrm{Z}} < \Delta M_{\mathrm{Z}}$

Initial line-shape study (all 4 channels). Use unpolarized cross-sections for now.





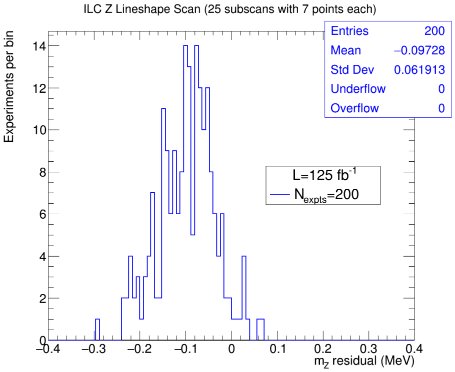
Uses $\sigma_{stat}/\sqrt{s}$ (%) $= 0.25/\sqrt{N_{\mu\mu}} \oplus 0.8/\sqrt{N_{\rm h}}$

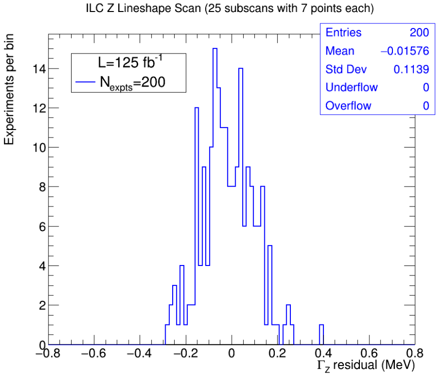
- Scan has 7 nominal $\sqrt{s}$ points, (peak,$\pm\Delta, \pm 2\Delta \pm 3\Delta$) with $\Delta = 1.05$ GeV
- 25 scans of 5 fb^{-1} per "experiment". $7 \times 25 \times 4 = 700$ $\sigma_{\rm tot}$ measurements.
- Assign luminosity per scan point in (2:1:2:1) ratio. (1 or 0.5 fb^{-1} each).
- Do LEP-style fit to $(M_{\rm Z}, \Gamma_{\rm Z}, \sigma^0_{\rm had}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER
- Model center-of-mass energy systematics and int. lumi syst. of 0.064%.
- Each scan-point (175 per expt.) shifted from $\sqrt{s}_{\rm nominal}$ by a 100% **correlated** overall scale systematic (here +100 keV) and by stat. component driven by stat. uncertainty of $\sqrt{s}$ measurement (typically 0.4 MeV/4.4 ppm).

Polarized Beams Z Scan for Z LineShape Study: WIP II

Ensemble tests with 200 experiments.

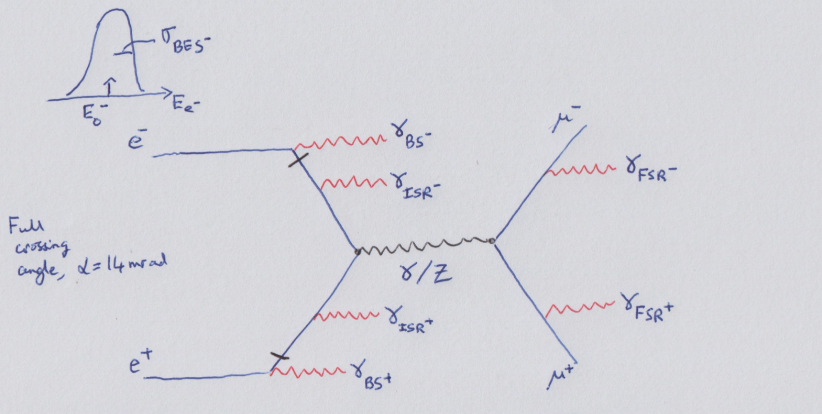
Currently, fit the 700 measured cross-sections (actually occuring at shifted $\sqrt{s}$) using assumed nominal $\sqrt{s}$. Ensemble mean χ^2 of 790 for 693 dof.





- As expected M_Z biased down by assumed scale error (here +100 keV) with stat. error of 50–60 keV.
- As expected Γ_Z bias small with stat. dominated error of 100–120 keV.
- Such an experiment has 1.9B hadronic Zs.

More Realism



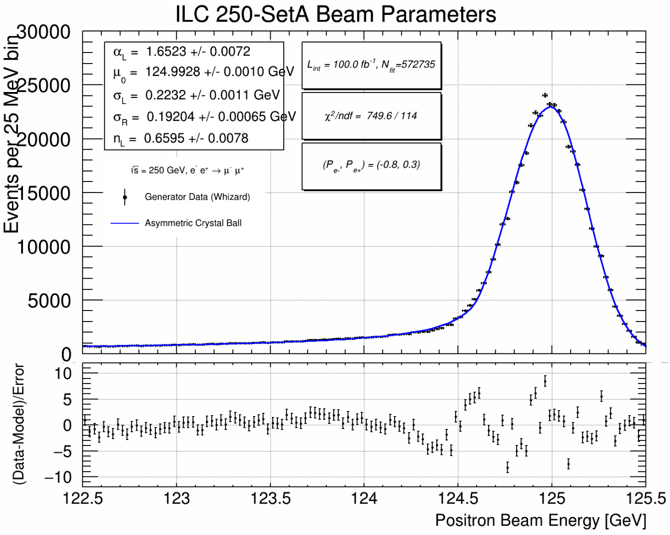
$$\sqrt{s}_{nominal} = 2\sqrt{E_0^- E_0^+} \cos(\alpha/2)$$

$$\sqrt{s}_{After\ BS} = 2\sqrt{(E_b^- - E_{BS}^-)(E_b^+ - E_{BS}^+)} \cos(\alpha/2)$$

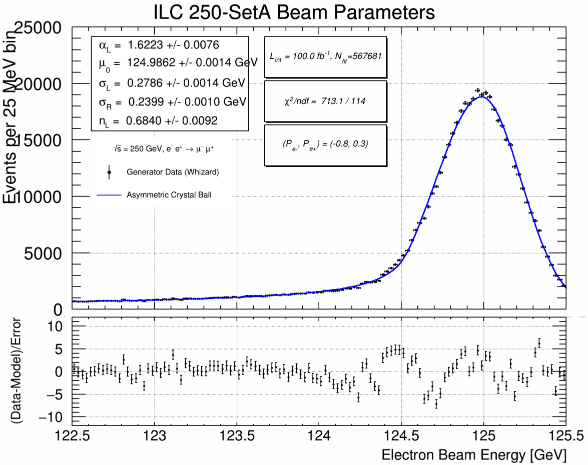
See backup for more detailed explanations

Positron Beam Energy (After Beamstrahlung)

Fits use asymmetric Crystal Ball with 5 parameters (details in backup)

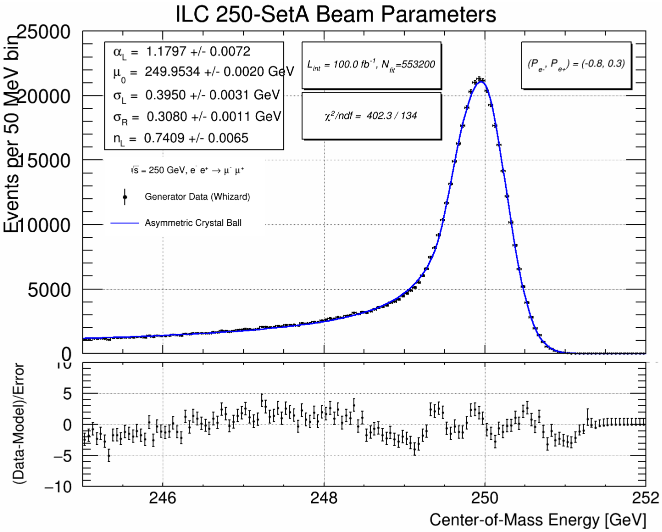


$$\sigma_{\rm R}/E = 0.1536 \pm 0.0005\% \text{ (cf } 0.152\% \text{ in TDR)}$$



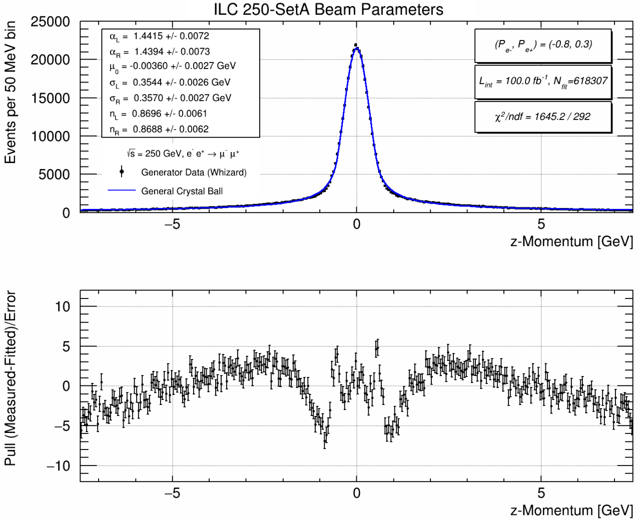
$$\sigma_{\rm R}/E = 0.1919 \pm 0.0008\% \text{ (cf } 0.190\% \text{ in TDR)}$$

Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.



$\sigma_{\rm R}/\sqrt{s} = 0.1232 \pm 0.0004\%$ (cf 0.122% in TDR (0.190% $\oplus$ 0.152%)/2)

z-Momentum of e^+e^- system (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

$\sqrt{s}_p$ with crossing angle

(More details in previous talks ...)
The outlined approach results in a quadratic equation in E_{ave}, ($AE_{\mathrm{ave}}^2 + BE_{\mathrm{ave}} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

$$C = (M_{12}^2)/4 + p_{12}^z \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2) - \overline{\Delta E_{\mathrm{b}}}^2 \cos^2(\alpha/2)$$

Based on this, there are three particular cases of interest to solve for E_{ave}.

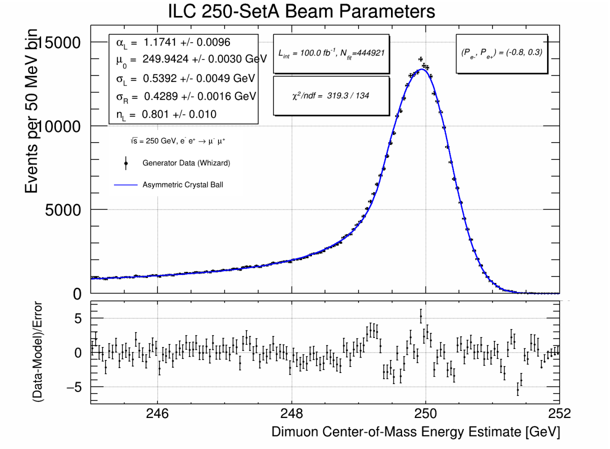
1. Zero crossing angle, $\alpha = 0$, and zero beam energy difference.
2. Crossing angle and zero beam energy difference.
3. Crossing angle and non-zero beam energy difference.

The original formula,

$$\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$$

arises trivially in the first case. In the rest of this talk I will use the $\sqrt{s}$ estimate from the largest positive solution of the second case as what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

Dimuon Estimate of Center-of-Mass Energy (After BS)



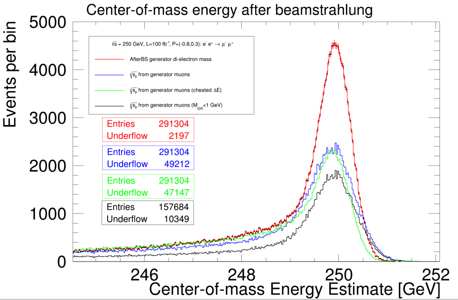
$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1716 \pm 0.0006\%$ (cf 0.1232% with true $\sqrt{s}$)

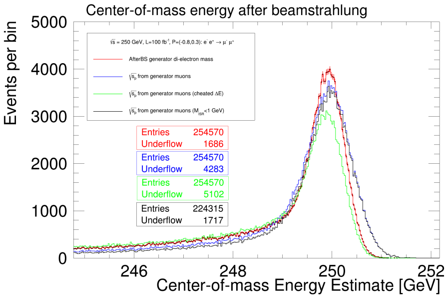
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Likely because some events violate the assumptions that $\overline{\Delta E_{\mathrm{b}}} = 0$ and $m_3 = 0$
- The former is no surprise given the p_z distribution
- The latter can be associated with events with 2 or more non-collinear ISR/FSR photons

Comparisons After BS

$50 < m_{\mu\mu}^{\text{gen}} < 150$ GeV

$m_{\mu\mu}^{\text{gen}} > 150$ GeV





- For lower dimuon mass events, only about half are reconstructed close to $\sqrt{s}$
- Most higher dimuon mass events reconstructed close to the original $\sqrt{s}$

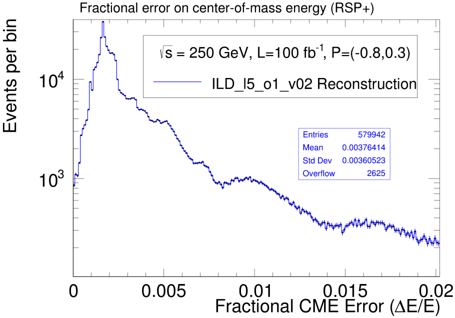
Conclusion

Lower dimuon mass events are more likely to violate the assumptions.

Event Selection Requirements

Currently rather simple.
Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** (<0.15%), **silver** ([0.15, 0.30]%), **bronze** ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %



Selection efficiencies for (80%/30%) beam polarizations:

- $\varepsilon_{-+} = 69.77 \pm 0.06$ %
- $\varepsilon_{+-} = 67.35 \pm 0.06$ %
- $\varepsilon_{--} = 69.47 \pm 0.05$ %
- $\varepsilon_{++} = 67.72 \pm 0.06$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Vertex Fit: Exploit ILC nanobeams

With well modeled track errors, and given that the 2 muons should originate from a common vertex consistent with the interaction point, we can perform:

- Vertex Fit: Constrain the two tracks to a common point in 3-d
- Beam-spot Constrained Vertex Fit

The ILC beam-spot size (no pinch) is $(\sigma_x, \sigma_y) = (515, 7.7)$ nm, $\sigma_z = 0.202$ mm

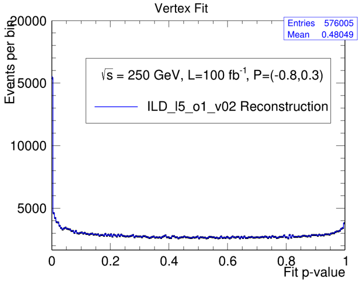
- Vertex fit (see AWLC2014 talk) implemented using the fully simulated and reconstructed data
- Also have explored beam-spot constraints

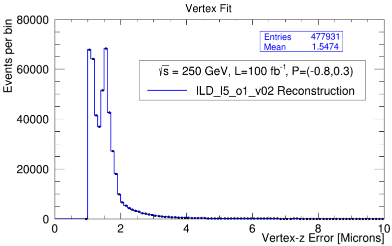
What good is this?

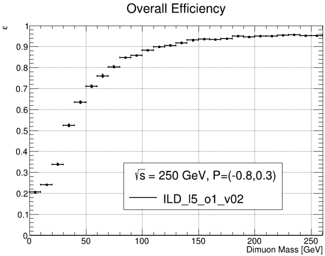
- Residual background rejection (eg. $\tau^+\tau^-$ reduced by factor of 20)
- Additional handle for rejecting or deweighting mis-measured events
- Some modest improvement in precision of di-muon kinematic quantities
- Also useful for $\mathrm{H} \to \mu^+\mu^-$ and for ZH recoil
- Interaction point measurement ($\mathcal{O}\,(1\mu\mathrm{m})$ resolution per event) **can** be used to correlate with (E_-, E_+) for understanding beamstrahlung effects

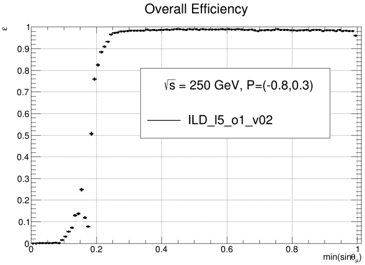
Note: simulated data does not currently simulate the transverse beam-spot ellipse

Event Selection Aspects: Vertex Fit and Overall Efficiency



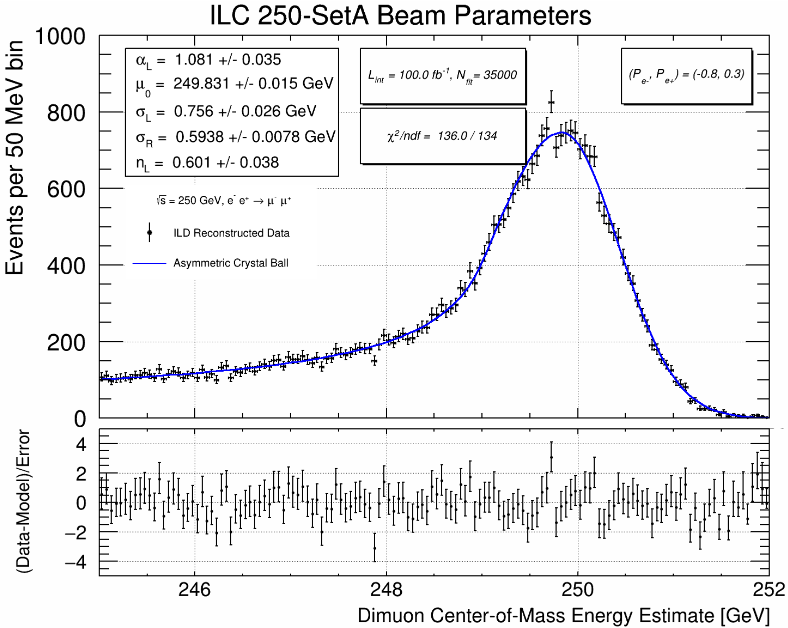




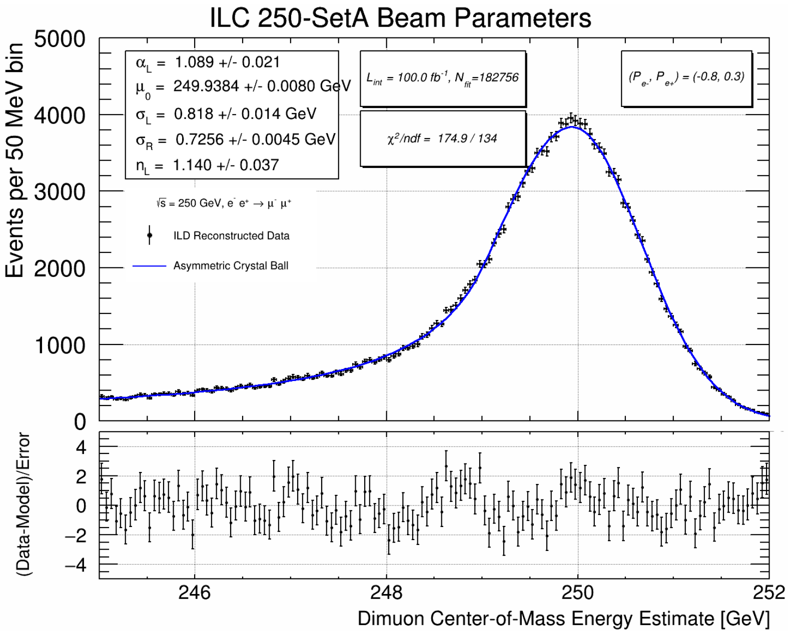


Efficiency rather mass dependent. Mostly due to geometrical acceptance.

Gold Quality Dimuon PFOs (After BS)



Silver Quality Dimuon PFOs (After BS)



Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The **luminosity spectrum** and **absolute center-of-mass energy** are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- **Beam energy spread** should be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- Working on parametrizing the underlying (E_-, E_+) distribution, with plan to model quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

Current Estimation Method

- Follow a similar approach to before, but using estimates of the statistical error on μ_0 for 5-parameter Crystal Ball fits to fully simulated data with the 4 shape parameters fixed to their best fit values. Fits are done in the various resolution categories (example gold, silver, bronze fits in backup slides).
- These estimates follow on the next slide

$\sqrt{s}$ Sensitivity Estimate at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

L_{int} [ab^{-1}]	Poln [%]	Gold	Silver	Bronze	G+S+B
0.9	$-80, +30$	6.5	3.1	8.5	2.7
0.9	$+80, -30$	7.7	3.4	9.6	3.0
0.1	$-80, -30$	26	12.1	33	10.4
0.1	$+80, +30$	29	13.0	41	11.4
2.0	–	4.8	2.2	6.2	**1.9**

Fractional errors on μ_0 parameter (mode of peak) when fitting with 5-parameter Crystal Ball function with all 4 shape parameters fixed to their best-fit values.

Also the $\mathrm{e}^+\mathrm{e}^-$ channel should be used. The additional benefit of the much larger statistics from more forward Bhabhas is offset by the poorer track momentum resolution at forward angles.

Stat. uncertainty at 250 GeV of 2 ppm far exceeds the 10 ppm requirement. Allows for 100 $\mu^+\mu^-$ sub-sets with 20 ppm stat. uncertainty each.

New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

1. Explore AP method using mainly $\mathrm{K}^0_\mathrm{S} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

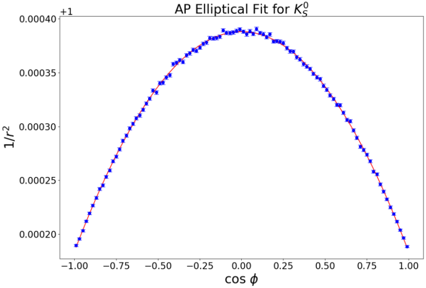
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

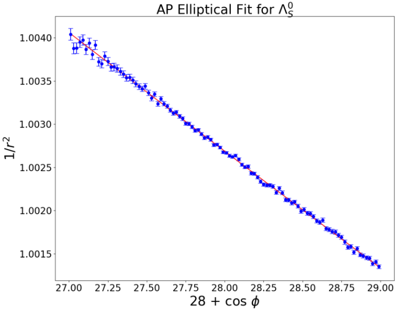
By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents (K^0_S and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work: typical existing experiments are at best at the 100 ppm level

Tracker momentum scale sensitivity estimate

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K^0_S}, \Lambda, \overline{\Lambda}$ in various momentum bins.





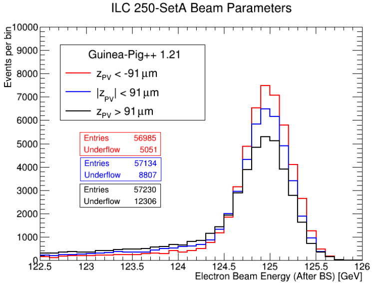
1. $m_{\mathrm{K^0_S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

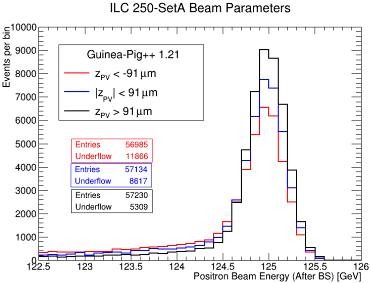
- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K^0_S}, \Lambda/\overline{\Lambda}, \pi^{\pm})$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z, ILC Z run has 400 such samples.

Beamstrahlung / z-Vertex Effects Explained

Divide interactions in 3 equi-probability parts according to z_{PV}. Preferentially

1. $\mathrm{e}^+\mathrm{e}^-$ collisions occuring more on the initial e^- side $(z < 0)$
2. $\mathrm{e}^+\mathrm{e}^-$ collisions mostly central
3. $\mathrm{e}^+\mathrm{e}^-$ collisions preferentially on the initial e^+ side $(z > 0)$



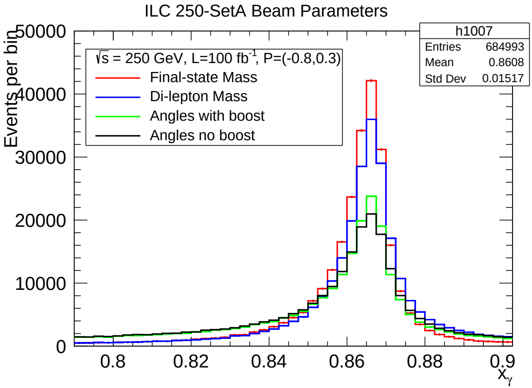


The beamstrahlung tail grows and the peak shrinks for e^- as z increases, and, for e^+ as z decreases. In both cases, the largest beamstrahlung tail occurs when the interacting e^- or e^+ has on average traversed more of the opposing bunch.

Thus both $\sqrt{s}$ and $p_z = E_- - E_+$ distributions depend on z. Likely needs to be taken into account for $\sqrt{s}$, $\mathrm{dL}/\mathrm{d}\sqrt{s}$, Higgs recoil, kinematic fits ...

Investigate $\sqrt{s}_A$ Limitations (WIP)

Generator-level study with crossing-angle



- Muon angles with boost, uses measured momenta to boost in x (small effect).
- Mass estimates use either the $\mu^+\mu^-$ mass or the final-state system mass (including FSR) and use the true ECM after BS emission.

For RR Z's. Lose a factor of 2 in potential stat. precision. (multiple radiation)

My Take on Appropriate/Relevant R&D Topics/Wish-List

- MDI/BDS: Assess and plan for global energy/luminosity spectrum/beam diagnostics analysis and insights.
- MDI/BDS: Upgrade beam-beam studies/generators to representative complete machine and variations thereof.
- MDI/BDS: Assess and plan for ultimate beam-spot/luminous region diagnostics including vertexing
- MDI/BDS: How do we deal with E-z correlations?
- MDI/BDS: Can we go beyond 100 ppm for energy spectrometers?
- PHYS/DET: Include all channels in physics center-of-mass energy estimates.
- DET: Assess and plan for ultimate tracker momentum-scale capability.
- DET: Assess and plan for ultimate polar angle systematic uncertainty.
- DET: Assess and plan for ultimate detector solenoid field-mapping capability.
- DET: Assess and plan for ultimate tracker alignment.
- DET: Incorporate more appropriate momentum reconstruction for high energy electrons (example: Gaussian Sum Filter a la CMS)

Concluding Remarks

Progress

- New high precision method for momentum-scale using especially $\mathrm{K^0_S}$ and Λ. Promises 2.5 ppm stat. uncertainty per 10M hadronic Z decays.
- More detailed investigation of dimuons for $\sqrt{s}$ and $\mathrm{dL/d}\sqrt{s}$ reconstruction
- Much higher confidence that 10 ppm is achievable for ILC250.
- Prospects for ILC precision polarized Z lineshape scan. Γ_{Z} to 0.1 MeV.
- Beamstrahlung energy/vertexing correlations look very promising

Conclusions

- ILC tracking detectors have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events)
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 2 ppm stat. precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not limit measurements such as M_{W}.
- Potential to improve M_{Z} by a factor of three using 250 GeV di-lepton data
- Applying the same techniques to running at the Z-pole enables a high precision electroweak measurement program for ILC. Takes advantage of absolute center-of-mass energy scale knowledge.
- With physics goals of energy to 10 ppm for $\sqrt{s} = 250$ GeV and of order 1 ppm at the Z, can we go beyond 100 ppm with spectrometer diagnostics?

Acknowledgments

- KU graduate student, Brendon Madison, now working on aspects of the center-of-mass energy studies including luminosity spectrum.
- Input from Michael Peskin on Z lineshape helped with the semi-empirical fit parametrization.
- Mikael Berggren for help with Guinea-PIG setup consistent with ILD's Whizard event generation.
- Feedback from Glen White and Kaoru Yokoya on some accelerator physics issues.
- Work done producing the central ILD simulated samples.
- arXiv:1909.12245 has related studies of energy calibration for FCC-ee.

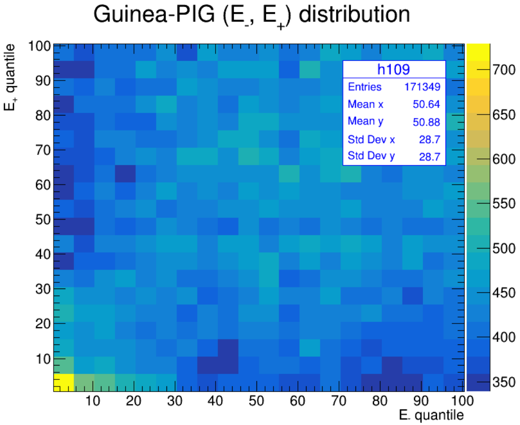
Backup Slides

Parametrizing the Luminosity Spectrum

CIRCE1 by Thorsten Ohl was a simple parametrization of the luminosity spectrum. Essentially 3-parameters: p_{peak} and the two parameters of a Beta distribution and the assumption of beam 1 being independent from beam 2.

$$\mathrm{Beta}(y; \alpha, \beta) \sim y^{\alpha-1}(1-y)^{\beta-1}$$

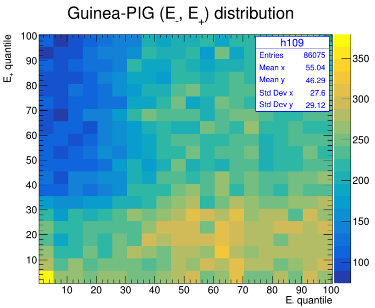
where y is the fractional energy loss.



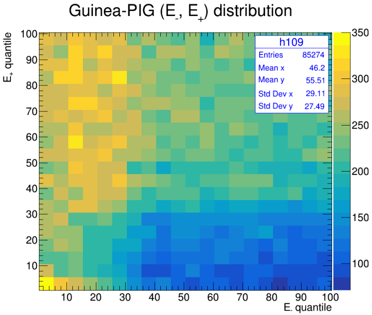
If independent, and the 1-d quantiles have equal probability (design here is 1%) each 2-d cell should have 0.25% of the entries.

Motivation for "CoPa" type parametrization (see Andre Sailer thesis).

For symmetric configurations, find that the distributions after BES and beamstrahlung can be reasonably modeled with a 10-parameter function.



$z_{\mathrm{PV}} < 0$



$z_{\mathrm{PV}} > 0$

In order to accommodate these obvious asymmetries associated with z_{PV}, have adopted a 15-parameter relatively parsimonious fit for this.

New 15-parameter model including BES

- Use four region probabilities: peak, arm1, arm2, body (slide 10). (4 - 1 = **3**)
- Each BS component has its own 2-parameter beta distribution. ($4 \times 2 =$ **8**)
- Model BES with a Gaussian for each beam, $z_i \sim \text{Ga}(\mu_i, \sigma_i)$. ($2 \times 2 =$ **4**)
- Model BS as a Beta distribution, $y_i \sim 1 - \text{Beta}(\mu, \text{rms})$. The convolved, $x_i = y_i z_i$ where $x_i = E_i / E_{\text{nom}}$. Use ($\mu$, rms) as fit parameters (not α, β).

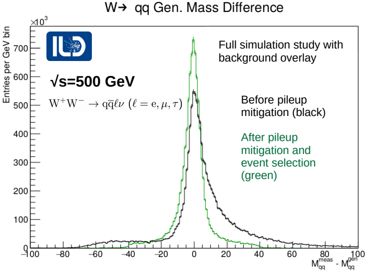
- The 4 region probabilities correspond to (BES, BES), (BES+BS, BES), (BES, BES+BS), and (BES+BS, BES+BS).
- dmu1, dmu2 are in units of 0.001.
- arm1 defined as BS for beam 1. e^- loses less energy than e^+ here.
- Find good reweighting fits using 10k quantiled cells to 171k events.

NAME	VALUE	ERROR
ppeak	0.18163	0.16265E-01
pbody	0.21808	0.12701E-05
parm1	0.22421	0.14074E-01
meanb1	0.25342E-01	0.42865E-03
rmsb1	0.39322E-01	0.52909E-03
meana1	0.11508E-01	0.73532E-03
rmsa1	0.26036E-01	0.80829E-03
meanb2	0.28197E-01	0.45942E-03
rmsb2	0.39870E-01	0.51985E-03
meana2	0.19601E-01	0.51195E-03
rmsa2	0.32457E-01	0.39896E-03
dmu1	-0.25977E-01	0.95168E-02
s1	0.19010E-02	0.67902E-05
dmu2	-0.18797E-01	0.11780E-01
s2	0.15164E-02	0.72661E-05

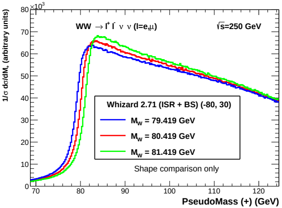
ILC250 $z_{\text{PV}} < 0$

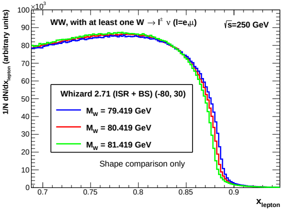
Would be great to have BES **and** BS in more MC generators. Also need reliable and appropriately configured beam-beam simulations (Guinea-PIG, CAIN).

M_{W}, Γ_{W} measurements concurrent with Higgs program



- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta M_{\mathrm{W}} = 2.4$ MeV for $1.6\ \mathrm{ab}^{-1}$ (-80%, +30%).
- Can be improved, but m_{had}-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set



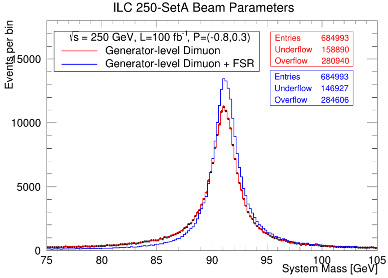


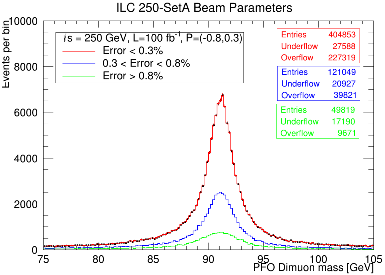
Sensitivity to M_{W} with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta M_{\mathrm{W}} = 4.4$ MeV for $2\ \mathrm{ab}^{-1}$ (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell / E_{\mathrm{b}}$. Exptl. systematics small.

Measuring M_Z using $m_{\mu^+\mu^-}$ with high energy running

Look at $\sqrt{s} = 250$ GeV running with latest beam parameters and full simulation





Adding in FSR photon(s) reduces the peak width to be consistent with Γ_Z. Improves statistical sensitivity on mode by 10–20%.

$m_{\mu^+\mu^-}$ resolution is much less than Γ_Z. Sensitivity estimates from prior study (slide n+2) with smeared MC will be reasonable.

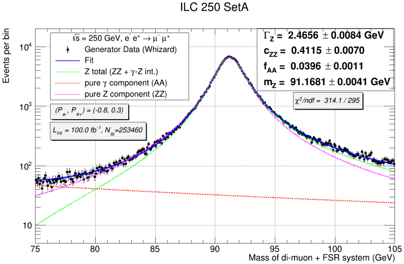
Main systematics:

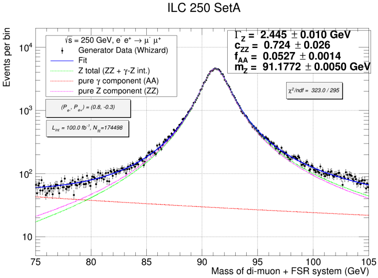
1. momentum-scale
2. FSR modeling/treatment
3. Electron p-scale in the e^+e^- channel

Also direct measurement of Γ_Z

Radiative return to the Z for M_Z and Γ_Z

Expected stat. precision on M_Z and Γ_Z is driven by the no. of events and Γ_Z.





Semi-empirical physics-based parametrization. Shape given by a relativistic Breit-Wigner with additional shape contributions from pure photon-exchange and $\gamma - Z$ interference using Born-level $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)$ at ISR reduced $\sqrt{s'}$. Fits generator-level distribution (after BS and ISR) surprisingly well.

Using similar fits to gen.-level distributions (but for dimuon events passing event selection criteria): uncertainty of 1.0 MeV on M_Z and 2.2 MeV on Γ_Z for 2 ab^{-1} at $\sqrt{s} = 250$ GeV (just $\mu^+\mu^-$ channel)

Measuring M_Z from $m_{\mu^+\mu^-}$

Revisited old study of $\sqrt{s}_p$ at $\sqrt{s} = 250, 350, 500, 1000$ GeV. Used smeared MC. Fitted $m_{\mu^+\mu^-} \in [75, 105]$ GeV with sum of two Voigtians. Statistical uncertainties on the peak parameter, M_Z, scaled to full ILC program using simulations with TDR beam parameters

Statistical uncertainties for $\mu^+\mu^-$ channel

$\sqrt{s}$ [GeV]	L_{int} [ab^{-1}]	Poln [%]	Sharing [%]	ΔM_Z [MeV]
250	2.0	80/30	(45,45,5,5)	1.20
350	0.2	80/30	(67.5,22.5,5,5)	5.99
500	4.0	80/30	(40,40,10,10)	2.55
1000	8.0	80/20	(40,40,10,10)	5.75
All	14.2	–	–	1.05

- Current PDG uncertainty on M_Z is 2.1 MeV
- FSR makes effective Breit-Wigner width larger and shifts the peak
- Treatment of FSR and especially inclusion of e^+e^- channel should decrease stat. uncertainty to **0.7 MeV**. Similarly Γ_Z to 1.5 MeV.
- Sensitivity dominated by $\sqrt{s} = 250$ GeV running
- Main systematic - tracker p-scale. Target at most 2.5 ppm in this context.