

# Searching for Right Handed Neutrinos using same sign leptons at ILC

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[arXiv:2203.06929](https://arxiv.org/abs/2203.06929)

S O K E N D A I

The text 'S O K E N D A I' is displayed in a black, sans-serif font. Below the text is a thick black line that starts at the left edge of the slide, dips down, then rises to form a jagged, step-like pattern under the letters 'K', 'E', 'N', and 'D', before rising again under 'A' and 'I'.

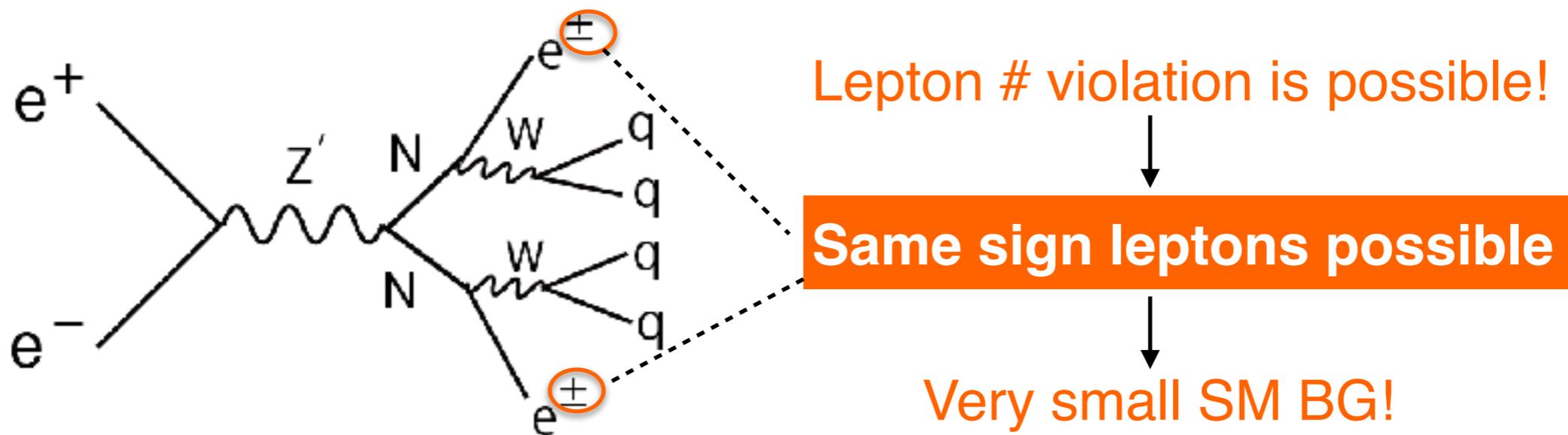
# Motivation and introduction

The Right Handed Neutrino (RHN) can address the following big questions

- Why does matter dominate anti-matter in our universe?
- Do quarks and leptons unify?
- Why is neutrino mass so small?

RHN is assumed to be

- a **Majorana** particle ( $N = \bar{N}$ )      gauge boson :  $Z'$
  - minimal  $U(1)_{B-L}$  model
- $G_{B-L} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$        $\uparrow$        $\rightarrow$       RHN **pair** production



# Study flow

Same sign electron

Same sign muon

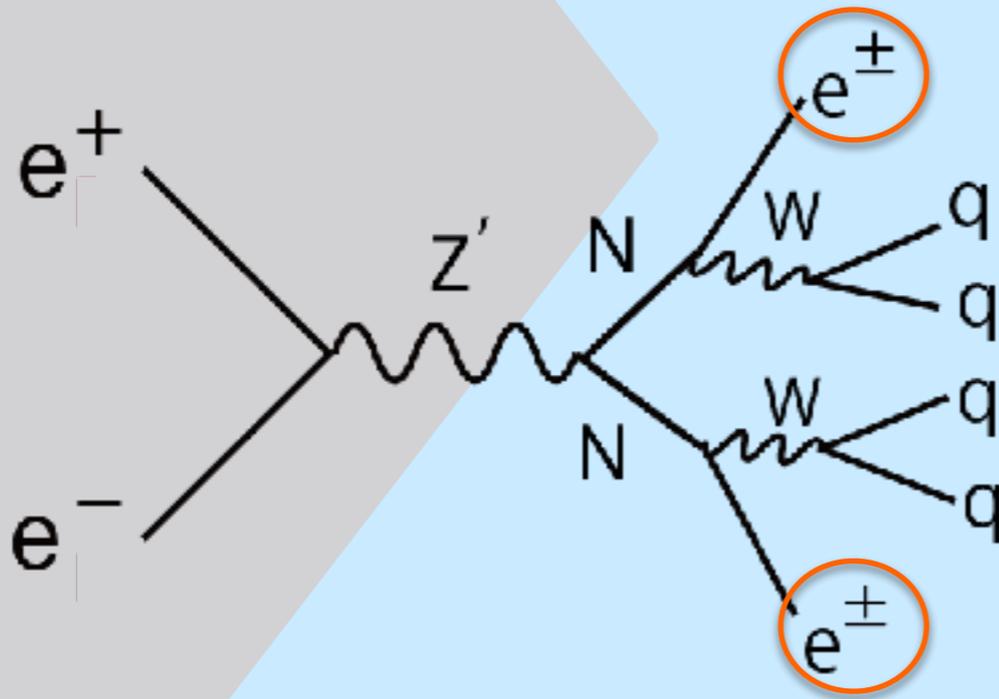
ILC 500

ILC 250

ILC 500

+

ILC 250



Almost done

(Most of today's talk)



Now

(Briefly report)



Future plan

ILC 500

# Benchmark points @ ILC 500

- $\text{Pol}(e^-, e^+) = (-0.8, +0.3), (+0.8, -0.3): \mathcal{L} = 1600 [\text{fb}^{-1}]$
- $\text{Pol}(e^-, e^+) = (-0.8, -0.3), (+0.8, +0.3): \mathcal{L} = 400 [\text{fb}^{-1}]$

$M_N$ [GeV] RHN mass	$M_{Z'}$ [TeV] Z' mass	$g_1'$ U(1) <sub>B-L</sub> coupling	$ V_{eN} ^2$ mixing angle	$BR(N \rightarrow eW)$	$\sigma_{LR}$ [fb] 100% polarisation	$\sigma_{RL}$
100	7	1	0.003	0.44	0.55	0.71
150	7	1	0.003	0.43	0.36	0.45
200	7	1	0.003	0.30	0.14	0.16
225	7	1	0.003	0.29	0.046	0.0052

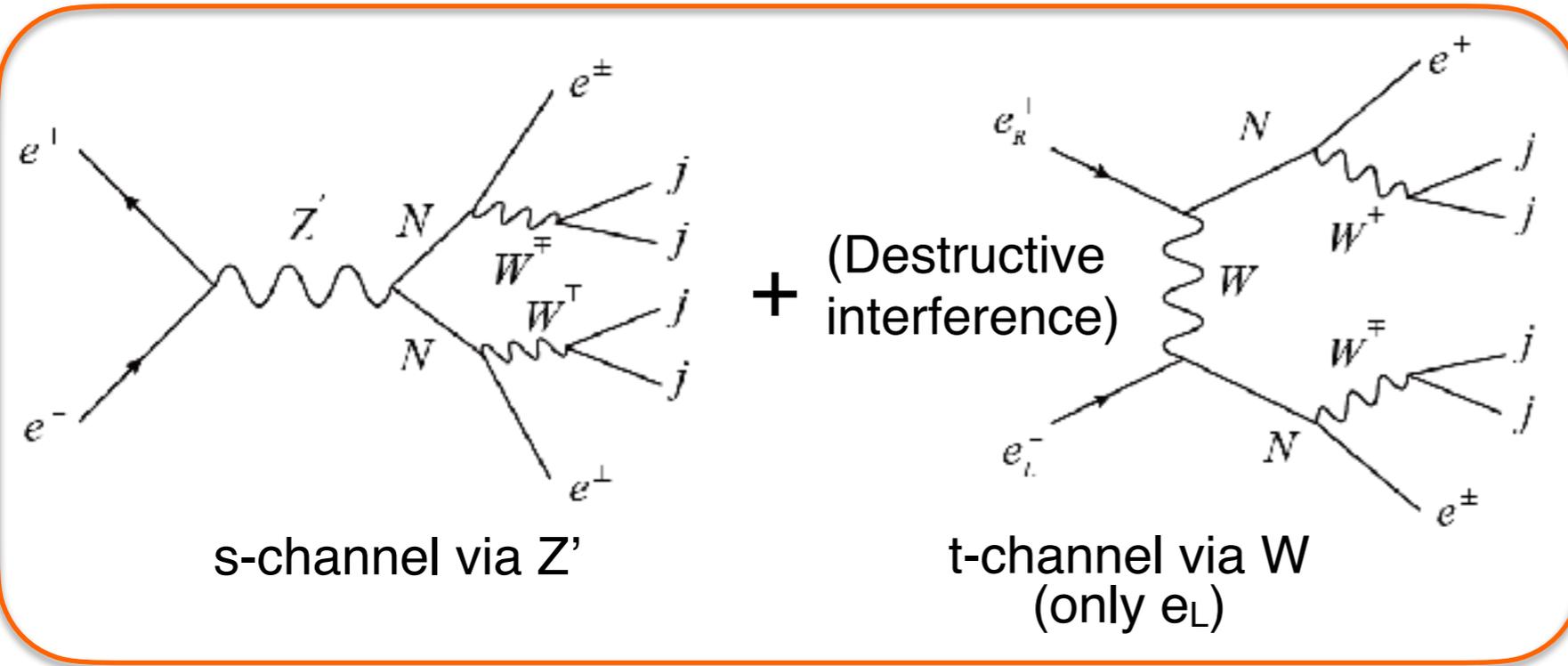
► minimal U(1)<sub>B-L</sub> model

► ILC 500 with initial state radiation (ISR) and beamstrahlung (BS)

# Analysis tool and signal + backgrounds

ILC500

Signal process:

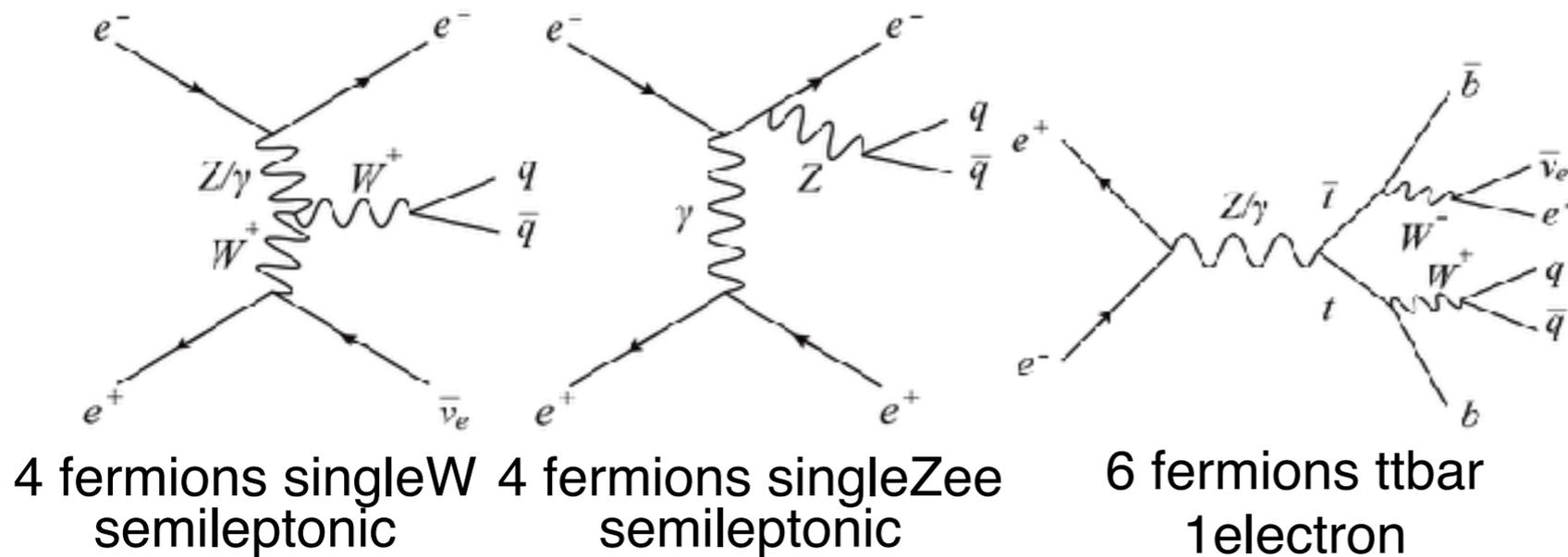


**WHIZARD** ver 2.8.5  
Make Events

**ILD Full Simulation & (Geant4) Reconstruction**

**miniDST**  
Events format

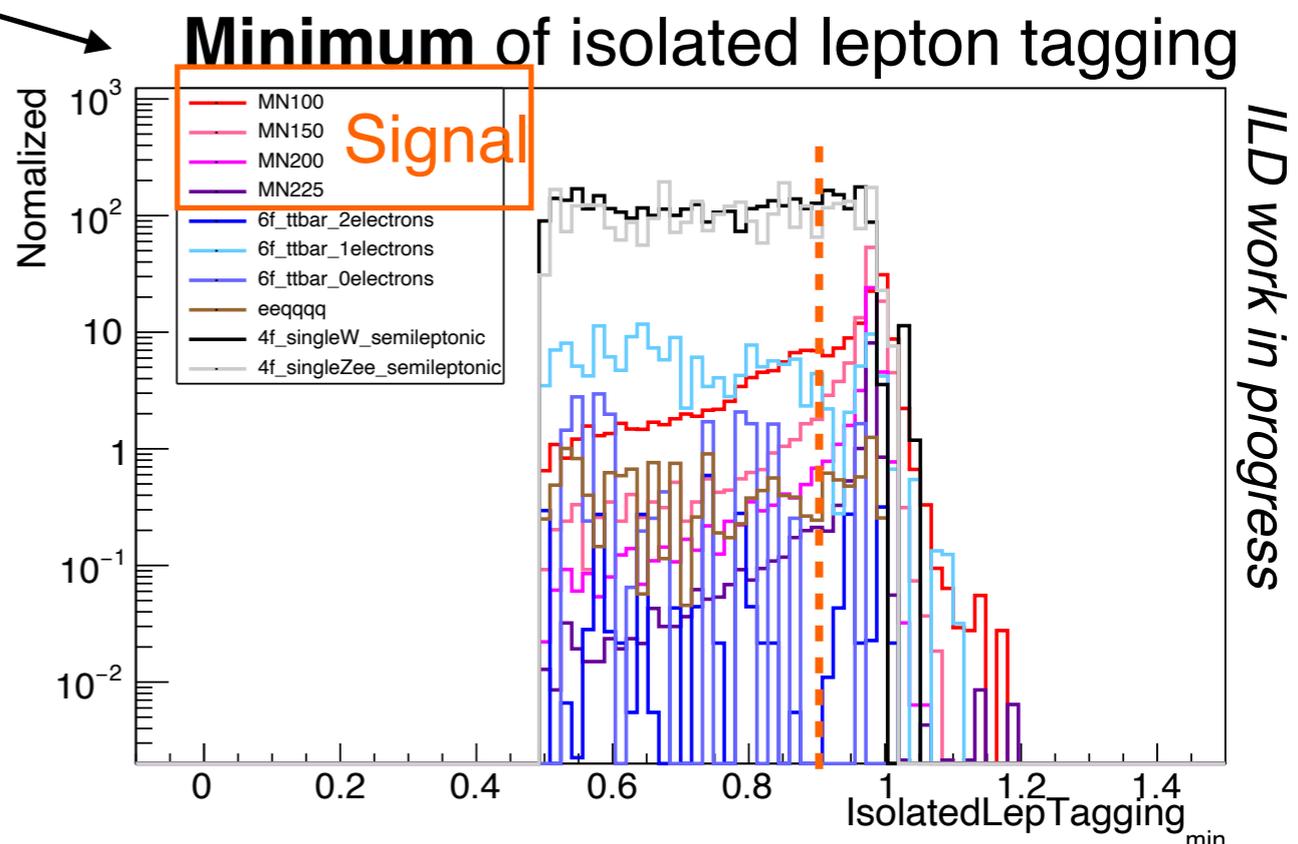
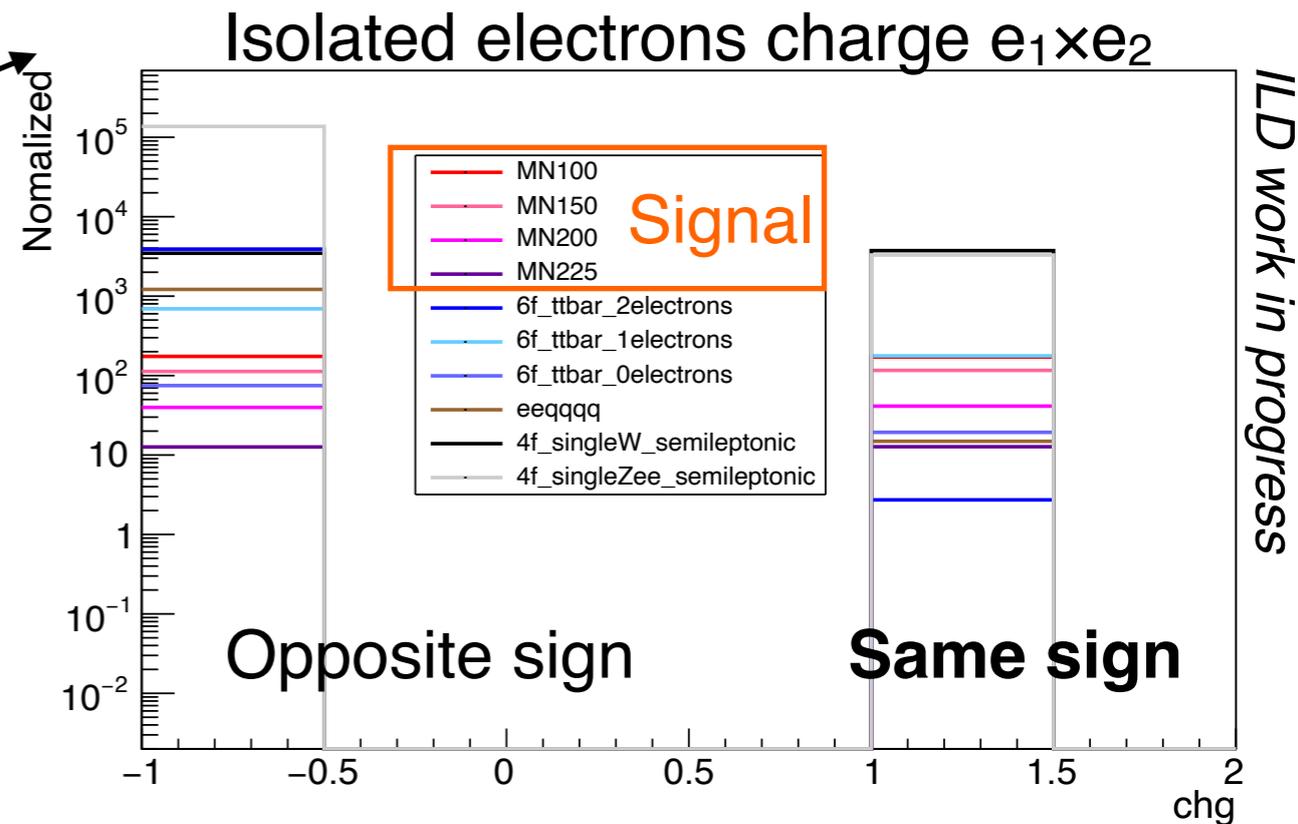
6f and 4f major background processes:



# Cut conditions to select signal events

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$

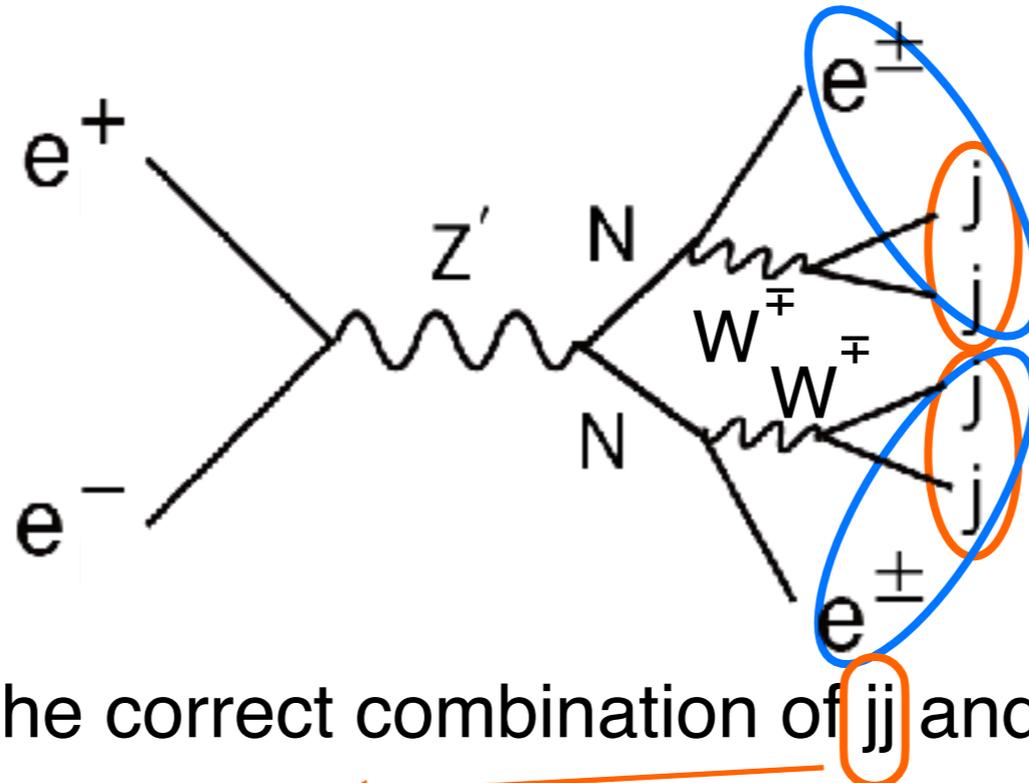
- Isolated  $e \# = 2$  && Isolated  $\gamma, \mu \# = 0$
- Same sign isolated electrons
- Isolated electron energies  $E_{\text{iso}} < 200$  [GeV]
- Isolated electron angles  $|\cos\theta_{\text{iso}e}| < 0.95$
- $\text{IsolatedLepTagging}(\text{min}) > 0.9$
- Jet clustering with Durham  $\log_{10}(y_{12}) > -1$
- $P_{\text{miss}} < 100$  [GeV]  
&& ( $P_{\text{miss}} < 40$  [GeV]  $\parallel |\cos\theta_{P_{\text{miss}}}| > 0.95$ )



Signal efficiency  $\sim$  **20%**  
 Remaining backgrounds  
 $\sim$  **150** (eLpR), **20** (eRpL)

# Reconstruction methods

After removing isolated electrons force into 4 jets (Durham)



Search for the correct combination of  $jj$  and  $jje$

Jet pair 1  $\rightarrow M_{jj1}$ , Jet pair 2  $\rightarrow M_{jj2}$

$$F_1 = (M_{jj1} - M_w)^2 + (M_{jj2} - M_w)^2$$

Best jet pair 1 + iso e  $\rightarrow M_{jje1}$

Best jet pair 2 + iso e  $\rightarrow M_{jje2}$

We expect for " $M_{jje1} = M_{jje2}$ "

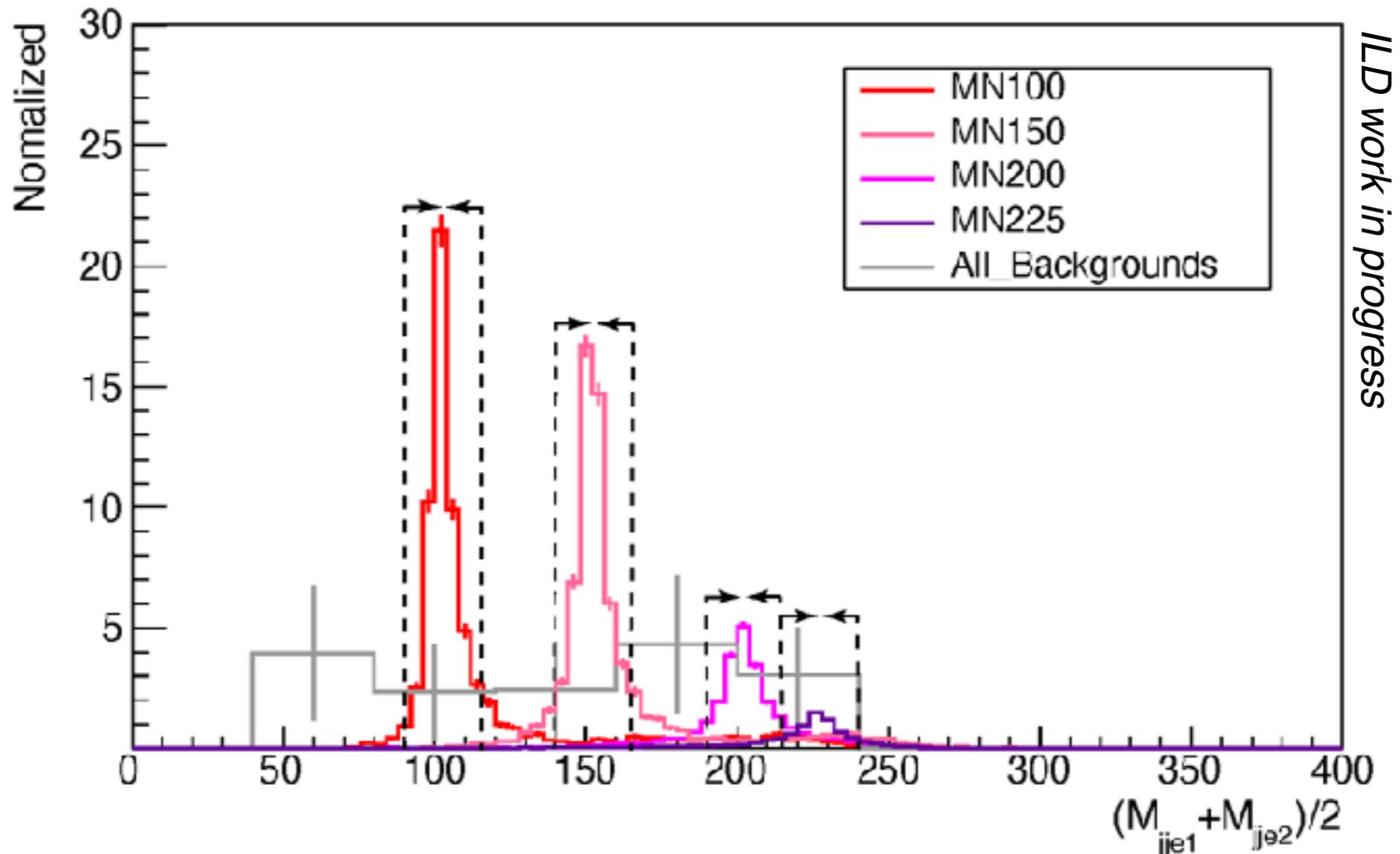
$$F_2 = (M_{jje1} - M_{jje2})^2$$

**Choose combination with minimum  $F_1, F_2$**

# Signal mass cut

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$   
 $\mathcal{L} = 1600 [\text{fb}^{-1}]$

For each  $M_N$ , mass window  $M_N-10, M_N+15$  [GeV]



Assume background distribution is flat

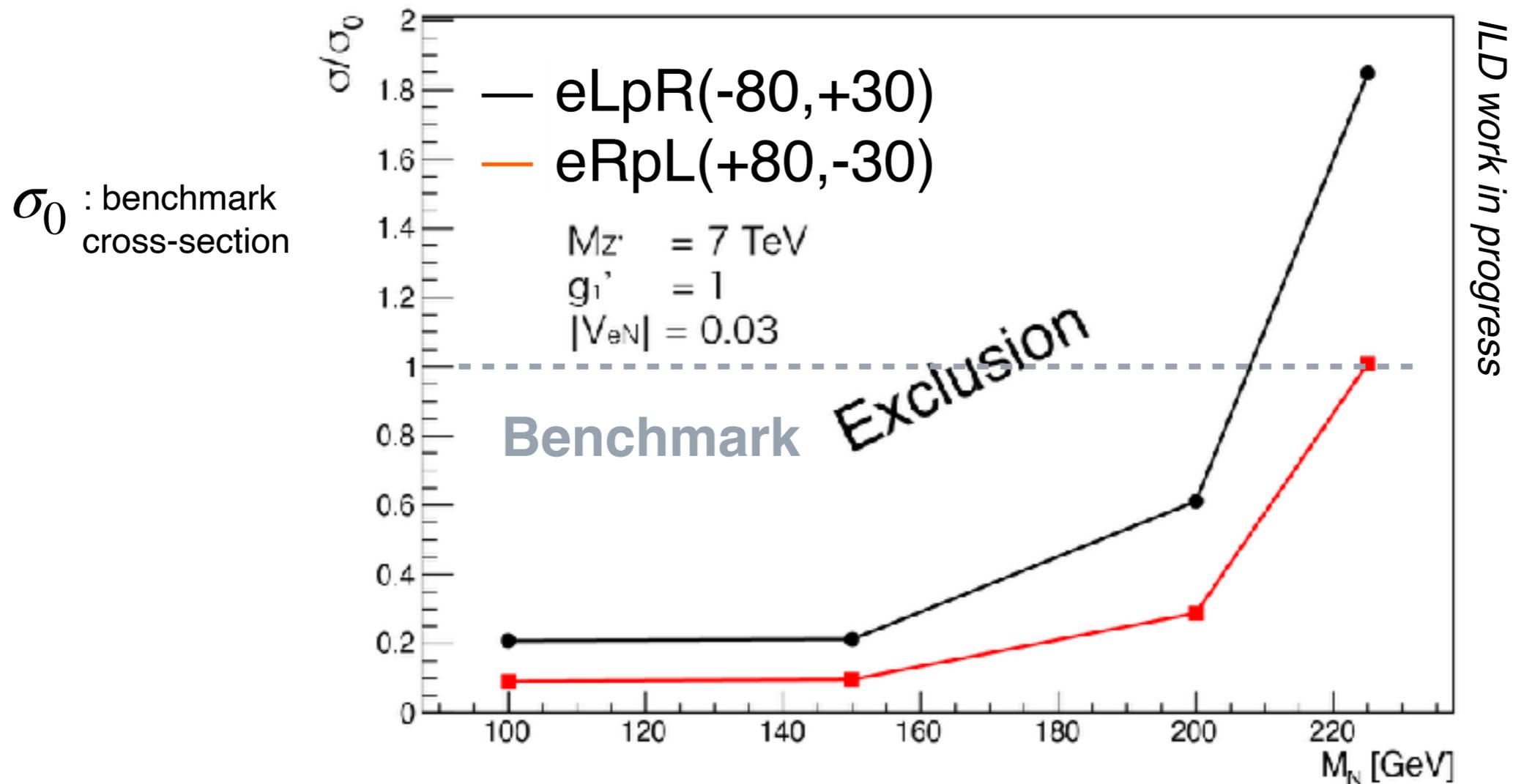
**20 (eLpR) and 3 (eRpL) background events remain in mass window**

# Results

	$M_N$ [GeV]	$N_S$ # of Signal After cut	$N_B$ # of BG After cut	$\frac{N_S}{\sqrt{N_B + N_S}}$ Significance	$\frac{\sigma^{95}}{\sigma_0}$
LR 80,30	100	53	20.12	6.25	0.21
	150	52		6.18	0.21
	200	18		2.95	0.61
	225	5		1.18	1.8
RL 80,30	100	66	3.24	7.98	0.0092
	150	63		7.77	0.097
	200	21		4.29	0.29
	225	6		1.99	1

# Exclusion plot on cross-section

Calculate 95% UL on cross-section



Exclude benchmark points and cross-sections up to 10x smaller

# Summary of ILC 500 case

- ★ We analyze “RHN pair production” by full simulation at ILC500.
- ★ 4 fermion semileptonic processes are dominant backgrounds.  
**Background is mostly removed, 20(eLpR) and 3(eRpL) events remain.**
- ★ Exclude benchmark points and cross-sections up to 10x smaller

Conclusion:

**Can use same sign lepton signature to set powerful limits on RHN at ILC!**

ILC 250

# Benchmark points @ ILC 250

$$\text{Pol}(e^+, e^-) = (-0.8, +0.3), (+0.8, -0.3) : \mathcal{L} = 900 [\text{fb}^{-1}]$$

$$\text{Pol}(e^+, e^-) = (-0.8, -0.3), (+0.8, +0.3) : \mathcal{L} = 100 [\text{fb}^{-1}]$$

$M_N$ [GeV] RHN mass	$M_{Z'}$ [TeV] Z' mass	$g_{1'}$ $U(1)_{B-L}$ coupling	$ V_{eN} ^2$ mixing angle	$BR(N \rightarrow eW)$	$\sigma_{LR}$ [fb] 100% polarisation	$\sigma_{RL}$
85	7	1	0.003	0.50	0.048	0.089
95	7	1	0.003	0.48	0.033	0.060
100	7	1	0.003	0.44	0.026	0.046
110	7	1	0.003	0.40	0.012	0.021
120	7	1	0.003	0.37	0.0021	0.0035

► minimal  $U(1)_{B-L}$  model

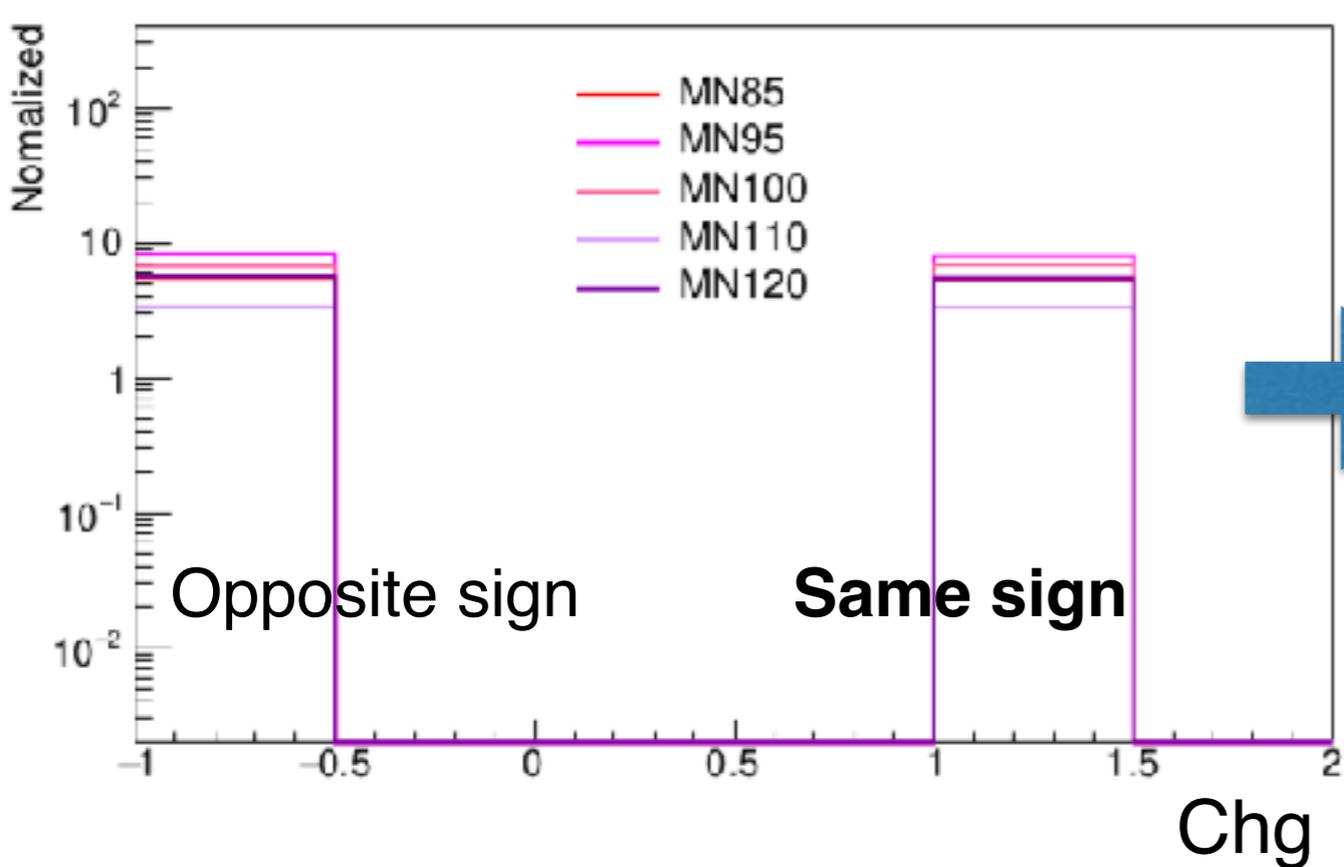
► ILC 250 with initial state radiation (ISR) and beamstrahlung (BS)

# Preliminary results

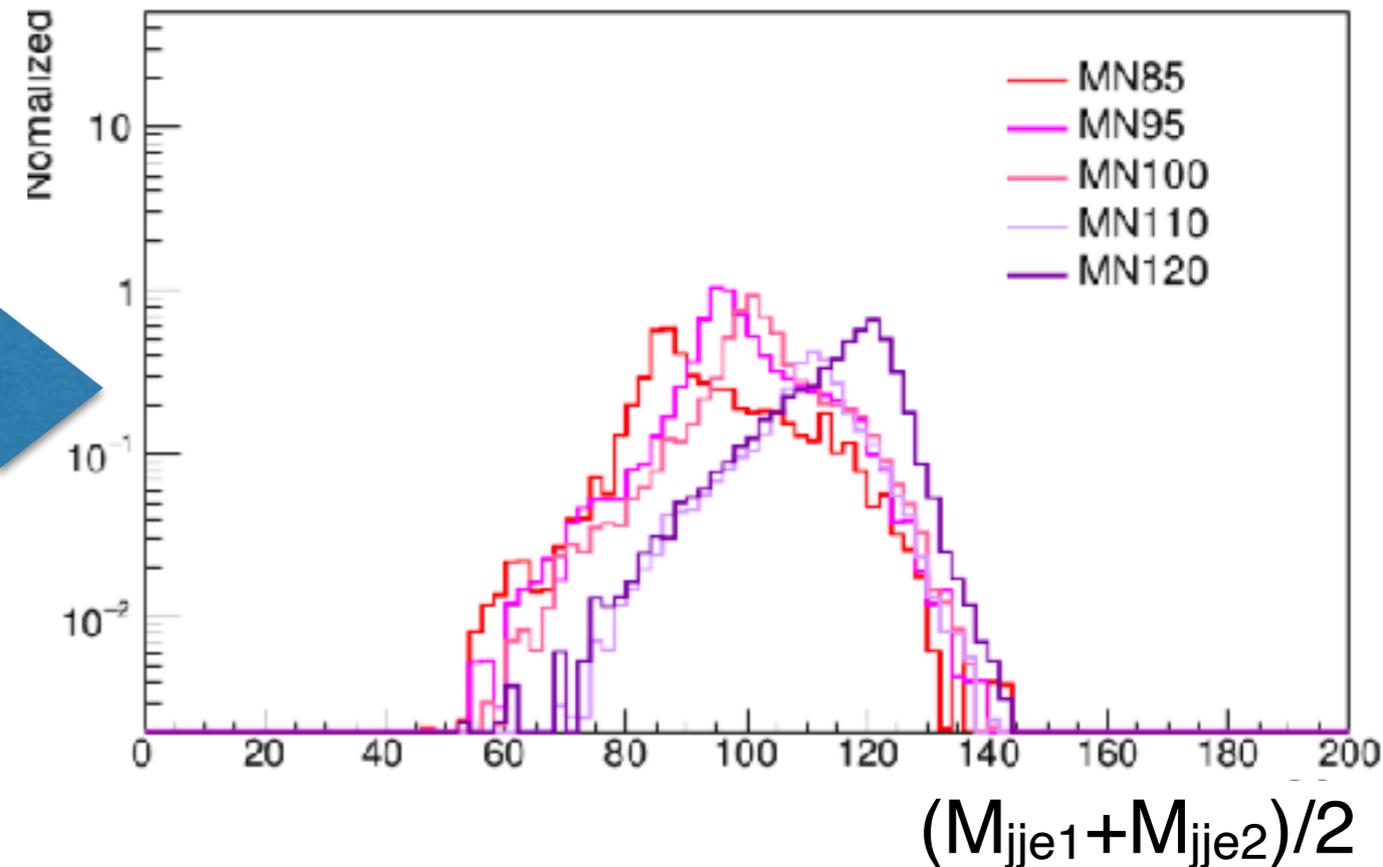
- ILC 250 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$

Isolation cut == Isolated e # = 2 && Isolated  $\gamma, \mu$  # = 0

Isolated electrons charge  $e_1 \times e_2$   
(Isolation cut)



Reconstruction RHN  
(Isolation + same sign cut)



Signal efficiency about 10 - 20 %  
(Remaining signal events < 10)

# Next step

★ I add background events for these signal events.

Ex) 4 and 6 fermions events in the final state

-> Shinichi.K is preparing minidst files of backgrounds for me.

★ I need to consider “cut condition”.

★ Finding the exclusion limit on cross-section

Back up

# Benchmark points

- $\text{Pol}(e^-, e^+) = (-0.8, +0.3), (+0.8, -0.3): \mathcal{L} = 1600 [\text{fb}^{-1}]$
- $\text{Pol}(e^-, e^+) = (-0.8, -0.3), (+0.8, +0.3): \mathcal{L} = 400 [\text{fb}^{-1}]$

$M_N$ [GeV] RHN mass	$M_{Z'}$ [TeV] Z' mass	$g_1'$ $U(1)_{B-L}$ coupling	$ V_{eN} ^2$ mixing angle	BR $(N \rightarrow eW)$	$\sigma(e_L^- e_R^+ \rightarrow NN)$ 100% polarization [fb]	Event # at ILC500 [4000fb <sup>-1</sup> ]
100	7	1	0.003	0.44	0.71	<b>1261</b>
150	7	1	0.003	0.33	0.45	<b>229</b>
200	7	1	0.003	0.30	0.16	<b>131</b>
225	7	1	0.003	0.29	0.052	<b>18</b>

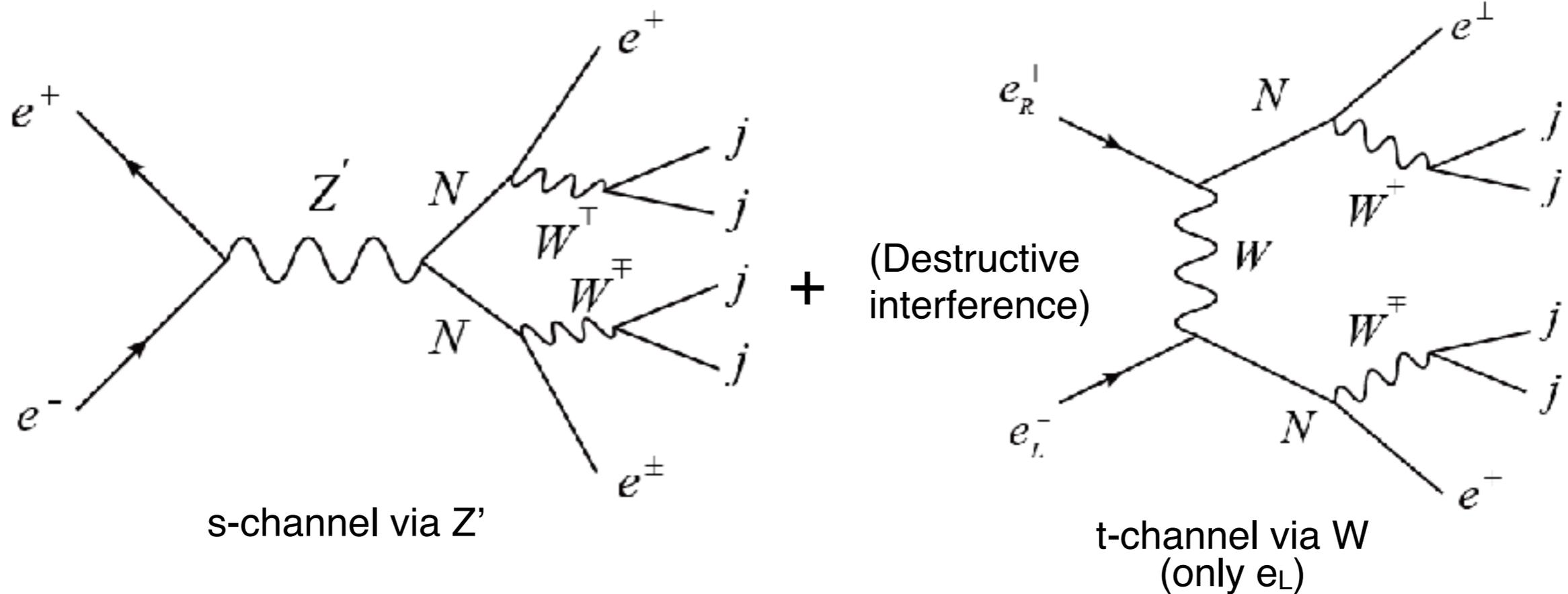
► minimal  $U(1)_{B-L}$  model

► ILC 500 with initial state radiation (ISR) and beamstrahlung (BS)

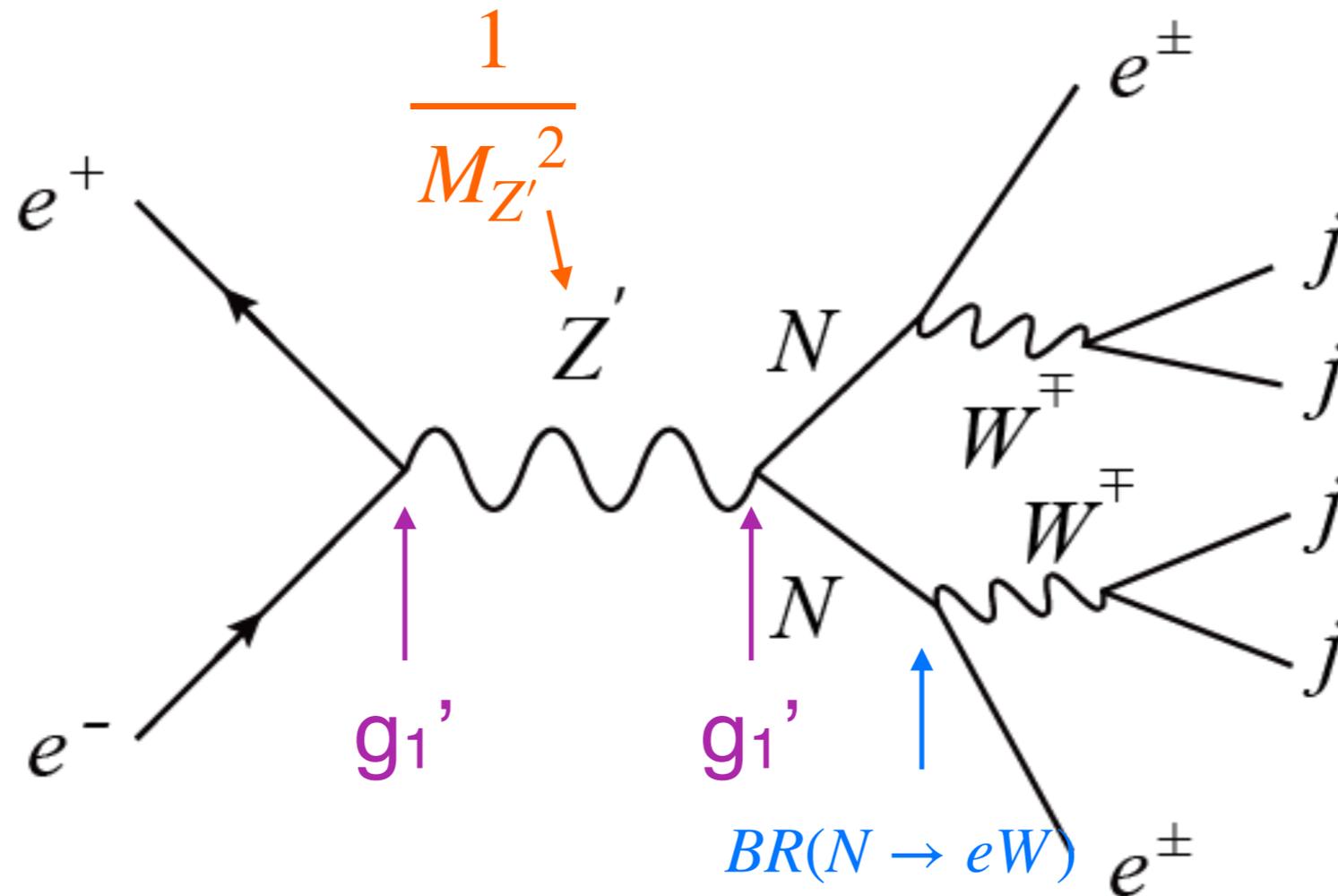
# Signal events

## ILC250

Signal process:



# Model parameters



$$\sigma = \sigma(ee \rightarrow NN) \times (BR(N \rightarrow e^\pm W^\mp))^2$$

$$\propto g_1'^4 \frac{1}{M_{Z'}^4} (BR(N \rightarrow e^\pm W^\mp))^2 \equiv \alpha$$

# Model : minimal $U(1)_{B-L}$

## Gauged B-L extension of Standard Model (SM)

The unique anomaly free global symmetry in the SM

$$G_{B-L} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

- ▶ Anomaly free requirement → **RHNs**
- ▶ **Seesaw mechanism** ← automatically included

Gauge boson :  $Z'$

If B-L symmetry breaks spontaneously →  $Z'$  becomes **massive**

minimal  $U(1)_{B-L}$  model : charge

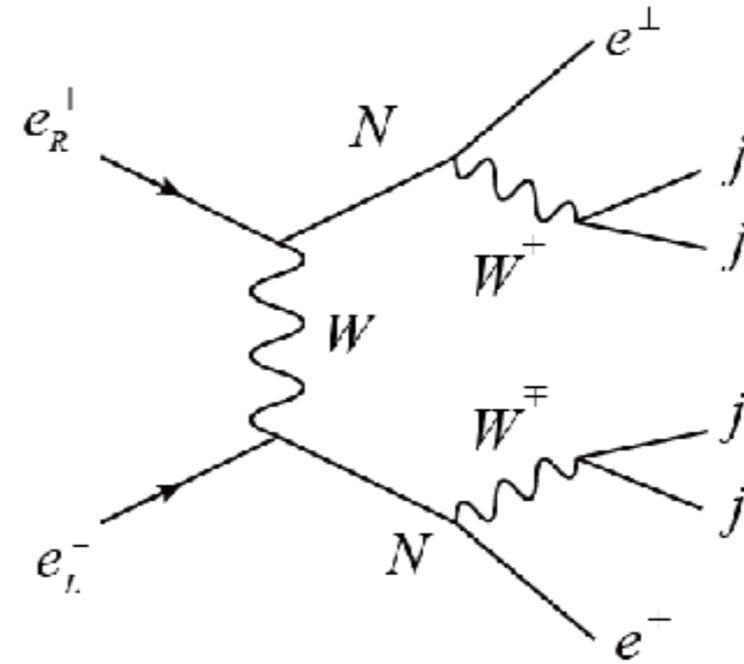
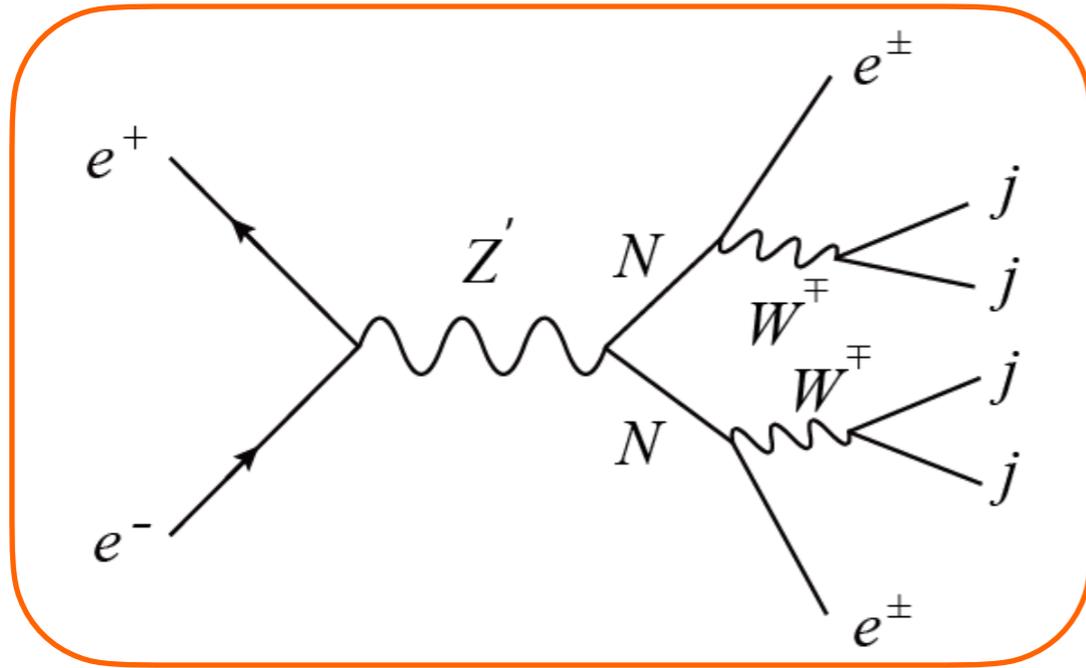
		$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
RHN	$N_R^i$	1	1	0	-1
New Higgs field	$\Phi$	1	1	0	2

$i=1,2,3$

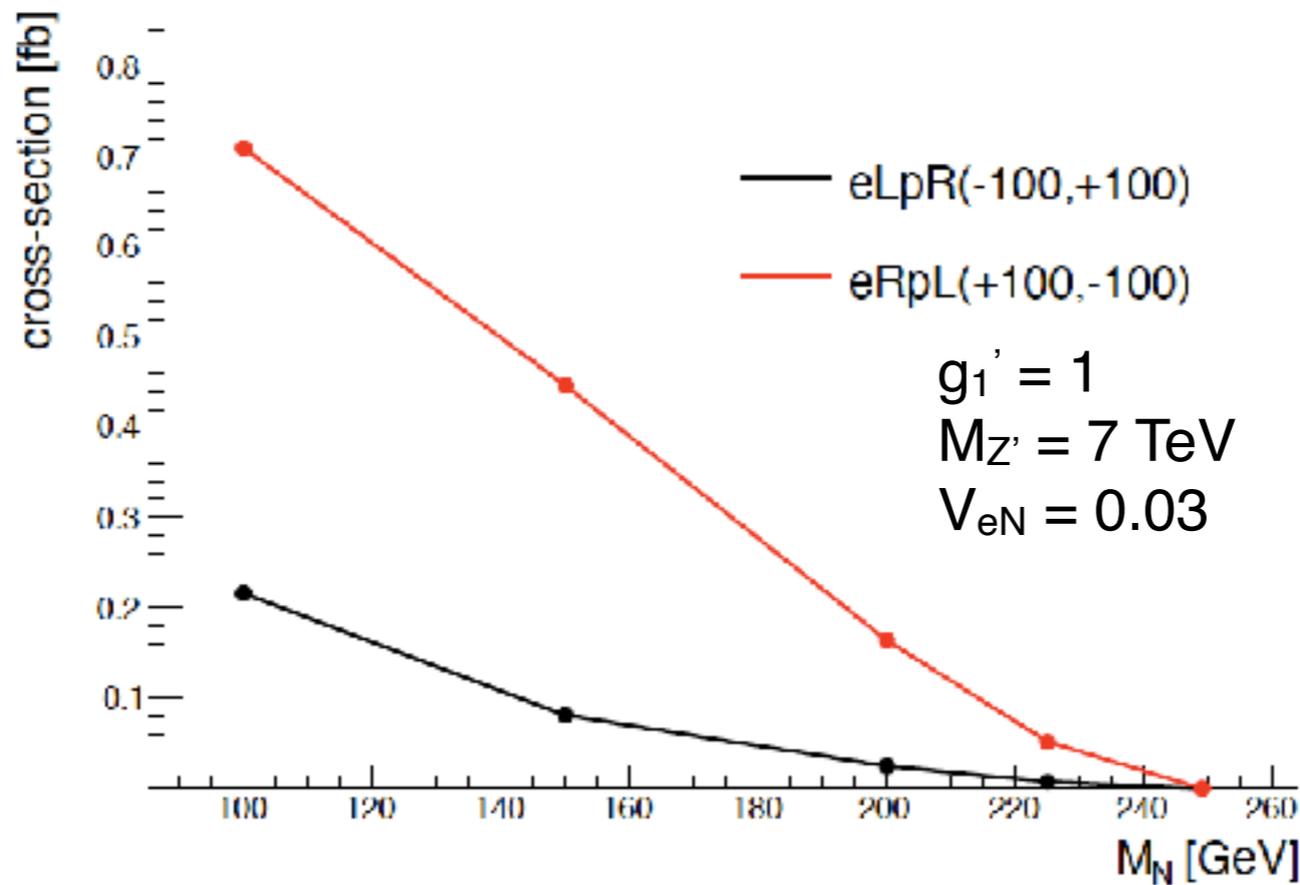
[arXiv\[1812.11931\]](https://arxiv.org/abs/1812.11931)

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# Signal



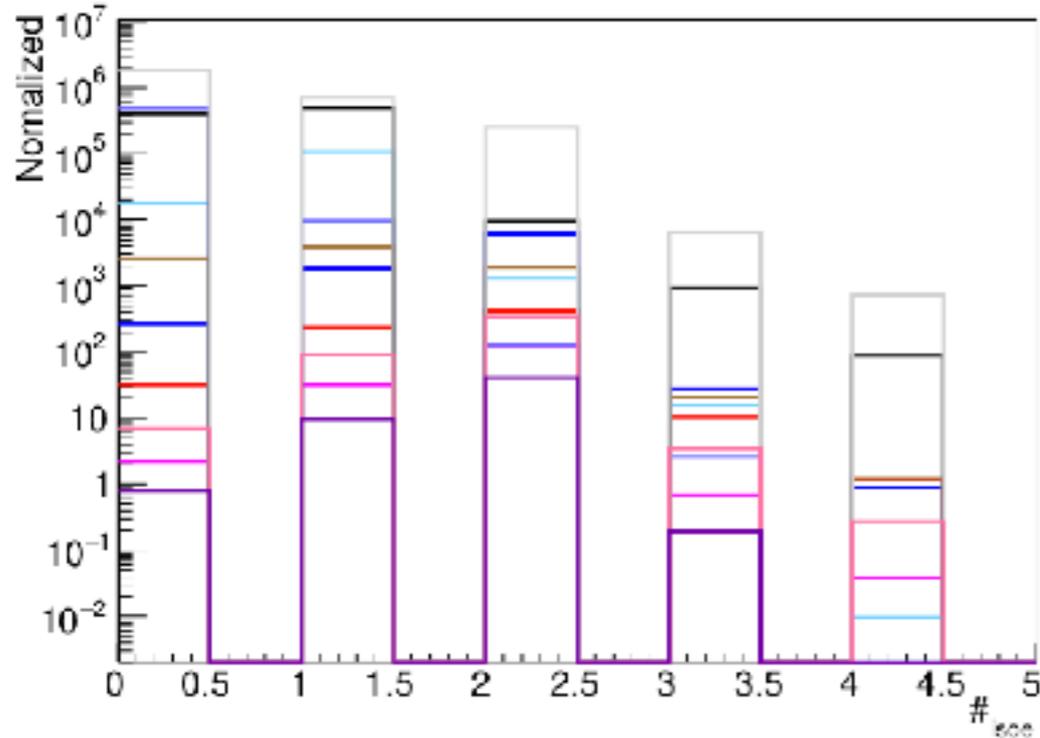
Destructive interference



# Isolated $e, \gamma, \mu$

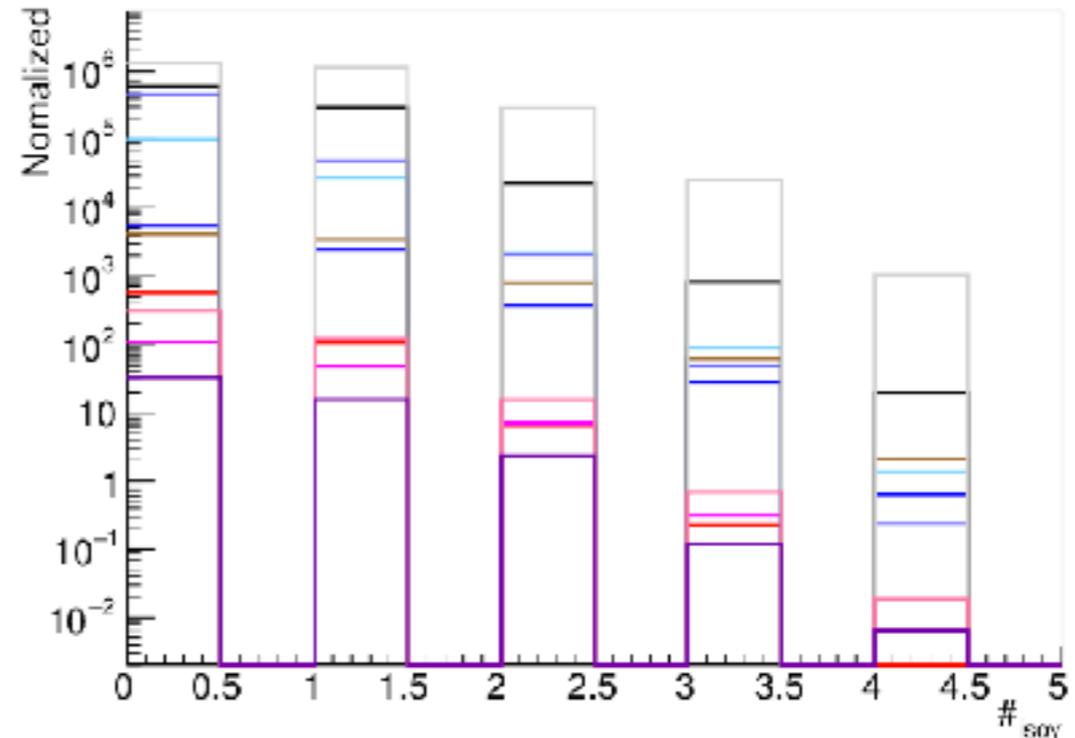
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$

Number of isolated  $e$



ILD work in progress

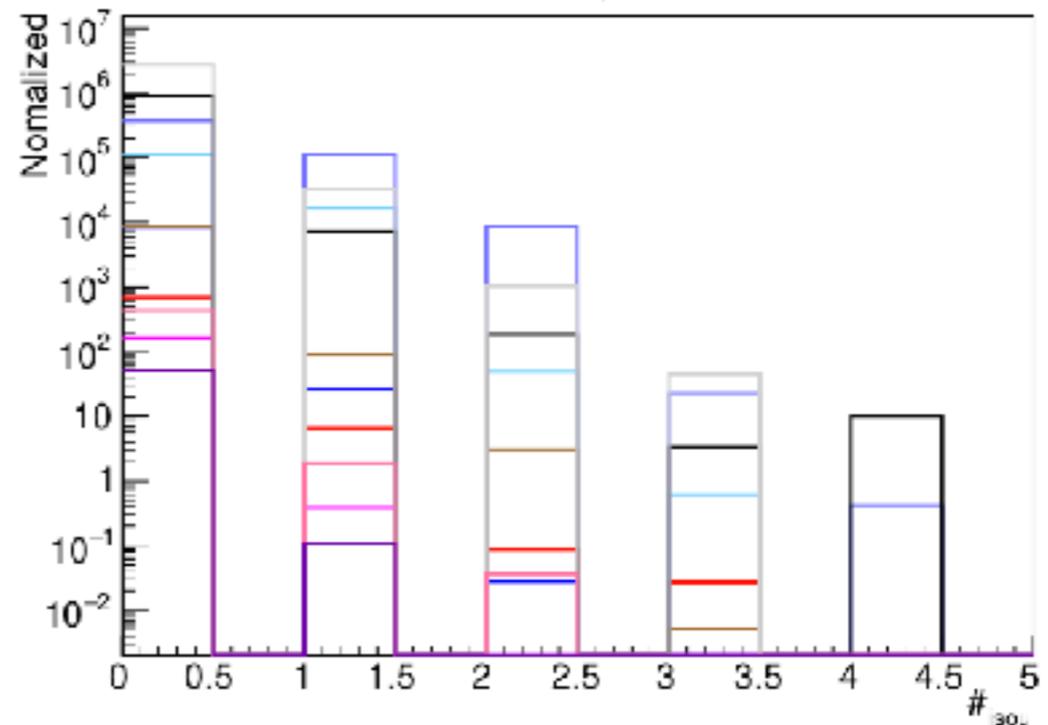
Number of isolated  $\gamma$



ILD work in progress

- |                 |                  |
|-----------------|------------------|
| — $M_N=100$ GeV | — BG_6f_ttbar_2e |
| — $M_N=150$ GeV | — BG_6f_ttbar_1e |
| — $M_N=200$ GeV | — BG_6f_ttbar_0e |
| — $M_N=225$ GeV | — BG_4f_sw_sl    |
|                 | — BG_4f_sze_sl   |
|                 | — BG_eeqqqq      |

Number of isolated  $\mu$

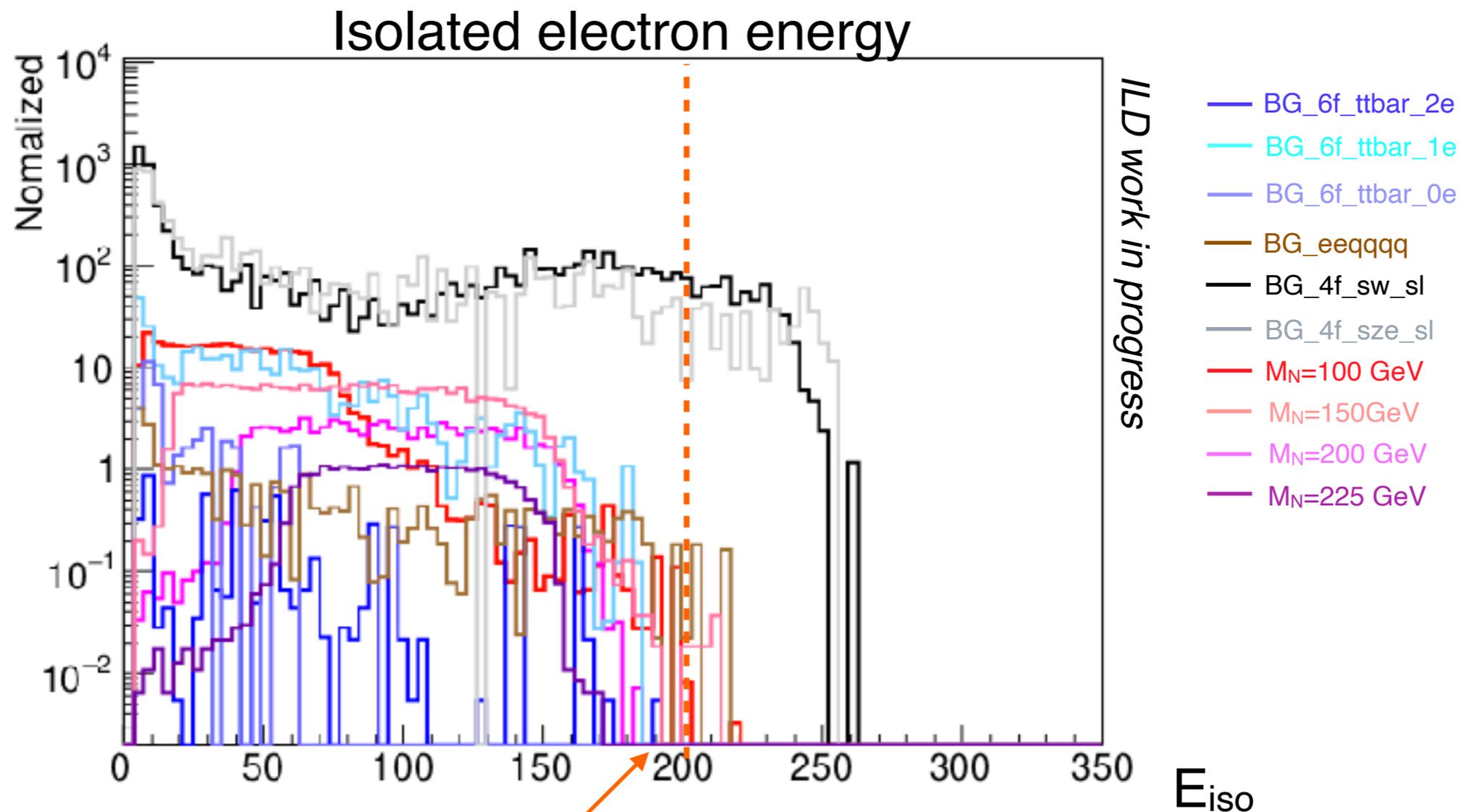


ILD work in progress

- Isolated  $e$  # = 2 && Isolated  $\gamma, \mu = 0$

# Distribution of Isolated electron energy

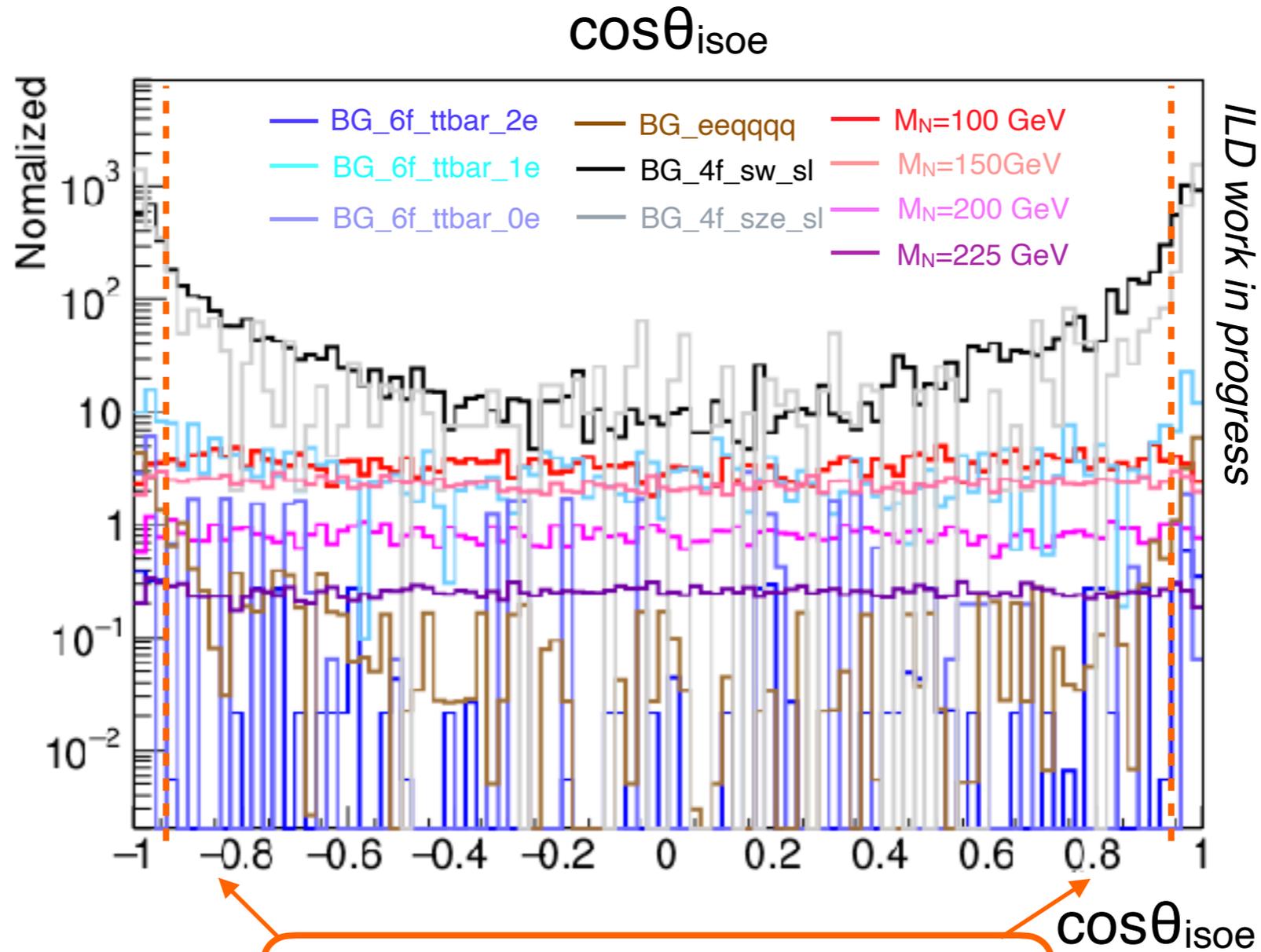
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



$E_{\text{iso}} < 200$  [GeV]

# Distribution of $\cos\theta_{\text{isoe}}$

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )

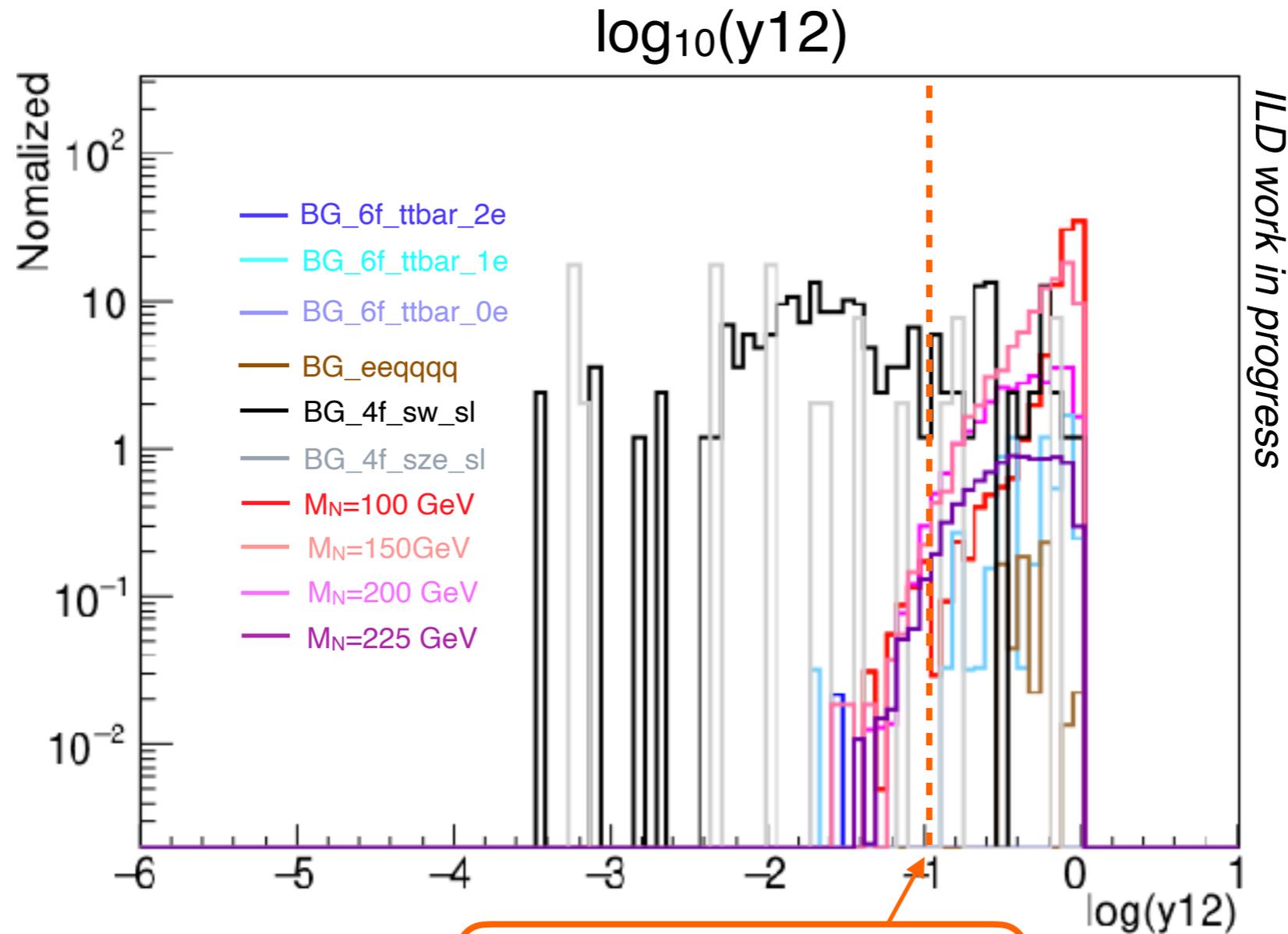


**$-0.95 < \cos\theta_{\text{isoe}} < 0.95$**

4 fermions semi leptonic processes in t-channel  $\rightarrow$  distributed in  $|\cos\theta_{\text{isoe}}| \sim 1$

# Distribution of $y_{12}$ (Durham)

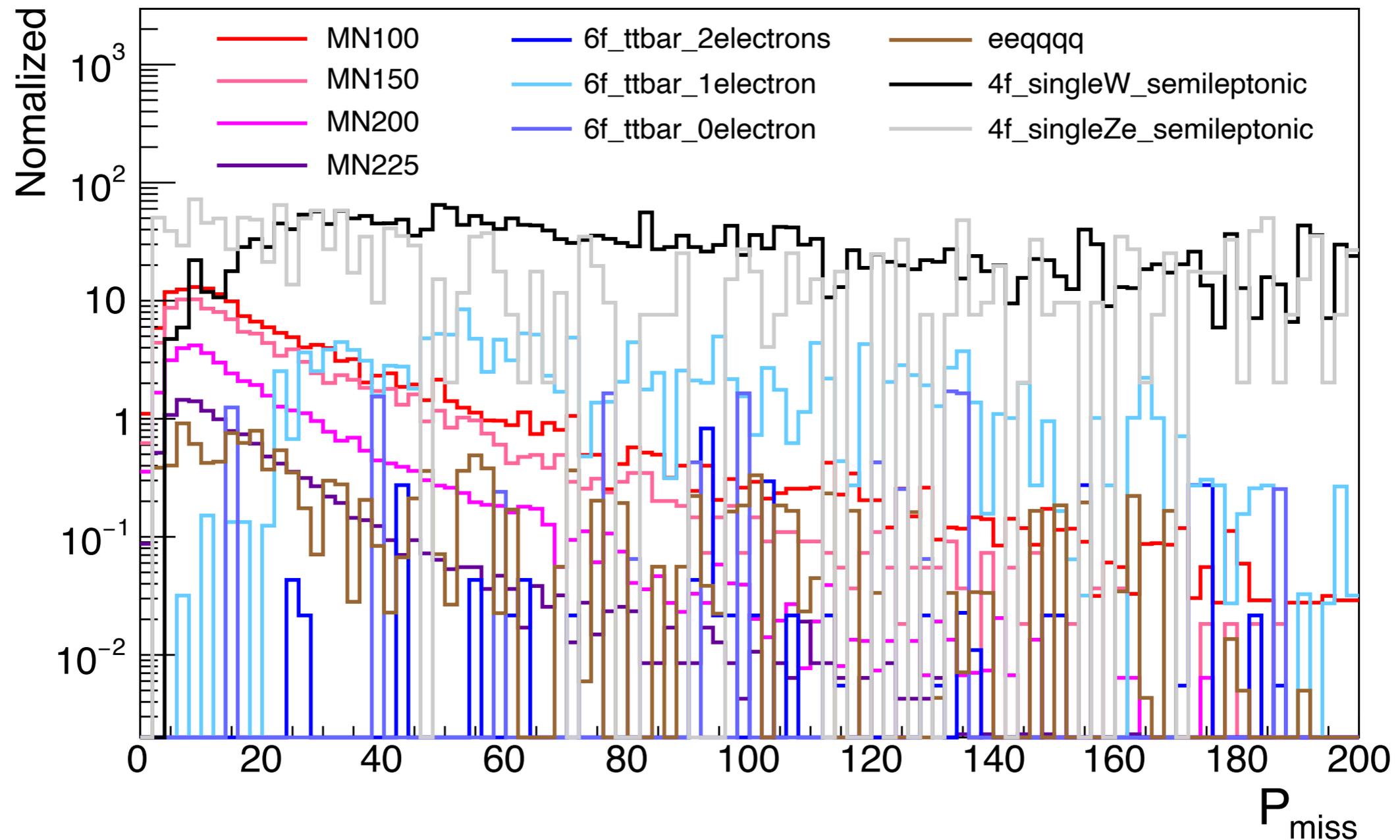
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



$\log_{10}(y_{12}) > -1$

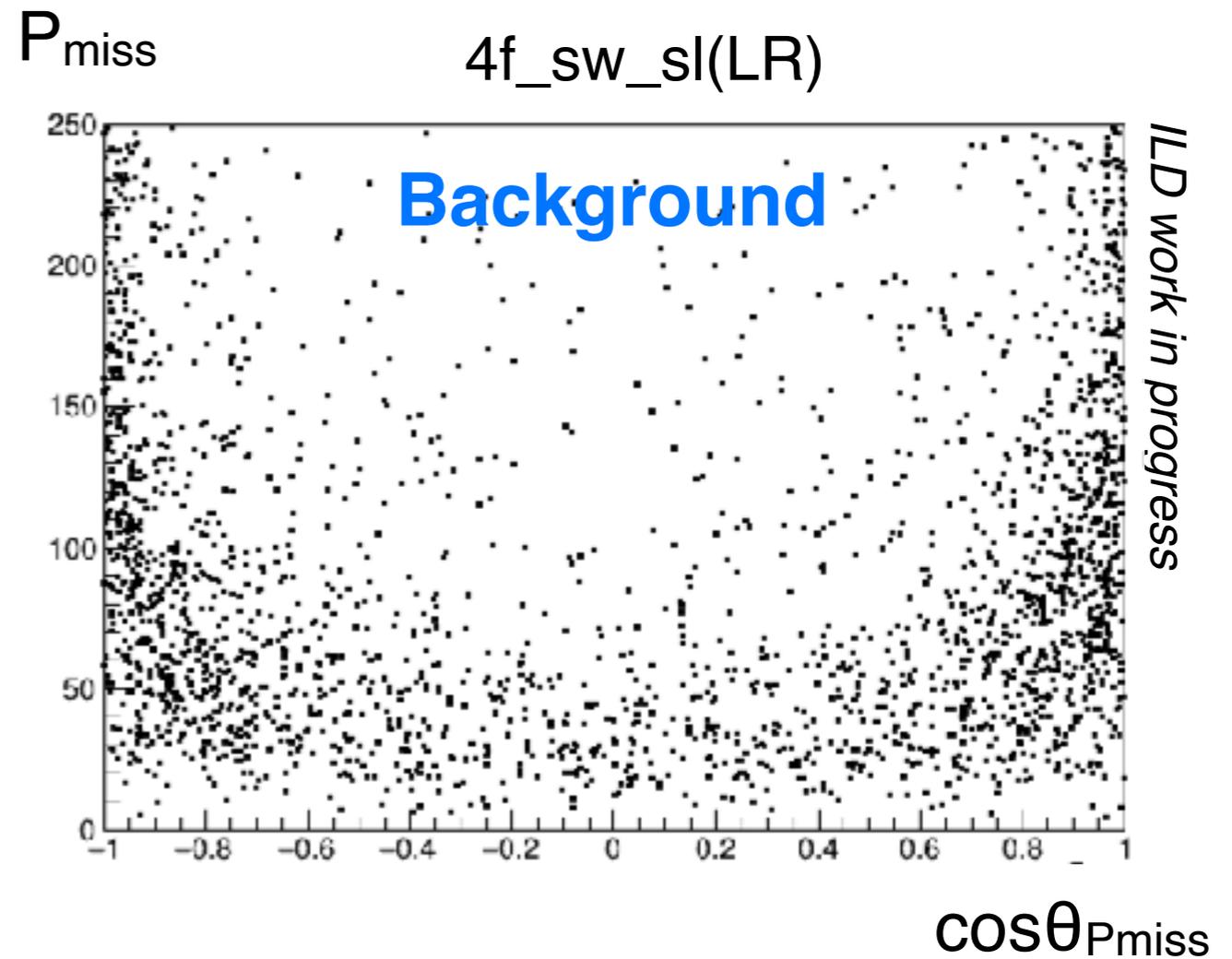
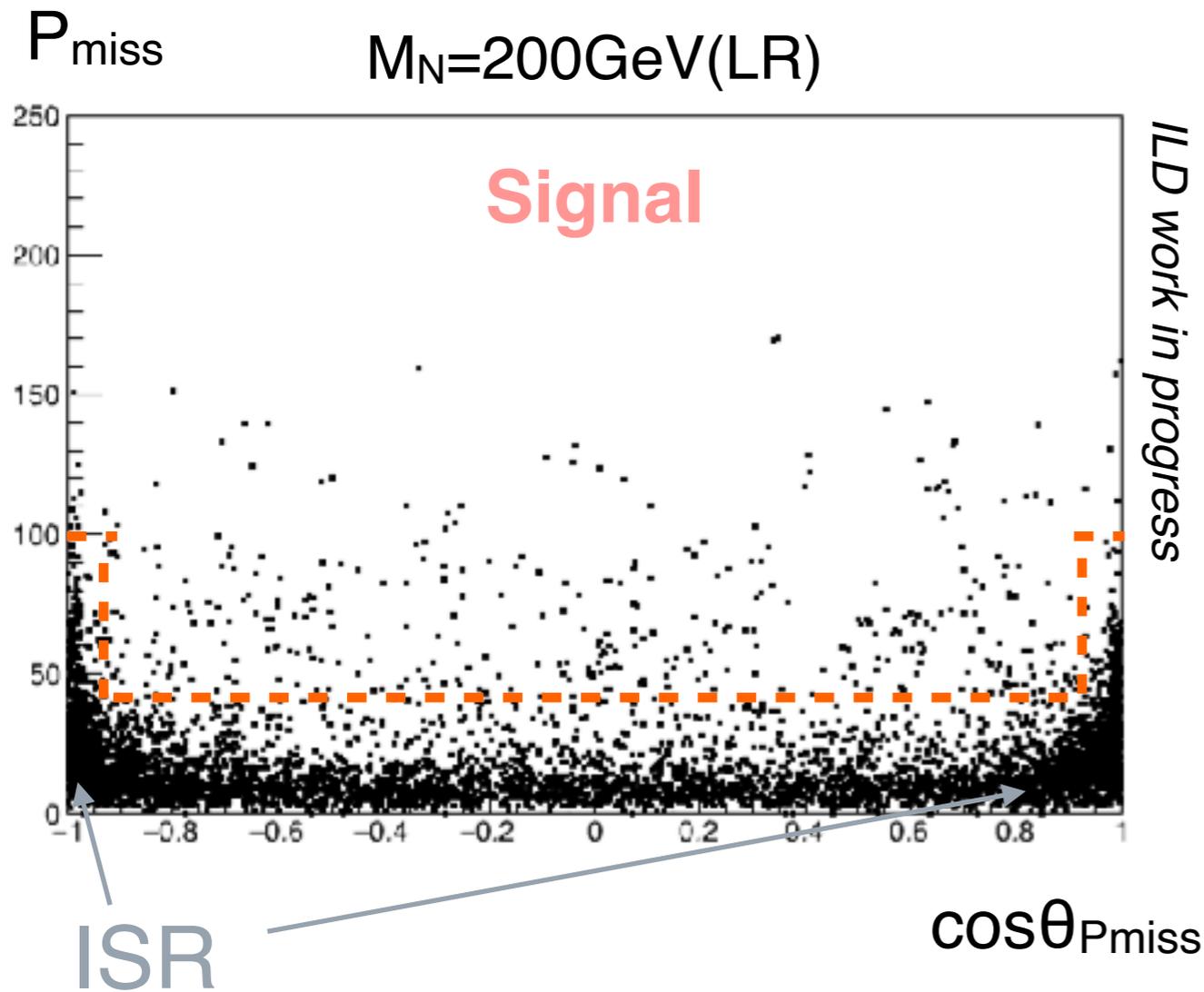
# Distribution of $P_{\text{miss}}$

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



# $\cos\theta_{P_{\text{miss}}}$ vs Magnitude of missing momentum $P_{\text{miss}}$

- ILC 500 with ISR / BS



$$P_{\text{miss}} < 100 \ \&\& \ ( P_{\text{miss}} < 40 \ \parallel \ |\cos\theta_{P_{\text{miss}}}| > 0.95 )$$

# Cut flow (eRpL)

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$   $\mathcal{L} = 1600 [\text{fb}^{-1}]$

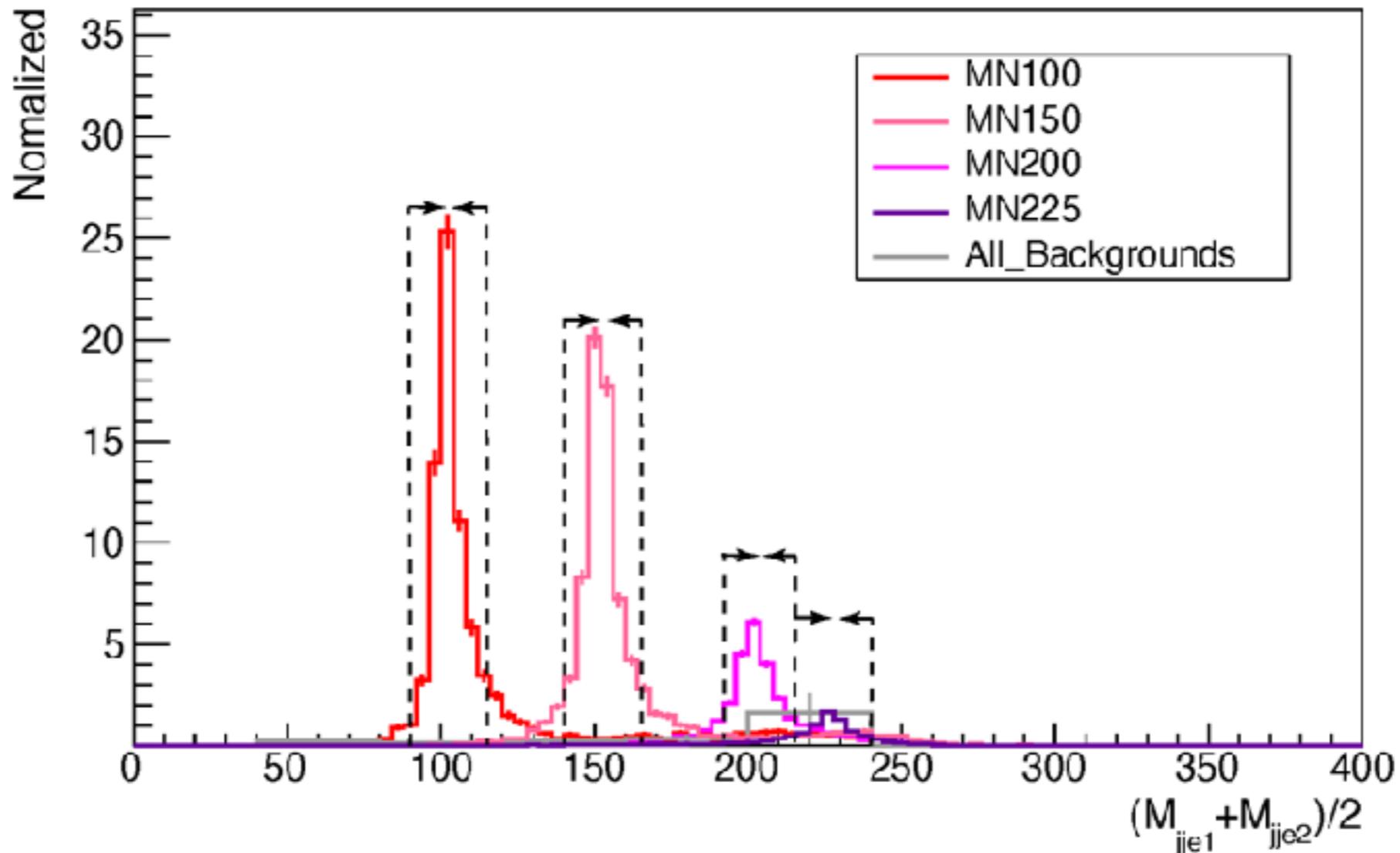
*ILD work in progress*

	Expected signal				Expected background					
	$M_N=100$	$M_N=150$	$M_N=200$	$M_N=225$	eeqqqq	4f_singleW _semileptonic	4f_singleZee_ semileptonic	6f_ttbar 2electrons	6f_ttbar 1electron	6f_ttbar 0electron
No cut	558	394	143	45	3925	258648	612455	7100	56233	4894
$e_{\text{iso}} \# = 2 \ \&\&$ $\gamma_{\text{iso}} \# = 0 \ \&\&$	420	343	126	40	1935	9426	249000	6142	1295	127
Same sign ( $e_{\text{iso}1} \times e_{\text{iso}2} = 1$ )	346	115	81	12	1231	7210	140176	3911	870	94
$E_{\text{iso}} < 200$ [GeV]	171	114	41	12	14	3741	3294	2	177	19
$-0.95 <$ $\cos\theta_{\text{isoe}} < 0.95$	158	103	37	11	3	1324	475	1	113	12
IsolatedLepTa gging <sub>min</sub> > 0.9	96	91	32	10	0	198	101	0	15	1
$\log_{10}(y_{12}) > -1$	88	90	30	9	0	199	86	0	6	0
$P_{\text{miss}} < 100 \ \&\&$ ( $P_{\text{miss}} < 40 \ \parallel$ $ \cos\theta_{P_{\text{miss}}}  >$ 0.95)	86	84	29	9	0	4	15	0	2	0

# Signal mass cut

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (+0.8, -0.3)$   
 $\mathcal{L} = 1600 [\text{fb}^{-1}]$

For each  $M_N$ , mass window  $M_N-10, M_N+15$  [GeV]



Assume background distribution is flat

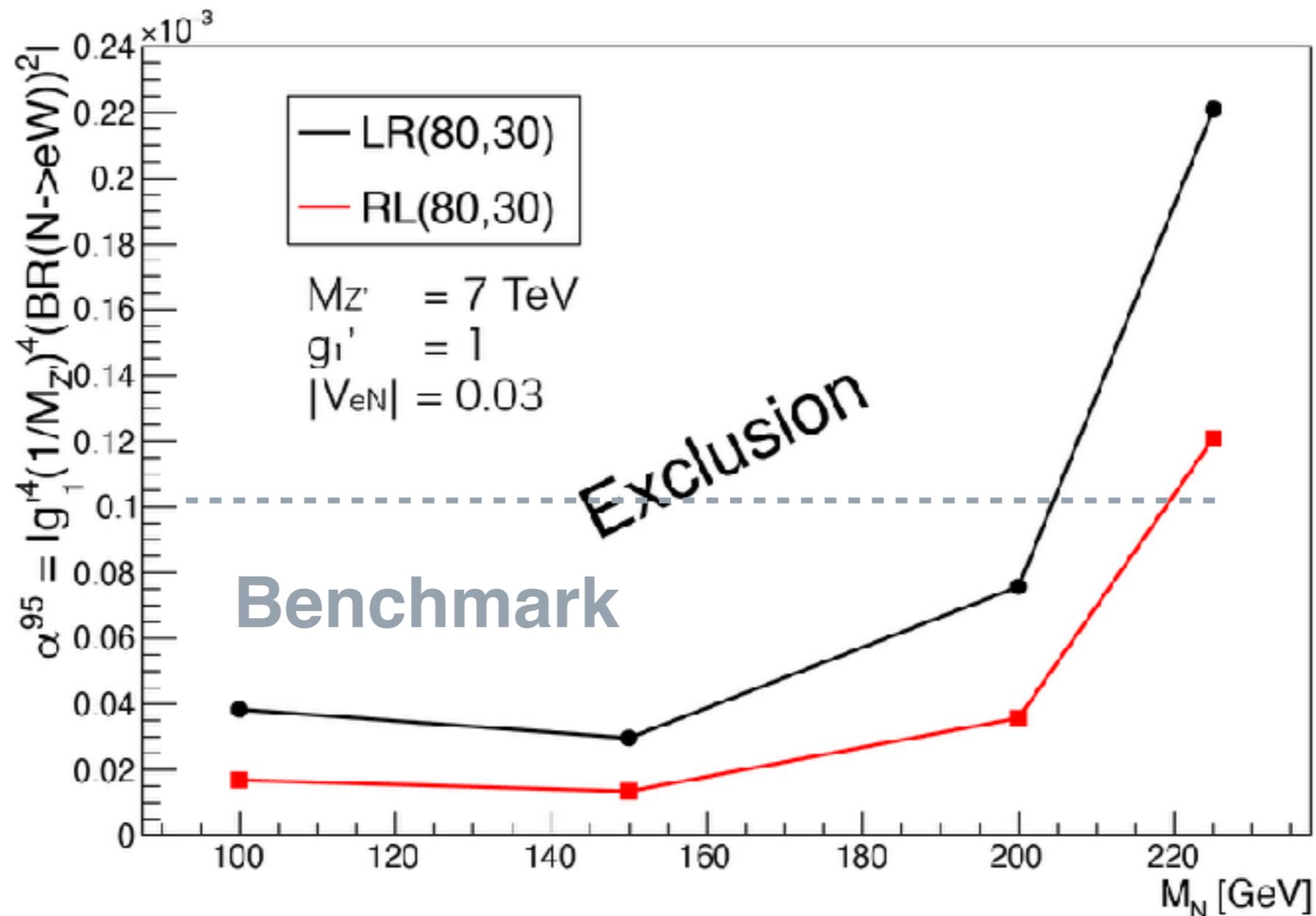
**20 (eLpR) and 3 (eRpL) background events remain in mass window**

# Results

	$M_N$ [GeV]	# of Signal After cut	# of BG After cut	Signal Significance	$\sigma_0$ [fb] Initial benchmark	$\sigma^{95}$ [fb] 95% exclusion limit	$\frac{\sigma^{95}}{\sigma_0}$	$\alpha^{95}$ [TeV <sup>-4</sup> ]
LR 80,30	100	53.64	20.12	6.25	0.55	0.12	0.21	3.83E-05
	150	52.73		6.18	0.36	0.076	0.21	2.96E-05
	200	18.30		2.95	0.14	0.086	0.61	7.57E-05
	225	5.51		1.18	0.046	0.085	1.8	2.21E-04
RL 80,30	100	66.75	3.24	7.98	0.71	0.065	0.092	1.69E-05
	150	63.41		7.77	0.45	0.043	0.097	1.35E-05
	200	21.23		4.29	0.16	0.047	0.29	3.57E-05
	225	6.077		1.99	0.052	0.052	1	1.21E-04

# Exclusion plot on $U(1)_{B-L}$ parameters

Translate to the  $U(1)_{B-L}$  model parameters



The benchmark points isn't excluded only at  $M_N = 225$  GeV

eLpR case

# Cut flow (eLpR)

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$   $\mathcal{L} = 1600 [\text{fb}^{-1}]$

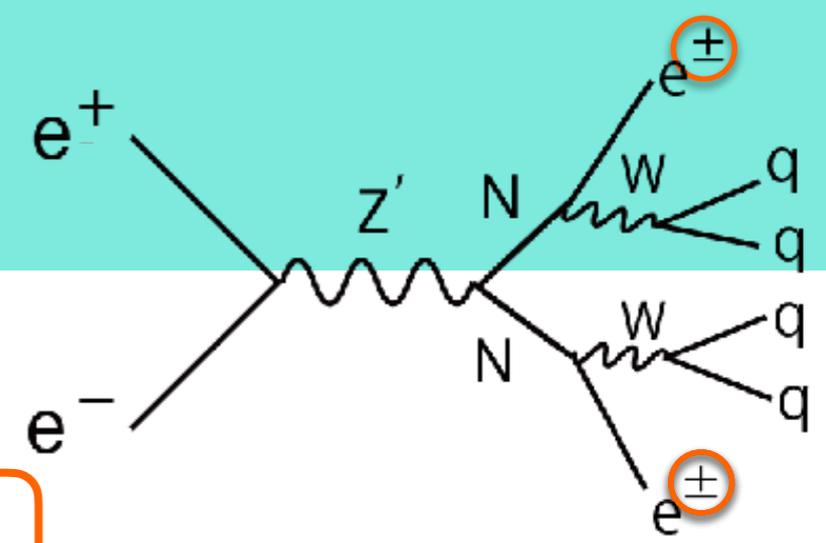
*ILD work in progress*

	Signal Entries				Background Entries					
	$M_N=100$	$M_N=150$	$M_N=200$	$M_N=225$	eeqqqq	4f_singleW _semileptonic	4f_singleZee_ semileptonic	6f_ttbar 2electrons	6f_ttbar 1electron	6f_ttbar 0electron
No cut	554	394	143	45	11898	2825010	699475	16425	129283	11028
$e_{\text{iso}} \# = 2 \ \&\&$ $\gamma_{\text{iso}} \# = 0 \ \&\&$	347	343	79	40	4721	90818	162774	9422	2271	201
Same sign ( $e_{\text{iso}1} \times e_{\text{iso}2} = 1$ )	176	115	39	12	39	46138	3800	8	439	25
$E_{\text{iso}} < 200$ [GeV]	175	114	39	12	39	41319	3557	8	439	25
$-0.95 <$ $\cos\theta_{\text{iso}e} < 0.95$	156	103	36	11	13	17506	623	4	266	15
IsolatedLepTa gging <sub>min</sub> > 0.9	94	91	31	10	2	2632	128	1	50	0
$\log_{10}(y_{12}) > -1$	94	90	31	9	2	2632	128	1	50	0
$P_{\text{miss}} < 100 \ \&\&$ ( $P_{\text{miss}} < 40 \ \parallel$ $ \cos\theta_{P_{\text{miss}}}  >$ 0.95)	84	84	28	9	1	79	30	0	9	0

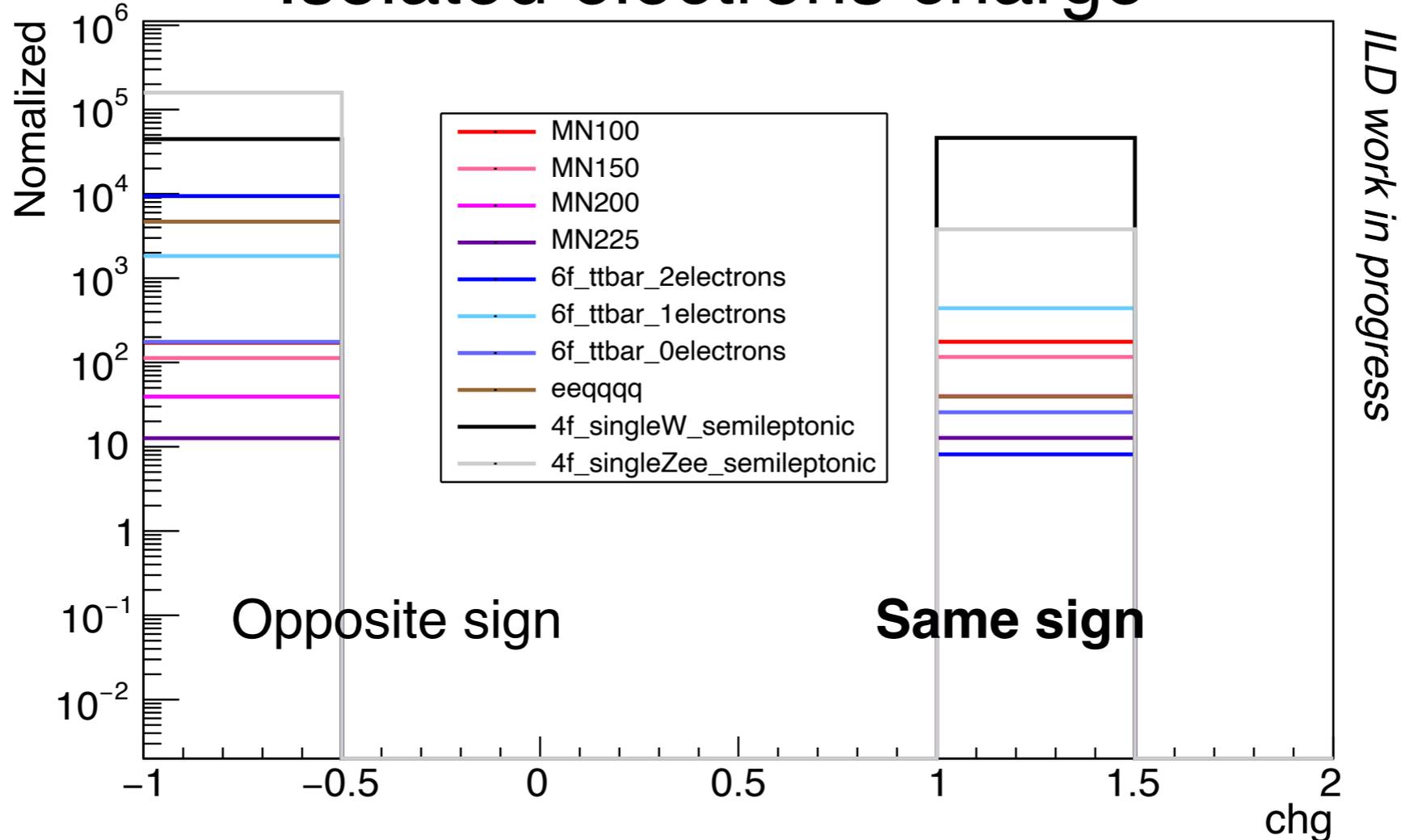
# Electron Charge

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$

- Isolated  $e^- \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$



## Isolated electrons charge



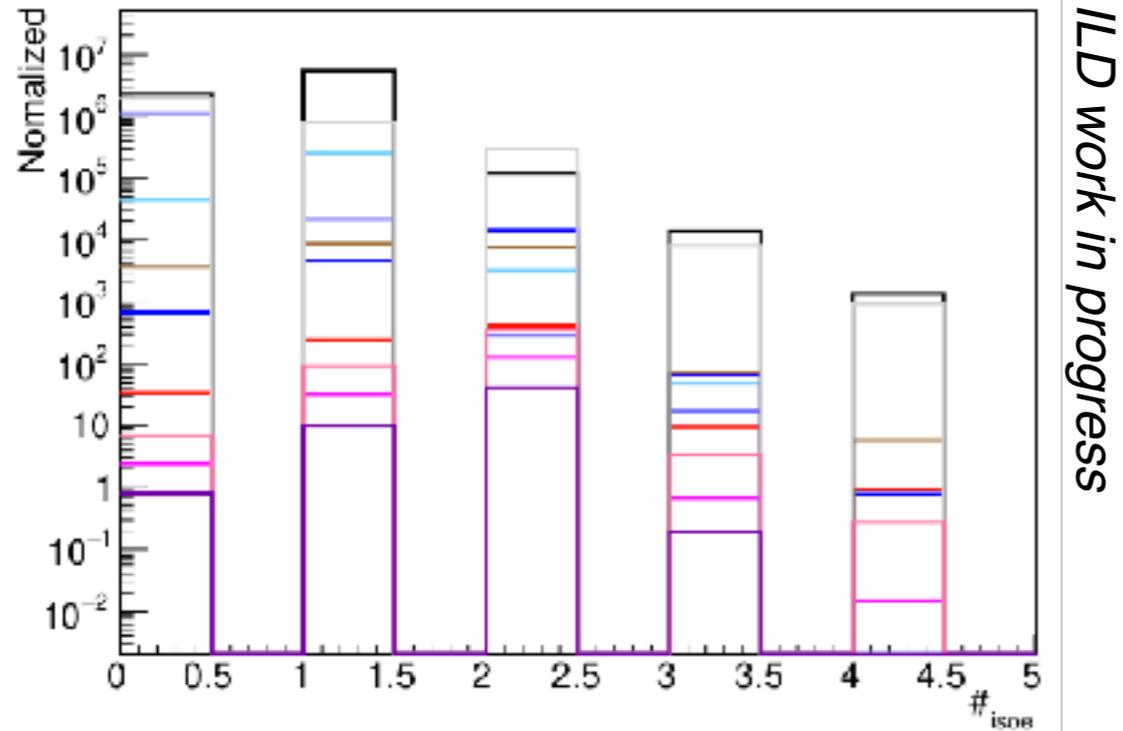
We use only same sign samples  $e_1 \times e_2 = 1$



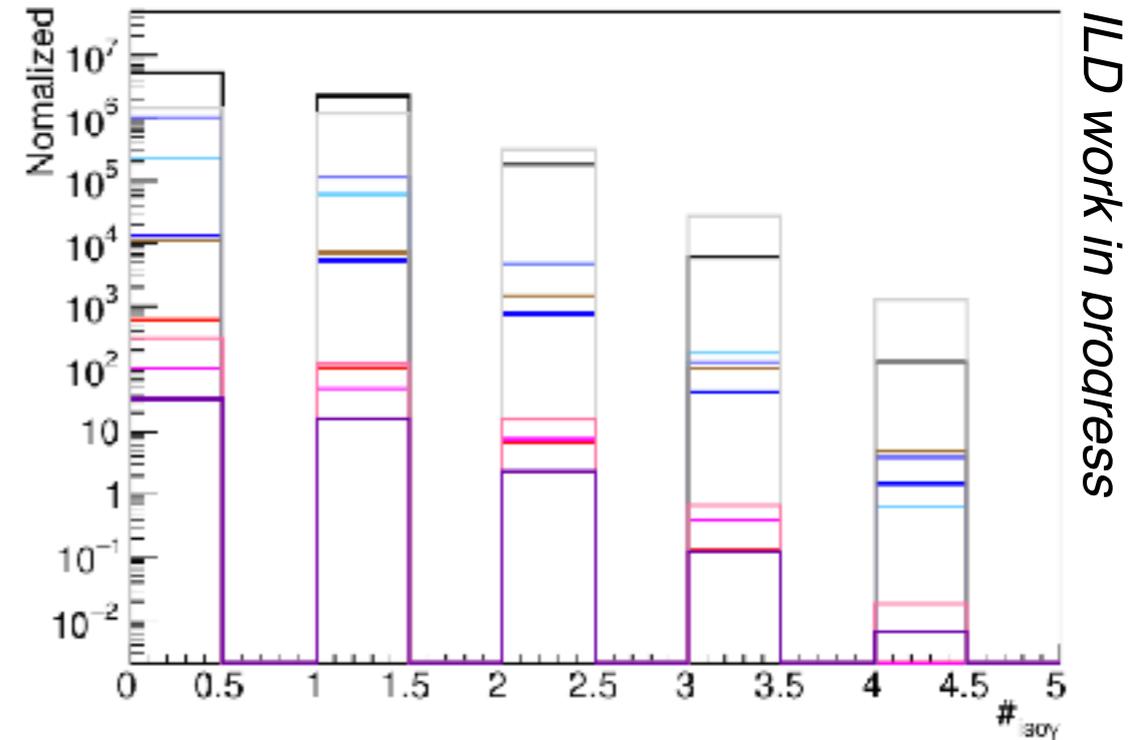
# Isolated $e, \gamma, \mu$

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$

Number of isolated  $e$

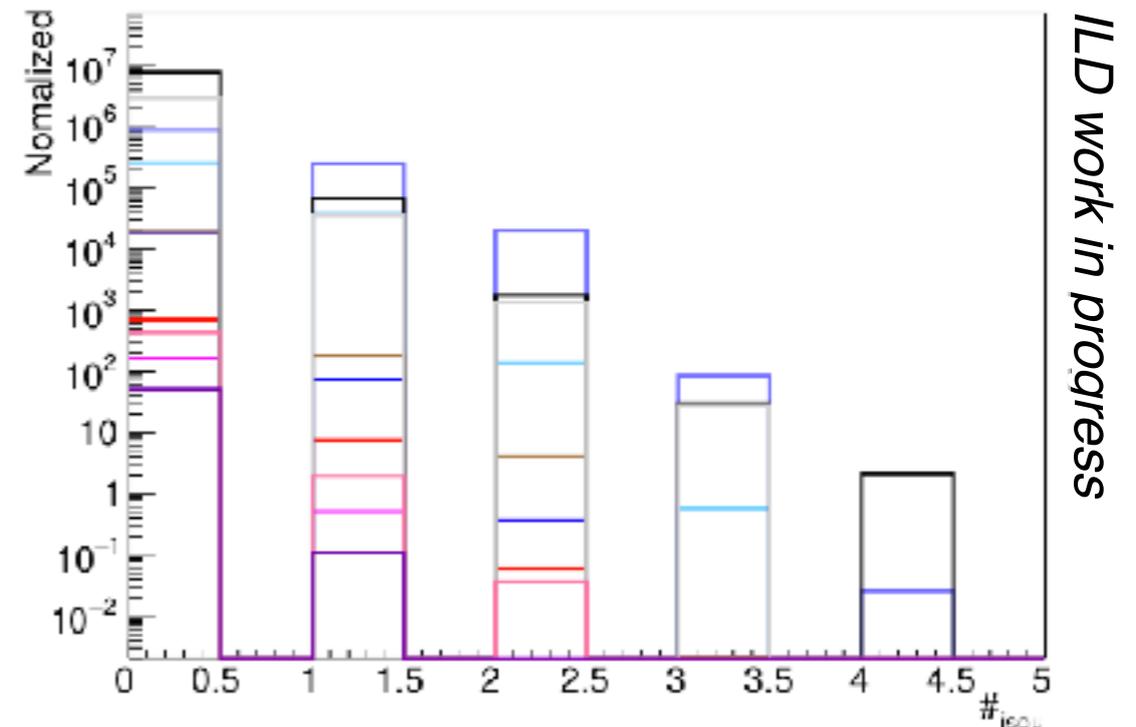


Number of isolated  $\gamma$



- |                 |                  |
|-----------------|------------------|
| — $M_N=100$ GeV | — BG_6f_ttbar_2e |
| — $M_N=150$ GeV | — BG_6f_ttbar_1e |
| — $M_N=200$ GeV | — BG_6f_ttbar_0e |
| — $M_N=225$ GeV | — BG_4f_sw_sl    |
|                 | — BG_4f_sze_sl   |
|                 | — BG_eeqqqq      |

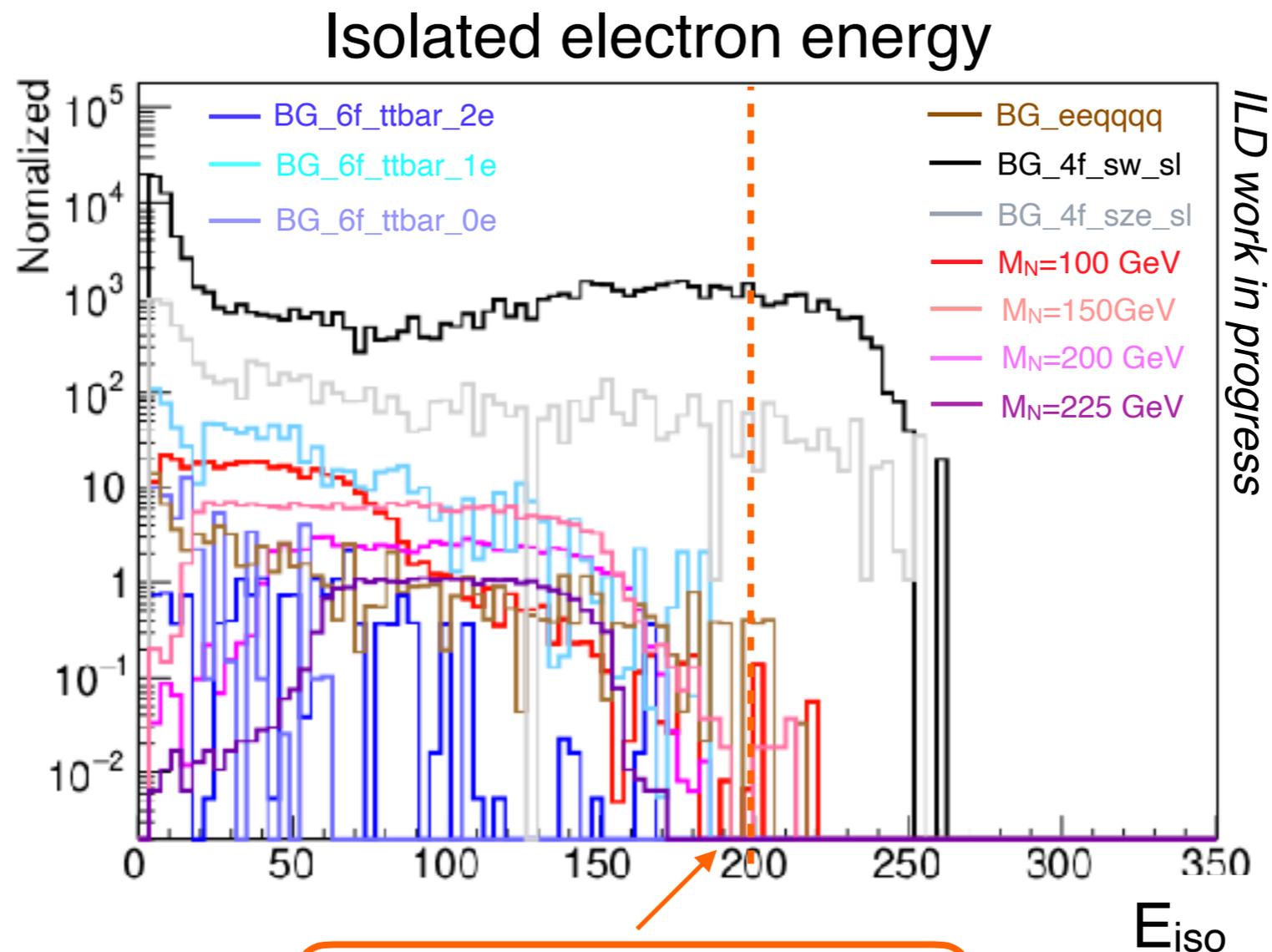
Number of isolated  $\mu$



- Isolated  $e$  # = 2 && Isolated  $\gamma, \mu = 0$

# Distribution of Isolated electron energy

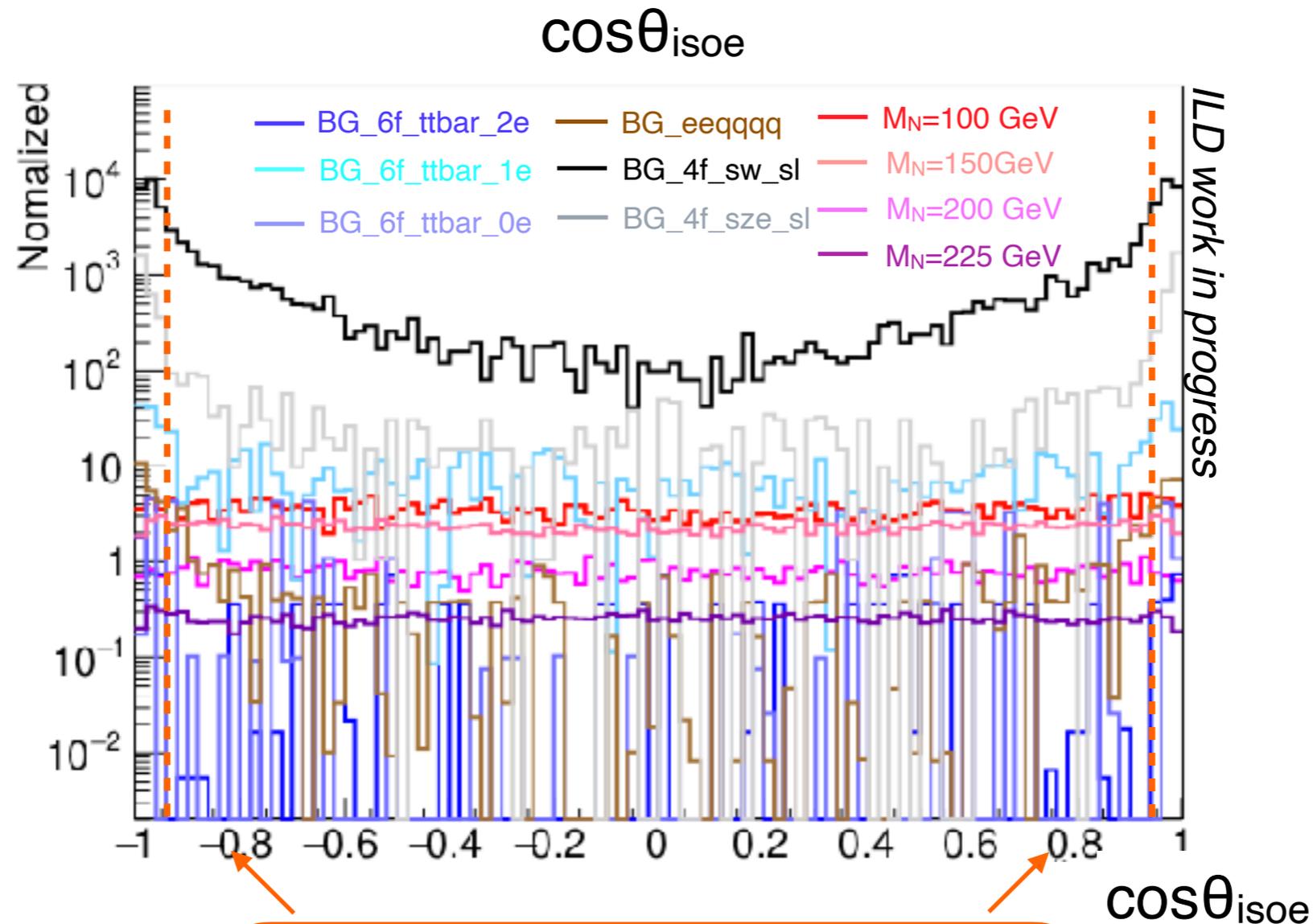
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



$E_{\text{iso}} < 200$  [GeV]

# Distribution of $\cos\theta_{\text{isoe}}$

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )

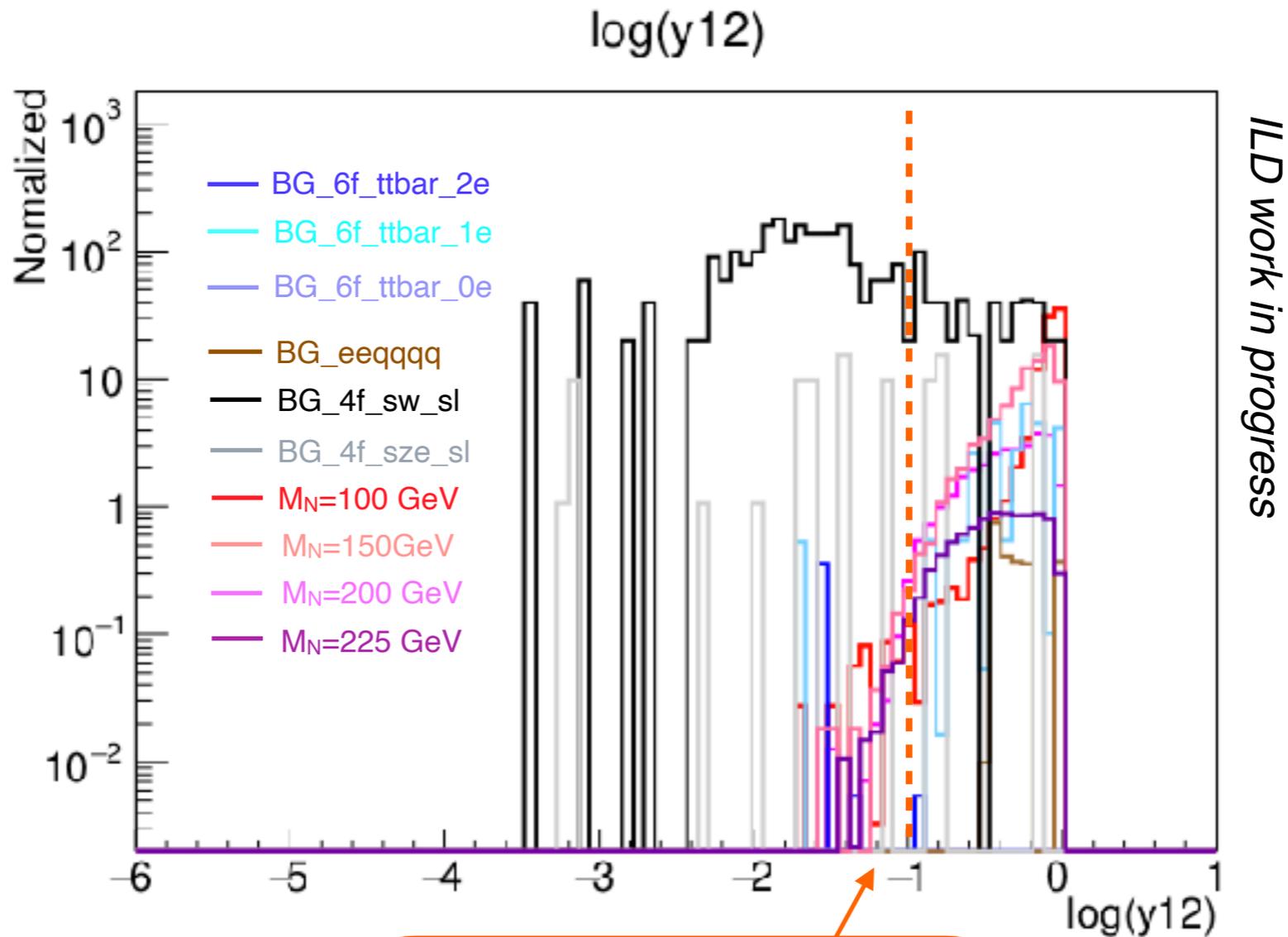


$$-0.95 < \cos\theta_{\text{isoe}} < 0.95$$

4 fermions semi leptonic processes in t-channel  $\rightarrow$  distributed in  $|\cos\theta_{\text{isoe}}| \sim 1$

# Distribution of $y_{12}$ (Durham)

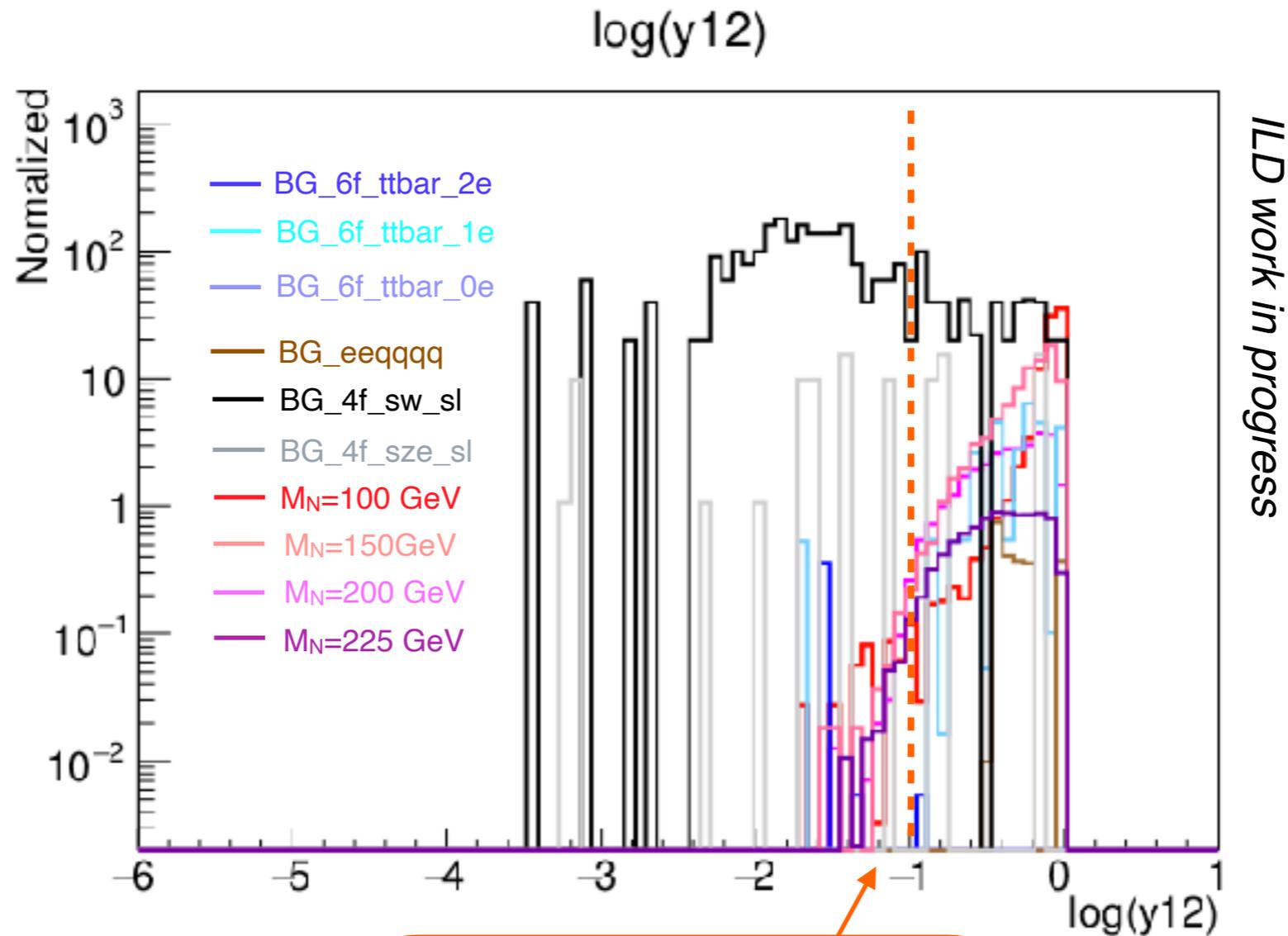
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



$\log_{10}(y_{12}) > -1$

# Distribution of $y_{12}$ (Durham)

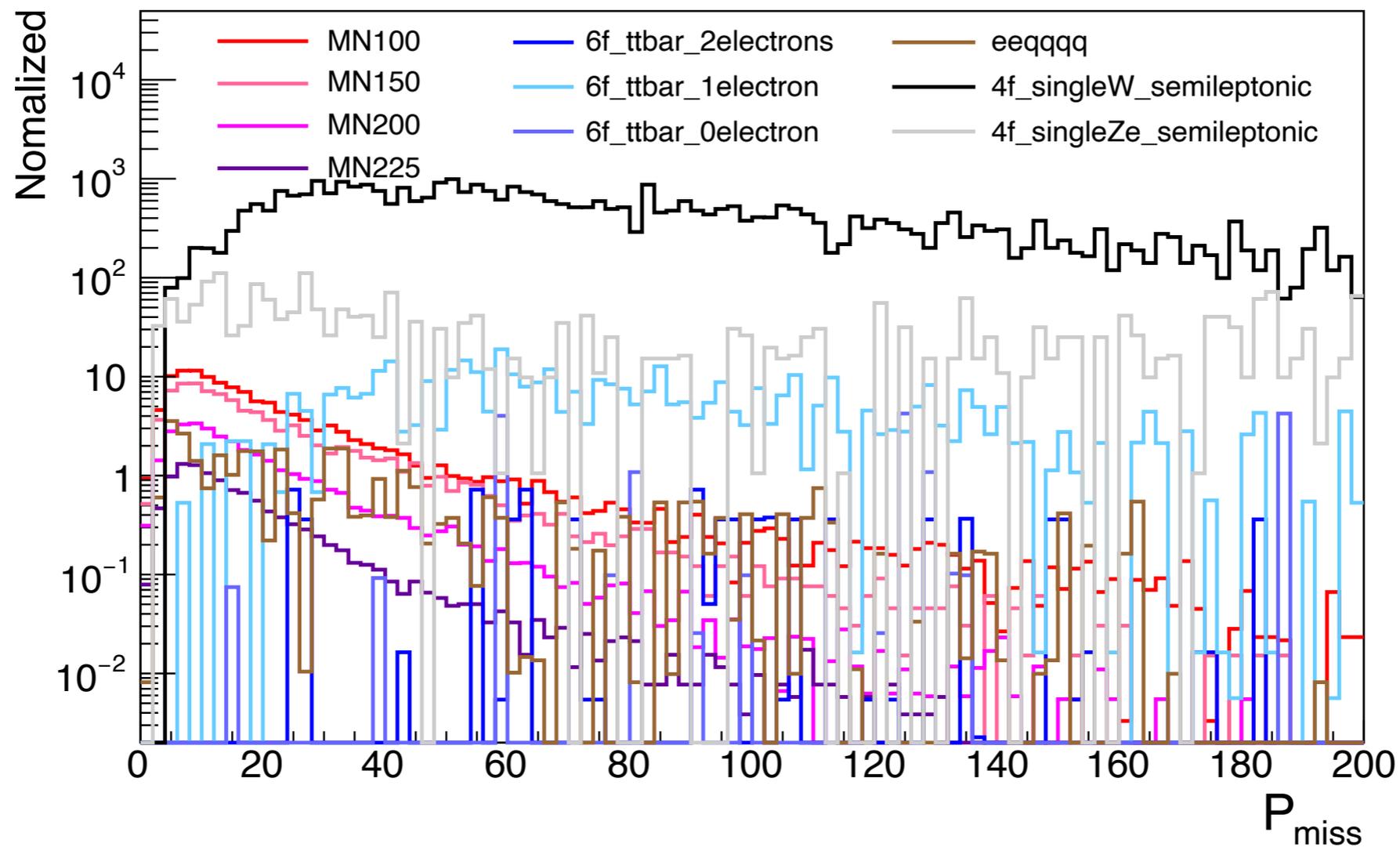
- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



$$\log_{10}(y_{12}) > -1$$

# Distribution of $P_{\text{miss}}$

- ILC 500 with ISR / BS
- $\text{Pol}(e^-, e^+) = (-0.8, +0.3)$
- Isolated  $e \# = 2$  && Isolated  $\gamma \# = 0$  && Isolated  $\mu \# = 0$
- Isolated  $e$  is same sign ( $e_1 \times e_2 = 1$ )



4f and 6f background information

# Cross section — BG

- ILC 500 with ISR / BS

(100%,100%)	eeqqqq			4f_singleW _semileptonic	4f_singleZee _semileptonic
	eexyyx	xxxxee	yyyyee	4f_sw_sl	4f_sze_sl
eLpR	1.64E+01	8.71E-02	1.45E-01	7.81E+03	1.96E+03
eRpL	3.64	4.62E-02	5.31E-02	2.28E+01	1.73E+03
eLpL	6.63	3.38E-02	2.20E-02	7.53E+02	1.78E+03
eRpR	6.61	3.30E-02	1.97E-02	7.50E+02	1.78E+03

# Cross section — BG

- ILC 500 with ISR / BS

6f\_ttbar

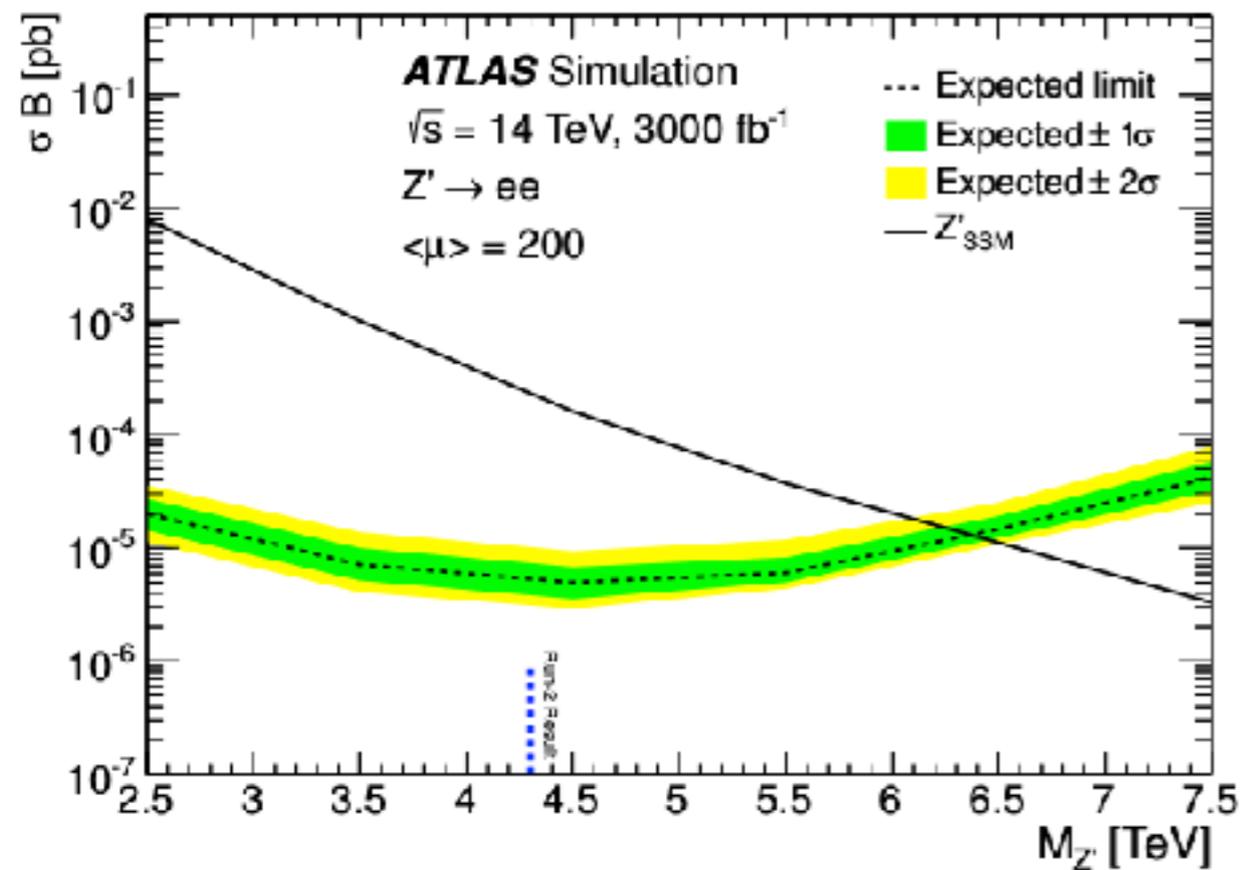
(100%, 100%)

	yyveev	yyvelv	yyvlev	yyveyx	yyxyev	yyvllv	yyvlyx	yyxylv	yyuyyu	yyuyyc	yycyuu	yycyuc
eLpR	2.01E+01	3.96E+01	3.96E+01	1.17E+02	1.17E+02	7.87E+01	2.32E+02	2.31E+02	1.67E+02	1.64E+02	1.65E+02	1.63E+02
eRpL	7.56E+00	1.50E+01	1.50E+01	4.45E+01	4.45E+01	3.01E+01	8.91E+01	8.89E+01	6.45E+01	6.44E+01	6.41E+01	6.07E+01
eLpL	1.08E-01	1.89E-01	/	5.46E-01	/	/	/	/	/	/	/	/
eRpR	1.09E-01	/	1.88E-01	5.42E-01	/	/	/	/	/	/	/	/

Information associated  
with  $U(1)_{B-L}$  model

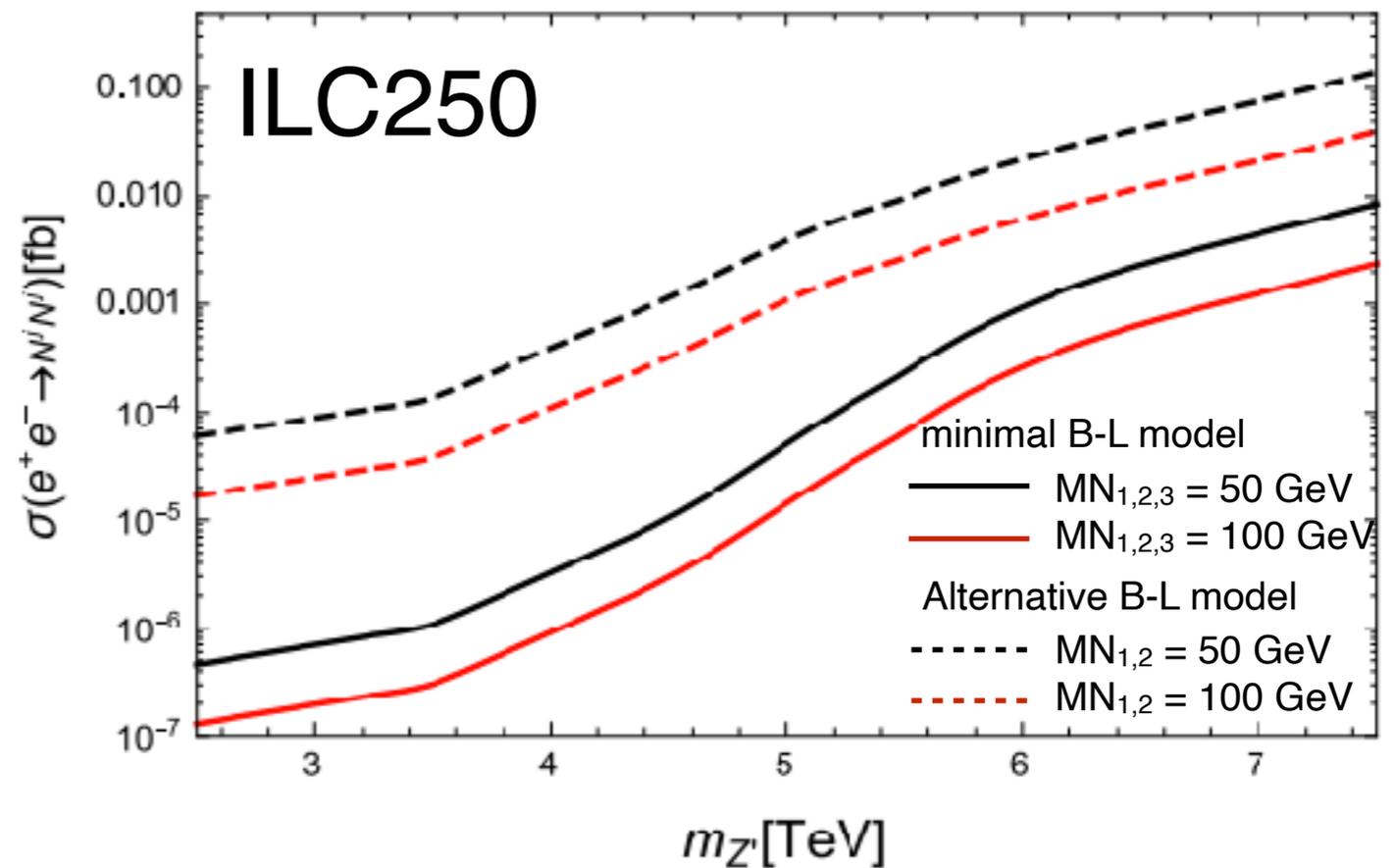
# Current limits - Z' mass

SM like Z' coupling



ATLAS-TDR-LHCC2017-2018

HL-LHC prospects limit for  $U(1)_{B-L}$  model

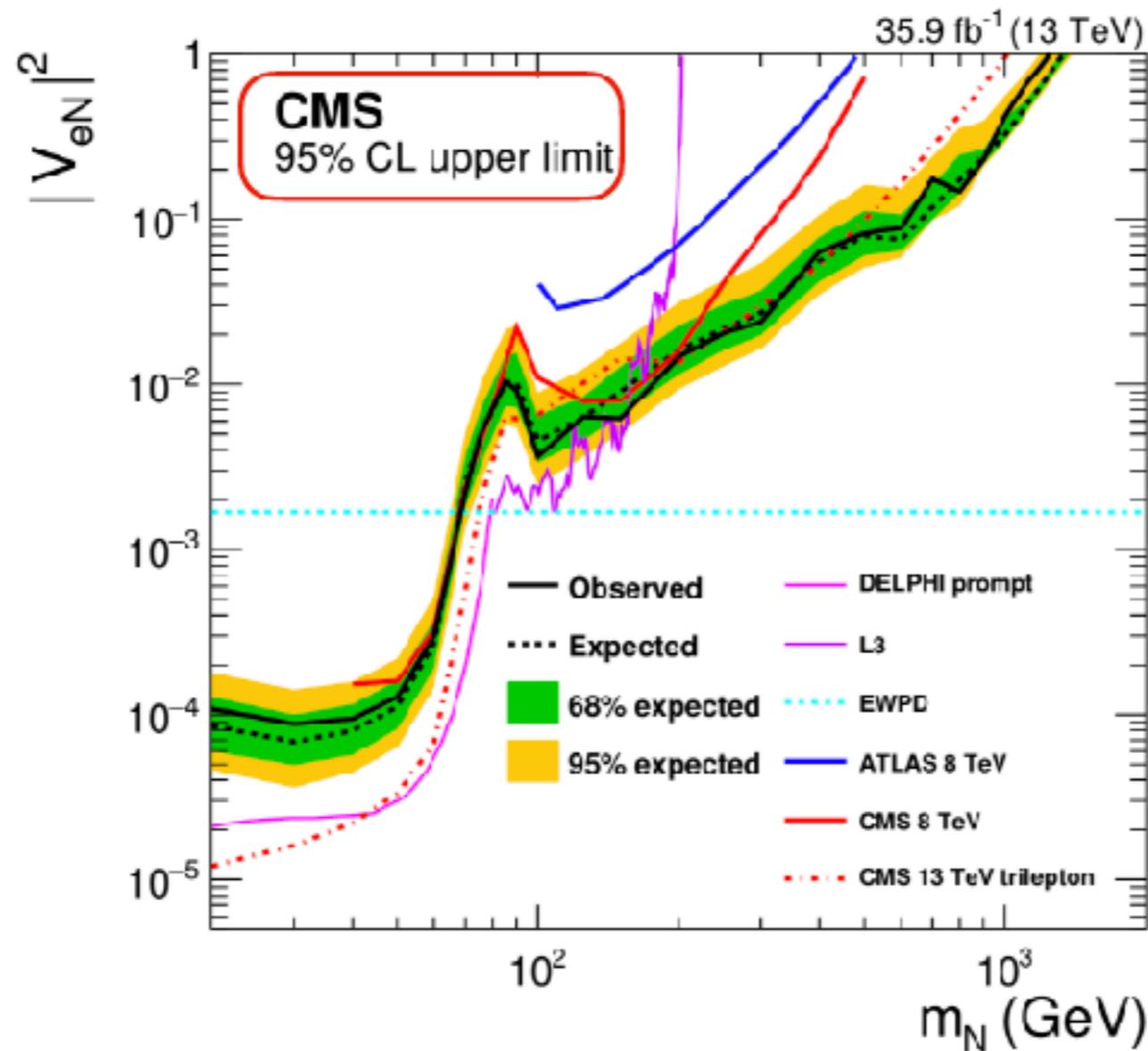


[arXiv\[1812.11931\]](https://arxiv.org/abs/1812.11931)

**The heavier Z' mass less constrained by LHC**

# Current limits $|V_{eN}|^2$

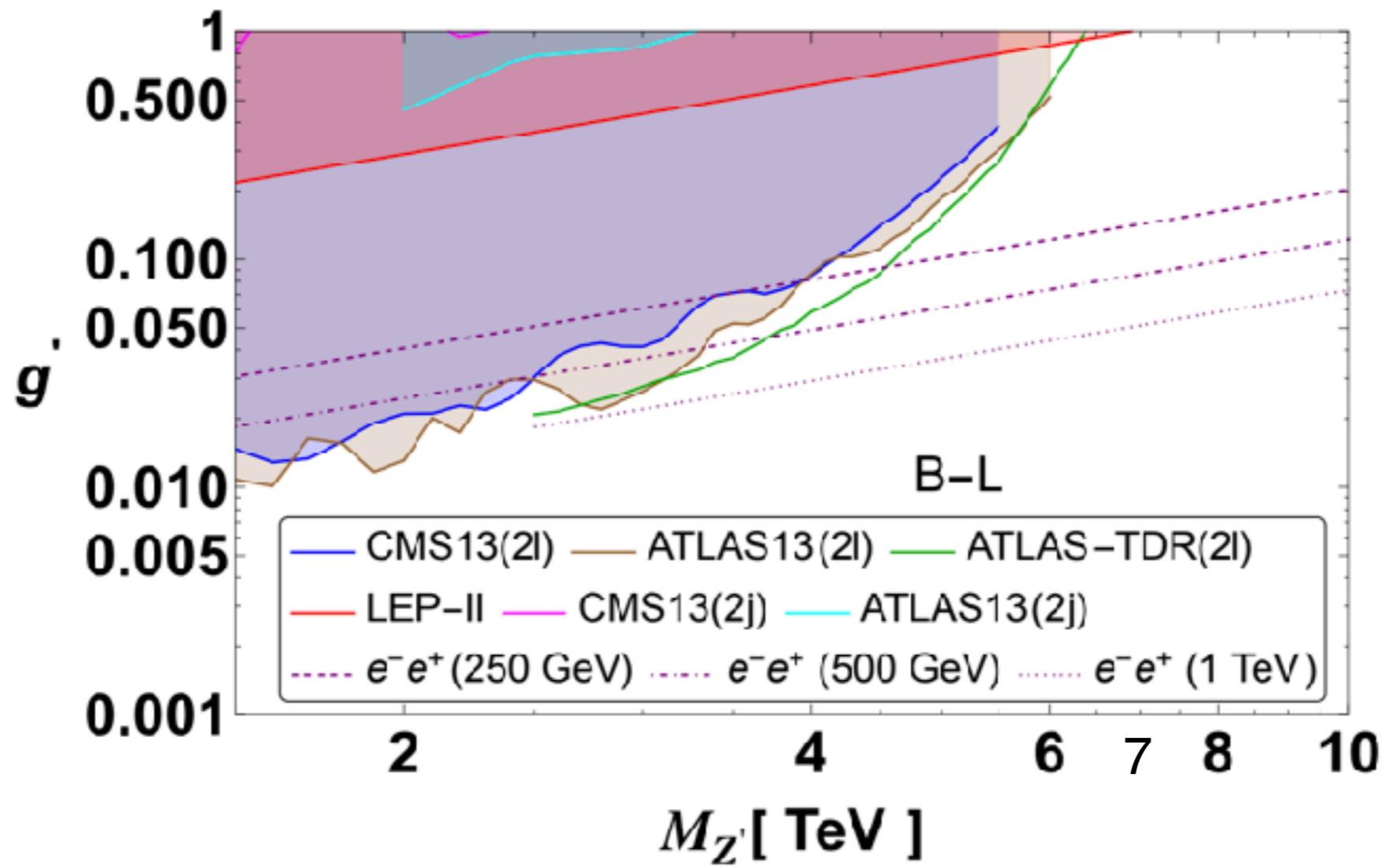
$|V_{eN}|^2$  : the “light-heavy” neutrino mixing matrix



<https://arxiv.org/pdf/1802.02965.pdf>

# Current Limits and prospects - $Z'$ mass, $g_1'$

$g_1'$  :  $U(1)_{B-L}$  gauge coupling constant



[arXiv:2203.06929](https://arxiv.org/abs/2203.06929)