A_LR measurement using e+e- to gamma Z

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Physics at ILC 250

- Primary physics aim of ILC 250: measuring the **coupling constants between the Higgs boson and various other standard model (SM) particles**
- The coupling constants can deviate from their SM values because of possible Beyond the Standard Model (BSM) effects.
- The size of corrections to the SM predictions for the Higgs boson couplings:

$$a\frac{m_H^2}{M^2}$$

M: new particle mass [GeV]

m_H: 125 [GeV]

a: coefficients of order 1

→ Need to measure the couplings with the precision ~1%

SM effective field theory (SMEFT)

- Lorentz invariant and SU(2) × U(1) gauge invariant dim-6 operators

$$
\begin{aligned}
\Delta\mathcal{L} = & \frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) + \frac{c_T}{2v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6\lambda}{v^2}(\Phi^\dagger\Phi)^3 \\
& + \frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2}\Phi^\dagger t^a\Phi W^a_{\mu\nu}B^{\mu\nu} \\
& + \frac{g'^2 c_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho W^{c\rho\mu} \\
& + i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\overline{L}\gamma_\mu L) + 4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu\Phi)(\overline{L}\gamma_\mu t^a L) \\
& + i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu\Phi)(\overline{e}\gamma_\mu e)
\end{aligned}
$$

ILC, Higgs related part

(CP-Conserving)

- The useful observables for the SMEFT global fit include not only those from the reactions that directly involve the Higgs boson, but also those from Electroweak Precision Observables (EWPOs) for W and Z bosons.
  For example, left-right polarization asymmetry A_{LR} of the Z-pole cross section is important.

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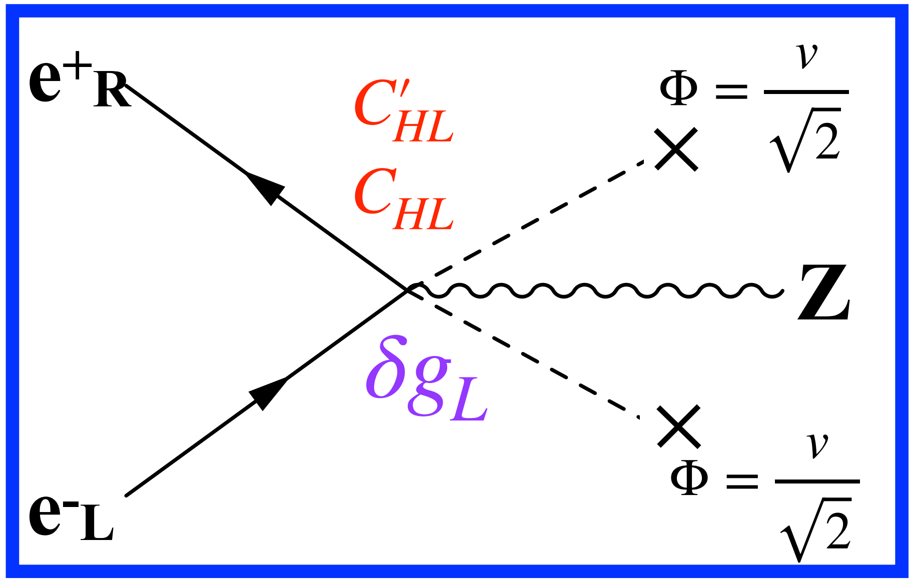
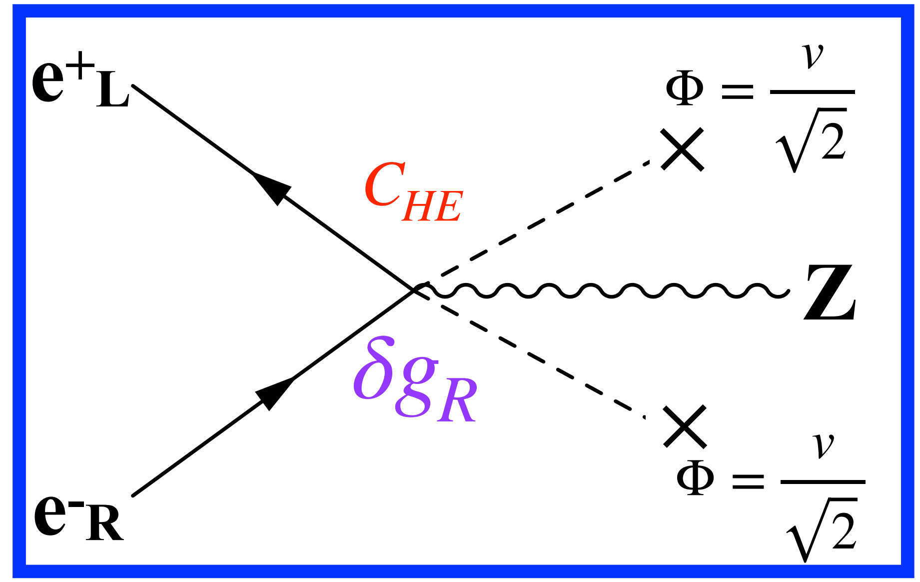
$(vZ^\mu v)$ $(vt^aZ^\mu v)$ $(vZ^\mu v)$





$$g_L = \frac{g}{c_W}[(-\frac{1}{2} + s_W^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}(c_{HL} + c\prime_{HL}) - s_W c_W \delta Z_{AZ}]$$

$$g_R = \frac{g}{c_W}[(+s_W^2)(1 + \frac{1}{2}\delta Z_Z) - \frac{1}{2}c_{HE} - s_W c_W \delta Z_{AZ}]$$

$$\delta Z_Z = c_W^2(8c_{WW}) + 2s_W^2(8c_{WB}) + \frac{s_W^4}{c_W^2}(8c_{BB})$$

$$\delta Z_{AZ} = s_W c_W((8c_{WW}) - (1 - \frac{s_W}{c_W^2})(8c_{WB}) - \frac{s_W}{c_W^2}(8c_{BB})$$

$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2(\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

$$\Delta\mathcal{L} = i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu L) + 4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu t^a L) + i\frac{\bar{c}_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{e}\gamma_\mu e)$$

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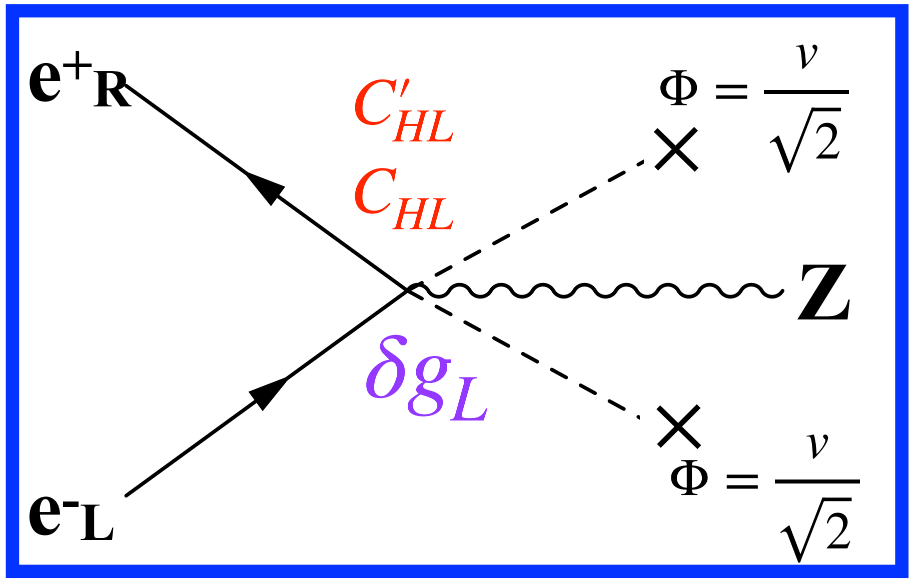
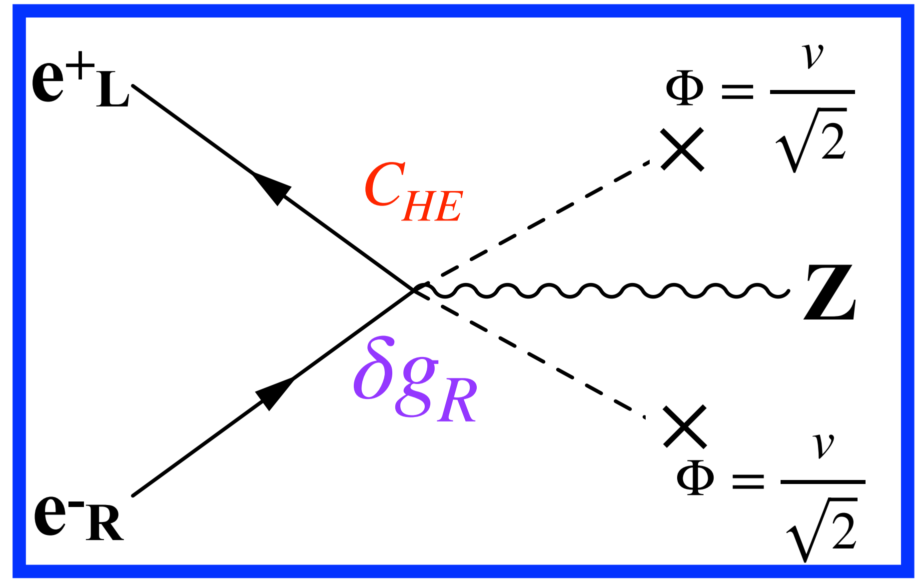




Current best measurement

A_{LR}= 0.1514 ± 0.0019 (statistic error) ± 0.0011(systematic error)

Need full detector simulation of radiative return events to check how much error we have at ILC 250

$$A_{LR} = A_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2}$$

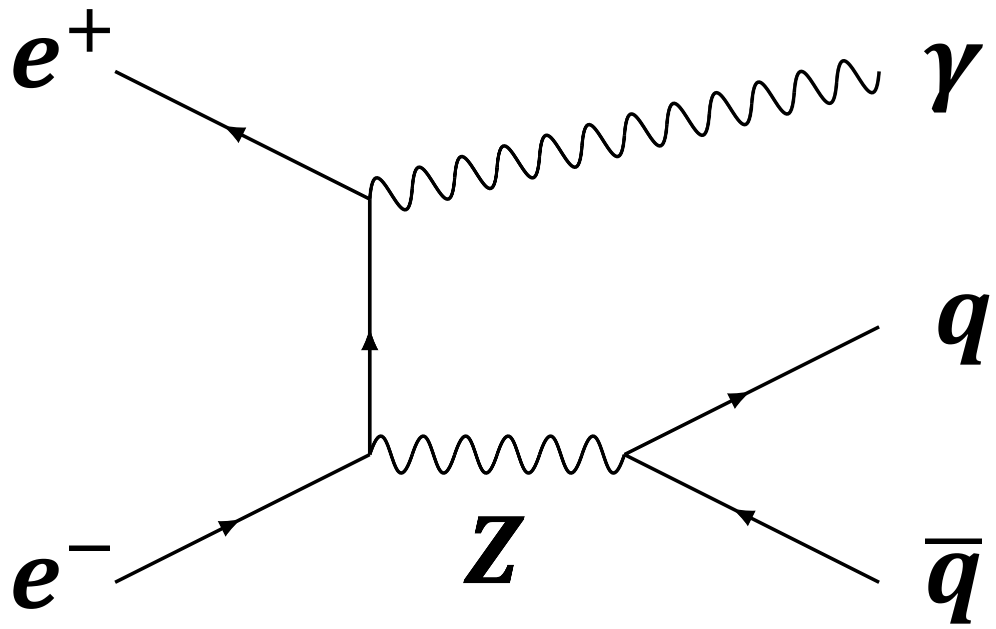
$$\delta A_e = \frac{4g_{Le}^2 g_{Re}^2(\delta g_{Le} - \delta g_{Re})}{g_{Le}^4 - g_{Re}^4}$$

Simulation Setup

Full simulation (MC-2020, Whizard 2.8.5)

- E_{CM}= 250 GeV
- $\int \mathscr{L} dt$ =900 fb^{-1} for each of the 2 polarization combinations
- sin θ_W = 0.22225 which is equivalent to A_{LR} = 0.219298.

Signal: e^+e^- -> Zγ -> 2 jets +γ



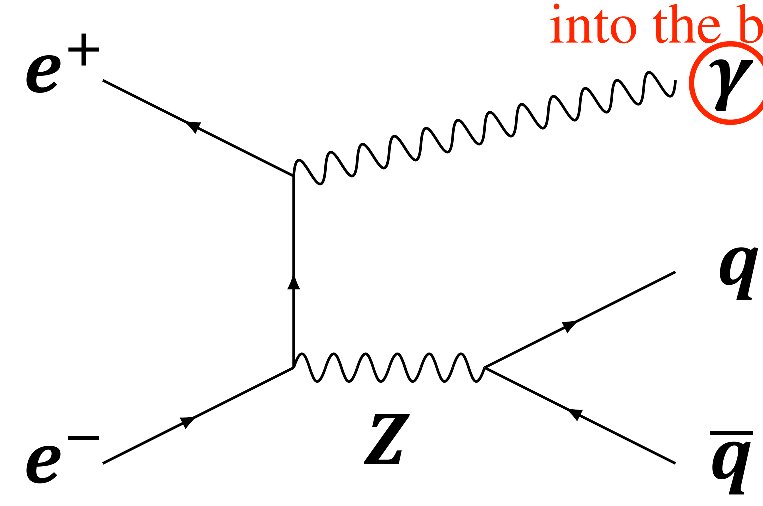
Background: All the events only with 2 jets in the final state

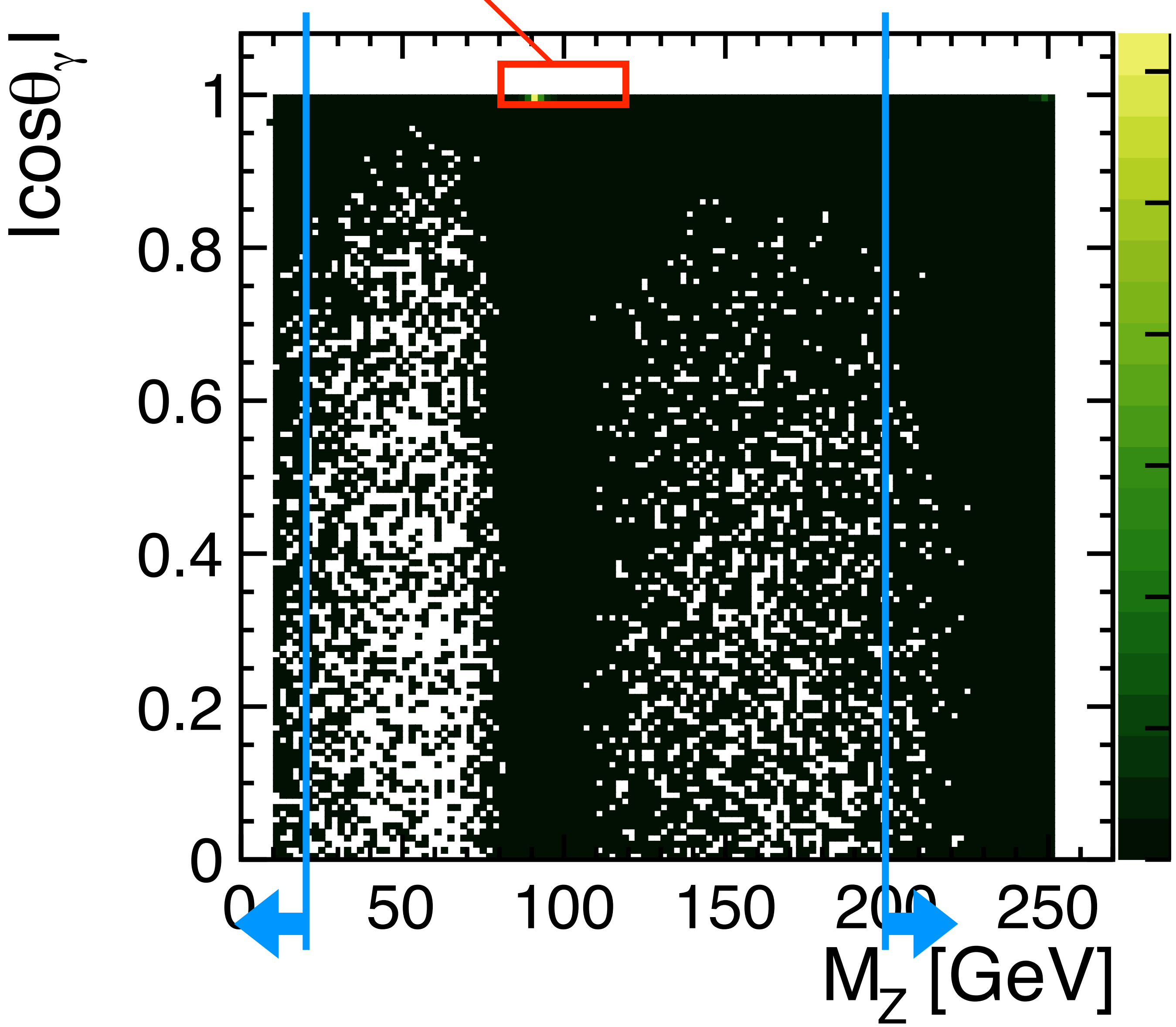
Signal event definition

Signal event: radiative return with photon escaping into beam pipe

A. 80 GeV < $M_{Z(truth)}$ < 120 GeV

B. $|\cos\theta_{\gamma(truth)}| > 0.999$





eLpR

Background

Non-signal 2f_z_h

$M_{Z(truth)}$ < 20 GeV, $M_{Z(truth)}$ > 200 GeV

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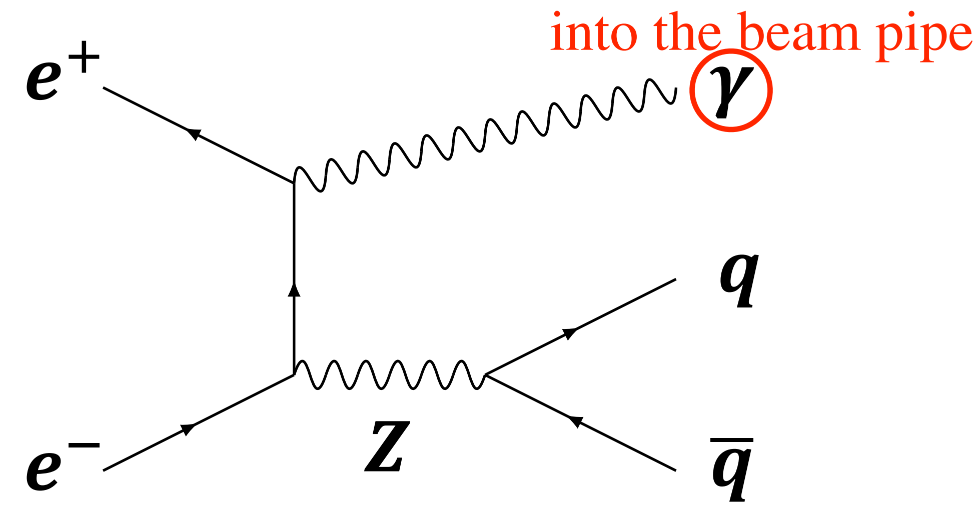
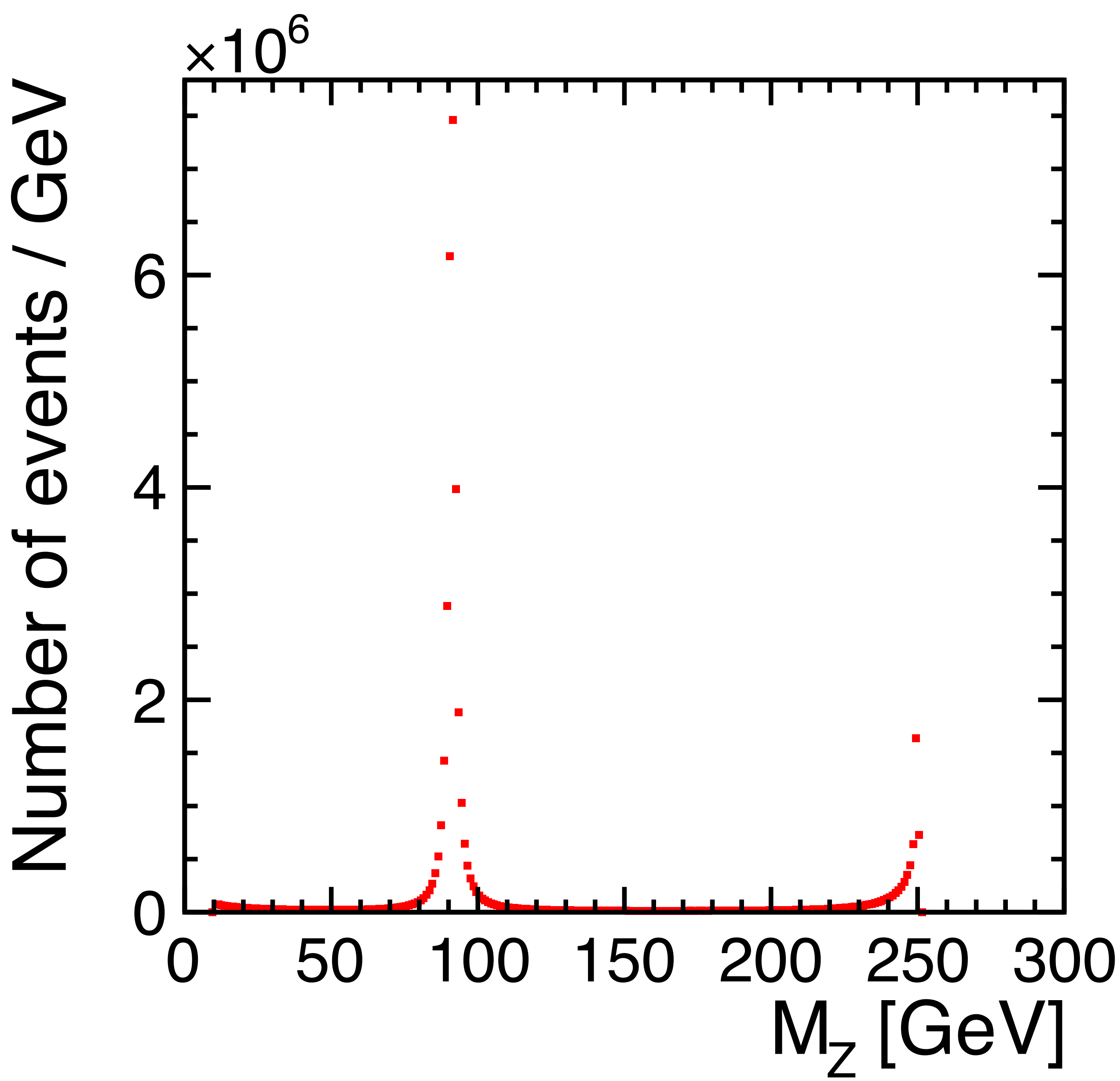
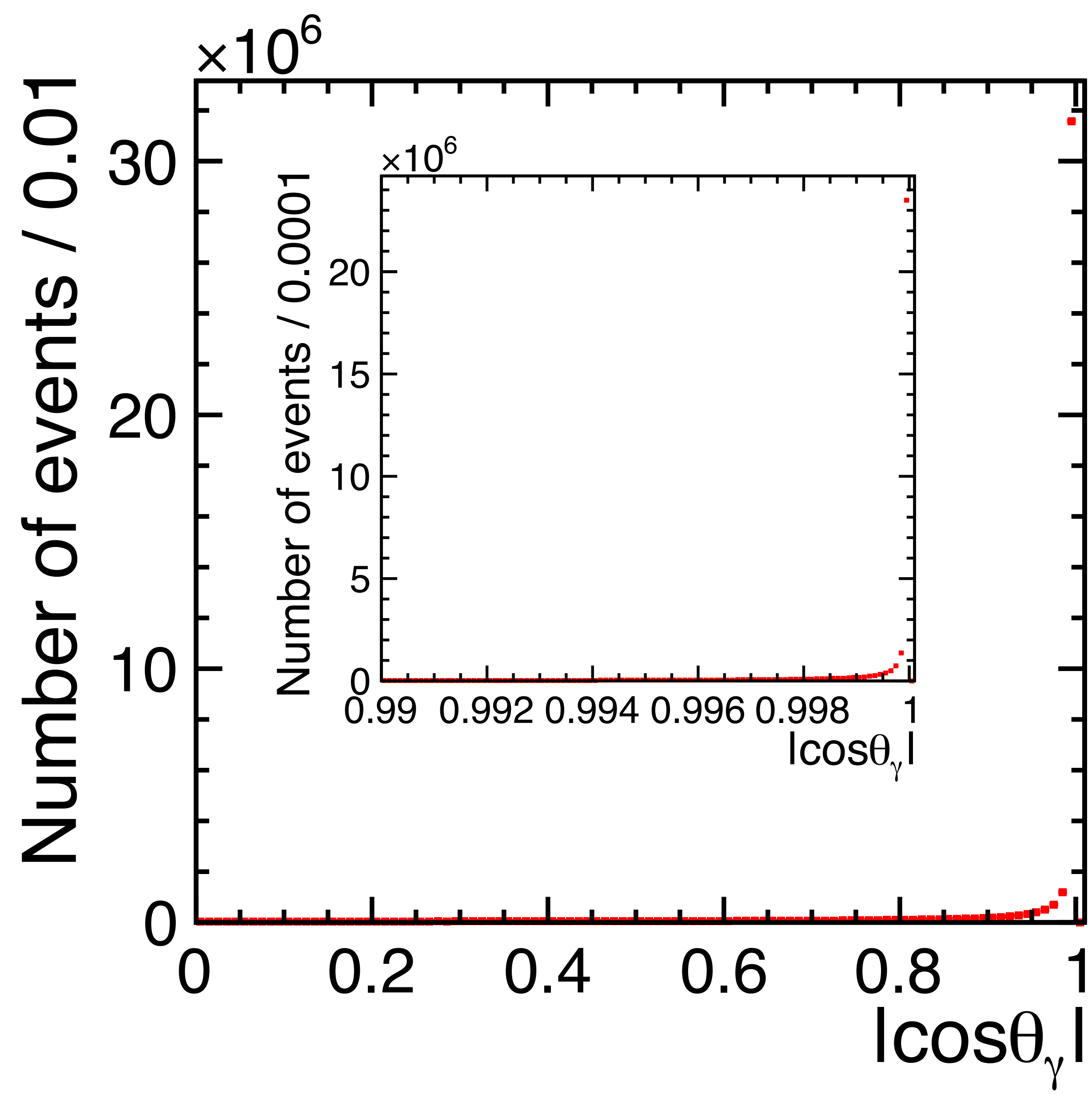
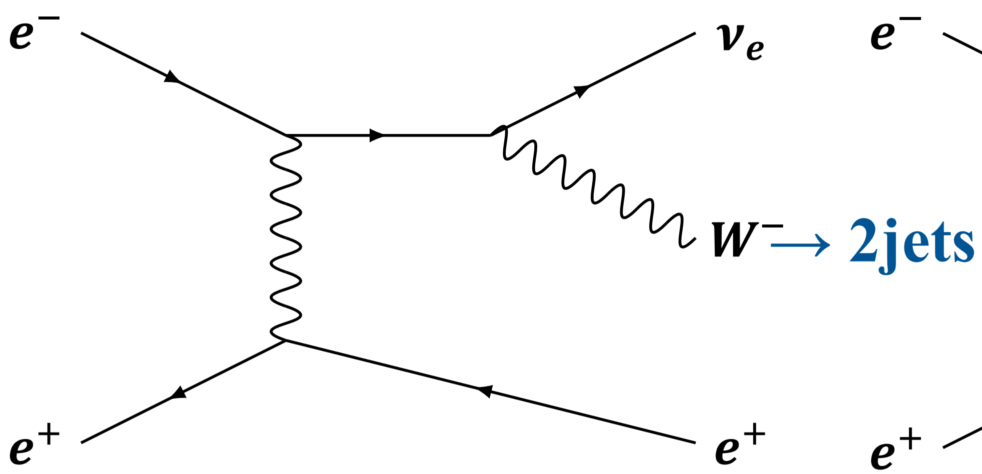




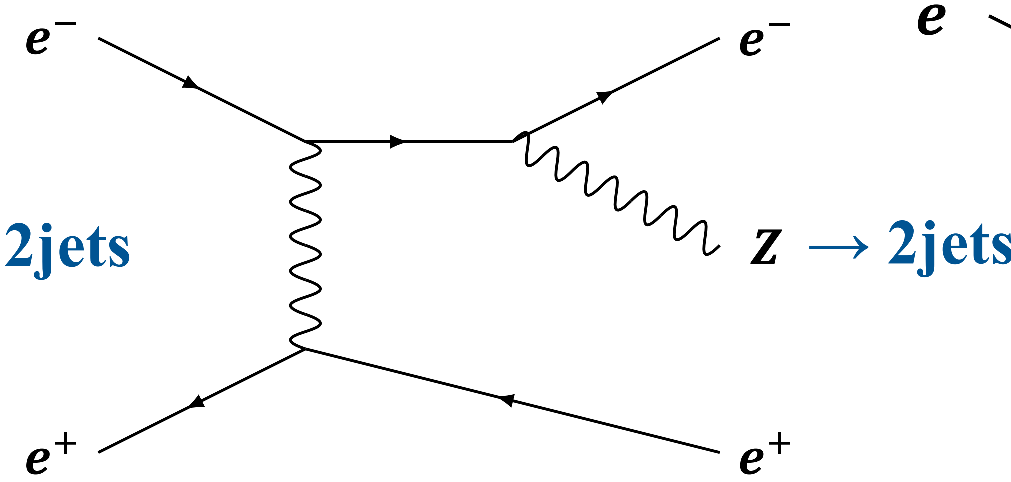


Major backgrounds

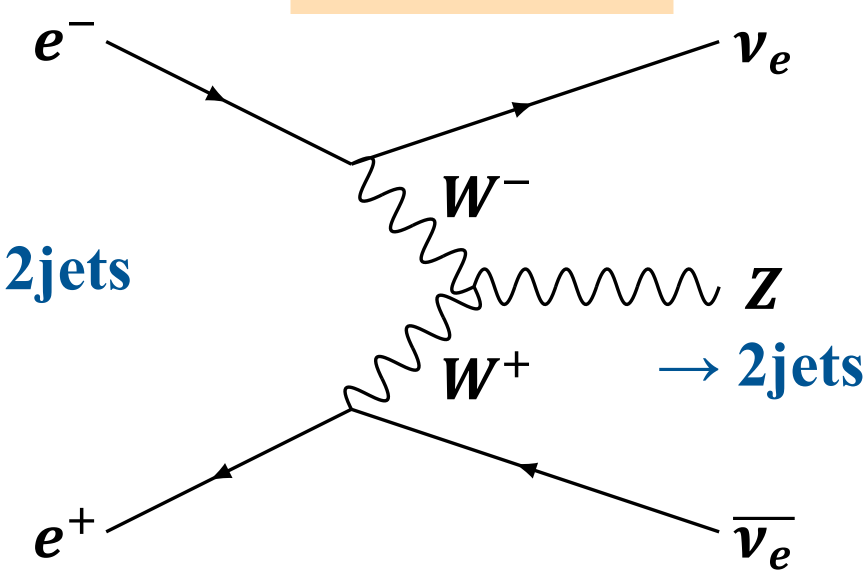
4f_sw_sl



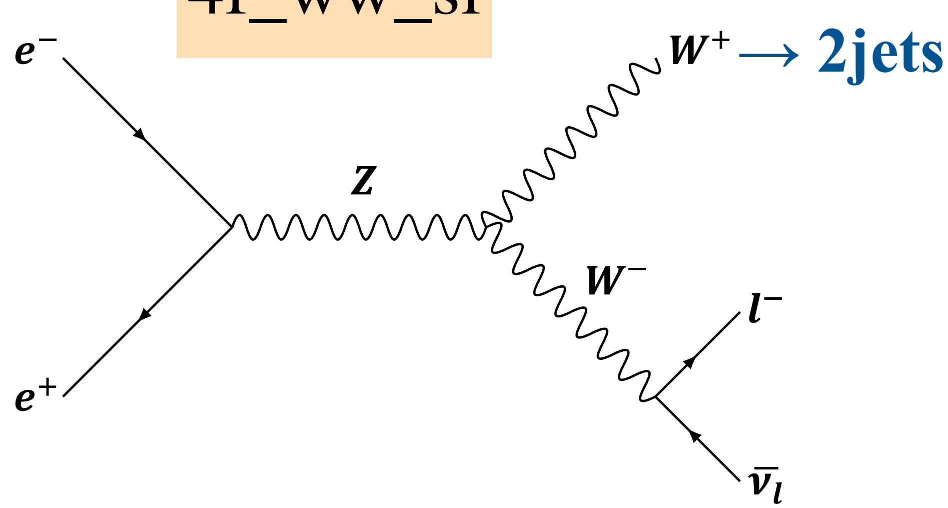
4f_sze_sl



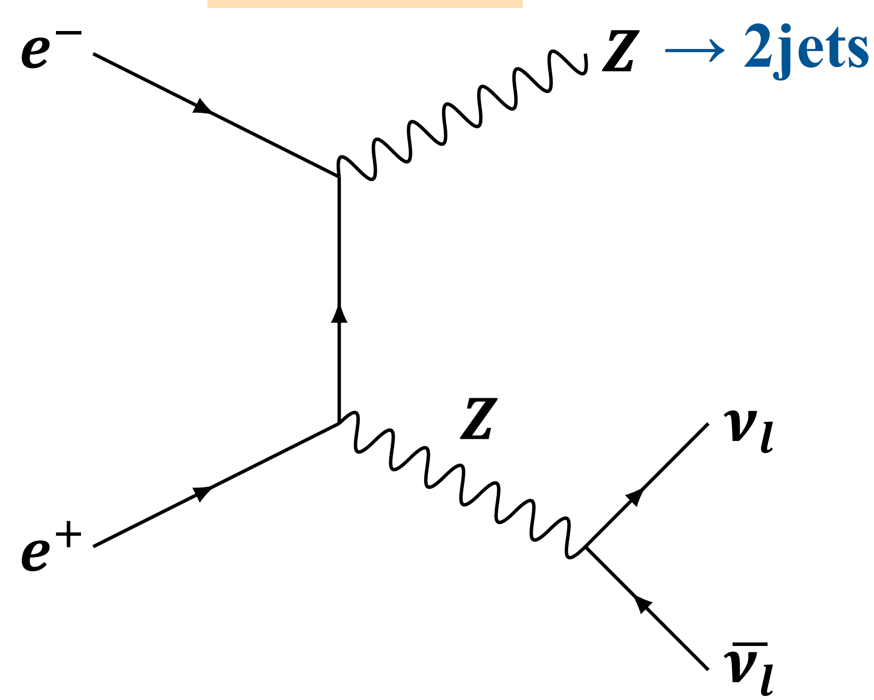
4f_sznu_sl



4f_ww_sl



4f_zz_sl



Event selection

Cut 1 $N_{\gamma(E>50\,GeV)} = 0$

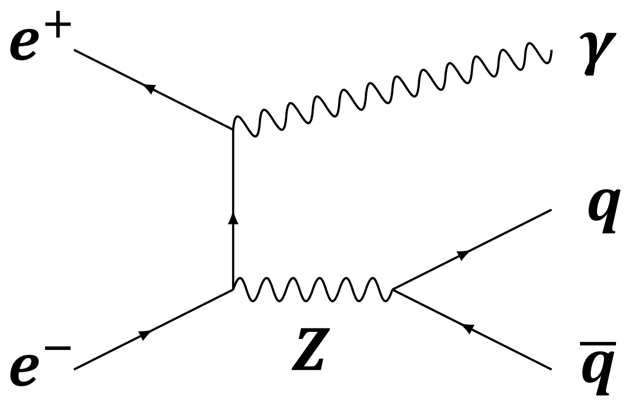
Cut 2 $120\,GeV < E_{vis} < 160\,GeV$

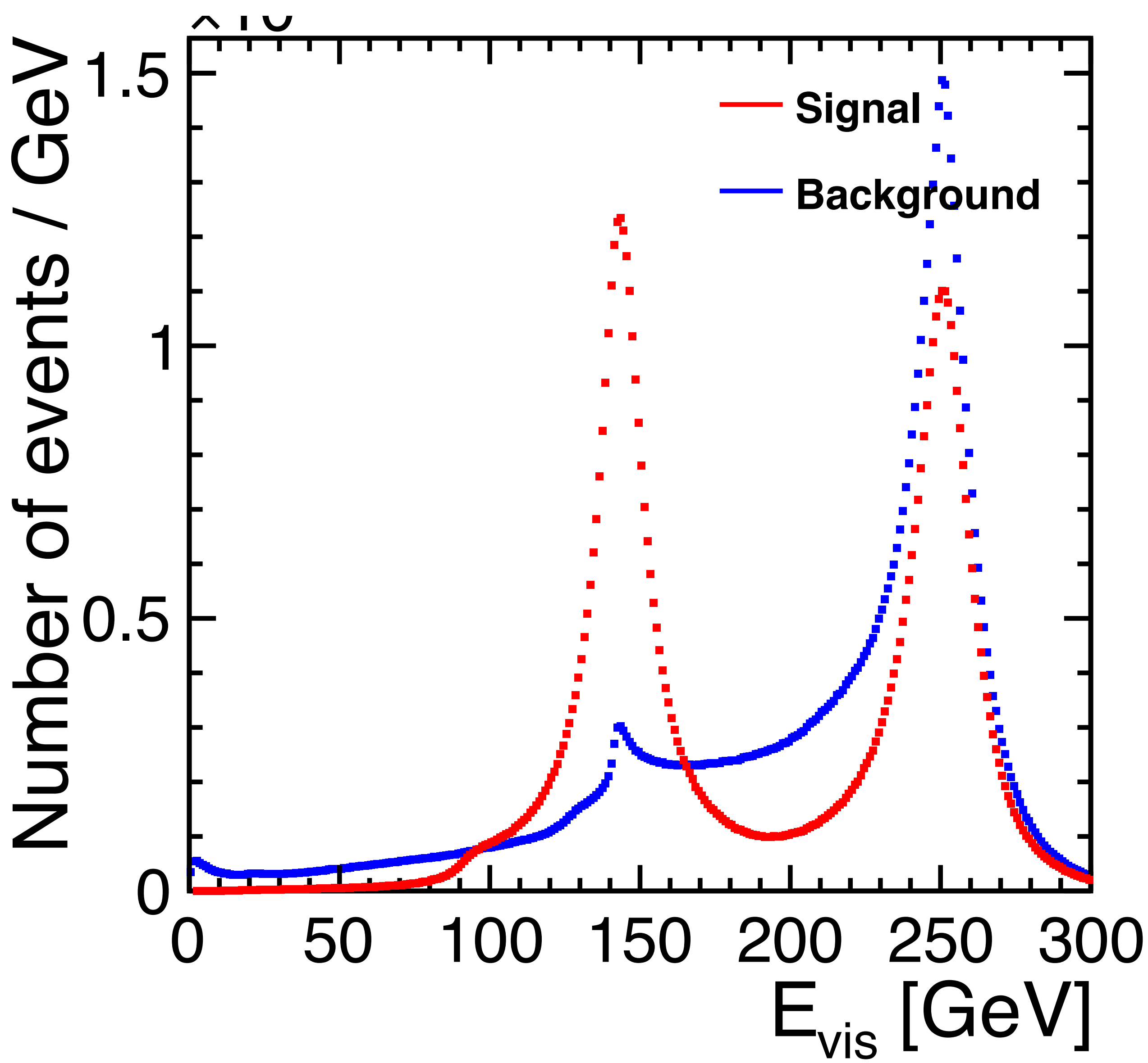
Cut 3 $|\cos\theta_{2j}| > 0.95$

Cut 4 $N_{J1}^{charged} > 4,\ N_{J2}^{charged} > 4$

Cut 5 $N_{J1}^{charged+neutral} > 6,\ N_{J2}^{charged+neutral} > 6$

Cut 6 $50\,GeV < M_{2j} < 160\,GeV$





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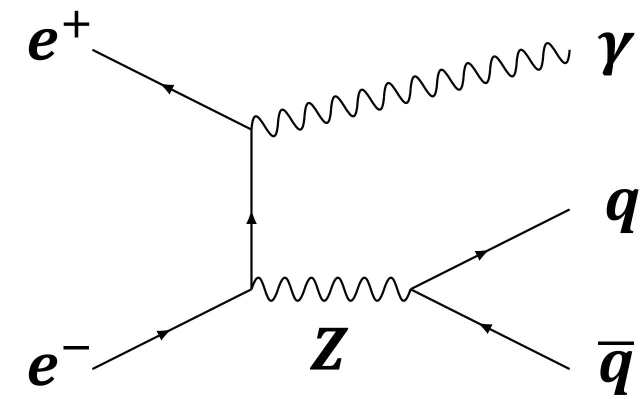

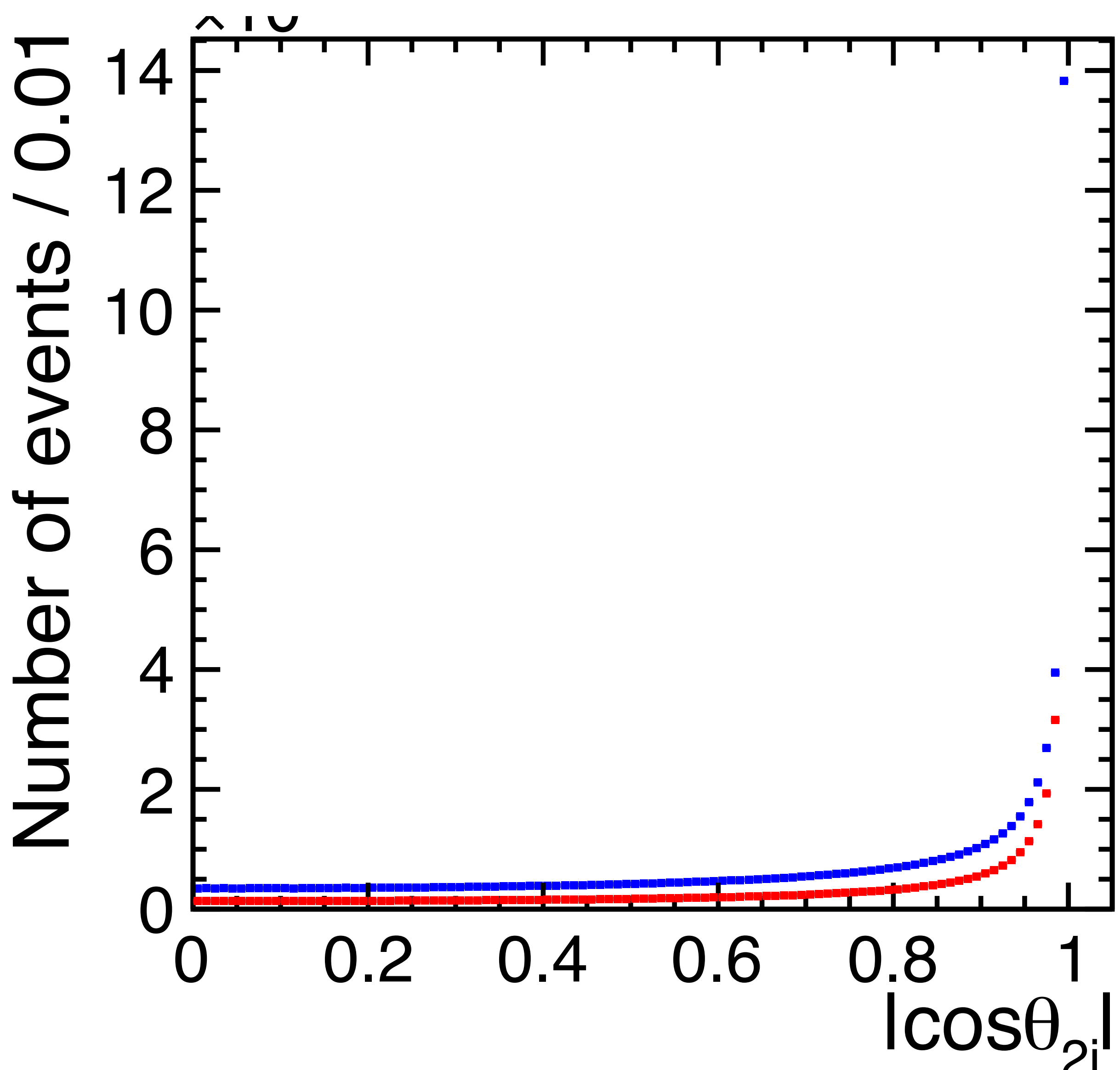

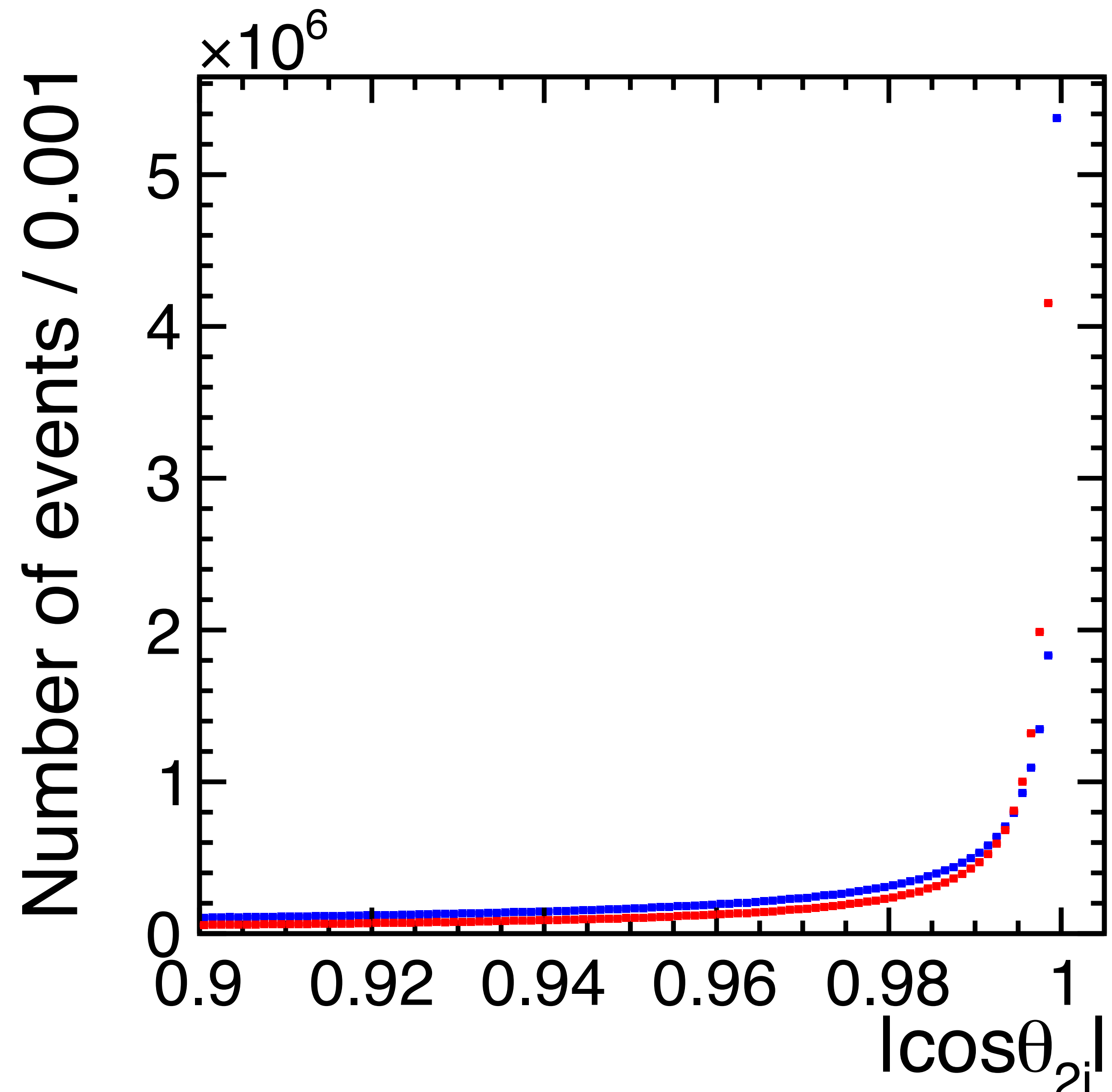

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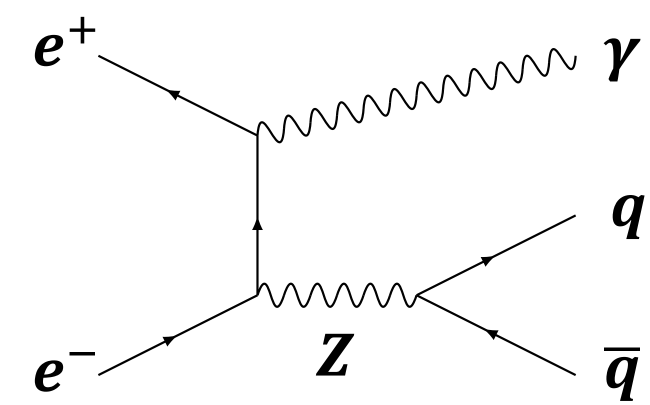

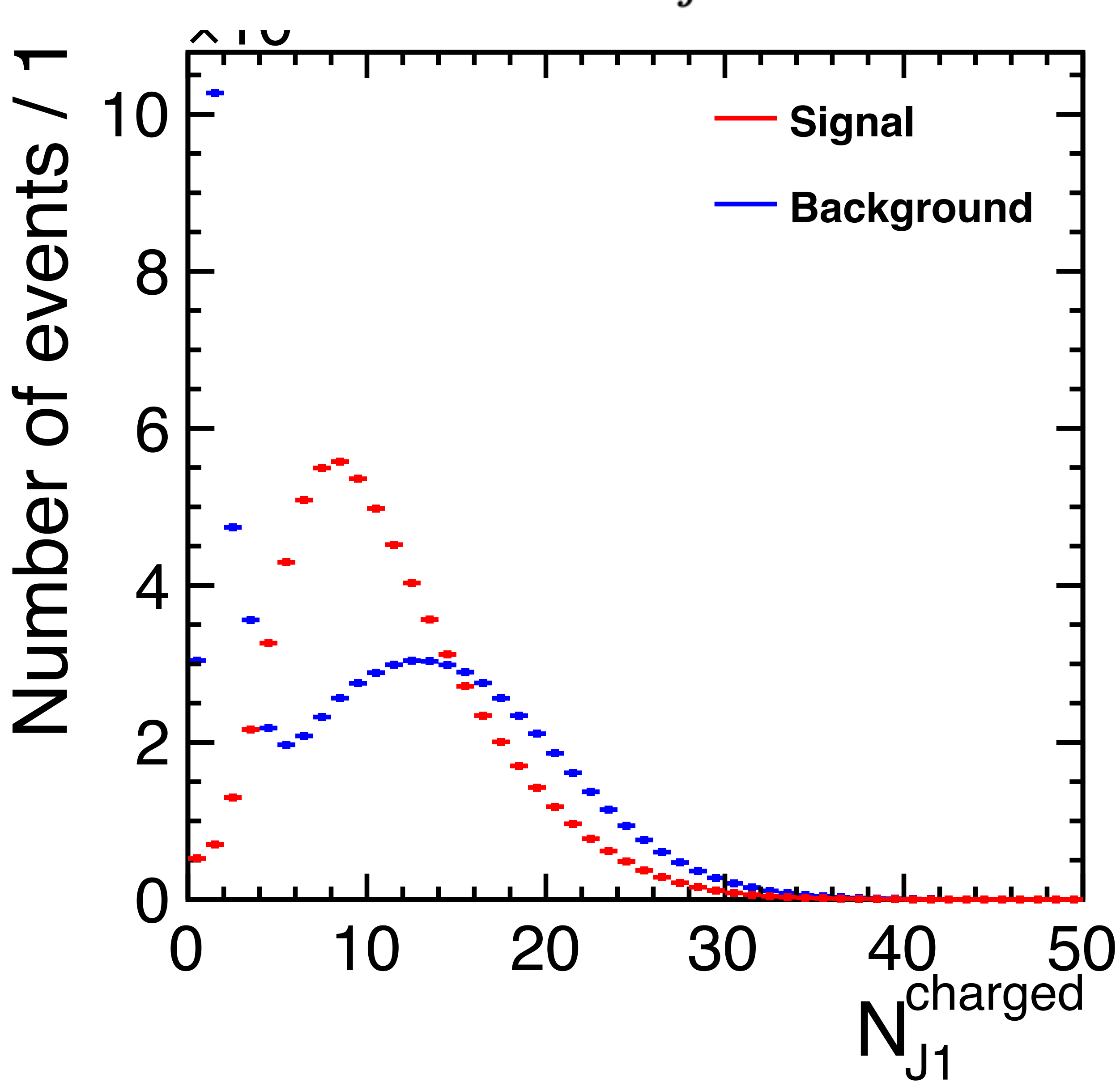

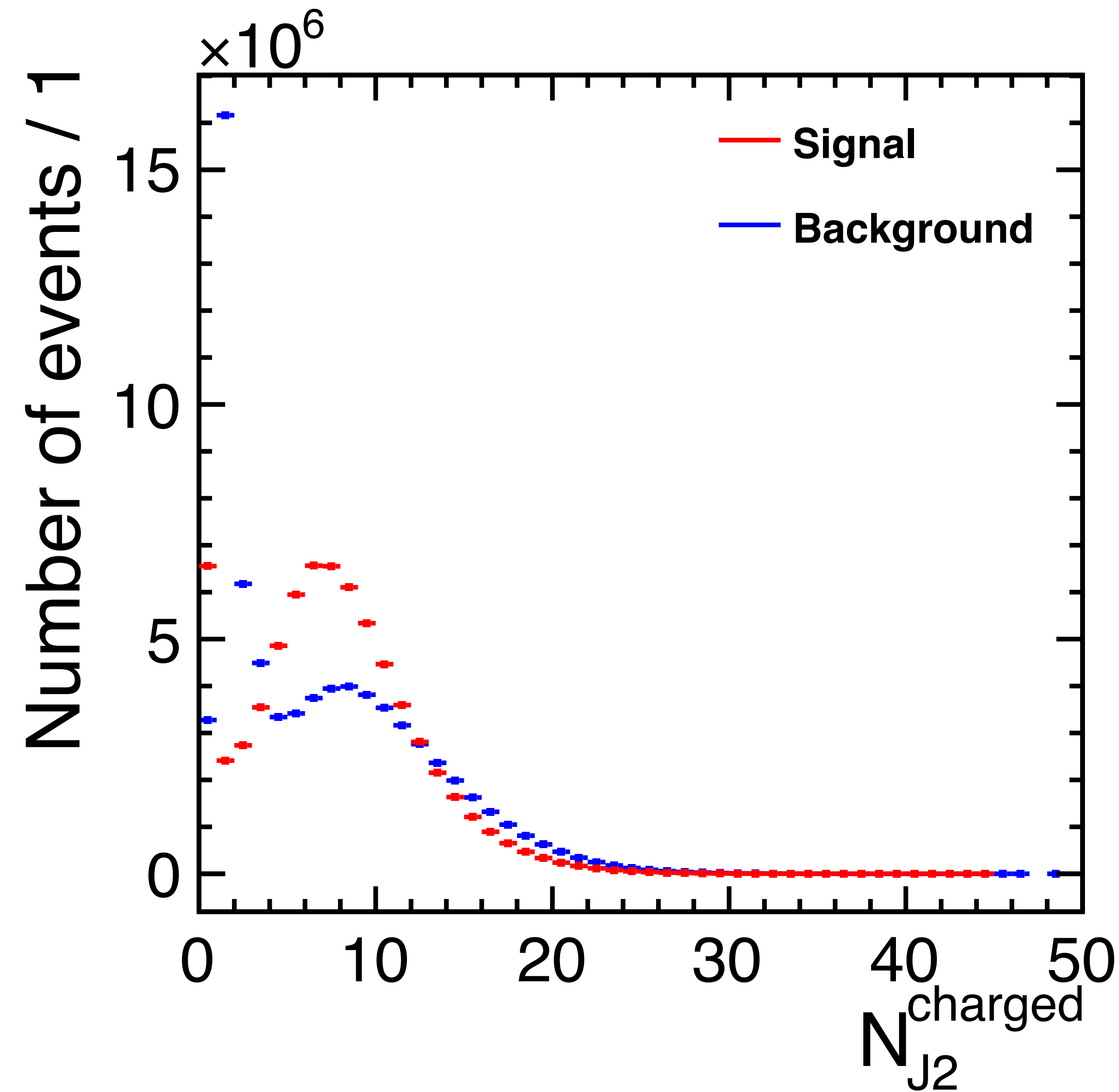

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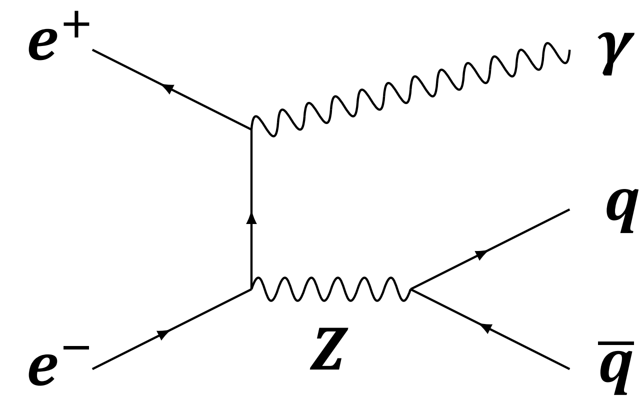

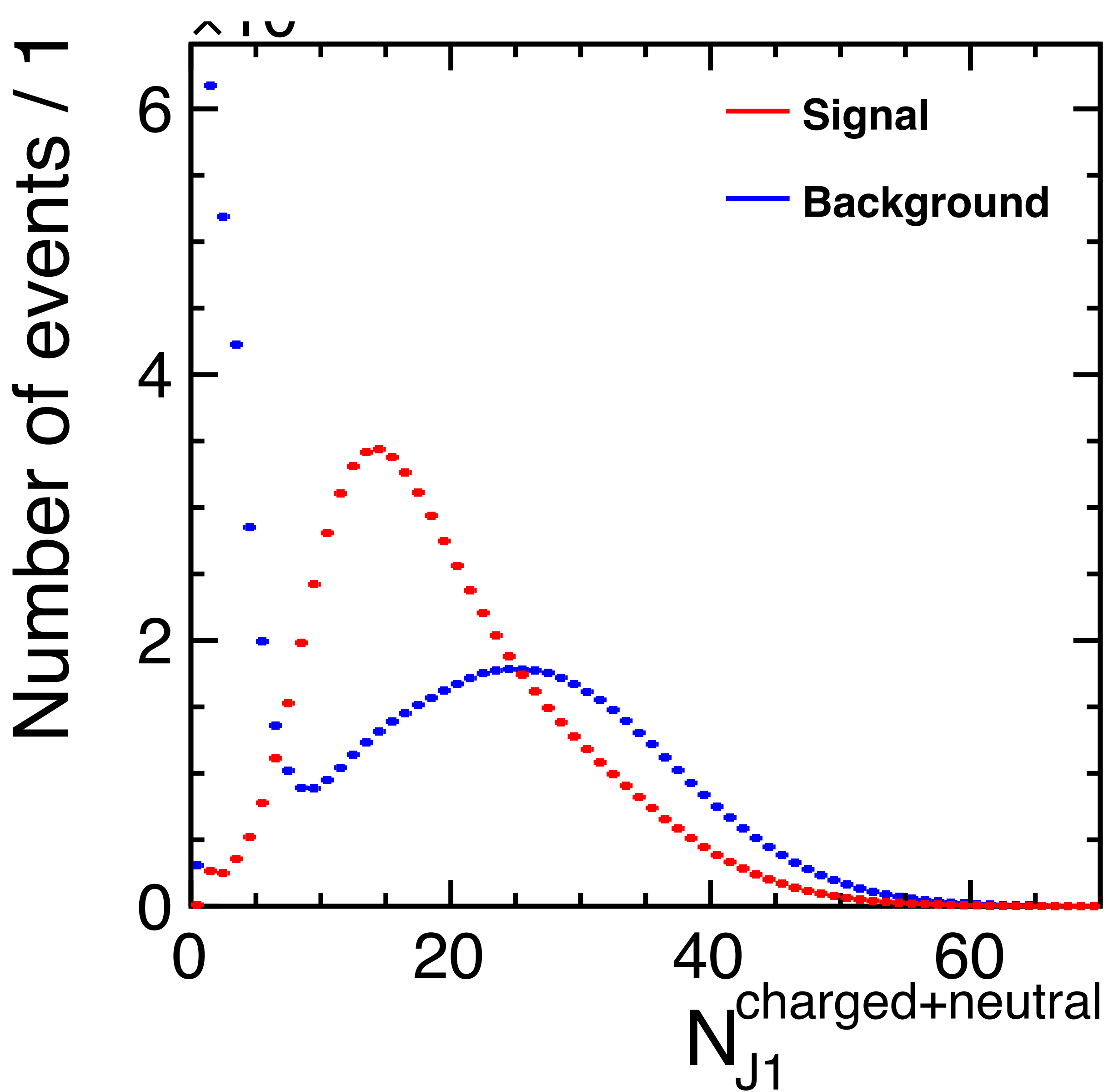

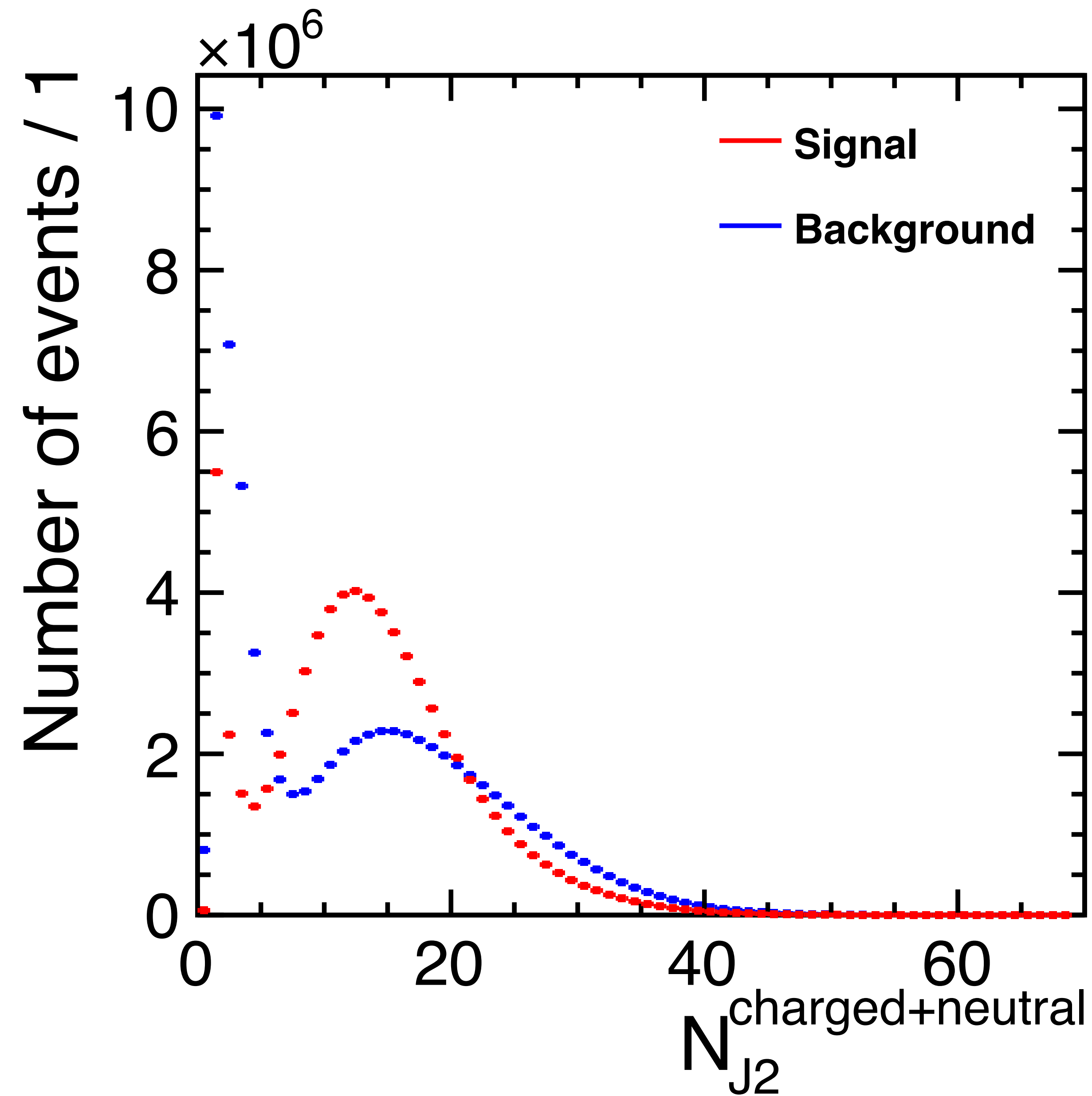

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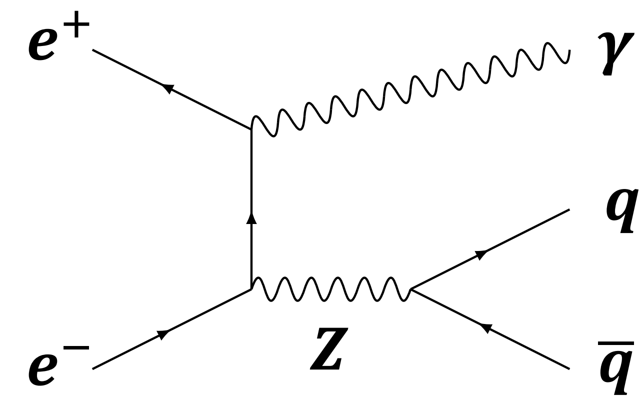

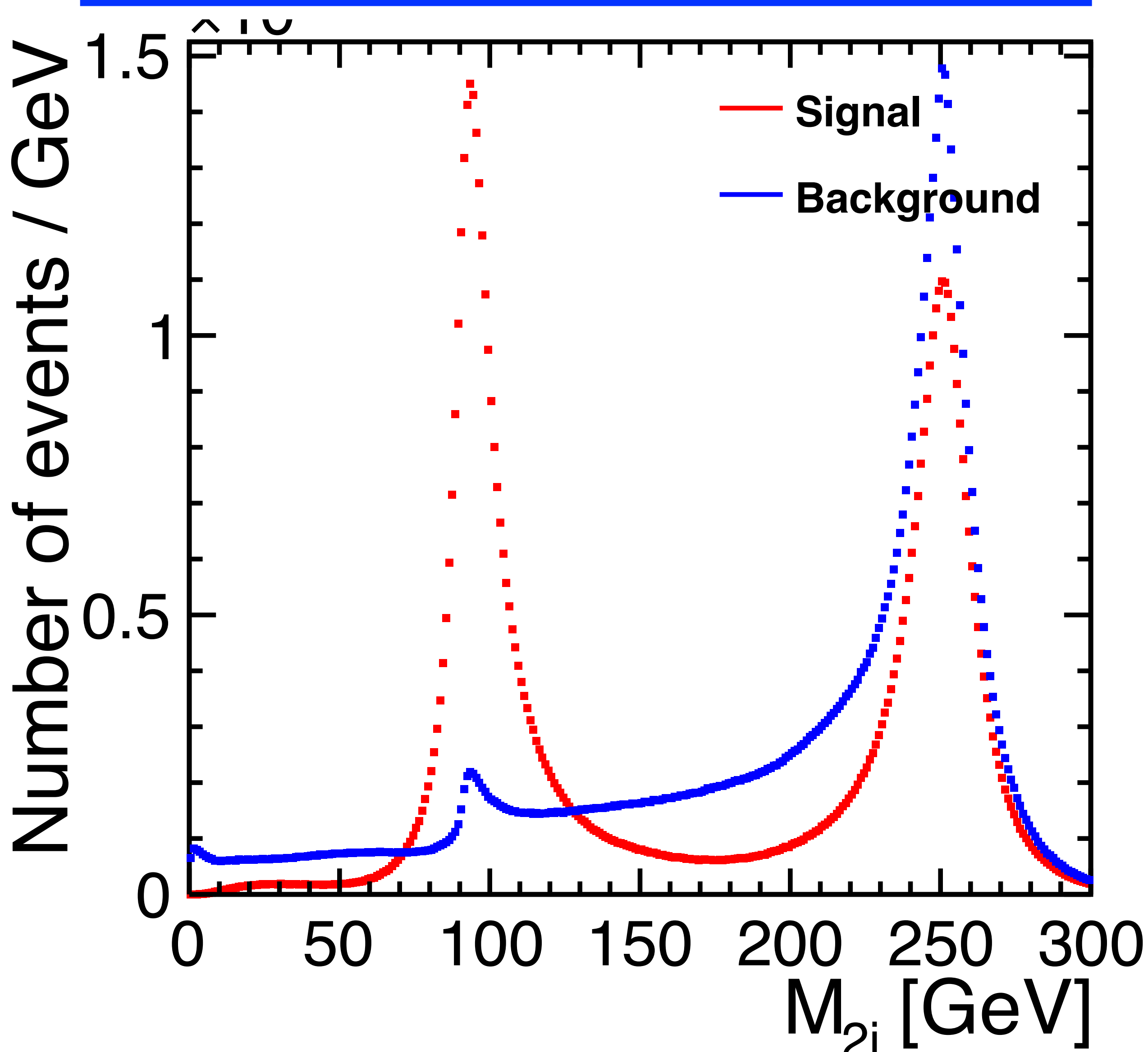

Cut Table

(−0.8, +0.3) polarization.

$\times 10^6$ events	Signal	2f_l	4f_l	4f_sl	4f_h	2f_h (non-Signal)	Background tot.
Expected	32.5	12.7	9.34	17.2	15.1	18.8	73.2
Cut 1	31.1	10.1	5.96	16.0	14.8	17.9	64.7
Cut 2	24.4	2.55	1.46	3.22	0.00422	0.0722	7.30
Cut 3	24.4	1.93	0.366	0.526	0.00352	0.0601	2.89
Cut 4	16.6	0.00328	0.000386	0.321	0.00299	0.00329	0.33
Cut 5	16.2	0.00163	0.000155	0.312	0.00295	0.00273	0.32
Cut 6	16.2	0.00163	0.000155	0.312	0.00295	0.00258	0.32

Estimation of A_{LR} precision (1)

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}.$$

L/R : 100% polarization

$$A_{LRobs} \equiv \frac{\sigma_{-+} - \sigma_{+-}}{\sigma_{-+} + \sigma_{+-}}$$

-/+ : Polarization at ILC

$$\sigma_{-+} = \frac{1}{4}(1+|P_-|)(1+|P_+|)\sigma_L + \frac{1}{4}(1-|P_-|)(1-|P_+|)\sigma_R$$

$$\sigma_{+-} = \frac{1}{4}(1-|P_-|)(1-|P_+|)\sigma_L + \frac{1}{4}(1+|P_-|)(1+|P_+|)\sigma_R$$

$$A_{LR} = A_{LRobs}\frac{1+|P_-||P_+|}{|P_-|+|P_+|} = A_{LRobs} \times f$$

The error of the A_{LR} can be expressed as

$$\left(\frac{\Delta A_{LR}}{A_{LR}}\right)^2 = \left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 + \left(\frac{\Delta f}{f}\right)^2$$

Estimation of A_{LR} precision (2)

Assume $\Delta|P_-|$ and $\Delta|P_+|$ are independent, then

$$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{|P_-|(1+|P_+|)(1-|P_+|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_-|}{|P_-|}\right)^2 + \left(\frac{|P_+|(1+|P_-|)(1-|P_-|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_+|}{|P_+|}\right)^2$$

As for the error of A_{LRobs}, defining

$$N_{-+} = \eta_{-+}L_{-+}\sigma_{-+} \qquad \alpha \equiv L_{-+}\eta_{-+}$$

$$N_{+-} = \eta_{+-}L_{+-}\sigma_{+-}, \qquad \beta \equiv L_{+-}\eta_{+-},$$

$$A_{LRobs} = \frac{\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}}{\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}},$$

N: number of events

η: selection efficiency

L: integrated luminosity

Correlated parts of the error of α and β cancel in A_{LRobs}.

$$\Delta A_{LRobs\_correlated} \simeq \frac{1}{2}\left(1 - A^2_{LRobs}\right) \times \left|\frac{\beta-\alpha}{\alpha}\right| \times \frac{\sqrt{c}}{\alpha}$$

$$= \frac{\sqrt{c}}{\alpha} \times 4.60 \times 10^{-5} \quad \text{(with current using sample)}$$

$$E = \begin{pmatrix} <(\Delta\alpha)^2> & <(\Delta\alpha)(\Delta\beta)> \\ <(\Delta\alpha)(\Delta\beta)> & <(\Delta\beta)^2> \end{pmatrix}$$

$$= \begin{pmatrix} u_\alpha & 0 \\ 0 & u_\beta \end{pmatrix} + \begin{pmatrix} c & c \\ c & c \end{pmatrix}$$

Uncorrelated **Correlated**

Estimation of A_{LR} precision (2)

Assume $\Delta|P_-|$ and $\Delta|P_+|$ are independent, then

$$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{|P_-|(1+|P_+|)(1-|P_+|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_-|}{|P_-|}\right)^2 + \left(\frac{|P_+|(1+|P_-|)(1-|P_-|)}{(|P_-|+|P_+|)(1+|P_-||P_+|)}\right)^2 \left(\frac{\Delta|P_+|}{|P_+|}\right)^2$$

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Correlated parts of the error of α and β cancel in A_{LRobs}.

-> Δα and Δβ below only refer to uncorrelated parts.

$$\left(\frac{\Delta A_{LRobs}}{A_{LRobs}}\right)^2 = \left(\frac{2\left(\frac{N_{-+}}{\alpha}\right)\left(\frac{N_{+-}}{\beta}\right)}{\left(\frac{N_{-+}}{\alpha} - \frac{N_{+-}}{\beta}\right)\left(\frac{N_{-+}}{\alpha} + \frac{N_{+-}}{\beta}\right)}\right)^2 \left(\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta N_{-+}}{N_{-+}}\right)^2 + \left(\frac{\Delta N_{+-}}{N_{+-}}\right)^2\right)$$

Results

If errors of η, L, and polarization are negligible,

$A_{LR} = 0.22827 \pm 0.00018$

If we add polarization error $\Delta f/f = 0.001$,

Absolute error of $A_{LR} = 0.00022$

If $\Delta\alpha/\alpha = \Delta\beta/\beta$ (uncorrelated) = **0.00017 (i.e. 0.017%)**,

Systematic error = 0.000179, same size with the statistical error

In this case,

Absolute error = 0.00026 (cf. Abs. error at SLC = 0.00219)

8.6 times better than the SLC

Conclusion of A_{LR} measurement

- As A_{LR} is useful to constrain SMEFT parameters, it is motivated to improve this observable at the ILC. In order to access how much we can improve the precision, full simulation study including $e^+e^- \to \gamma Z$ process and various background processes are performed.
- In order to exclude the background processes, cut conditions are considered.
- The **statistical error of the A_{LR}** is estimated to be 1.8×10^{-4} i.e. **12 times better than the overall error at the SLC** and in order to keep the statistical error same size as statistical error, we need to keep the uncorrelated part of the error on product of efficiency and luminosity below **0.017%**. In this case, total error is 2.6×10^{-4} i.e. 8.6 times better precision than SLC,