W boson mass measurement prospects at e^+e^- colliders

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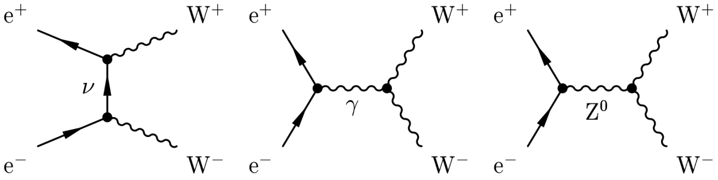
May 11, 2022



Outline of Topics

Introduction

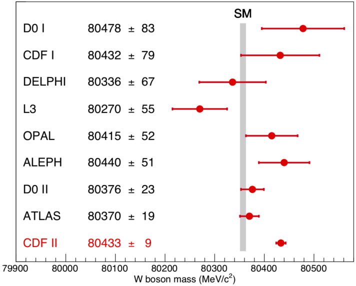
1. Comments on current m_{W} picture
2. Measuring m_{W} in $\mathrm{e}^{+}\mathrm{e}^{-}$ collisions
3. Mini-review of LEP2 measurements

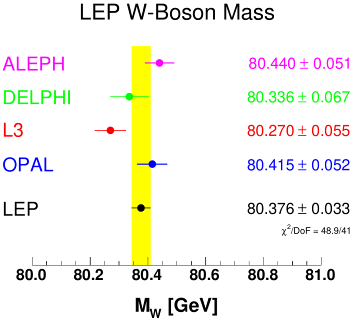
Future $\mathrm{e}^{+}\mathrm{e}^{-}$ measurements

1. Cross-Sections
2. Single W
3. ILC and Run Plan
4. $\sqrt{s}$ and beamstrahlung
5. Constrained Reconstruction
6. Leptonic Observables
7. Threshold
8. Conclusions

INTRODUCTION

What to think of m_W measurements?

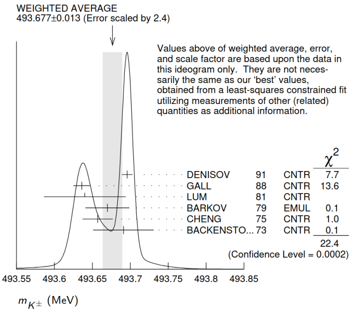




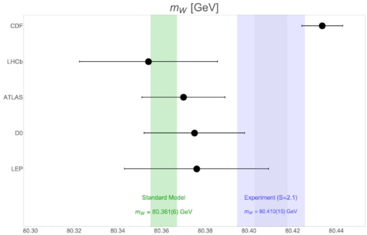
- The LEP results are based on 42 separate measurements with a **healthy** χ^2.
- The LEP-combined (33 MeV), LHCb (32 MeV), D0 Run II (23 MeV), ATLAS (19 MeV) and CDF Run II (9.4 MeV) measurements have a $\chi^2/\text{DoF} = 17.1/4$, with p-value of **0.2%** for compatibility (neglecting correlations).
- So reasonably strong evidence that the ensemble of experimental results are **inconsistent with each other** independent of any SM prediction.
- The standard PDG procedure is to add a scale factor "democratically" to all measurements to parametrize our ignorance.

PDG scale factors

(What can happen with supposed high precision measurements)
The new world average m_{W} uncertainty should be scaled up by about 2.1 leading to an uncertainty of 15 MeV in PDG-2022 compared with 12 MeV in PDG-2020.



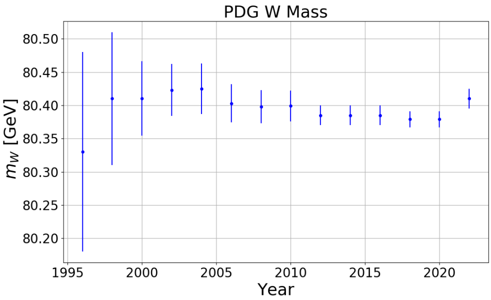
The charged kaon mass has been in this scale-factored state for 30 years!



Plot from Resonaances blog (Adam Falkowski). Independently I had done the same thing and concluded that the scale-factored world-average is $+3.2\sigma$ off the SM value used by CDF $(80\,357 \pm 4 \pm 4$ MeV$)$

My guess: perhaps one or more experiments has underestimated uncertainties. Also may be difficult to measure the same thing in $p\bar{p}$, pp, and $\mathrm{e}^+\mathrm{e}^-$ collisions.

PDG m_{W} World Average History



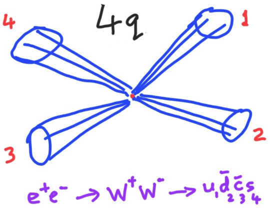
Last point (with latest CDF measurement) is unofficial (my appraisal of what the PDG will do) and has a scale-factor of 2.1.

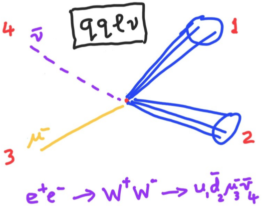
Advice to ambulance chasers from an experimentalist:

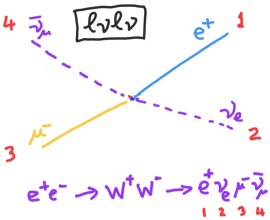
- Please don't take the CDF central value and uncertainty as the best experimental estimate of m_{W}
- As we see here the world-average measurements of m_{W} have historically been rather consistent over time
- Maybe we do only know m_{W} to 15 MeV at this time

WW Topologies

fully hadronic $q\bar{q}q\bar{q}$	semi-leptonic $q\bar{q}\ell\nu_\ell$	fully leptonic $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$







$B_h^2 = 45.4\%$	$6B_\ell B_h = 43.9\%$	$9B_\ell^2 = 10.6\%$

- Here we take $\ell = e, \mu, \tau$. Events with τ leptons are of some use even for m_{W}.
- 100% of the WW final states are potentially useful for m_{W} in $\mathrm{e^+e^-}$ collisions not just the 22% of the W final state used in hadron colliders.
- Much of the power of an $\mathrm{e^+e^-}$ collider is that one measures the mass of the W decay products either directly or by imposing kinematic constraints.

W Mass

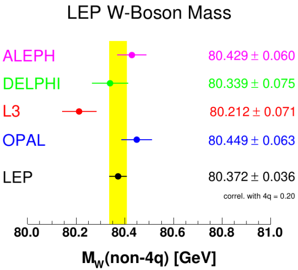
$m_{\rm W}$ is an experimental challenge. Especially so for hadron colliders.

There are several promising approaches to measuring $m_{\rm W}$ at an $\rm e^+e^-$ collider:

1. Constrained Reconstruction Kinematically-constrained reconstruction of $\rm W^+W^-$ using constraints from **four-momentum conservation** and optionally mass-equality: the LEP2 work-horse. Primarily using semileptonic events. Color reconnection assumed to dog fully hadronic - really?
2. Hadronic Mass Direct measurement of the **hadronic mass**. This can be applied particularly to single-W events decaying hadronically or to the hadronic system in semi-leptonic $\rm W^+W^-$ events (especially for $q\bar{q}\tau\nu_\tau$).
3. Lepton Endpoints The 2-body decay of each W leads to endpoints in the lepton (or jet) **energy** at $E_\ell = E_{\rm b}(1 \pm \beta)/2$ where β is the W velocity. These can be used to infer $m_{\rm W}$. Can use for WW events with ≥ 1 prompt lepton.
4. Fully Leptonic Reconstruction Pseudomass method (Apply 5 constraints).
5. Polarized Threshold Scan Measurement of the $\rm W^+W^-$ cross-section near **threshold** with longitudinally polarized beams. Requires dedicated luminosity well below Higgs threshold; can it not be done well enough in other ways?

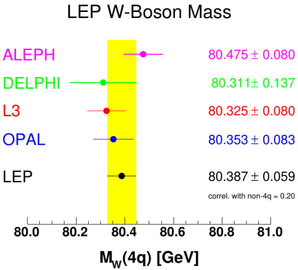
Mini Review of LEP2 m_W Results (arXiv:1302.3415

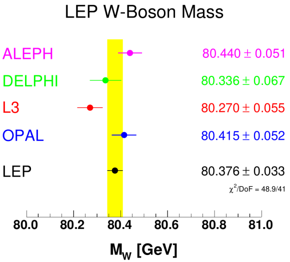
Data-taking 1996–2000, with $\sqrt{s}$ =161–209 GeV



Threshold Analysis	
Experiment	m_W[GeV]
ALEPH	80.20 ± 0.34
DELPHI	$80.45^{+0.45}_{-0.41}$
L3	$80.78^{+0.48}_{-0.42}$
OPAL	$80.40^{+0.46}_{-0.43}$

OPAL $(\ell\nu_\ell\ell'\bar{\nu}_{\ell'})$: $80.41 \pm 0.41 \pm 0.13$ GeV





Constrained Reconstruction of m_{W} in WW events

$$P_s(m_{\mathrm{W}}, \Gamma_{\mathrm{W}}, m_{i,\mathrm{rec}}) = S(m_{\mathrm{W}}, \Gamma_{\mathrm{W}}, m_i, s') \otimes ISR(s', s) \otimes R(m_i, m_{i,\mathrm{rec}})$$

- Main LEP2 results were based on applying kinematic constraints to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$ events.
- Here 5C fit. $(E, \vec{p}) = (\sqrt{s}, \vec{0})$ and $m_{W^+} = m_{W^-}$
- OPAL uses a convolution fit (CV), a reweighting MC template technique (RW) and a Breit-Wigner fit (BW). All 3 applied separately to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$.
- CV fit is most powerful - uses per event resolution function.

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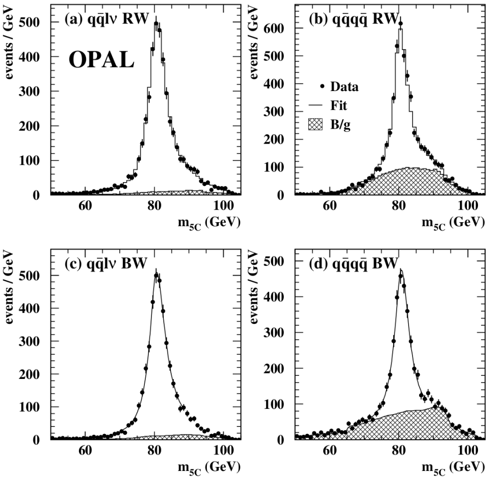


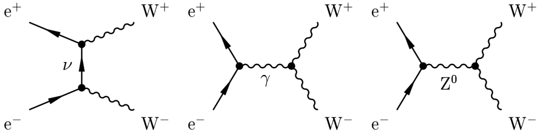
LEP Combined m_W Systematics

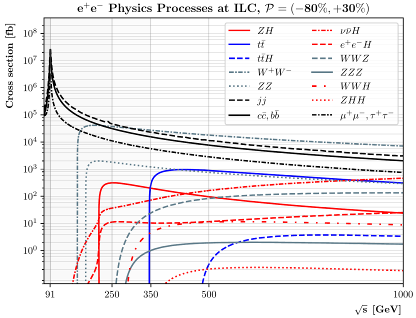
Source	Systematic Uncertainty in MeV			
	on m_W			on Γ_W
	$q\bar{q}\ell\nu_\ell$	$q\bar{q}q\bar{q}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	–	35	8	27
Bose-Einstein Correlations	–	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

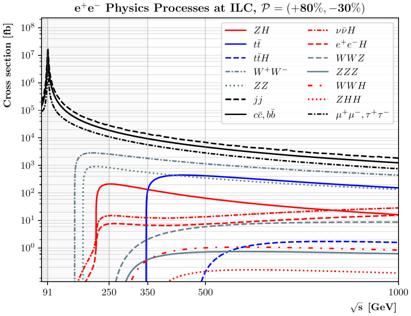
- $q\bar{q}q\bar{q}$ events benefit in fitted mass resolution from all 4 fermions being visible and detectable, but they also have combinatorial ambiguities.
- The color reconnection (CR) phenomenon (well established in other systems) is thought to be a severe limitation for using the $q\bar{q}q\bar{q}$ channel to progress on m_W at future e^+e^- colliders. LEP2 results assume no CR.

FUTURE e^+e^- MEASUREMENTS OF m_W

(Polarized) Cross-Sections







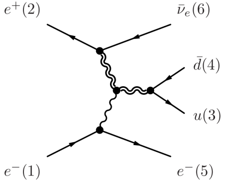
$\sigma_{WW}\ (\sqrt{s} = 250\ \text{GeV}) = 37\ \text{pb}$

$\sigma_{WW}\ (\sqrt{s} = 250\ \text{GeV}) = 3\ \text{pb}$

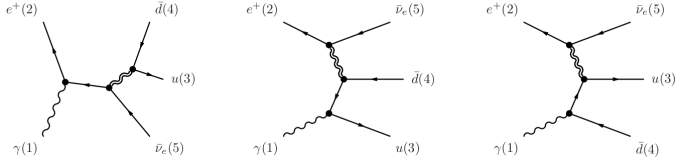
For (-80%, +30%) expect 75M W bosons per ab^{-1} at $\sqrt{s} = 250$ GeV.

Single W production ($e^+e^- \to We\nu_e$)

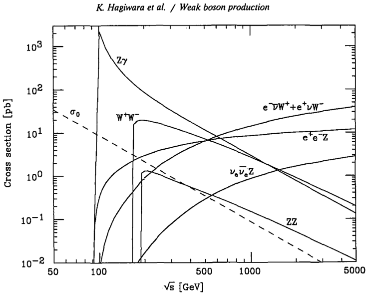
4f final state, $ff'e^+\nu_e$ or $ff'e^-\bar{\nu}_e$ with $W \to ff'$. (CC20 diagrams for $W \to q\bar{q}$)



1 $\gamma e^+ \to u\bar{d}\bar{\nu}_e$



- At higher $\sqrt{s}$, opportunity to produce W and Z in t-channel processes where typically an electron has minimal p_T and is undetected
- Can use hadronic W decays to reconstruct the mass
- Could use hadronic Z decays with similar kinematics for control
- Some benefit from polarization



See ILC paper for Snowmass for latest on ILC accelerator, detectors and physics

Quantity	Symbol	Unit	Initial	$\mathcal{L}$ Upgrade	Z pole	Upgrades		
Centre of mass energy	$\sqrt{s}$	GeV	250	250	91.2	500	250	1000
Luminosity	$\mathcal{L}$	$10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.35	2.7	0.21/0.41	1.8/3.6	5.4	5.1
Polarization for e^-/e^+	$P_-(P_+)$	%	80(30)	80(30)	80(30)	80(30)	80(30)	80(20)
Repetition frequency	f_{rep}	Hz	5	5	3.7	5	10	4
Bunches per pulse	n_{bunch}	1	1312	2625	1312/2625	1312/2625	2625	2450
Bunch population	N_{e}	10^{10}	2	2	2	2	2	1.74
Linac bunch interval	Δt_{b}	ns	554	366	554/366	554/366	366	366
Beam current in pulse	I_{pulse}	mA	5.8	8.8	5.8/8.8	5.8/8.8	8.8	7.6
Beam pulse duration	t_{pulse}	μs	727	961	727/961	727/961	961	897
Average beam power	P_{ave}	MW	5.3	10.5	1.42/2.84*)	10.5/21	21	27.2
RMS bunch length	σ^*_{z}	mm	0.3	0.3	0.41	0.3	0.3	0.225
Norm. hor. emitt. at IP	$\gamma\epsilon_{\mathrm{x}}$	μm	5	5	5	5	5	5
Norm. vert. emitt. at IP	$\gamma\epsilon_{\mathrm{y}}$	nm	35	35	35	35	35	30
RMS hor. beam size at IP	σ^*_{x}	nm	516	516	1120	474	516	335
RMS vert. beam size at IP	σ^*_{y}	nm	7.7	7.7	14.6	5.9	7.7	2.7
Luminosity in top 1 %	$\mathcal{L}_{0.01}/\mathcal{L}$		73 %	73 %	99 %	58.3 %	73 %	44.5 %
Beamstrahlung energy loss	δ_{BS}		2.6 %	2.6 %	0.16 %	4.5 %	2.6 %	10.5 %
Site AC power	P_{site}	MW	111	138	94/115	173/215	198	300
Site length	L_{site}	km	20.5	20.5	20.5	31	31	40

Table 4.1: Summary table of the ILC accelerator parameters in the initial 250 GeV staged configuration and possible upgrades.

Note: $\sqrt{s}$, luminosities, polarizations, BS energy loss. Potential to run at all center-of-mass energies from 91 to 1000 GeV.

ILC and Run Plan

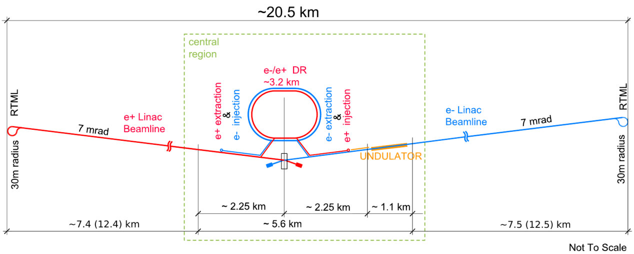
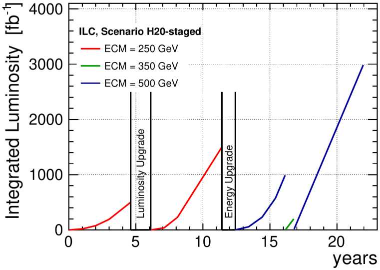


Figure 4.1: Schematic layout of the ILC in the 250 GeV staged configuration.



- (2.0, 0.2, 4.0) ab^{-1} at $\sqrt{s} =$ (250, 350, 500) GeV
- Room for dedicated runs at Z and at WW threshold prior to energy upgrade
- Potential for upgrading to higher energies

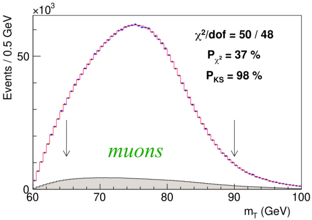
General Remarks

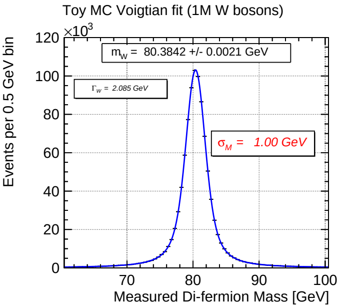
- It is not straightforward to project the performance for measurements that are probably **systematics** limited with ab^{-1} data sets.
- ILC data sets benefit from much **better detectors** than at LEP2, the advantages of beam **polarization**, and an experimental environment conducive to precision measurement (trigger, bunch structure, hermeticity, detector material).
- Measurements of W mass, were already quite complex at LEP2. Getting to a **realistic** estimate of the eventual performance at ILC is difficult.
- We can make educated guesses and identify salient issues.
- In some simpler cases, like the polarized WW threshold scan and purely leptonic observables, we can be relatively confident of the experimental projections.

Sensitivity to m_{W} at hadron and $\mathrm{e^+e^-}$ colliders

Hadron colliders rely on the $m_T(\ell, \nu)$ and $p_T(\ell)$ in leptonic decays of singly produced W bosons. In contrast, $\mathrm{e^+e^-}$ colliders can reconstruct the mass of the W boson decay products: measure directly $(m_{\mathrm{W}}, \Gamma_{\mathrm{W}})$ from the B-W lineshape.

CDF Run II
2.4M $W \to \mu\nu_\mu$ decays

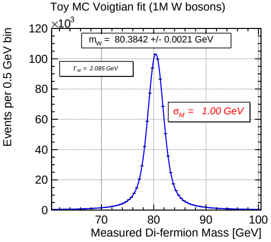


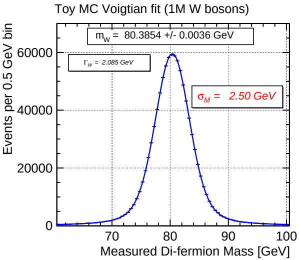


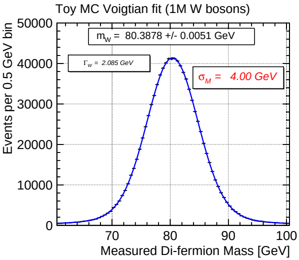
$m_{\mathrm{W}}(m_T) = 80\,446.1 \pm 9.2 \pm 7.3$ MeV

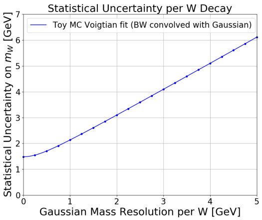
Fit with Breit-Wigner $\otimes$ Gaussian

Ultimate sensitivity of a future $\mathrm{e^+e^-}$ collider depends on the techniques, channels, mass resolution, and statistics. Could achieve the same m_{W} stat. sensitivity as this CDF plot with **only** 2.2% of the W decays for $\sigma_M = 1.0$ GeV (optimistic).







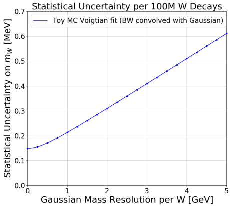


Basic sensitivity

$$\sigma_{m_{\mathrm{W}}} = \frac{f(\sigma_M, \Gamma_{\mathrm{W}})}{\sqrt{N_W}}$$

We will use both:

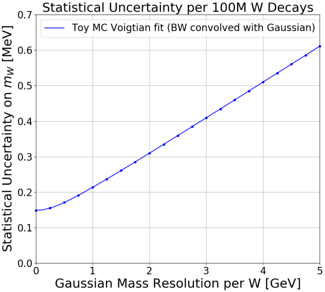
- Per decay m_{W} estimators (m_{ij}).
- Per event estimators, eg. average mass, $\frac{1}{2}(m_{12} + m_{34})$ $\{N_{WW}\}$

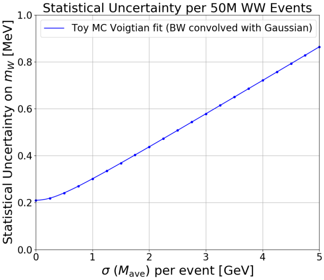


Scaled to ILC-like statistics

Decays or Events

To a very good approximation, the distribution of the averaged mass, follows the same Breit-Wigner distribution. So apply the same curve to WW events.





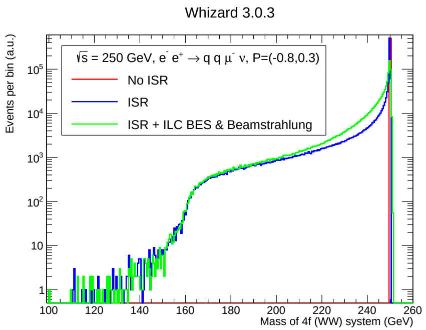
σ_M (GeV)	Δm_W (MeV)	$\Delta\Gamma_W^a$ (MeV)	$\Delta\Gamma_W^b$ (MeV)
1.0	0.21	0.41	0.63
2.5	0.35	0.63	1.0
4.0	0.50	0.89	1.6

- Fits with 100M W decays and 1, 2 or 3 parameters fitted (m_{W}, Γ_{W}, σ_M).
- Statistical uncertainties only. Note that individual W's and event-averaged masses will have very different resolutions (some excellent).

Beamstrahlung

Beam-beam interaction leads to energy loss (radiated photons).
Two main issues (more important at high $\sqrt{s}$).

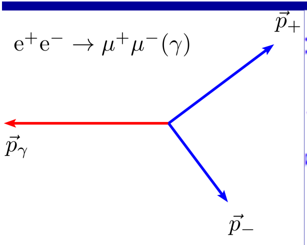
1. worsening of the validity of the kinematic constraints (similar to ISR).
2. presence of "overlay" events from soft $\gamma\gamma$ collisions akin to pile-up.



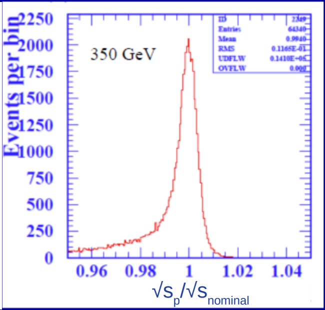
- Idealized: $< M >= 250.0$ GeV
- ISR only: $< M >= 242.9$ GeV
- ISR+BES+BS: $< M >= 240.3$ GeV

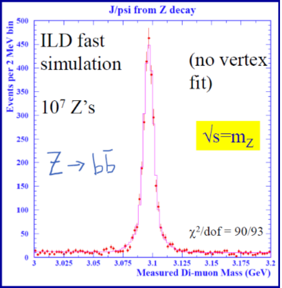
$\sqrt{s_p}$ Method for Absolute Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.



Measure $\sqrt{s_p}$ using, $(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$



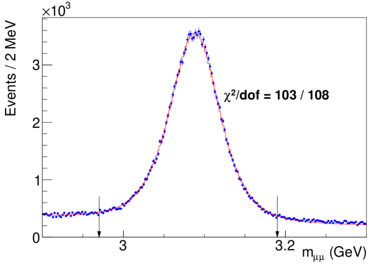


Tie detector p-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

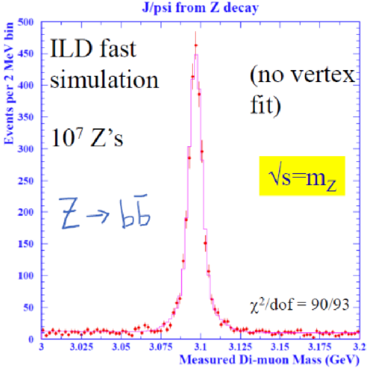
Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc). (Links to more details in backup)

Compare J/ψ Mass Resolution (CDF vs ILD for ILC)



Source	J/ψ (ppm)	Υ (ppm)	Correlation (%)
QED	1	1	100
Magnetic field non-uniformity	13	13	100
Ionizing material correction	11	8	100
Resolution model	10	1	100
Background model	7	6	0
COT alignment correction	4	8	0
Trigger efficiency	18	9	100
Fit range	2	1	100
$\Delta p/p$ step size	2	2	0
World-average mass value	4	27	0
Total systematic	29	34	16 ppm
Statistical NBC (BC)	2	13(10)	0
Total	29	36	16 ppm



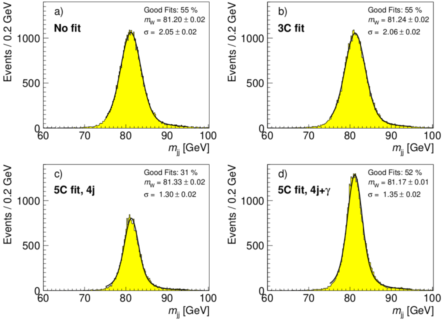
Much better mass resolution at ILC. Can measure momentum scale to 1 ppm stat. with 4.2B hadronic Z's. Systematics should be better than CDF (eg. no trigger). Previous "conservative" estimate of 10 ppm for ILC seems too conservative.

Constrained Fits

Some ideas and progress

1. Photon radiation treatment in kinematic fits (M. Beckmann, B. List and J. List) arXiv:1006.0436 Applied to $q\bar{q}q\bar{q}$ at $\sqrt{s} = 500$ GeV.
2. Jet specific energy resolution studies (Wilson, IWLC 2010).
3. Studies on "ErrorFlow" ie. parametrizing jet uncertainties. A. Ebrahimi
4. Kinematic Fitting for Particle Flow Detectors at Future Higgs Factories (Y.Radkhorrami, J.List), arXiv:2111.14775

- BLL - do simplified study of $q\bar{q}q\bar{q}$ reconstruction at $\sqrt{s} = 500$ GeV without "overlay".
- Shown is the average di-jet mass and its resolution (Voigtian fit).
- 4j+γ method adds an ISR photon as an additional "measured" object with large error
- Estimate 1.35 GeV mass resolution for 52% of events.

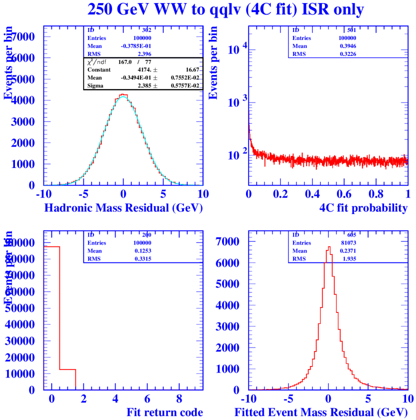


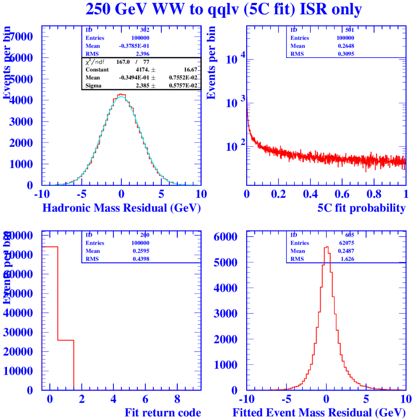
Toy study of constrained fitting for $q\bar{q}\ell\nu_\ell$

- Looked at $\mathrm{e^+e^-} \to u\bar{d}\mu^-\bar{\nu}_\mu$ events generated with Whizard 3.0.3.
- 3 configurations examined: no ISR, ISR only, ISR + ILC-BES&BS
- Used jet energy and angular resolution parametrization from D. Ward and W. Yan (from 2009)
- Hadronic mass resolution about 2.4 GeV.
- Neglected quark/jet masses
- Used APLCON (V. Blobel) implementation
- Treat neutrino as unmeasured
- With 4C it is a (4-3)=1 dof fit.
- With 5C it is a (5-3)=2 dof fit.
- Method works perfectly with no ISR.
- Lots of room for improvement by using event-by-event fitted uncertainties.
- (tried the BLL photon method - suspect it may not work so well with many fewer constraints ..)

Fit $q\bar{q}\ell\nu_\ell$ ($\ell = e, \mu$) with ISR only (not even BES)

Successful fits defined as converging and having $p_{\mathrm{fit}} > 0.02$



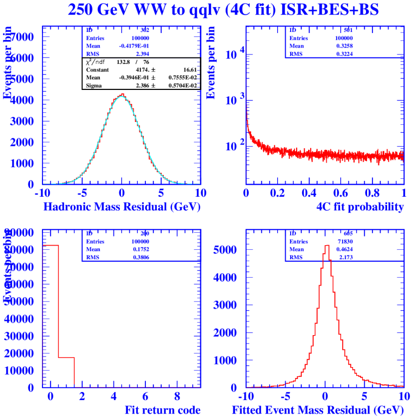


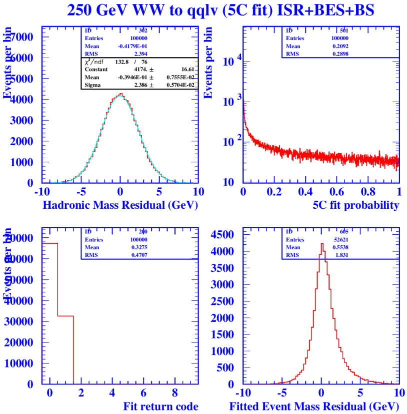
$\varepsilon_{\mathrm{fit}} = 81\%$, "$\sigma$"=1.94 GeV

$\varepsilon_{\mathrm{fit}} = 62\%$, "$\sigma$"=1.63 GeV

Fit $q\bar{q}\ell\nu_\ell$ ($\ell = e, \mu$) with ILC beam effects

Successful fits defined as converging and having $p_{\rm fit} > 0.02$

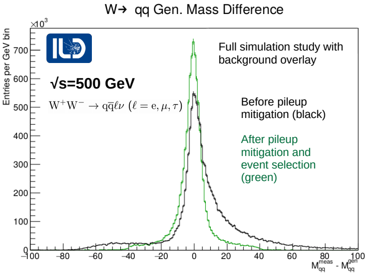




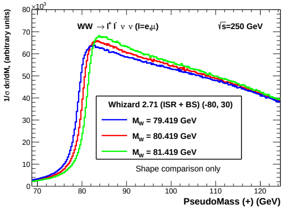
$\varepsilon_{\rm fit} = 72\%$, "$\sigma$"=2.17 GeV

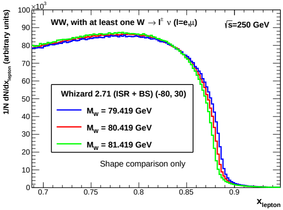
$\varepsilon_{\rm fit} = 55\%$, "$\sigma$"=1.83 GeV

On average, the fit does not improve much over the hadronic mass



- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta m_{\mathrm{W}} = 2.4$ MeV for $1.6\ \mathrm{ab}^{-1}$ (-80%, +30%).
- Can be improved, but m_{had}-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set





Sensitivity to m_{W} with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta m_{\mathrm{W}} = 4.4$ MeV for 2 ab^{-1} (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell/E_{\mathrm{b}}$. Exptl. systematics small.

m_{W} Measurement Using Leptons

One complementary method for measuring $\mathrm{M_W}$ at LEP was the measurement by OPAL (hep-ex/020326) using $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$ events. Results were modest. Limited by the integrated luminosity of 0.67 fb^{-1} (unpolarized), and the poor momentum resolution ($\Delta p/p$). ILC will be much better for L, P and $\Delta p/p$. Disadvantages: higher $\sqrt{s}$ and beamstrahlung.

Method uses lepton $\vec{\mathrm{p}}$ measurement:

- The prompt (e, μ)-lepton energy spectrum in ee, $\mu\mu$, eμ, eτ, $\mu\tau$ events with endpoints at $E_\pm = \frac{1}{2}\, E_\mathrm{b}(1 \pm \beta)$. Can also apply to $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$.
- The positive pseudo-mass (M_+) solution in ee, $\mu\mu$, eμ events.

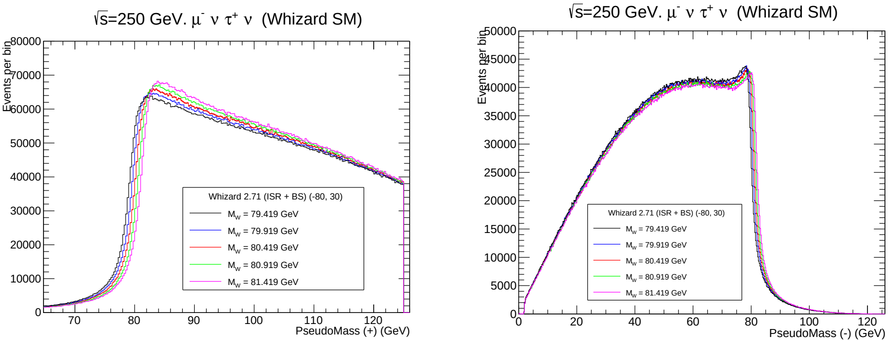
Latter assumes 4-momentum conservation, equal (l-ν) masses, and guesses that the neutrinos are in the same plane as the di-lepton.

$$M_\pm^2 = \frac{2}{|\vec{p}_\ell + \vec{p}_{\ell'}|^2}\Big((P\ \vec{p}_{\ell'} - Q\ \vec{p}_\ell)\cdot(\vec{p}_\ell + \vec{p}_{\ell'}) \pm \sqrt{|\vec{p}_\ell \times \vec{p}_{\ell'}|^2 [|\vec{p}_\ell + \vec{p}_{\ell'}|^2 (E_\mathrm{b} - E_\ell)^2 - (P+Q)^2]} \Big), \quad (1)$$

where

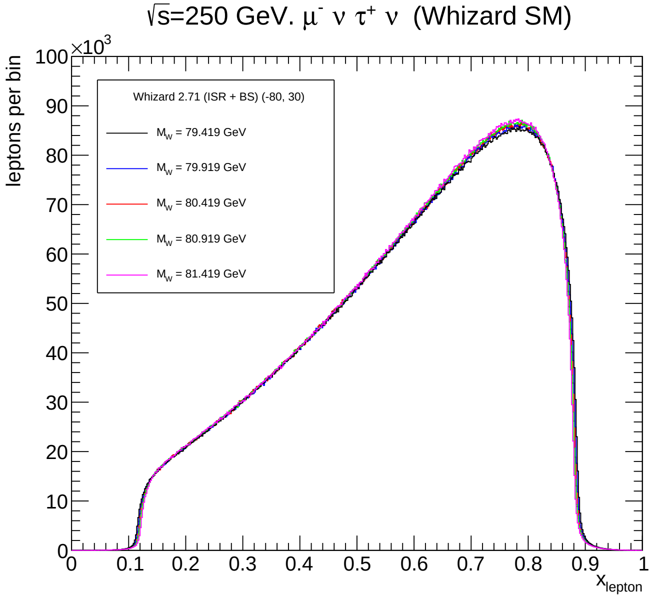
$$P = E_\mathrm{b} E_\ell - E_\ell^2 + \frac{1}{2} m_\ell^2, \qquad Q = -E_\mathrm{b} E_{\ell'} - \vec{p}_\ell \cdot \vec{p}_{\ell'} + \frac{1}{2} m_{\ell'}^2.$$

PseudoMasses (10M events per sample) (-80,+30)



- Study just uses changes in the shape. The total cross sections should be relatively insensitive to m_{W} well above threshold (depends on SM parameter scheme implementation though ....).
- Plots are at generator level (no detector smearing).
- Find that **both** pseudomasses are sensitive to m_{W}.

Lepton Endpoint (20M leptons per sample) (-80,+30)

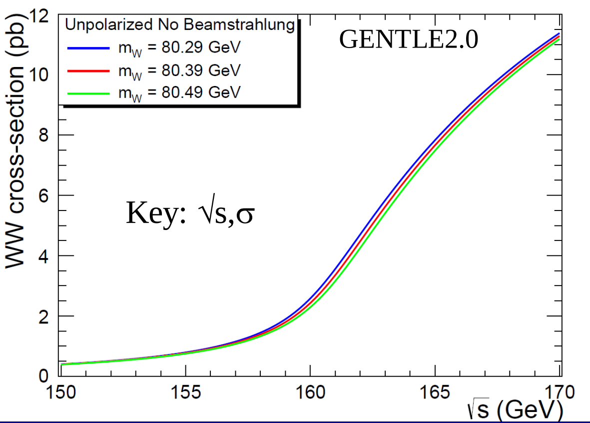


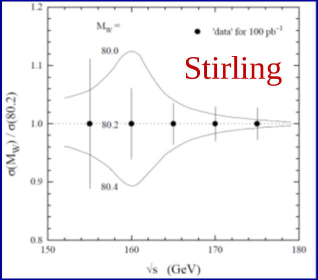
Estimated m_{W} statistical uncertainties from leptons

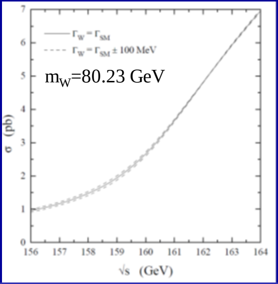
Based on 2.0 ab^{-1} with all beam polarizations (45/45/5/5) at generator level at $\sqrt{s} = 250$ GeV. Now with beamstrahlung. Detector resolution neglected. Estimates based on ensemble test fits.

1. M_{+}: 1.50M prompt dilepton events = 8.8 MeV
2. M_{-}: 1.50M prompt dilepton events = 11.2 MeV
3. Pseudomasses combined: 1.50M prompt dilepton events = 6.9 MeV (assuming uncorrelated)
4. Endpoints: 4.50M leptons (from dileptons)= 11.0 MeV
5. Combined: Fully leptonic (M and endpoints) = 5.9 MeV (neglects possible correlation (+11% in OPAL case))
6. Semi-leptonic endpoints (12.6M leptons) = 6.6 MeV
7. Grand total = 4.4 MeV

m_W from cross-section close to threshold





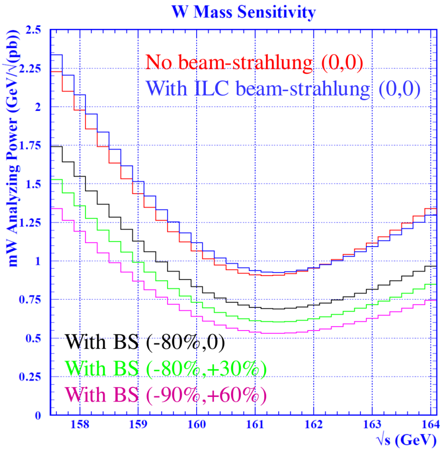


$\sigma_t \sim \beta$ $\sigma_s \sim \beta^3$

$$\Delta M_{\text{sys}}^{\text{bkgd}} = 470 \text{ MeV} \left[\frac{\Delta\sigma}{1 \text{ pb}}\right]$$

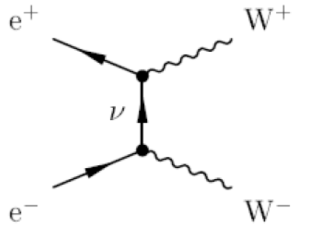
Threshold sensitivity to m_{W}

$$\Delta M_{\mathrm{stat}} = \left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1} \Delta\sigma = \left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1} \sqrt{\frac{\sigma}{\varepsilon\mathcal{L}}} = \frac{K}{\sqrt{\varepsilon\mathcal{L}}}$$



- Following Stirling, Nucl. Phys. B456 (1995) 3
- Plot shows $K = \sqrt{\sigma}\left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1}$
- For $\varepsilon = 100\%$, $\mathcal{L} = 100\ \mathrm{fb}^{-1}$ and (-80%, +30%) polarizations, find $\Delta M_{\mathrm{stat}} = 1.9$ MeV at the optimum
- Polarization of e^- and e^+ beams at ILC (necessarily with beamstrahlung) offers **much** better sensitivity per unit of integrated luminosity than the LEP-like unpolarized case

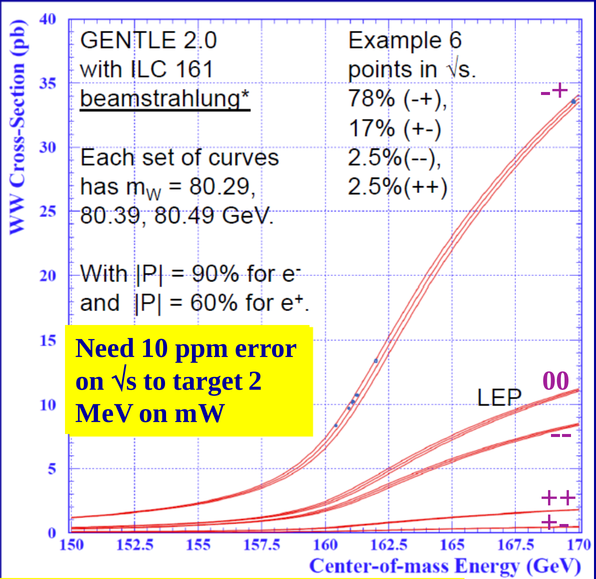
ILC Polarized Threshold Scan



Use (-+) helicity combination of e⁻ and e⁺ to enhance WW.

Use (+-) helicity to suppress WW and measure background.

Use (--) and (++) to control polarization (also use 150 pb Z-like events)



Experimentally very robust. Measure pol., bkg. in situ

ILC Polarized Scan Counting Experiment

Example: 6 point scan (index i), (90% e-, 60% e+ polarization) with -+, +-, ++ and - - helicity combinations (index k)

Count events in 3 WW candidate categories (lvlv, qqlv, qqqq – index j) with expectation μ_{ijk} and one Z-like category (radiative return and f fbar) with expectation ν_{ik}.

96 event counts

Data could also be taken with other helicity combinations (00, -0,+0,0-,0+) if warranted. (eg. further checks of polarization model)

$\sqrt{s}$ (GeV)	L (fb^{-1})	f	$\lambda_{e^-}\lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	−+	2752	11279	12321	926968
		0.1704	+−	20	67	158	139932
		0.0254	++	2	19	27	6661
		0.0254	−−	21	100	102	8455
161.2	21.739	0.7789	−+	16096	67610	73538	4635245
		0.1704	+−	98	354	820	697141
		0.0254	++	37	134	130	33202
		0.0254	−−	145	574	622	42832
161.4	21.739	0.7789	−+	17334	72012	77991	4639495
		0.1704	+−	100	376	770	697459
		0.0254	++	28	104	133	33556
		0.0254	−−	135	553	661	42979
161.6	21.739	0.7789	−+	18364	76393	82169	4636591
		0.1704	+−	81	369	803	697851
		0.0254	++	43	135	174	33271
		0.0254	−−	146	618	681	42689
162.2	4.348	0.7789	−+	4159	17814	19145	927793
		0.1704	+−	16	62	173	138837
		0.0254	++	10	28	43	6633
		0.0254	−−	46	135	141	8463
170.0	26.087	0.7789	−+	63621	264869	270577	5560286
		0.1704	+−	244	957	1447	838233
		0.0254	++	106	451	466	40196
		0.0254	−−	508	2215	2282	50979

Table 7: Illustrative example of the numbers of events in each channel for the standard 100 fb^{-1} 6-point ILC scan with 4 helicity combinations.

Fit the Event Counts to Model Expectations

$$x \equiv |P(\mathrm{e}^-)|, \;\; y \equiv |P(\mathrm{e}^+)|$$

Event count expectations:

$$\mu_{ijk} = \left(f_S^k(x,y,A_{LR}^{WW})\,\sigma_i(m_W,\alpha_S)\,\varepsilon_j B_j + g_B^k(x,y,A_{LR}^B)\,\sigma_B^j \right) f_l L_{ik}$$

$$\nu_{ik} = g_Z^k(x,y,A_{LR}^Z)\,\sigma_Z^i\, f_l L_{ik}$$

Signal, background, and Z-control sample spin factors:

$$f_S^{-+}(x,y,A) = 1 + xy + A(x+y)$$
$$f_S^{+-}(x,y,A) = 1 + xy - A(x+y)$$
$$f_S^{++}(x,y,A) = 1 - xy - A(x-y)$$
$$f_S^{--}(x,y,A) = 1 - xy + A(x-y)$$

$$g_{B,Z}^{-+}(x,y,A) = 1 + xy + A(x+y)$$
$$g_{B,Z}^{+-}(x,y,A) = 1 + xy - A(x+y)$$
$$g_{B,Z}^{++}(x,y,A) = 1 - xy - A(x-y)$$
$$g_{B,Z}^{--}(x,y,A) = 1 - xy + A(x-y)$$

Set A=0.99 for WW (estimate of 0.992 (Wopper), 0.988 (Racoon))

Results from updated ILC study (arXiv:1603.06016)

Fit essentially includes experimental systematics. Main one: **background** determination.

Fit parameter	Value	Error
m_W (GeV)	80.388	3.77×10^{-3}
f_l	1.0002	0.924×10^{-3}
ε (lvlv)	1.0004	0.969×10^{-3}
ε (qqlv)	0.99980	0.929×10^{-3}
ε (qqqq)	1.0000	0.942×10^{-3}
σ_B (lvlv) (fb)	10.28	0.92
σ_B (qqlv) (fb)	40.48	2.26
σ_B (qqqq) (fb)	196.37	3.62
A_{LR}^B (lvlv)	0.15637	0.0247
A_{LR}^B (qqlv)	0.29841	0.0119
A_{LR}^B (qqqq)	0.48012	4.72×10^{-3}
$\lvert P(e^-)\rvert$	0.89925	1.27×10^{-3}
$\lvert P(e^+)\rvert$	0.60077	9.41×10^{-4}
σ_Z (pb)	149.93	0.052
A_{LR}^Z	0.19062	2.89×10^{-4}

Example 6-point ILC scan with 100 fb^{-1}

Note 125 inv fb/yr now feasible! (1908.08212, Yokoya, Kubo, Okogi). 2-point scan estimates

$\lvert P(e^-)\rvert$	$\lvert P(e^+)\rvert$	100 fb^{-1}	500 fb^{-1}
80 %	30 %	6.02	**2.88**
90 %	30 %	5.24	2.60
80 %	60 %	4.05	2.21
90 %	60 %	3.77	2.12

Total m_{W} experimental uncertainty (MeV)

High $\lvert P(e^+)\rvert$ very helpful!

$$\Delta m_{\mathrm{W}}(\mathrm{MeV}) = 2.4\,(\mathrm{stat}) \oplus 3.1\,(\mathrm{syst}) \oplus 0.4\,(\sqrt{\mathrm{s}}) \oplus \mathrm{theory}$$

($\sqrt{s}$ uncertainty revised to 5 ppm given recent developments)

Other Methods

Fully hadronic channel has huge statistical power, but thought plagued by color reconnection (CR) systematics.

Christiansen and Sjöstrand (arXiv:1506.09085) show that CR effects could be diagnosed using W mass measurements at various $\sqrt{s}$.

Table 2 Systematic W mass shifts at center-of-mass energies of 240 and 350 GeV, respectively. The $\langle \delta \overline{m}_W \rangle$ is the mass shift in the CR models relative to the no-CR result. The Monte Carlo statistical uncertainty is 5 MeV

Method	$\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm} = 240$ GeV)						
	SK-I	SK-II	SK-II′	GM-I	GM-II	GM-III	CS
1	+95	+29	+25	−74	+400	+104	+9
2	+87	+26	+24	−68	+369	+93	+8
3	+95	+30	+26	−72	+402	+105	+10
Method	$\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm} = 350$ GeV)						
	SK-I	SK-II	SK-II′	GM-I	GM-II	GM-III	CS
1	+72	+18	+16	−50	+369	+60	+4
2	+70	+18	+15	−50	+369	+60	+4
3	+71	+18	+16	−50	+369	+60	+3

But this is not really at all well established.

Note that jet reconstruction in the 4q channel normally tries to reduce the potential size of such effects

Updated Prospects Tables

1: Polarized threshold scan

ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	161	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480	500
$P(e^-)$ [%]	0	90	90	80
$P(e^+)$ [%]	0	60	60	30
statistics	200	2.4	1.1	
background		2.0	0.9	
efficiency		1.2	0.9	
luminosity		1.8	1.2	
polarization		0.9	0.4	
systematics	70	3.0	1.6	
experimental total	210	3.9	1.9	3.0
beam energy	13	0.4	0.4	0.4
theory	-	1.0	1.0	1.0
total	210	4.0	2.2	3.2

Table 10: Current and preliminary anticipated uncertainties in the measurement of M_W at e^+e^- colliders close to WW threshold.

- Changes wrt Snowmass 2013
- Update with current ILC run plan integrated luminosities
- Halve beam energy uncertainty (10 ppm $\rightarrow$ 5 ppm)
- Include guessed theory uncertainty in threshold total

2: $q\bar{q}\ell\nu_\ell$

ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	172-209	250	350	500
$\mathcal{L}$ [fb^{-1}]	3.0	2000	200	4000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
beam energy	9	0.4	0.55	0.8
luminosity spectrum	N/A	1.0	1.4	2.0
hadronization	13	1.3	1.3	1.3
radiative corrections	8	1.2	1.5	1.8
detector effects	10	1.0	1.0	1.0
other systematics	3	0.3	0.3	0.3
total systematics	21	2.3	2.7	3.3
statistical	30	0.75	2.8	0.9
total	36	2.4	3.9	3.4

Table 6: Current and preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from kinematic reconstruction in the $q\bar{q}\ell\nu_\ell$ channel with $\ell = e, \mu$.

3: Hadronic mass

ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	2000	200	4000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	0.75	2.0	0.5	0.5
total	3.5	4.0	3.5	3.9

Table 8: Preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from direct reconstruction of the hadronic mass in single-W and WW events where one W decays hadronically. Does not include WW with $q\bar{q}\ell\nu_\ell$ where $\ell = e, \mu$.

Summary

- ILC can advance our knowledge of electroweak precision physics
- Several methods to measure the W mass with precisions in the few MeV range. Systematics are to some extent complementary. Estimate overall experimental uncertainty of 2.0 MeV. This could be reduced further to about 1.5 MeV combined with dedicated 500 fb^{-1} run at threshold.
- Scope for complementary m_{W} measurements with similar precision from standard ILC running.
- Fully leptonic events statistical estimate is 5.9 MeV.
- Constrained reconstruction - very promising - but needs more detailed study.
- Experimental strategies for controlling systematics associated with $\sqrt{s}$, polarization, luminosity spectrum are worked out.
- Momentum scale is a key. Enabled by precision low material tracker. Can also open up a measurement of m_{Z}.
- An accelerator is needed. Let's make this happen!

Backup Slides

Recent studies related to $\sqrt{s}_p$ method

- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

Recently,

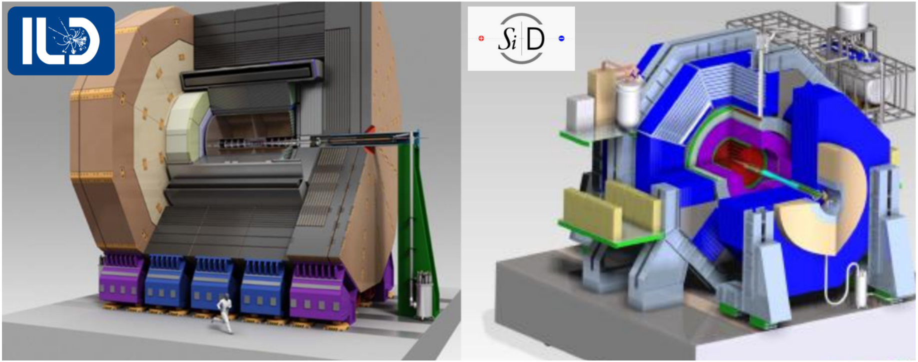
- Several talks on $\sqrt{s}_p$ and $\sqrt{s}$ issues. Latest ones, ILCX, ILC-WG3 and ILC-MDI
- Includes a more careful look at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$. Include crossing angle, full simulation and reconstruction with ILD, track error matrices, vertex fitting, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- Also a look at colliding beam-energy/interaction-vertex correlations and more of a focus on $\mathrm{dL}/\mathrm{d}\sqrt{s}$ issues.
- Prospects for Z lineshape with a polarized scan including energy systematics.

ILC Detectors

Modern detectors designed for ILC [5]

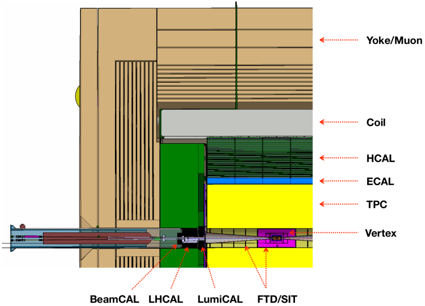
ILD = International Large Detector
(also ILD Interim Design Report (IDR) [6])

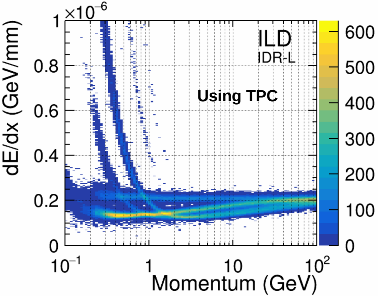
SiD = Silicon Detector



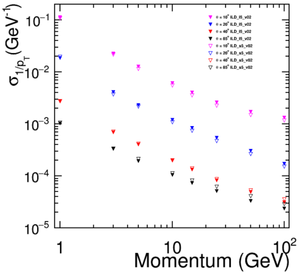
- B=3.5–5T. Particle-flow for hadronic jets. Very hermetic.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

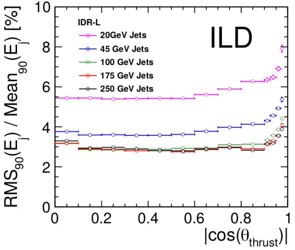
ILD Detector (See IDR)

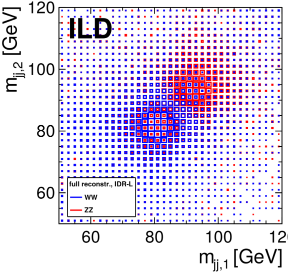




Momentum Resolution







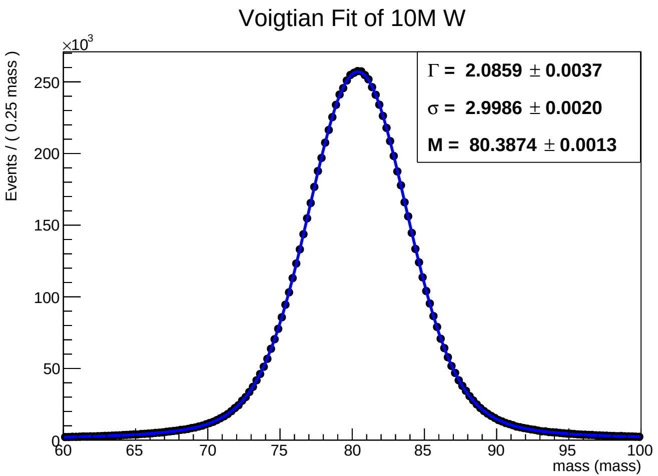
Fits to W Lineshape (M, Γ, σ_M)

Higgs factory machines like ILC likely systematics dominated for m_{W} and Γ_{W}. Statistical uncertainties for m_{W} and Γ_W for 10^7 W bosons.

σ_M (GeV)	Δm_{W} (MeV)	$\Delta\Gamma_W^a$ (MeV)	$\Delta\Gamma_W^b$ (MeV)
1.0	0.67	1.3	2.0
2.0	0.98	1.7	2.7
2.5	1.1	2.0	3.2
3.0	1.3	2.3	3.7
4.0	1.6	2.8	5.0

Estimated from a simple parametric fit of the Breit-Wigner lineshape convolved with a range of constant Gaussian experimental mass resolutions, σ_M. The m_{W} uncertainty is evaluated with a one parameter fit with the width and mass resolution fixed. The corresponding uncertainties on the Γ_W width are evaluated either with the mass resolution fixed and known perfectly from a 2-parameter fit (Γ_W^a), or more realistically, from a 3-parameter fit (Γ_W^b) that also fits for the mass resolution.

Toy MC Example. (Has $\chi^2/\text{ndf} = 152/157.$)



I had wrongly assumed that one needed to know σ very well to extract Γ, but this is not the case. Of course with no constraint on σ, the uncertainty on Γ is larger. In reality, σ varies from W to W. So for a similar approach to work, one needs well understood event by event errors. Use by categorizing events with varying quality levels.

Kinematic Reconstruction in Fully Leptonic Events

See Appendix B of Hagiwara et al., Nucl. Phys. B. 282 (1987) 253 for full production and decay 5-angle reconstruction in fully leptonic events ($\ell \nu_\ell \ell' \bar{\nu}_{\ell'}$) without taus as motivated by TGC analyses.

The technique applies energy and momentum conservation. One solves for the anti-neutrino 3-momentum, decomposed into its components in the dilepton plane, and out of it. Additional assumptions are:

- the energies of the two W's are equal to E_{b}, so $\mathrm{m}(\mathrm{W}^+) = \mathrm{m}(\mathrm{W}^-)$.
- a specified value for m_{W}

$$\vec{p}_{\bar{\nu}} = a\ \vec{p}_\ell + b\ \vec{p}_{\ell'} + c\ \vec{p}_\ell \times \vec{p}_{\ell'}$$

By specifying, m_{W}, one can find a, b and c^2, so there are two solutions.

The alternative pseudomass technique, does not assume m_{W}, but sets $c = 0$, and similarly has two solutions $(a_+,\ b_+)$ and $(a_-,\ b_-)$.

Hadronization Systematics

How does a W, Z, H, t decay hadronically?

Models like PYTHIA, HERWIG etc have been tuned extensively to data. Not expected to be a complete picture.

Inclusive measurements of **identified particle rates** and **momenta spectra** are an essential ingredient to describing hadronic decays of massive particles.

ILC could provide comprehensive measurements with up to 1000 times the published LEP statistics and with a much better detector with Z running.

High statistics with W events.

Why?

Measurements based on hadronic decays, such as hadronic mass, jet directions underlie much of what we do in energy frontier experiments.

Key component of understanding jet energy scales and resolution.

Important to also understand flavor dependence: u-jets, d-jets, s-jets, c-jets, b-jets, g-jets.

Momentum Scale Calibration (essential for $\sqrt{s}$)

Most obvious: use $J/\psi \to \mu^+\mu^-$. Event rate limited unless sizeable Z running.

Particle	$n_{Z\mathrm{had}}$	Decay	BR (%)	$n_{Z\mathrm{had}} \cdot$ BR	Γ/M	PDG ($\Delta M/M$)
J/ψ	0.0052	$\mu^+\mu^-$	5.93	0.00031	3.0×10^{-5}	1.9×10^{-6}
$\mathrm{K^0_S}$	1.02	$\pi^+\pi^-$	69.2	0.71	1.5×10^{-14}	2.6×10^{-5}
Λ	0.39	$\pi^-\mathrm{p}$	63.9	0.25	2.2×10^{-15}	5.4×10^{-6}
$\mathrm{D^0}$	0.45	$\mathrm{K}^-\pi^+$	3.88	0.0175	8.6×10^{-13}	2.7×10^{-5}
$\mathrm{K^+}$	2.05	various	-	-	1.1×10^{-16}	3.2×10^{-5}
π^+	17.0	$\mu^+\nu_\mu$	100	-	1.8×10^{-16}	2.5×10^{-6}

Candidate particles for momentum scale calibration and abundances in Z decay

Sensitivity of mass-measurement to p-scale (α) depends on daughter masses and decay

$$m_{12}^2 = m_1^2 + m_2^2 + 2p_1p_2\left[(\beta_1\beta_2)^{-1} - \cos\psi_{12}\right]$$

Particle	Decay	$<\alpha>$	max α	σ_M/M	$\Delta p/p$ (10 MZ)	$\Delta p/p$ (GZ)	PDG limit
J/ψ	$\mu^+\mu^-$	0.99	0.995	7.4×10^{-4}	13 ppm	1.3 ppm	1.9 ppm
$\mathrm{K^0_S}$	$\pi^+\pi^-$	0.55	0.685	1.7×10^{-3}	1.2 ppm	0.12 ppm	38 ppm
Λ	$\pi^-\mathrm{p}$	0.044	0.067	2.6×10^{-4}	3.7 ppm	0.37 ppm	80 ppm
$\mathrm{D^0}$	$\mathrm{K}^-\pi^+$	0.77	0.885	7.6×10^{-4}	2.4 ppm	0.24 ppm	30 ppm

Estimated momentum scale statistical errors ($p = 20$ GeV)

Use of J/ψ would decouple $\sqrt{s}$ determination from m_Z knowledge.

Opens up possibility of improved m_Z measurements.

Full Simulation + Kalman Filter

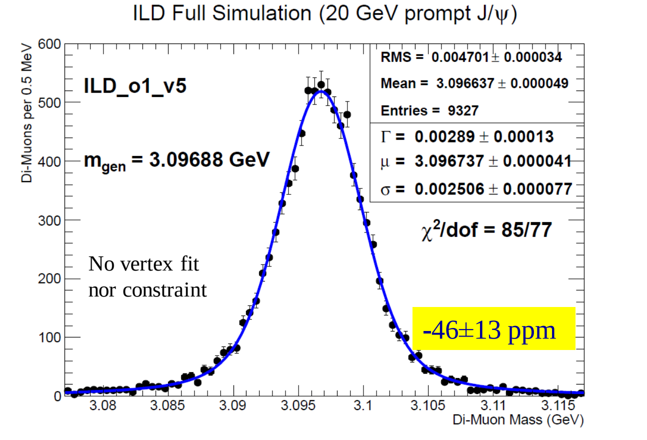
$\sqrt{s}=m_Z$

10k "single particle events"

Work in progress – likely need to pay attention to issues like energy loss model and FSR.

Preliminary statistical precision similar.

More realistic material, energy loss and multiple scattering.



Empirical Voigtian fit.

Need consistent material model in simulation AND reconstruction

m_W Prospects

1. Polarized Threshold Scan
2. Kinematic Reconstruction
3. Hadronic Mass

Method 1: Statistics limited.

Method 2: With up to 1000 the LEP statistics and much better detectors. Can target factor of 10 reduction in systematics.

Method 3: Depends on di-jet mass scale. Plenty Z's for 3 MeV.

1

ΔM_W [MeV]	LEP2	ILC	ILC
$\sqrt{s}$ [GeV]	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480
$P(e^-)$ [%]	0	90	90
$P(e^+)$ [%]	0	60	60
statistics	200	2.4	1.1
background		2.0	0.9
efficiency		1.2	0.9
luminosity		1.8	1.2
polarization		0.9	0.4
systematics	70	3.0	1.6
experimental total	210	3.9	1.9
beam energy	13	0.8	0.8
theory	-	(1.0)	(1.0)
total	210	4.0	2.1

2

ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	172-209	250	350	500
$\mathcal{L}$ [fb^{-1}]	3.0	500	350	1000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
beam energy	9	0.8	1.1	1.6
luminosity spectrum	N/A	1.0	1.4	2.0
hadronization	13	1.3	1.3	1.3
radiative corrections	8	1.2	1.5	1.8
detector effects	10	1.0	1.0	1.0
other systematics	3	0.3	0.3	0.3
total systematics	21	2.4	2.9	3.5
statistical	30	1.5	2.1	1.8
total	36	2.8	3.6	3.9

3

ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

See Snowmass document for more details

Bottom-line: 3 different methods with prospects to measure mW with error < 5 MeV

OPAL m_W Systematics

Source	$q\bar{q}\ell\nu$ CV	$q\bar{q}\ell\nu$ RW	$q\bar{q}\ell\nu$ BW	$q\bar{q}q\bar{q}$ $p_{2.5}$ CV	$q\bar{q}q\bar{q}$ $p_{2.5}$ RW	$q\bar{q}q\bar{q}$ $p_{2.5}$ BW	$q\bar{q}q\bar{q}$ J_0 CV	$q\bar{q}q\bar{q}$ $\kappa_{-0.5}$ CV	Comb. CV
Jet energy scale	7	1	2	4	4	4	5	4	6
Jet energy resolution	1	1	1	0	1	3	1	0	0
Jet energy linearity	9	9	12	2	2	4	2	1	6
Jet angular resolution	0	0	0	0	0	0	0	0	0
Jet angular bias	4	4	4	7	7	6	6	7	5
Jet mass scale	10	7	6	5	11	3	5	5	8
Electron energy scale	9	6	8	-	-	-	-	-	6
Electron energy resolution	2	2	6	-	-	-	-	-	1
Electron energy linearity	1	1	2	-	-	-	-	-	1
Electron angular resolution	0	0	0	-	-	-	-	-	0
Muon energy scale	8	7	7	-	-	-	-	-	6
Muon energy resolution	2	2	3	-	-	-	-	-	1
Muon energy linearity	2	2	2	-	-	-	-	-	1
Muon angular resolution	0	0	0	-	-	-	-	-	0
WW event hadronisation	14	8	16	20	26	18	6	19	16
Colour reconnection	-	-	-	41	41	32	125	228	14
Bose-Einstein correlations	-	-	-	19	18	21	35	64	6
Photon radiation	11	11	10	9	8	8	9	9	10
Background hadronisation	2	1	2	20	12	32	17	24	8
Background rates	1	0	5	6	2	7	4	7	3
LEP beam energy	8	9	9	10	11	10	10	10	9
Modelling discrepancies	4	0	0	15	0	0	10	11	8
Monte Carlo statistics	2	3	3	2	3	3	2	2	2
Total systematic error	28	22	29	58	56	56	133	240	32
Statistical error	56	58	64	60	64	73	51	73	42
Total error	63	62	70	83	85	92	142	251	53

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