Estimate of TPC distortions at tera-Z

- Methods and Toy MC Results -

2022/11/24 Keisuke Fujii @ LCTPC WP Mtg

Poisson's equation

The E field in a region (D) is the sum of the E field (E0) without space charge in the corresponding region defined by the field shaping strips and the two terminating plates and the field (Eion) calculated with space charge in the virtual grounded conducting boundary of D.

$$\triangle\phi_0(\boldsymbol{x}) = 0$$

$$\triangle\phi_{\text{ion}}(\boldsymbol{x}) = -4\pi\,\rho_{\text{ion}}(\boldsymbol{x})$$

in $\boldsymbol{x} \in D$

Boundary Conditions

$$\phi_0(\boldsymbol{x}) = V_i \qquad \boldsymbol{x} \in C_i$$

$$\phi(\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \phi_{\text{ion}}(\boldsymbol{x})$$

$$\longrightarrow \quad \boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_{\text{ion}} = \boldsymbol{E}_0 - \nabla\phi_{\text{ion}}(\boldsymbol{x})$$

$$\phi_{\text{ion}}(\boldsymbol{x}) = 0 \qquad \boldsymbol{x} \in \partial D$$

All we need is Green's function for

$$\triangle G(\boldsymbol{x},\boldsymbol{x}') = -4\pi\delta(\boldsymbol{x}-\boldsymbol{x}')$$

$$G(\boldsymbol{x},\boldsymbol{x}') = 0 \qquad \boldsymbol{x} \in \partial D$$

E-field distortion is then given by superposition:

$$\phi_{\text{ion}}(\boldsymbol{x}) = \int_D d^3\boldsymbol{x}\, G(\boldsymbol{x},\boldsymbol{x}')\,\rho_{\text{ion}}(\boldsymbol{x}')$$

Superposition makes life easy!

Green's function

Since the boundaries are most naturally expressed in the cylindrical coordinates (rin=a, rout=b, z=0, Z=L), the corresponding Green function is most conveniently expanded in terms of modified Bessel function as follows:

$$G(r,\varphi,z;r',\varphi',z') = \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} g_{mn}(r,r')\frac{1}{2\pi}e^{im(\varphi-\varphi')}\frac{2}{L}\sin(\beta_n z)\sin(\beta_n z')$$

where

$$g_{mn}(r,r') = \frac{4\pi\,[K_m(\beta_n a)I_m(\beta r_<) - I_m(\beta_n a)K_m(\beta_n r_<)]\,[K_m(\beta_n b)I_m(\beta r_>) - I_m(\beta_n b)K_m(\beta_n r_>)]}{\beta_n r'\,[I_m(\beta_n a)K_m(\beta_n b) - I_m(\beta_n b)K_m(\beta_n a)]\,[K_m(\beta_n r')I'_m(\beta_n r') - K'_m(\beta_n r')I_m(\beta_n r')]}$$

$$\beta_n = n\pi/L \qquad r_< := \min(r,r'), \quad r_> := \max(r,r')$$

If the charge distribution is uniform in phi, the phi-integral is trivial and we get

$$\phi_{\rm ion}(r,z) = \sum_{n=1}^{\infty}\frac{8\pi}{\beta_n}\int_a^b dr'\,\frac{[K_0(\beta_n a)I_0(\beta r_<) - I_0(\beta_n a)K_0(\beta_n r_<)]\,[K_0(\beta_n b)I_0(\beta r_>) - I_0(\beta_n b)K_0(\beta_n r_>)]}{[I_0(\beta_n a)K_0(\beta_n b) - I_0(\beta_n b)K_0(\beta_n a)]\,[K_0(\beta_n r')I'_0(\beta_n r') - K'_0(\beta_n r')I_0(\beta_n r')]}$$
$$\sin(\beta_n z)\int_0^L \frac{dz'}{L}\,\sin(\beta_n z')\boxed{\rho_{\rm ion}(r',z')}$$

← *no ϕ-dependence*

Derivatives of the modified Bessel functions can be rewritten in terms of those of different orders:

$$I'_0(x) = I_1(x) \quad \text{and} \quad K'_0(x) = -K_1(x)$$

Using these and differentiating $\phi_{ion}(r,z)$ with respect to r we get the following for Er:

$$E_r(r,z) = -8\pi \sum_{n=1}^{\infty} \frac{\sin(\beta_n z)}{I_0(\beta_n a)K_0(\beta_n b) - I_0(\beta_n b)K_0(\beta_n a)}$$

$$\Bigg[[K_0(\beta_n b)I_1(\beta r) + I_0(\beta_n b)K_1(\beta_n r)] \int_a^r dr' \, \frac{K_0(\beta_n a)I_0(\beta r') - I_0(\beta_n a)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')} \int_0^L \frac{dz'}{L} \, \sin(\beta_n z')\rho_{ion}(r',z')$$

$$+ [K_0(\beta_n a)I_1(\beta r) + I_0(\beta_n a)K_1(\beta_n r)] \int_r^b dr' \, \frac{K_0(\beta_n b)I_0(\beta r') - I_0(\beta_n b)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')} \int_0^L \frac{dz'}{L} \, \sin(\beta_n z')\rho_{ion}(r',z') \Bigg]$$

z'-integral now inside r'-integral

where

$$\beta_n = n\pi/L$$

In the practical calculations, we have to sum up the series up to **high enough "n"**, which is determined by the ratio of the shortest and the longest scales that specify the charge distribution and the geometry of the boundary of the region in question.

For a thin disk or in the MPGD-gate gap, summation up to 500 or more is necessary, which in turn requires quadruple precision calculations for the modified Bessel functions.

Principle (continued)

E0, if parallel with the B field, will not contribute to the ExB effect. (c.f.) the Langevin Equation:

$$\boldsymbol{\omega} := \frac{(-e)\boldsymbol{B}}{mc}$$

$$\omega\tau \simeq 10 \text{ for T2K gas at B=3.5T}$$

$$\langle \boldsymbol{v} \rangle = \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[1 + (\omega\tau)\hat{\boldsymbol{B}} \times +(\omega\tau)^2 \hat{\boldsymbol{B}}\,\hat{\boldsymbol{B}}\cdot \right] \frac{e}{m}\boldsymbol{E}$$

If we write down the distortion of the velocity due to the distortion of the E-field in the longitudinal and transverse directions, we get

$$\Delta \langle \boldsymbol{v} \rangle = \frac{e}{m} \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[(1+(\omega\tau)^2)\Delta\boldsymbol{E}_{\parallel} + \boldsymbol{E}_{\perp} - (\omega\tau)\boldsymbol{E}_{\perp} \times \hat{\boldsymbol{B}} \right]$$

Numerically integrating this over the drift time by noting $\delta l_i = \langle v_{\parallel} \rangle \delta t_i$, we get the following formula for the distortion:

$$\begin{aligned} \langle \Delta \boldsymbol{x} \rangle &= \sum_{i=1}^{n} \frac{\Delta \langle \boldsymbol{v} \rangle_i}{\langle v_{\parallel} \rangle_i} \delta l_i \\ &\simeq \sum_{i=1}^{n} \delta l_i \left[-\frac{\Delta \boldsymbol{E}_{\parallel i}}{E_0} - \left(\frac{1}{1+(\omega\tau)^2} \right) \frac{\boldsymbol{E}_{\perp i}}{E_0} + \left(\frac{\omega\tau}{1+(\omega\tau)^2} \right) \frac{\boldsymbol{E}_{\perp i} \times \hat{\boldsymbol{B}}}{E_0} \right] \end{aligned}$$

Key point: distortion is linear w.r.t. E-field distortion, and hence also w.r.t. space charge for a drift from the same z to the anode: Superposition makes life easy!

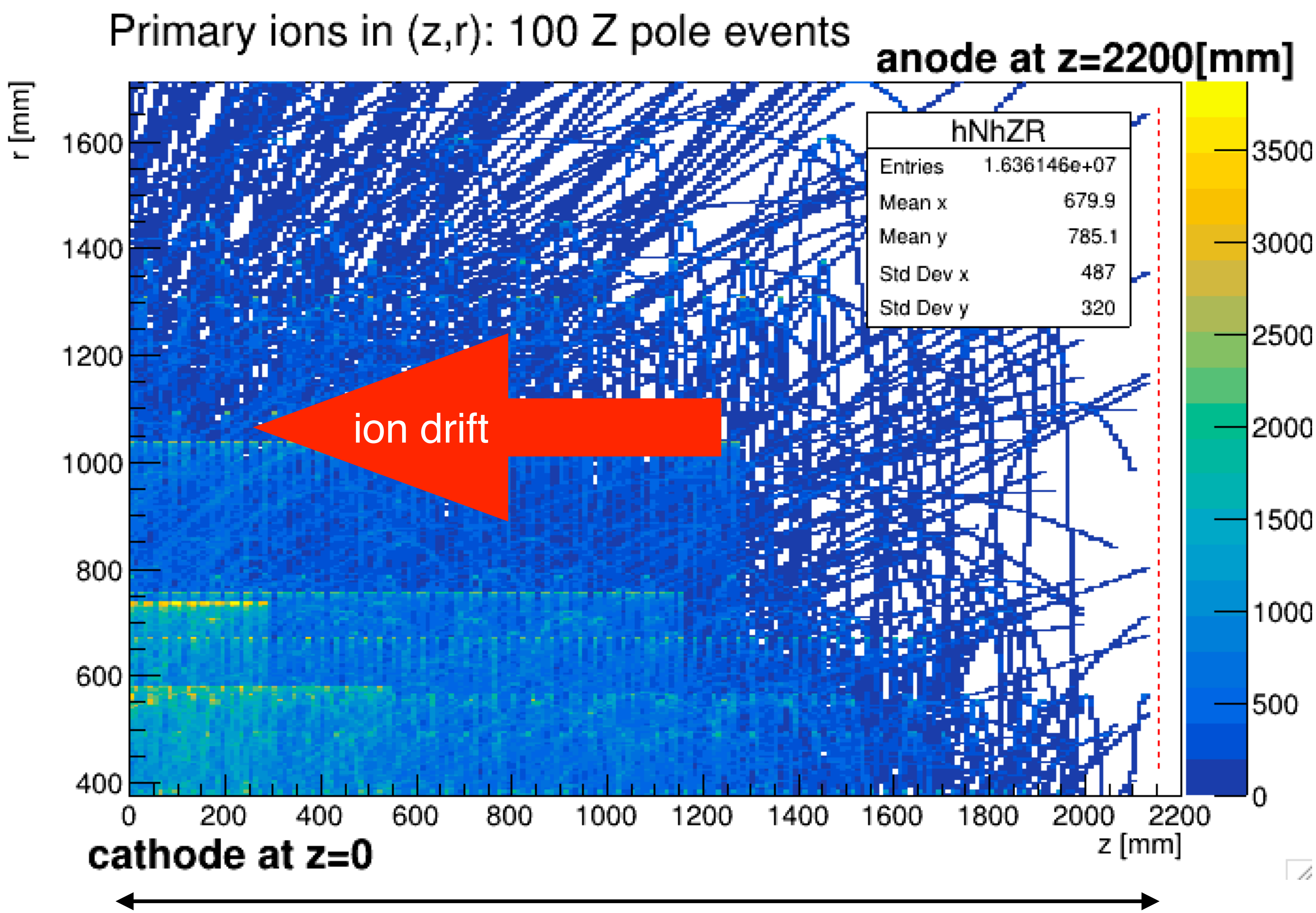
Primary Ions accumulated for 100 Z pole events in the 0.44 sec time frame

Toy MC
using Pythia8

no energy loss while curling, truncated after 200 turns

dE/dx simulated p-dependently for pure Ar (Alison-Cobb) w/o Landau fluctuation (U_{ion}=26[eV]) with ions distributed uniformly along each track.

100 events
in the time frame
in this example



Time frame width = len/v_{ion} = 2.2[m]/5[m/s] = 0.44[s]

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T], v_{ion} = 5[m/s]

Ions even if created at the farthest point from the cathode (.i.e. near the end plane) must have been absorbed by the cathode if they were created before this 0.44[s] time frame.

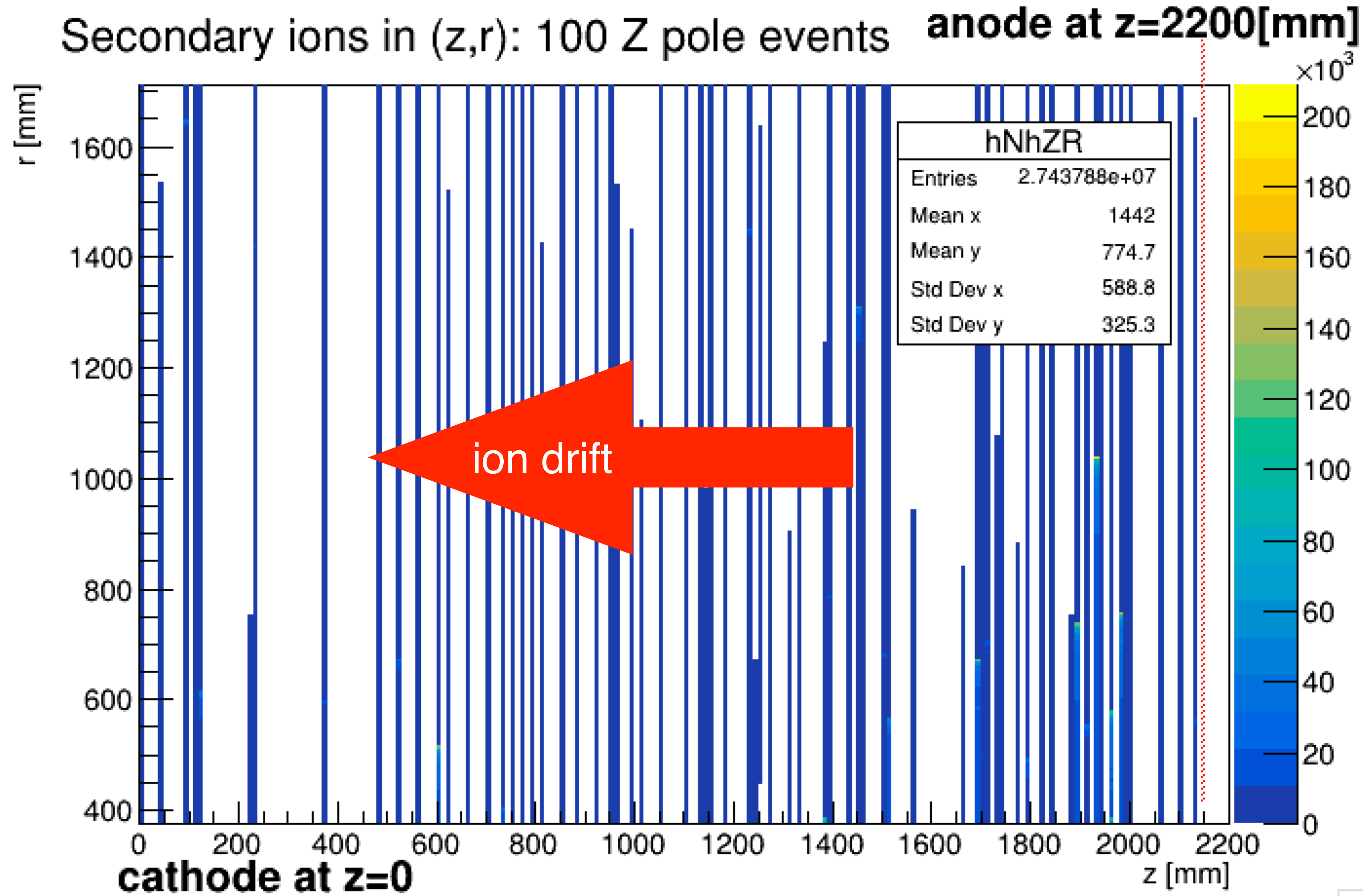
Secondary Ions flowed back from the anode accumulated for 100 Z pole events in the 0.44 sec time frame

Toy MC using Pythia8

no energy loss while curling, truncated after 200 turns.

dE/dx simulated p-dependently for pure Ar (Alison-Cobb) w/o Landau fluctuation (U_{ion}=26[eV]) with ions distributed uniformly along each track.

100 events in the time frame in this example



Time frame width = len/v_{ion} = 2.2[m]/5[m/s] = 0.44[s]

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T]
v_{ion} = 5[m/s]
v_{elec} = 75[mm/μs]

Secondary ions are quasi-continuously produced at the end plane within len/v_{elec} = 30[μs] after each event, forming an ion disk of the event image compressed in z-direction by a factor of v_{ion}/v_{elec}, flow back into the drift volume, and stay there for 0.44[s] until being absorbed by the cathode.

Ions accumulated for 22k Z pole events in the 0.44 sec time frame

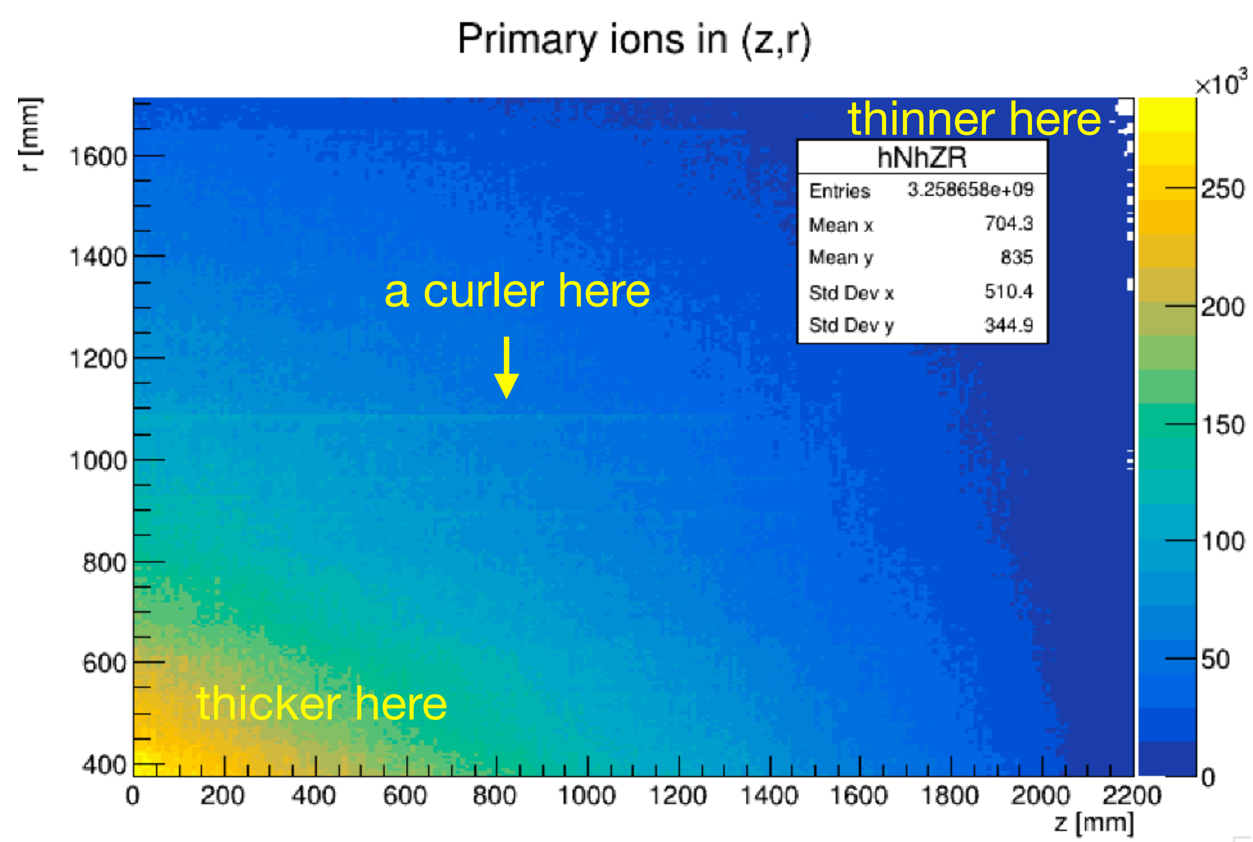
Z pole: **50 [kHz]**

Toy MC using Pythia8

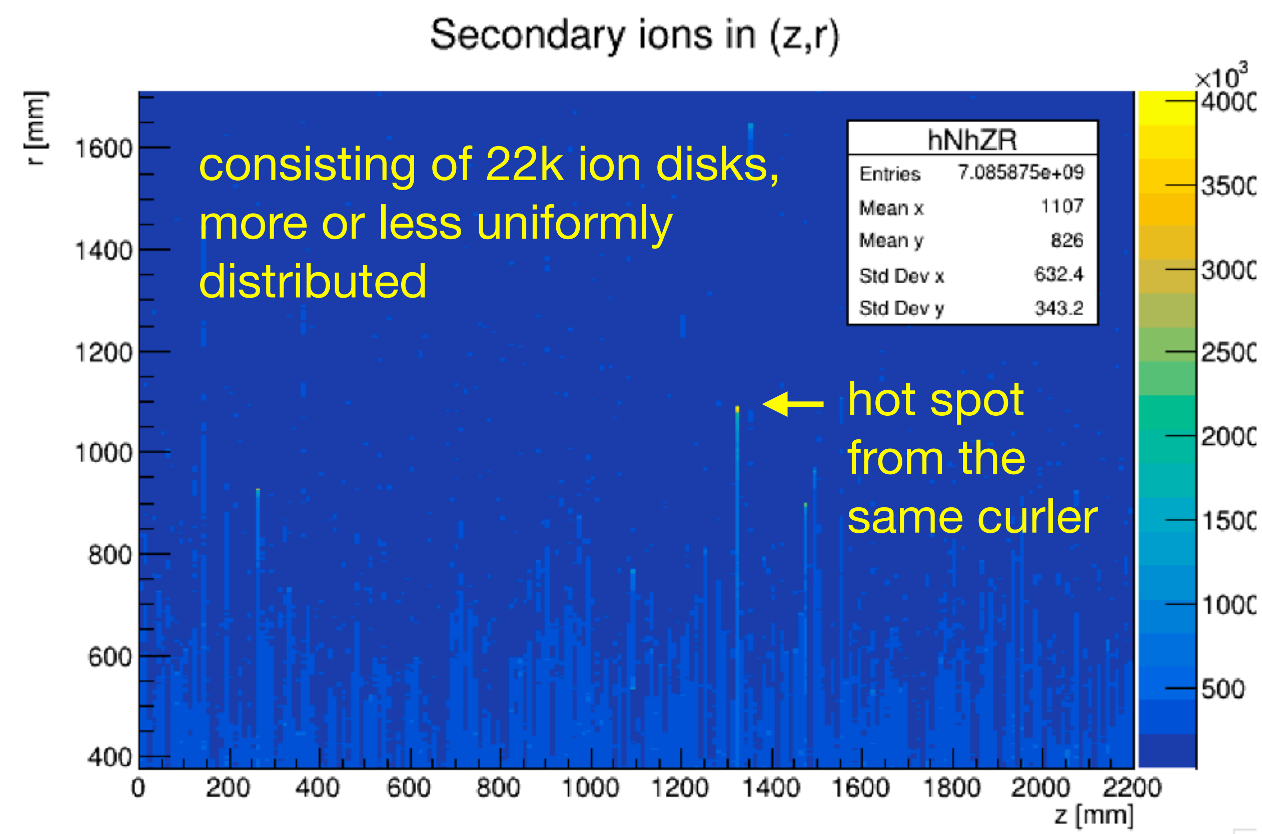
IBF=1

IBF:=# back flow ions / # seed electrons

Primary Ions



Ion Back Flow



cathode at z=0 anode at z=2.2[m] cathode at z=0 anode at z=2.2[m]

bin size: $(\Delta z, \Delta r)$=(1[cm], 0.5[cm])

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T], v_{ion} = 5[m/s]

Conversion from ZR hist. to $\rho_{ion}(r,z)$

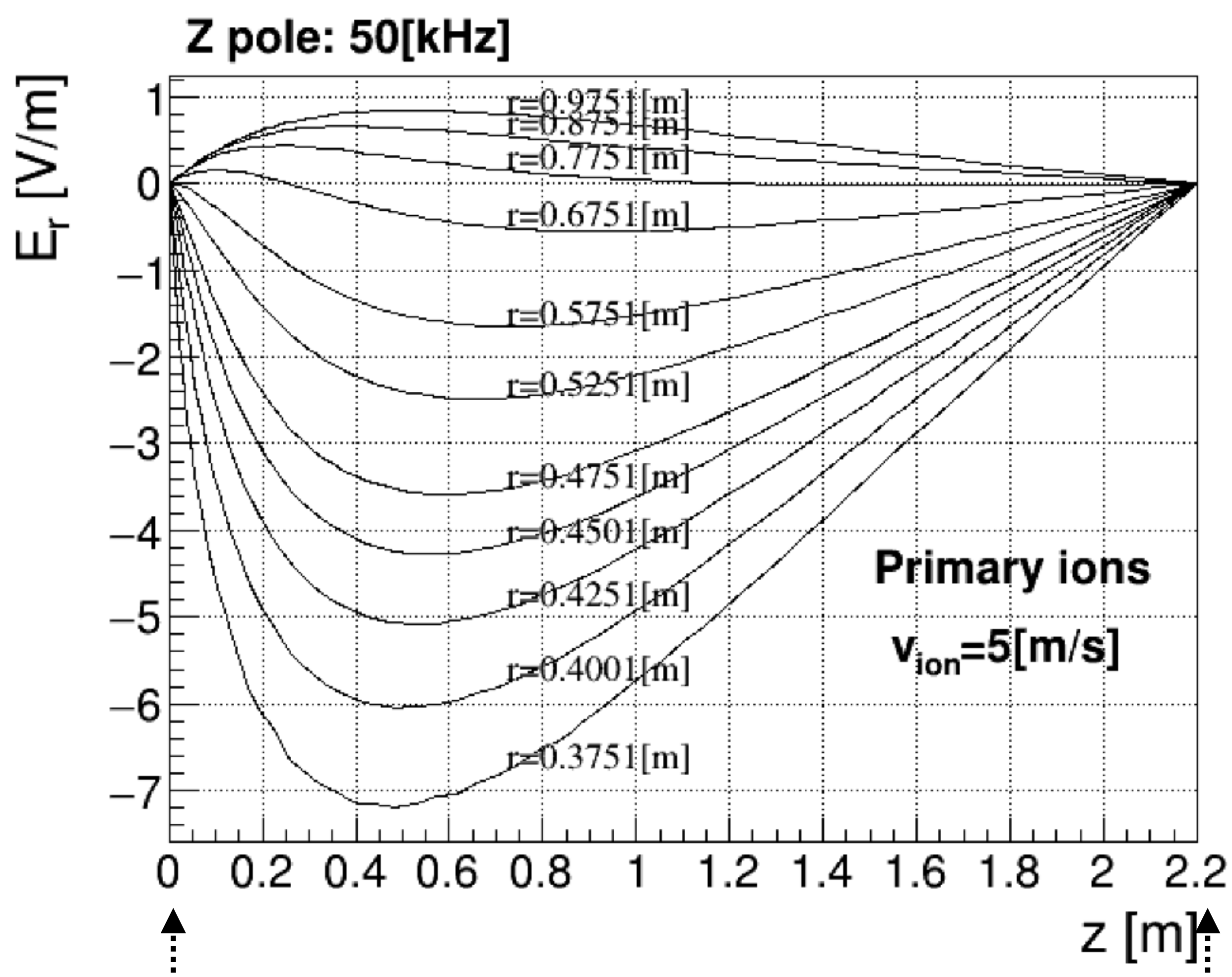
$$\rho_{ion}(r,z) \simeq e \cdot \mathrm{hNhZR}(z,r)/(2\pi r \Delta r \Delta z)$$

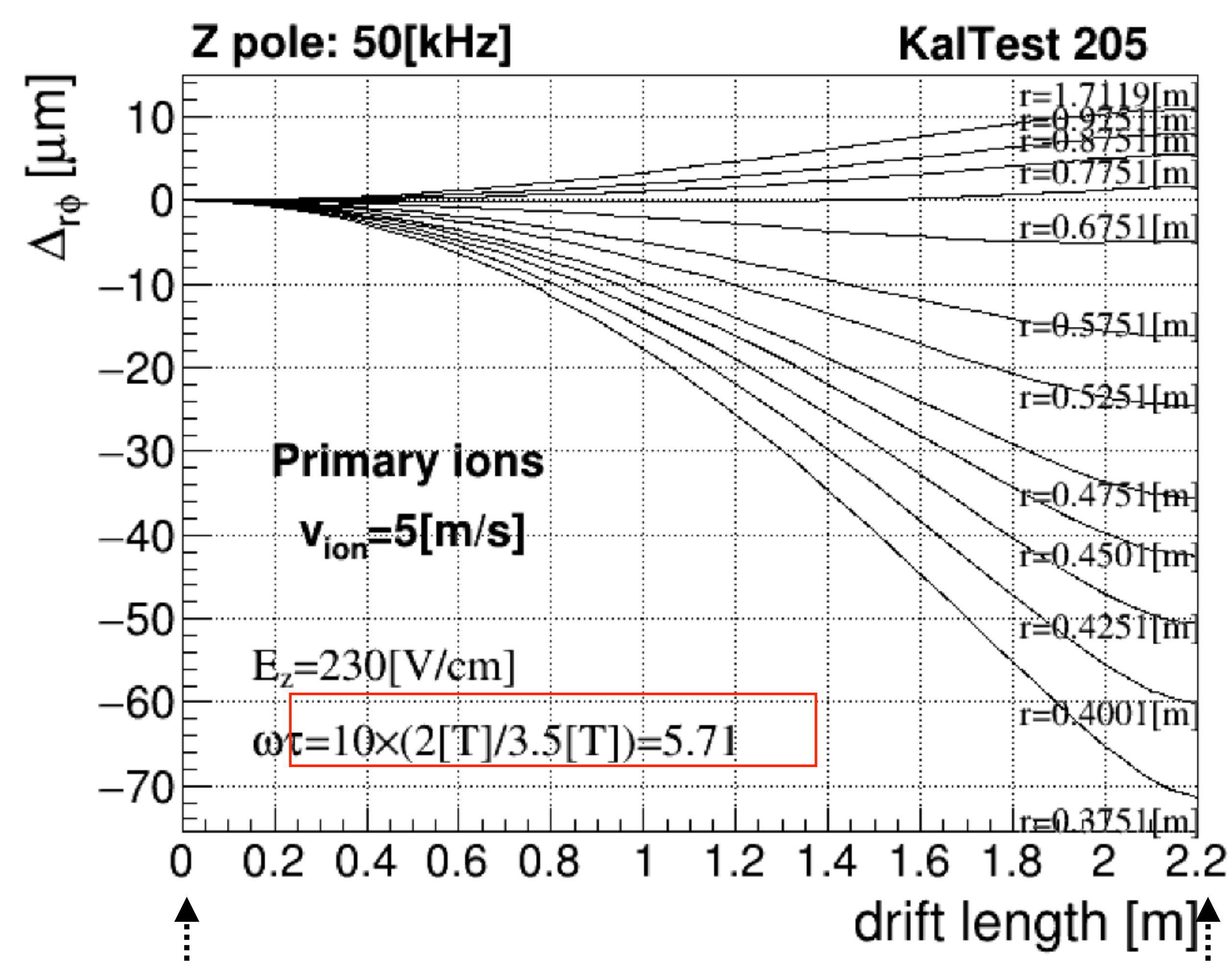
Note: ϕ-symmetry must be broken by curlers

Primary Ions (22k Z pole events)

Z pole run: hadronic Z event rate: **50 [kHz]** (toy MC using pythia8)

v_{ion} = 5 [m/s]





cathode at z=0 anode at z=2.2[m] anode cathode

bin size: (Δz, Δr)=(1[cm], 0.5[cm])

Note: E⊥ is constrained to be zero at anode and cathode (conductors).

Ez=230[V/cm] **ωτ=5.71** B=2[T]

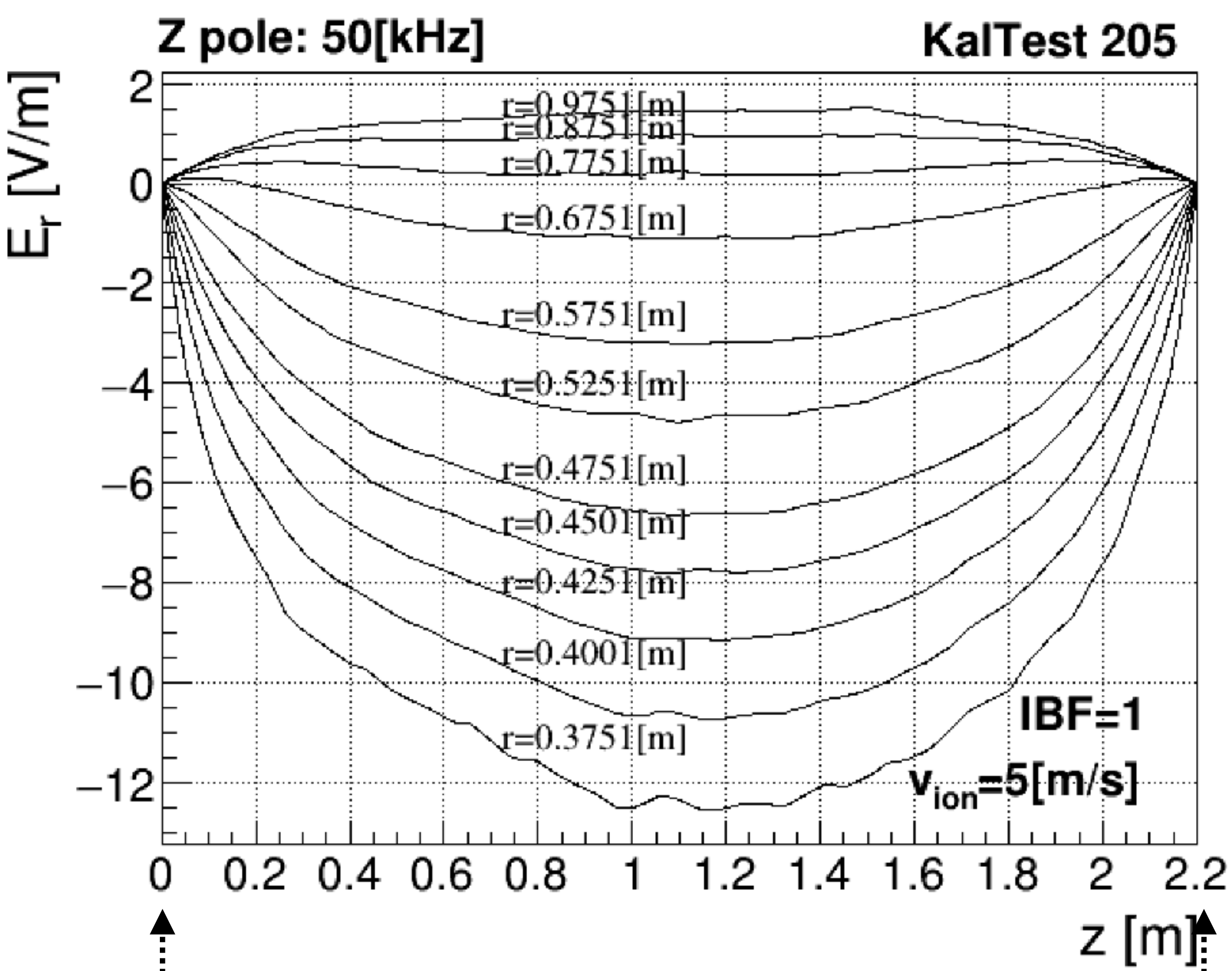
Maximum distortion ~70 [μm] at the innermost region for hadronic Z rate of 50 [kHz]

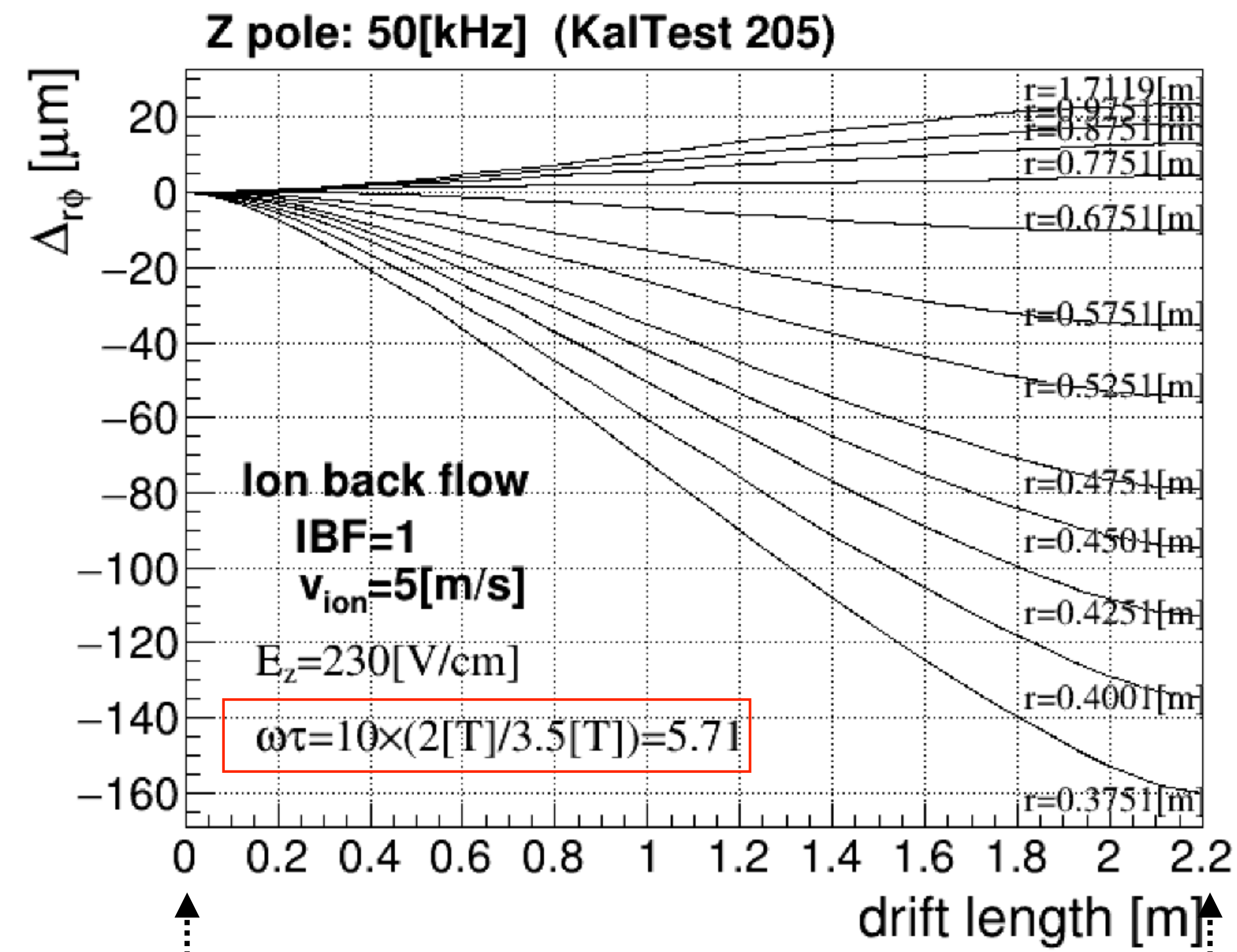
Positive Ion Back Flow (22k Z pole events)

Z pole run: hadronic Z event rate: **50 [kHz]** (toy MC using pythia8)

v_{ion} = 5 [m/s]

IBF = 1





cathode at z=0 anode at z=2.2[m] anode cathode

bin size: (Δz, Δr)=(1[cm], 0.5[cm])

Ez=230[V/cm] **$\omega\tau$=5.71** B=2[T]

Glitches correspond to hot spots in ρ_{ion}, which seem to be averaged out in Δrφ

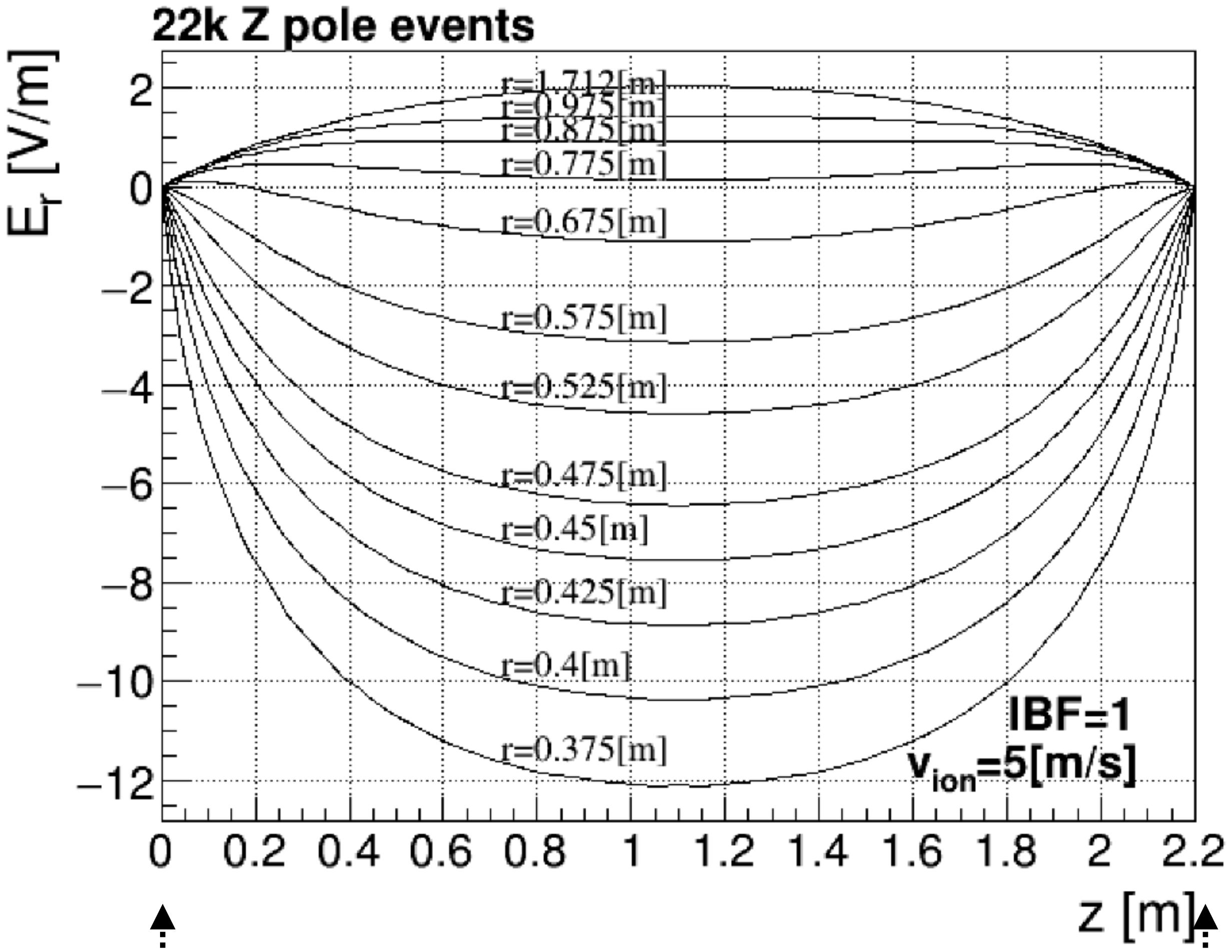
Maximum distortion ~160 [μm] at the innermost region for hadronic Z rate of 50 [kHz]

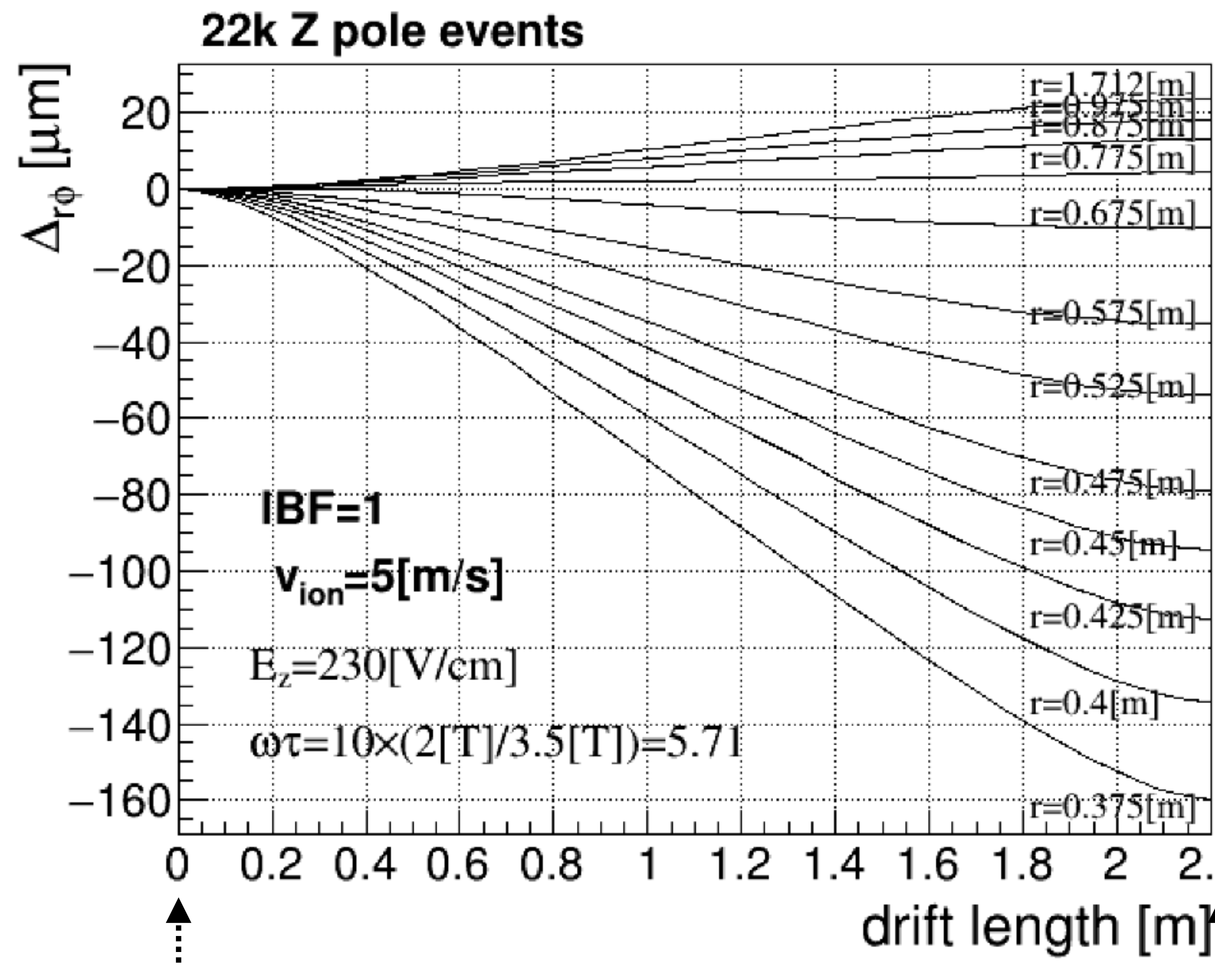
Positive Ion Back Flow (smoothed by proy)

Z pole run: hadronic Z event rate: 50 [kHz] (pythia8)

v_{ion} = 5 [m/s]

IBF = 1





cathode at z=0 anode at z=2.2[m] anode cathode

bin size: (Δz, Δr)=(1[cm], 0.5[cm])

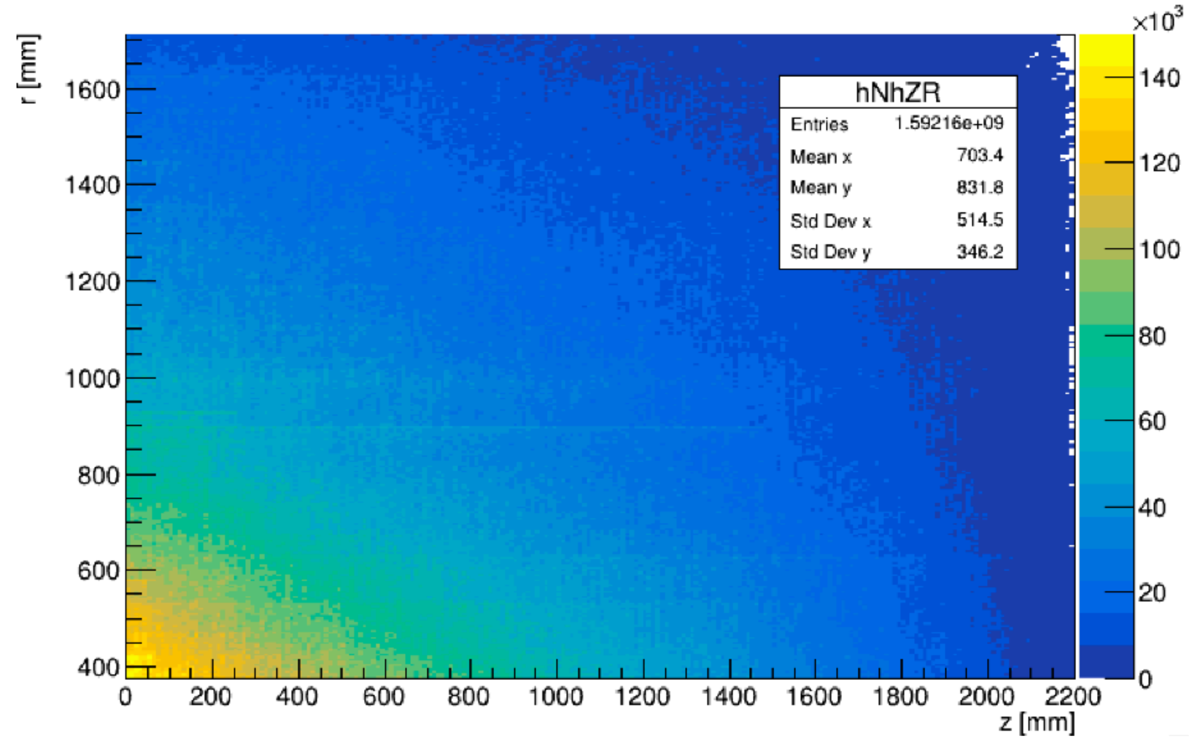
Ez=230[V/cm] **ωτ=5.71** B=2[T]

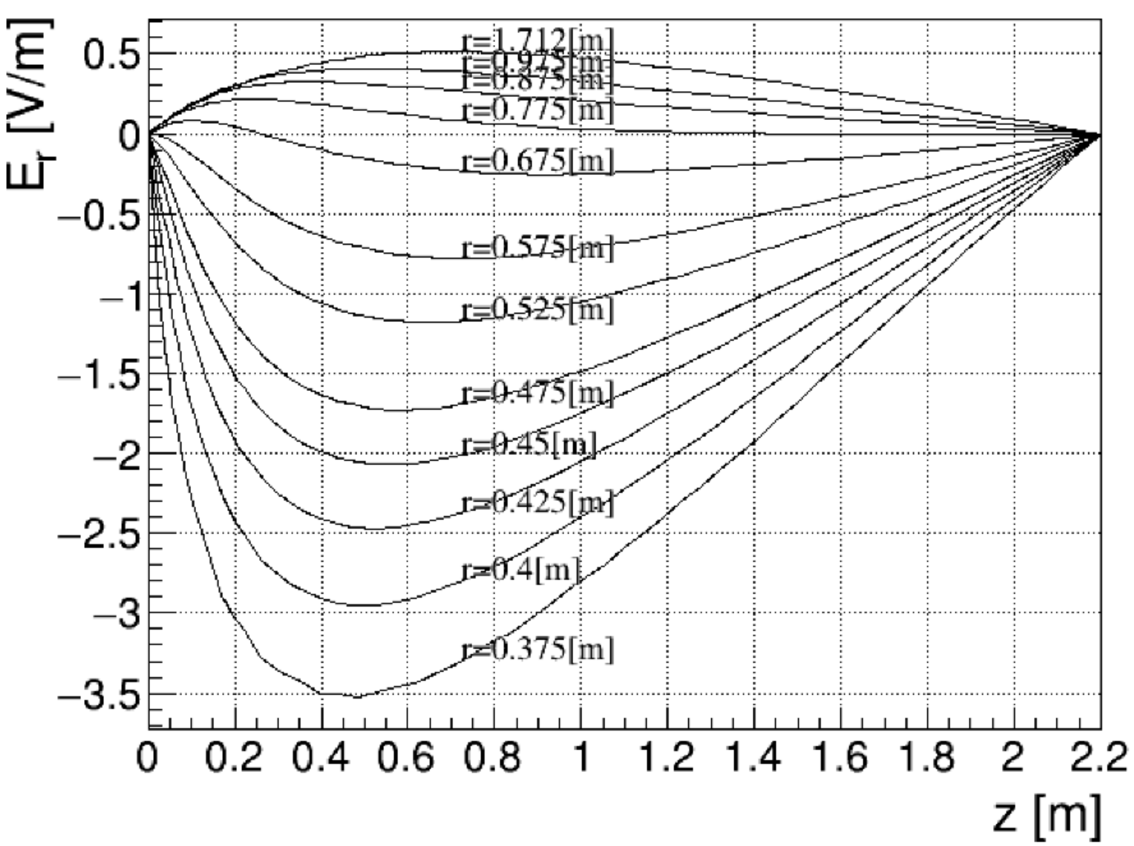
Glitches smoothed as expected

No visible difference in Δrφ!

What happens if the event rate is halved? (11k Z pole events)

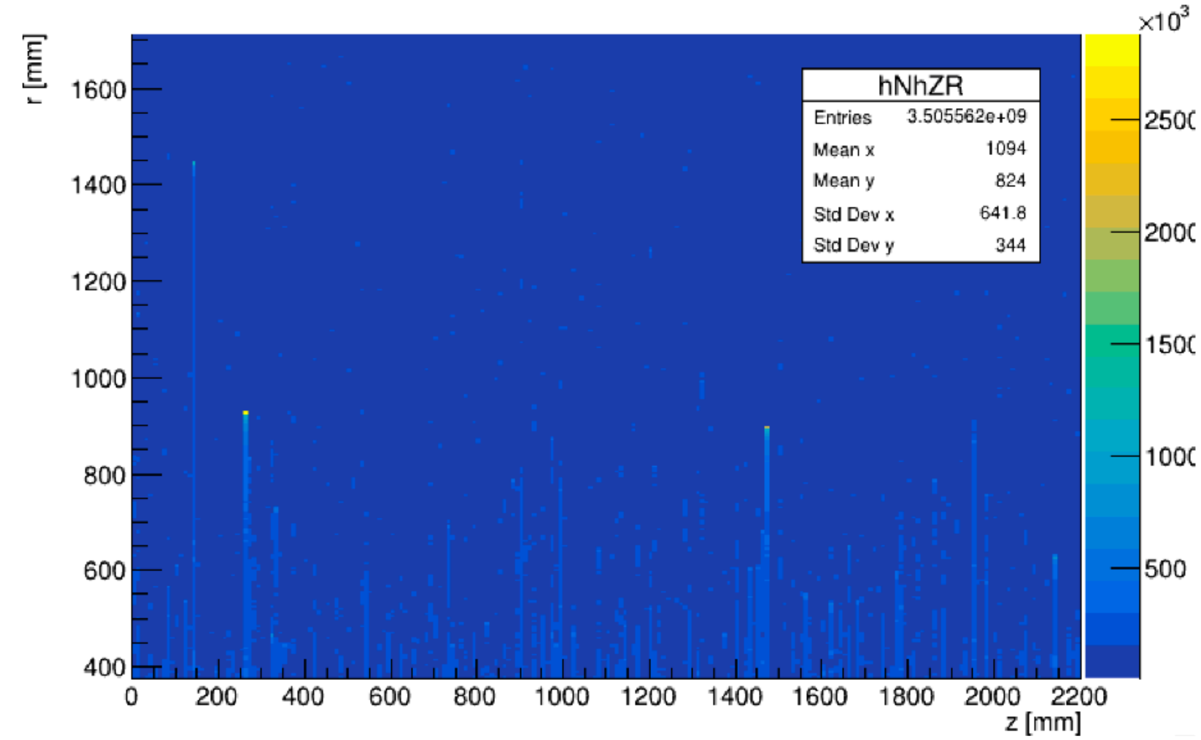
Primary Ions

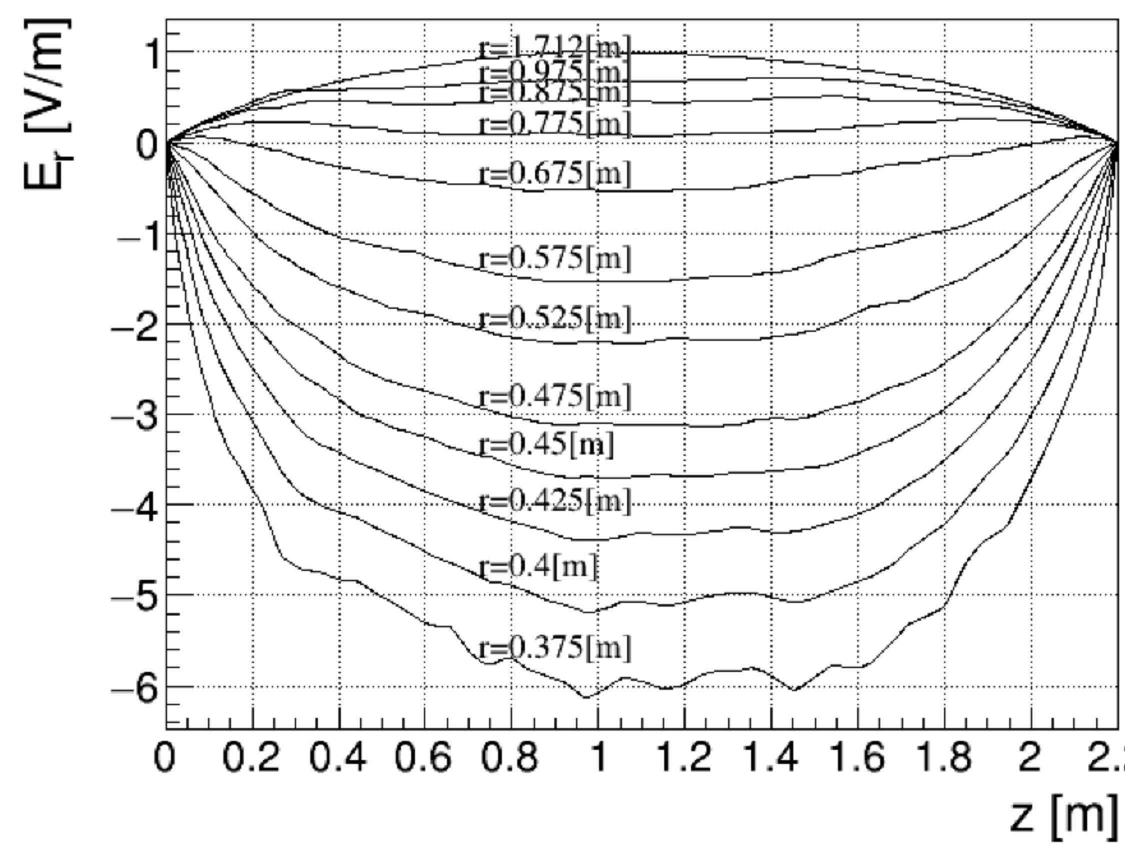




Er halved

Ion Back Flow



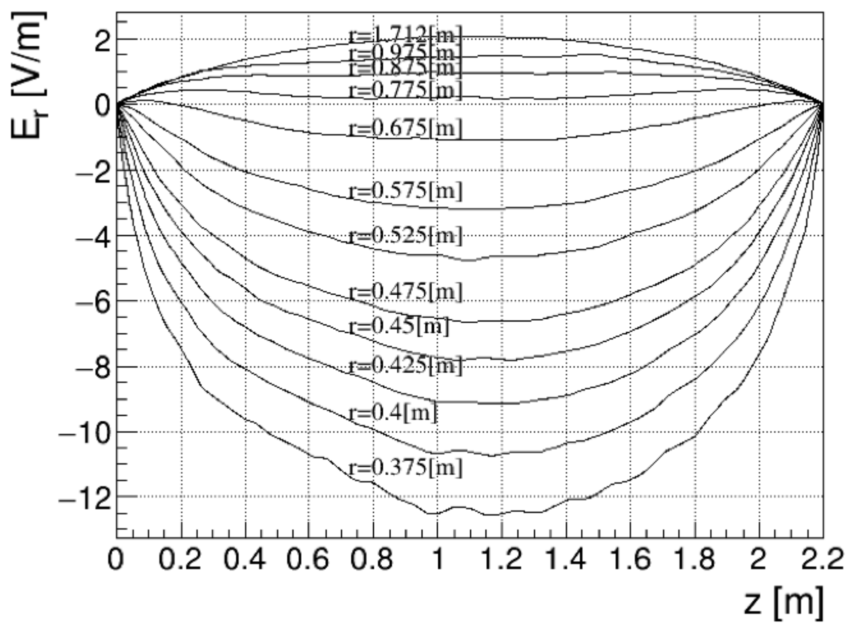


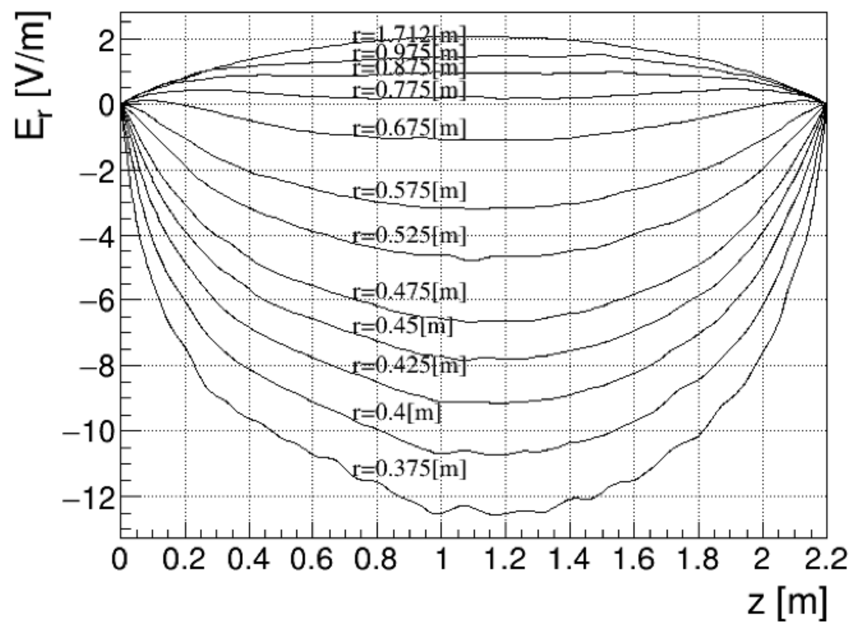
Er halved, more glitches

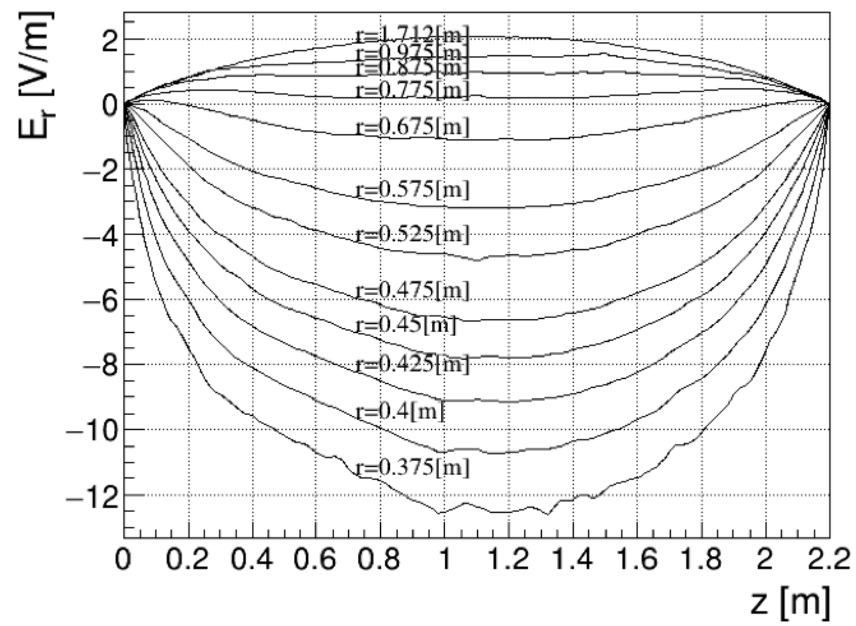
Positive Ion Back Flow (22k Z's): n and n_z high enough?

22k Z pole events

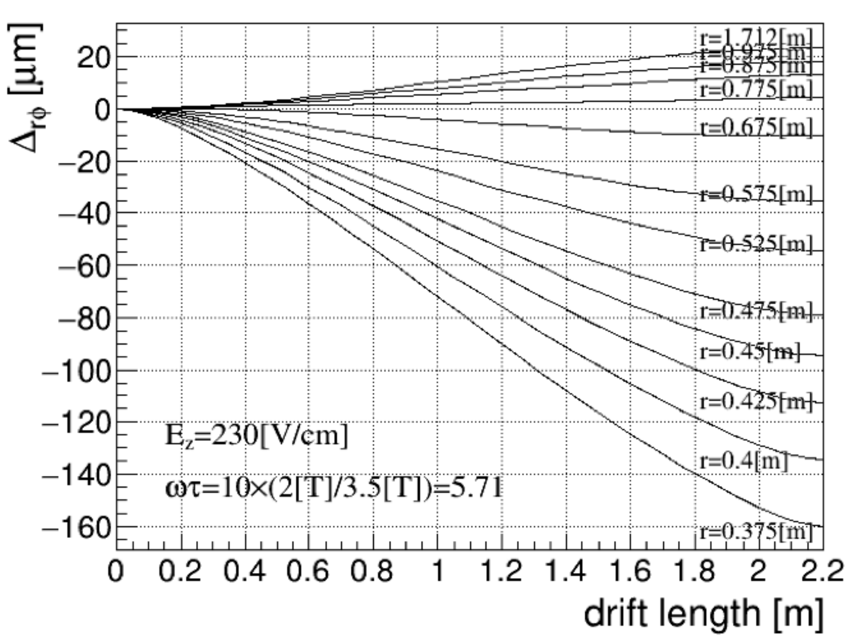
n = 40, nz = 50 → **n = 40, nz = 110** → **n = 220, nz = 110**

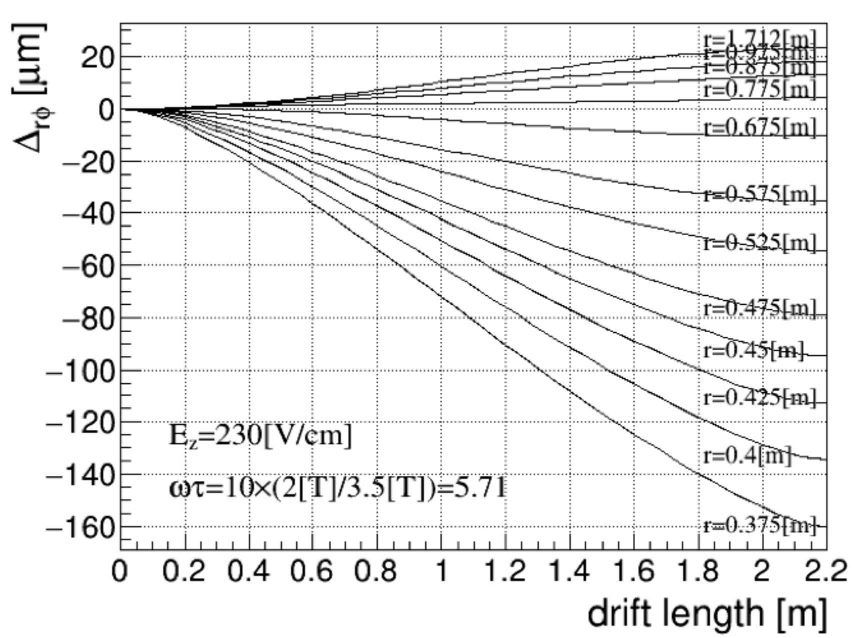


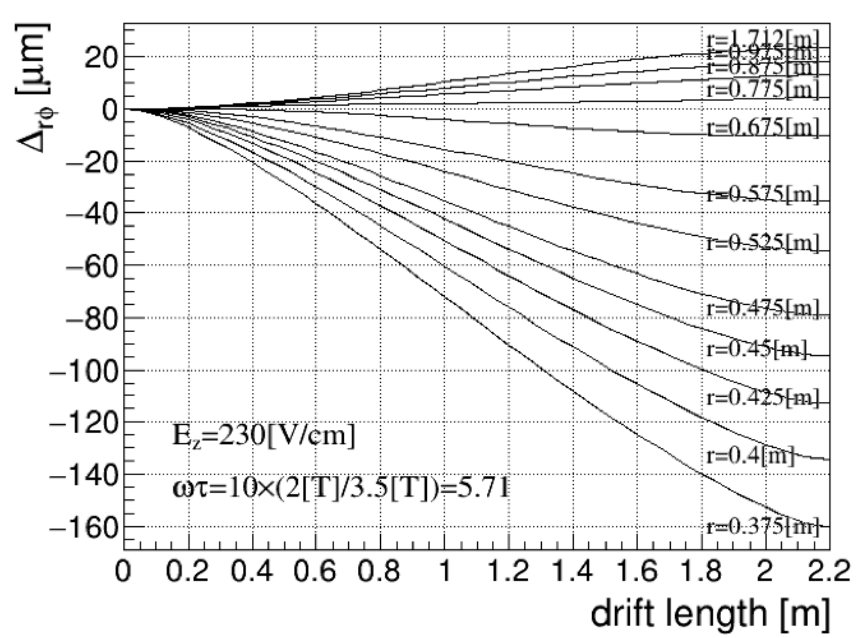




Higher n seems necessary to catch fine structure in Er







Nevertheless, glitches in Er seem to be averaged out in Δrϕ

→ For order of mag. Δrϕ estimate, n=40 and nz=50 seem OK.

Estimate of TPC distortions at tera-Z

- Full Simulation Results -

2022/11/23 ***Daniel Jeans*** @ ILD Software & Analysis Meeting

https://agenda.linearcollider.org/event/9876/contributions/51617/attachments/38548/60641/tpc-teraz-nov2022-jeans.pdf

Simulation Conditions

Keisuke just showed estimates based on a toy MC

I will use ILD full simulation to estimate ion densities,
and Keisuke's code to calculate the resulting distortions

qq (uds) events at 91 GeV
no bg, beamstrahlung, or beam en spread (JER calibration sample)
`E91-nobeam.Pqq.Gwhizard-1_95.e0.p0.I110025.${n}.stdhep`

simulated in ILD model ILD_l5_v02 with reduced B-field: 3.5 → 2T

keep "LowPt" TPC hits (default is not to keep in output file)

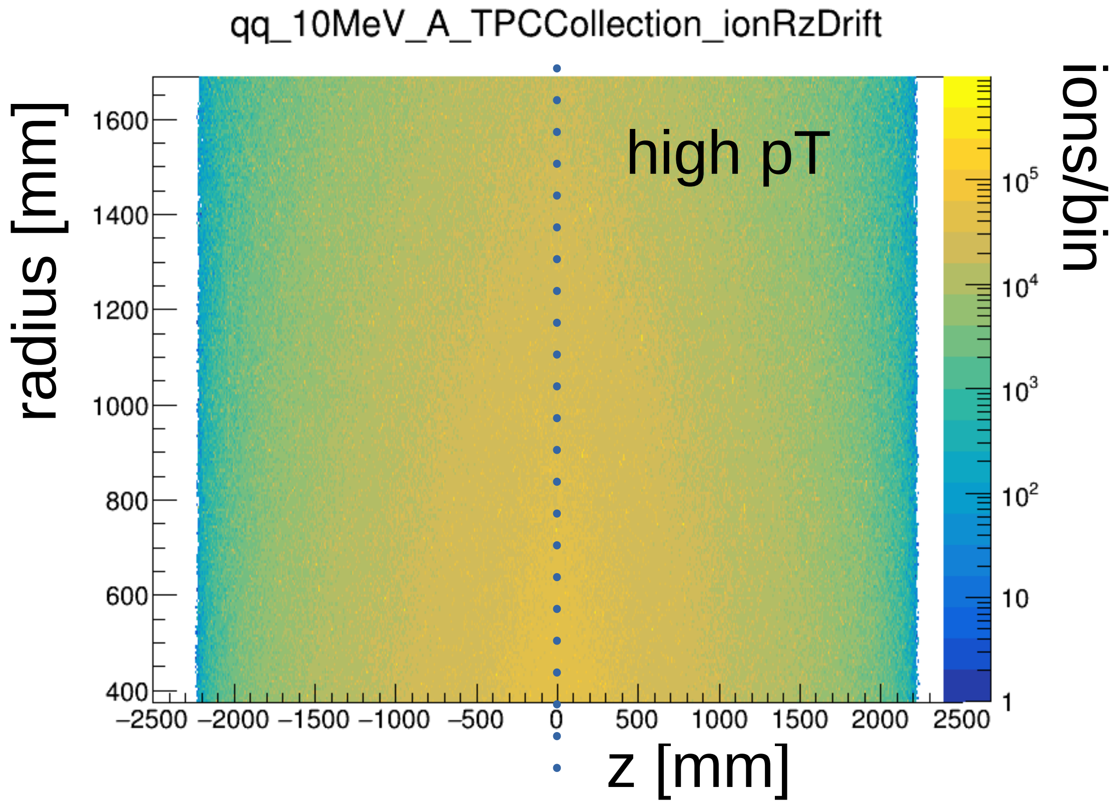
(+ some small fixes of TPCSDAction.cc :
defines how to go from G4 steps → SimTrackerHits)

Primary Ions

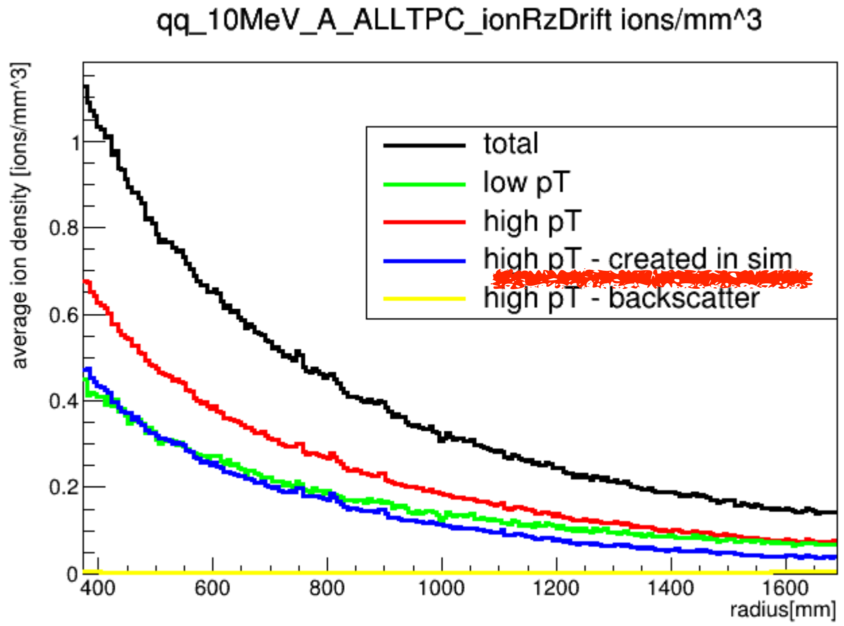
assume 26 eV energy deposit in TPC gas → one primary ion
→ average primary ions/event = 0.68 M (high pT) + 0.49 M (low pT)

also assume
ion drift vel = 5 m/s
max drift length = 2.2 m
→ max ion drift time = 0.44 s
hadronic Z event rate: 50 kHz → 22k events over 0.44 s

high pT primary ion distr
integrate over ~22k events
ion drift & absorption at
cathode (@ z=0)



average radial distribution
of ion density



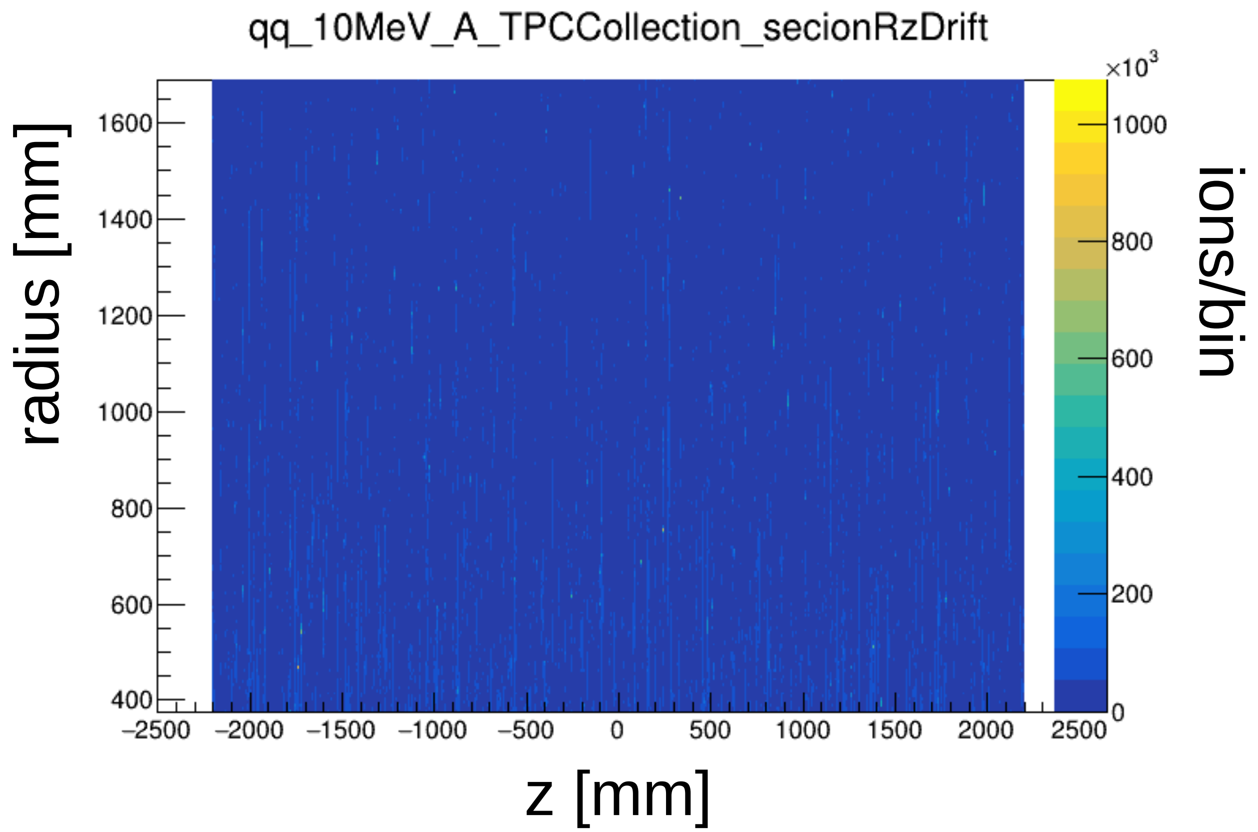
~1/3 from particles created in simulation

Secondary Ions (Ion Back Flow)

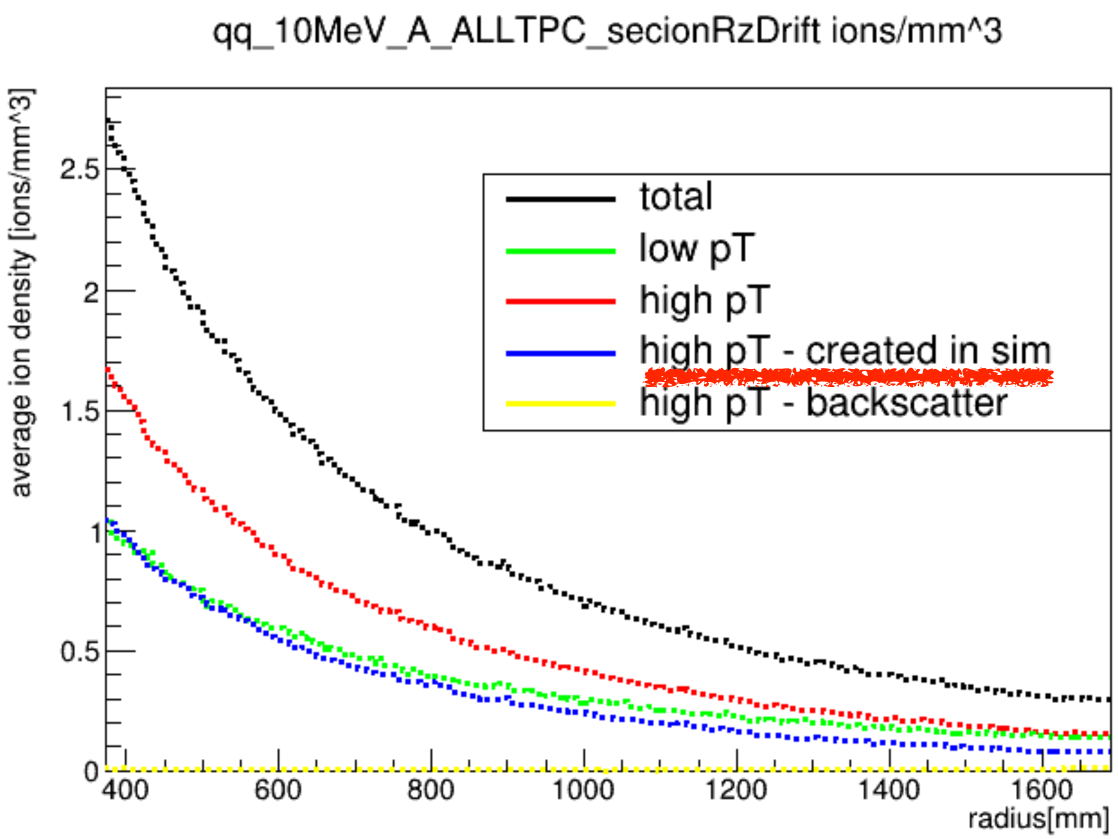
also consider Ion Back Flow (IBF) “secondary ions”

assume electrons from one event arrive at anode ~instantaneously, produce thin disk of ions
calculate for IBF=1 (one electron in → one ion out)
populate drift volume with ~22k such disks.

ions per bin



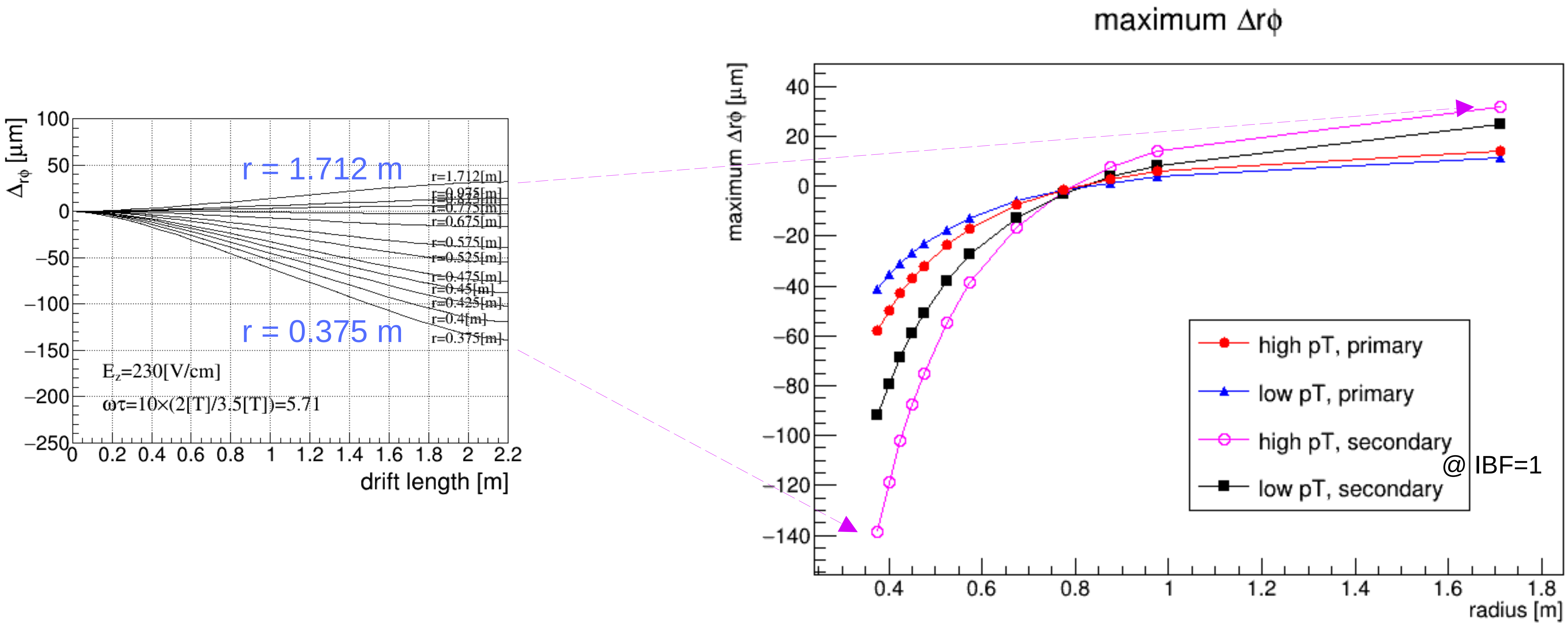
ion density



~1/3 from particles created in simulation

Maximum Distortions

maximum distortions (i.e. for maximum drift)



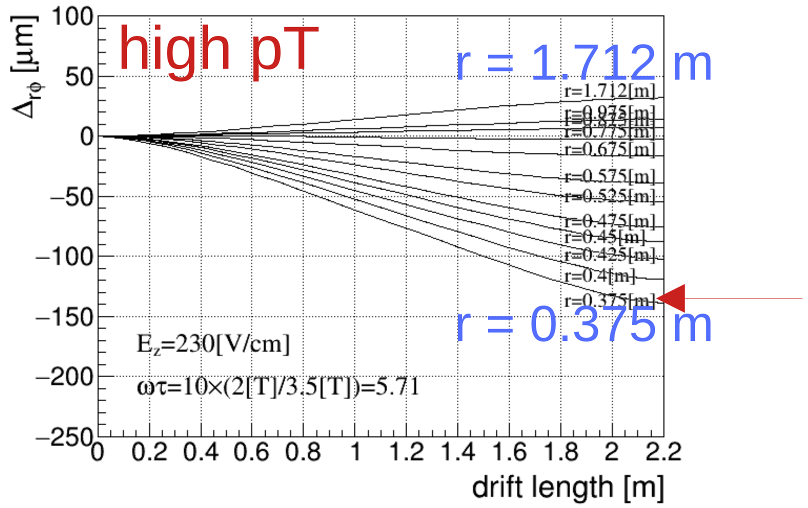
these distortions can be linearly summed to get total :

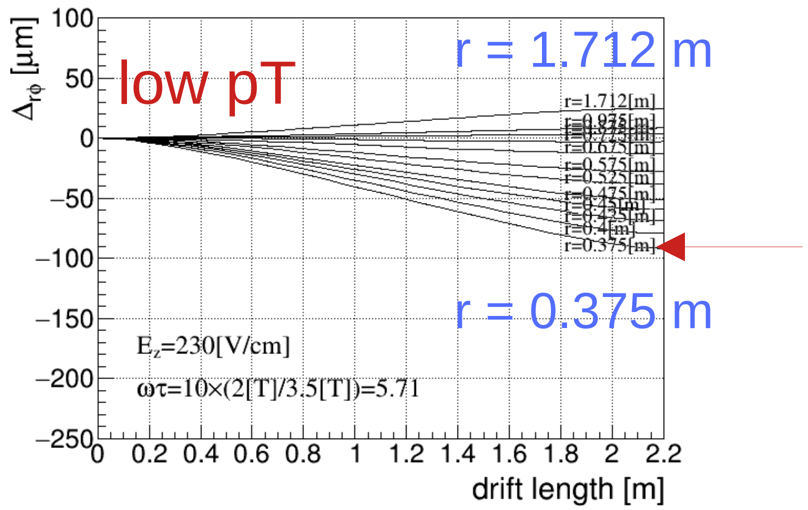
primaries	~100 micron	@ 0.375m
secondaries	~230 micron	@ 0.375 m & IBF=1
total	~330 micron	@ 0.375 m & IBF=1

max Δrϕ ~ 100 + (IBF × 230) µm

Comparison with Toy MC Results

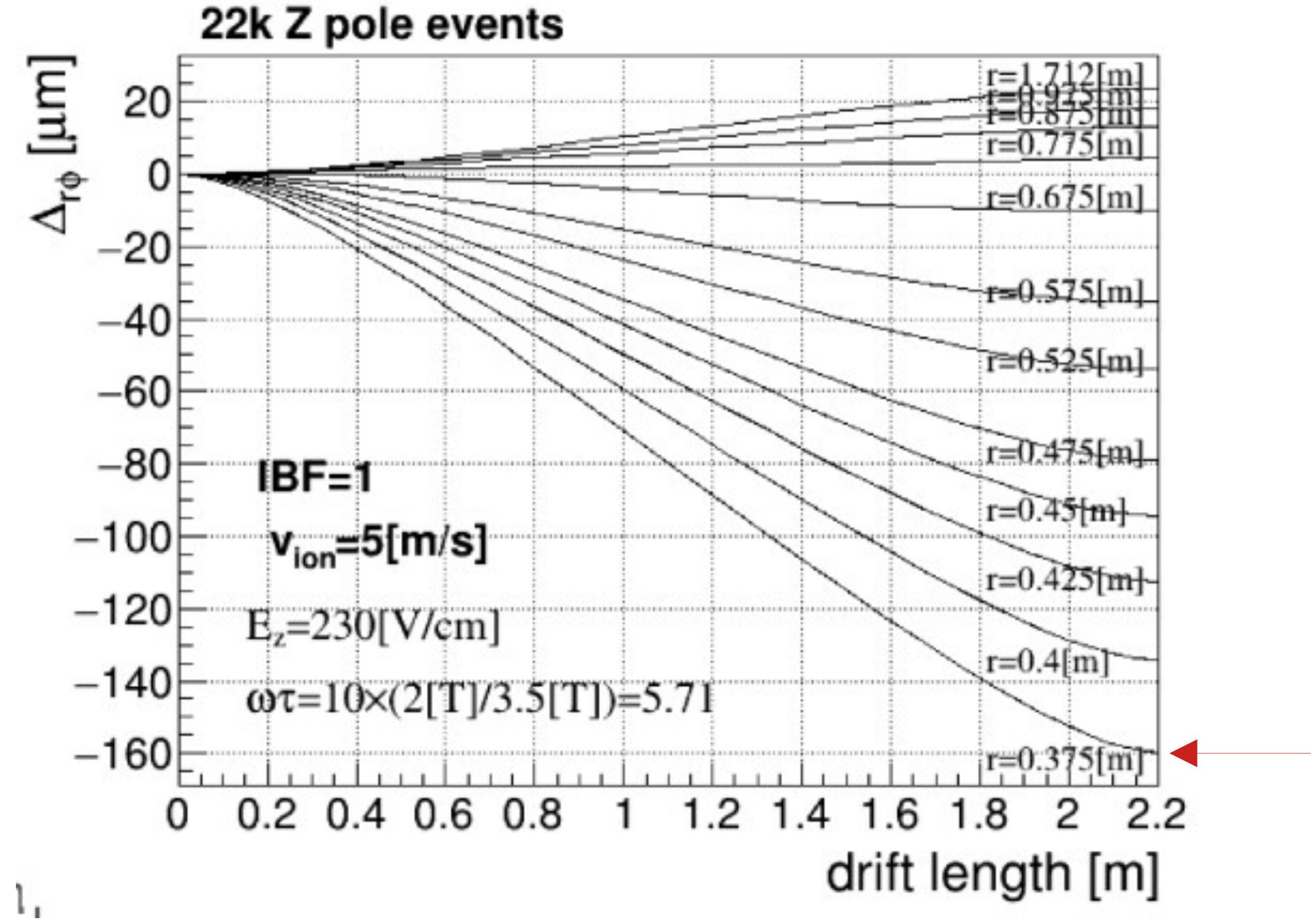
my estimate (from secondaries)





max total distortion (z=2.2m, r=0.375m)
~ 91+139 = 240 micron

Keisuke's estimate (from secondary ions)



max total distortion (z=2.2m, r=0.375m)
~ 160 micron

effect of additional particles created in simulation?

1/3 of ions from particles created in detector simulation

→ Δrφ from generator particles ~ 2/3 × 240 = 160 µm (seems consistent)

Summary and Conclusion

- Updated the distortion calculator to allow non-factorizable $\rho_{ion}(r,z)$ and used it to estimate $\Delta r\phi$ due to primary and secondary ions in the TPC drift volume.

- Primary ion contribution is much smaller (~44% if IBF=1) than that from secondary ions.

- Full ILD simulation by Daniel showed
  - particles created in detector simulation contribute significantly (~1/3) to the total ion density.
  - maximum distortion

    $$\textbf{max } \Delta r\phi \sim 100 + (\text{IBF} \times 230) \text{ [}\mu\text{m]}$$

    for 22k Z pole events in a time frame of ILD TPC with B=2 [T].

- Actual size of distortion depends on the choice of gas ($\omega\tau$, vion (velec)), drift field (E0), and TPC geometries (rin, rout, len), as well as the Z event rate,

- and of course other machine and beam-induced backgrounds.