Estimate of TPC distortions at tera-Z (I)

- Methods -

2022/11/23 Keisuke Fujii (2022/11/24: corrected and more comments added)

Poisson's equation

The E field in a region (D) is the sum of the E field (E0) without space charge in the corresponding region defined by the field shaping strips and the two terminating plates and the field (Eion) calculated with space charge in the virtual grounded conducting boundary of D.

$$\triangle \phi_0(\boldsymbol{x}) = 0$$

$$\triangle \phi_{\mathrm{ion}}(\boldsymbol{x}) = -4\pi\, \rho_{\mathrm{ion}}(\boldsymbol{x})$$

in $\boldsymbol{x} \in D$

Boundary Conditions

$$\boxed{\begin{array}{l} \phi_0(\boldsymbol{x}) = V_i \\ \qquad \boldsymbol{x} \in C_i \end{array}}$$

$$\phi(\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \phi_{\mathrm{ion}}(\boldsymbol{x})$$

$$\longrightarrow \quad \begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0 + \boldsymbol{E}_{\mathrm{ion}} \\ &= \boldsymbol{E}_0 - \nabla \phi_{\mathrm{ion}}(\boldsymbol{x}) \end{aligned}$$

$$\boxed{\begin{array}{l} \phi_{\mathrm{ion}}(\boldsymbol{x}) = 0 \\ \qquad \boldsymbol{x} \in \partial D \end{array}}$$

All we need is Green's function for

$$\boxed{\triangle G(\boldsymbol{x}, \boldsymbol{x}') = -4\pi \delta(\boldsymbol{x} - \boldsymbol{x}')}$$

$$\boxed{\begin{array}{l} G(\boldsymbol{x}, \boldsymbol{x}') = 0 \\ \qquad \boldsymbol{x} \in \partial D \end{array}}$$

E-field distortion is then given by superposition:

$$\boxed{\phi_{\mathrm{ion}}(\boldsymbol{x}) = \int_D d^3\boldsymbol{x}\, G(\boldsymbol{x}, \boldsymbol{x}')\, \rho_{\mathrm{ion}}(\boldsymbol{x}')}$$

Superposition makes life easy!

Green's function

Since the boundaries are most naturally expressed in the cylindrical coordinates (rin=a, rout=b, z=0, Z=L), the corresponding Green function is most conveniently expanded in terms of modified Bessel function as follows:

$$G(r,\varphi,z;r',\varphi',z') = \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} g_{mn}(r,r') \frac{1}{2\pi} e^{im(\varphi-\varphi')} \frac{2}{L} \sin(\beta_n z) \sin(\beta_n z')$$

where

$$g_{mn}(r,r') = \frac{4\pi \left[K_m(\beta_n a) I_m(\beta r_<) - I_m(\beta_n a) K_m(\beta_n r_<)\right] \left[K_m(\beta_n b) I_m(\beta r_>) - I_m(\beta_n b) K_m(\beta_n r_>)\right]}{\beta_n r' \left[I_m(\beta_n a) K_m(\beta_n b) - I_m(\beta_n b) K_m(\beta_n a)\right] \left[K_m(\beta_n r') I'_m(\beta_n r') - K'_m(\beta_n r') I_m(\beta_n r')\right]}$$

$$\beta_n = n\pi/L \qquad\qquad r_< := \min(r,r'), \quad r_> := \max(r,r')$$

If the charge distribution is uniform in phi, the phi-integral is trivial and we get

$$\phi_{\text{ion}}(r,z) = \sum_{n=1}^{\infty} \frac{8\pi}{\beta_n} \int_a^b dr' \frac{\left[K_0(\beta_n a) I_0(\beta r_<) - I_0(\beta_n a) K_0(\beta_n r_<)\right] \left[K_0(\beta_n b) I_0(\beta r_>) - I_0(\beta_n b) K_0(\beta_n r_>)\right]}{\left[I_0(\beta_n a) K_0(\beta_n b) - I_0(\beta_n b) K_0(\beta_n a)\right] \left[K_0(\beta_n r') I'_0(\beta_n r') - K'_0(\beta_n r') I_0(\beta_n r')\right]}$$
$$\sin(\beta_n z) \int_0^L \frac{dz'}{L} \sin(\beta_n z') \boxed{\rho_{\text{ion}}(r',z')} \leftarrow \textit{no } \phi\textit{-dependence}$$

Derivatives of the modified Bessel functions can be rewritten in terms of those of different orders:

$I'_0(x) = I_1(x)$ and $K'_0(x) = -K_1(x)$

Using these and differentiating $\phi_{ion}(r, z)$ with respect to r we get the following for Er:

$$
\begin{aligned}
E_r(r,z) = -8\pi \sum_{n=1}^{\boxed{\infty}} & \frac{\sin(\beta_n z)}{I_0(\beta_n a)K_0(\beta_n b) - I_0(\beta_n b)K_0(\beta_n a)} \\
& \Big[[K_0(\beta_n b)I_1(\beta r) + I_0(\beta_n b)K_1(\beta_n r)] \int_a^r dr' \, \frac{K_0(\beta_n a)I_0(\beta r') - I_0(\beta_n a)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')} \\
& \qquad \boxed{\int_0^L \frac{dz'}{L} \, \sin(\beta_n z')\rho_{ion}(r', z')} \\
& + [K_0(\beta_n a)I_1(\beta r) + I_0(\beta_n a)K_1(\beta_n r)] \int_r^b dr' \, \frac{K_0(\beta_n b)I_0(\beta r') - I_0(\beta_n b)K_0(\beta_n r')}{K_0(\beta_n r')I_1(\beta_n r') + K_1(\beta_n r')I_0(\beta_n r')} \\
& \qquad \boxed{\int_0^L \frac{dz'}{L} \, \sin(\beta_n z')\rho_{ion}(r', z')} \Big]
\end{aligned}
$$

z'-integral now inside r'-integral

where

$$\beta_n = n\pi/L$$

In the practical calculations, we have to sum up the series up to **high enough "n"**, which is determined by the ratio of the shortest and the longest scales that specify the charge distribution and the geometry of the boundary of the region in question.

For a thin disk or in the MPGD-gate gap, summation up to 500 or more is necessary, which in turn requires quadruple precision calculations for the modified Bessel functions.

Principle (continued)

E0, if parallel with the B field, will not contribute to the ExB effect. (c.f.) the Langevin Equation:

$$\boldsymbol{\omega} := \frac{(-e)\boldsymbol{B}}{mc}$$

$$\omega\tau \simeq 10 \text{ for T2K gas at B=3.5T}$$

$$\langle \boldsymbol{v} \rangle = \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[1 + (\omega\tau)\hat{\boldsymbol{B}} \times + (\omega\tau)^2 \hat{\boldsymbol{B}}\,\hat{\boldsymbol{B}} \cdot \right] \frac{e}{m} \boldsymbol{E}$$

If we write down the distortion of the velocity due to the distortion of the E-field in the longitudinal and transverse directions, we get

$$\Delta \langle \boldsymbol{v} \rangle = \frac{e}{m} \left(\frac{\tau}{1+(\omega\tau)^2} \right) \left[(1+(\omega\tau)^2)\Delta\boldsymbol{E}_{\parallel} + \boldsymbol{E}_{\perp} - (\omega\tau)\boldsymbol{E}_{\perp} \times \hat{\boldsymbol{B}} \right]$$

Numerically integrating this over the drift time by noting $\delta l_i = \langle v_{\parallel} \rangle \delta t_i$, we get the following formula for the distortion:

$$\boxed{\begin{aligned} \langle \Delta \boldsymbol{x} \rangle &= \sum_{i=1}^{\boxed{n}} \frac{\Delta \langle \boldsymbol{v} \rangle_i}{\langle v_{\parallel} \rangle_i} \delta l_i \\ &\simeq \sum_{i=1}^{\boxed{n}} \delta l_i \left[-\frac{\Delta \boldsymbol{E}_{\parallel i}}{E_0} - \left(\frac{1}{1+(\omega\tau)^2} \right) \frac{\boldsymbol{E}_{\perp i}}{E_0} + \boxed{\left(\frac{\omega\tau}{1+(\omega\tau)^2} \right)} \frac{\boldsymbol{E}_{\perp i} \times \hat{\boldsymbol{B}}}{E_0} \right] \end{aligned}}$$

Key point: distortion is linear w.r.t. E-field distortion, and hence also w.r.t. space charge for a drift from the same z to the anode: Superposition makes life easy!

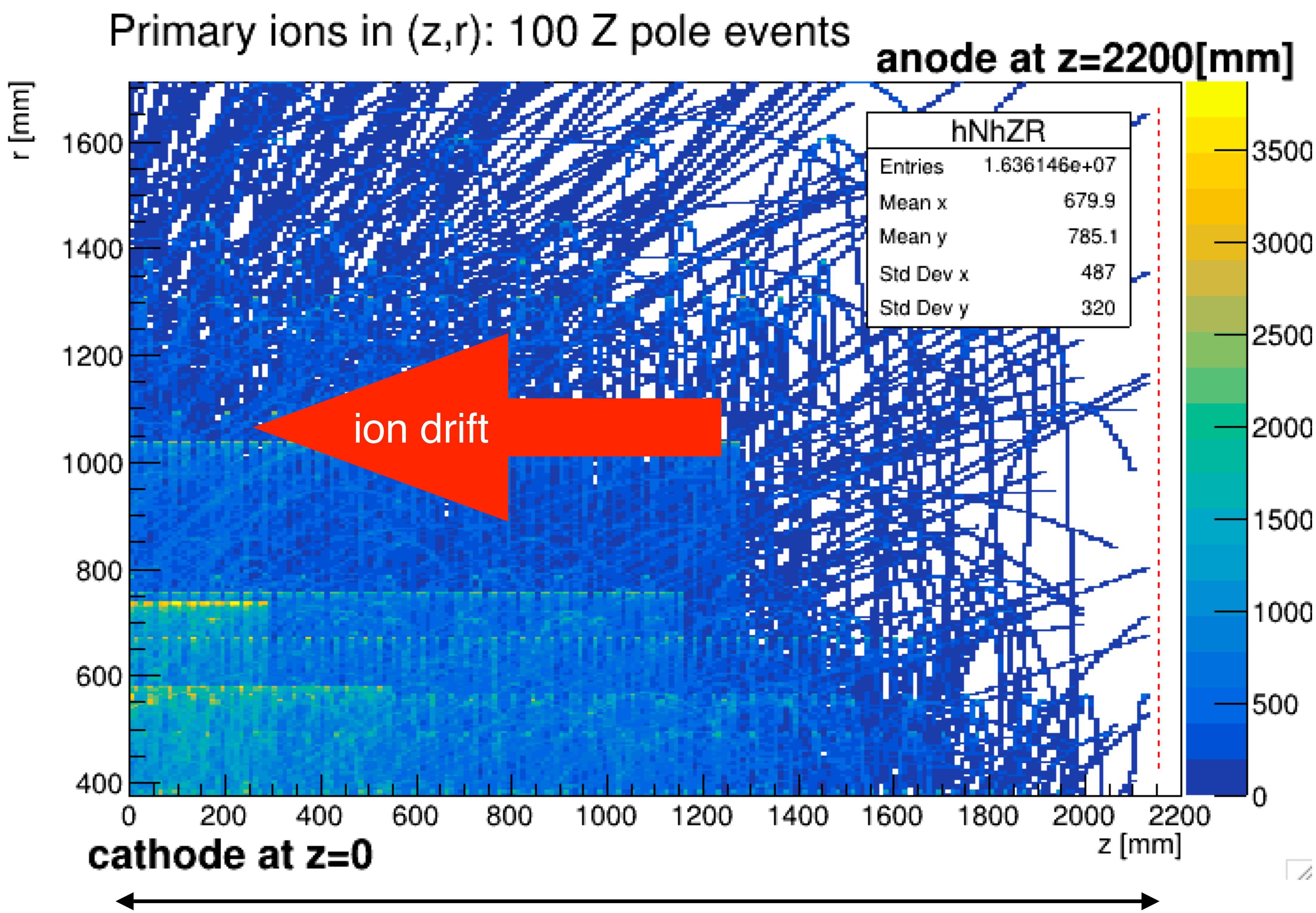
Primary Ions accumulated for 100 Z pole events in the 0.44 sec time frame

Toy MC using Pythia8

no energy loss while curling, truncated after 200 turns

dE/dx simulated p-dependently for pure Ar (Alison-Cobb) w/o Landau fluctuation (U_{ion}=26[eV]) with ions distributed uniformly along each track.

100 events in the time frame in this example



Time frame width = len/v_{ion} = 2.2[m]/5[m/s] = 0.44[s]

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T], v_{ion} = 5[m/s]

Ions even if created at the farthest point from the cathode (.i.e. near the end plane) must have been absorbed by the cathode if they were created before this 0.44[s] time frame.

Secondary Ions flowed back from the anode accumulated for 100 Z pole events in the 0.44 sec time frame

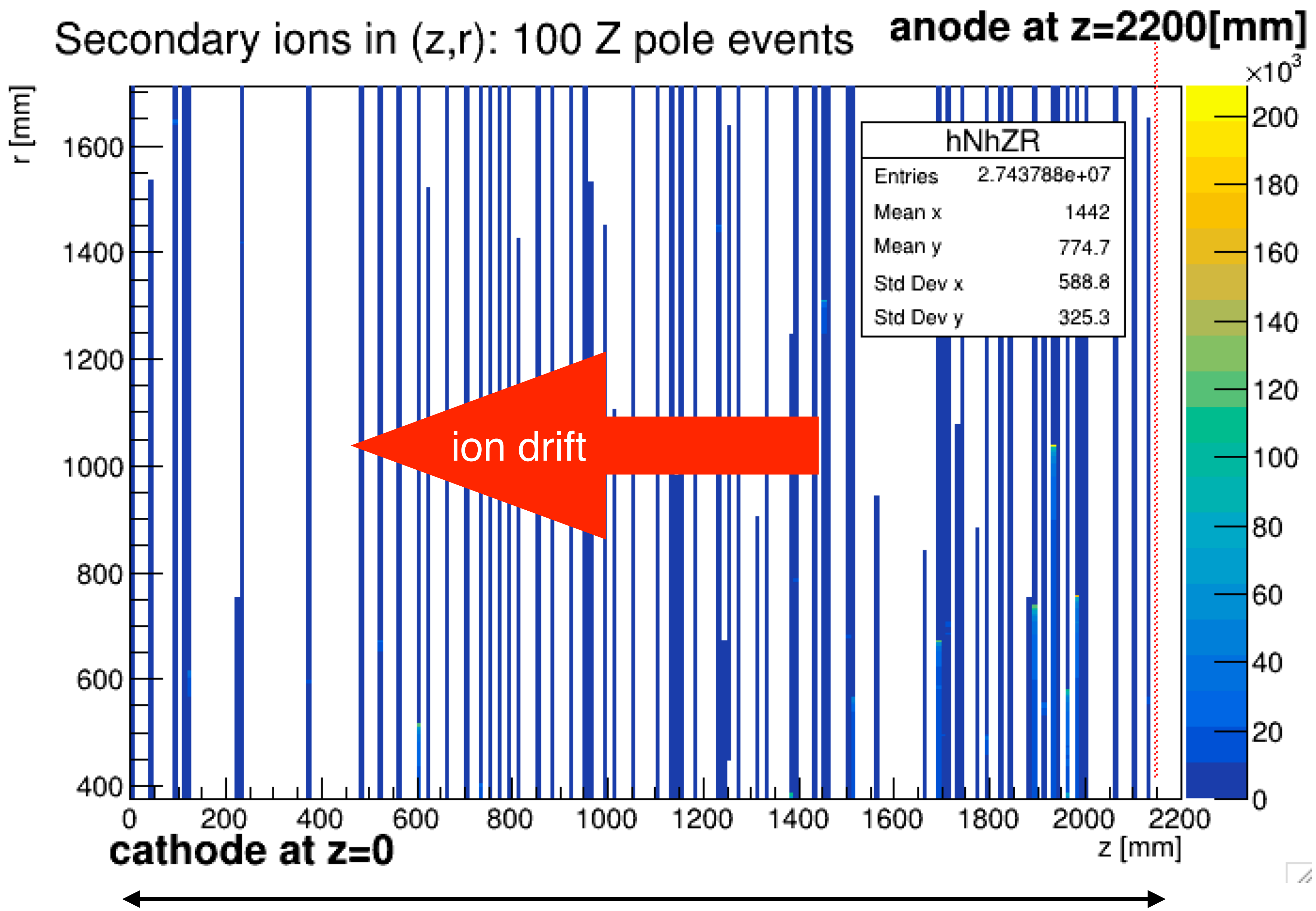
Toy MC using Pythia8

no energy loss while curling, truncated after 200 turns.

dE/dx simulated p-dependently for pure Ar (Alison-Cobb) w/o Landau fluctuation (U_{ion}=26[eV]) with ions distributed uniformly along each track.

100 events in the time frame in this example

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T]
v_{ion} = 5[m/s]
v_{elec} = 75[mm/μs]



Time frame width = len/v_{ion} = 2.2[m]/5[m/s] = 0.44[s]

Secondary ions are quasi-continuously produced at the end plane within len/v_{elec} = 30[μs] after each event, forming an ion disk of the event image compressed in z-direction by a factor of v_{ion}/v_{elec}, flow back into the drift volume, and stay there for 0.44[s] until being absorbed by the cathode.

Ions accumulated for 22k Z pole events in the 0.44 sec time frame

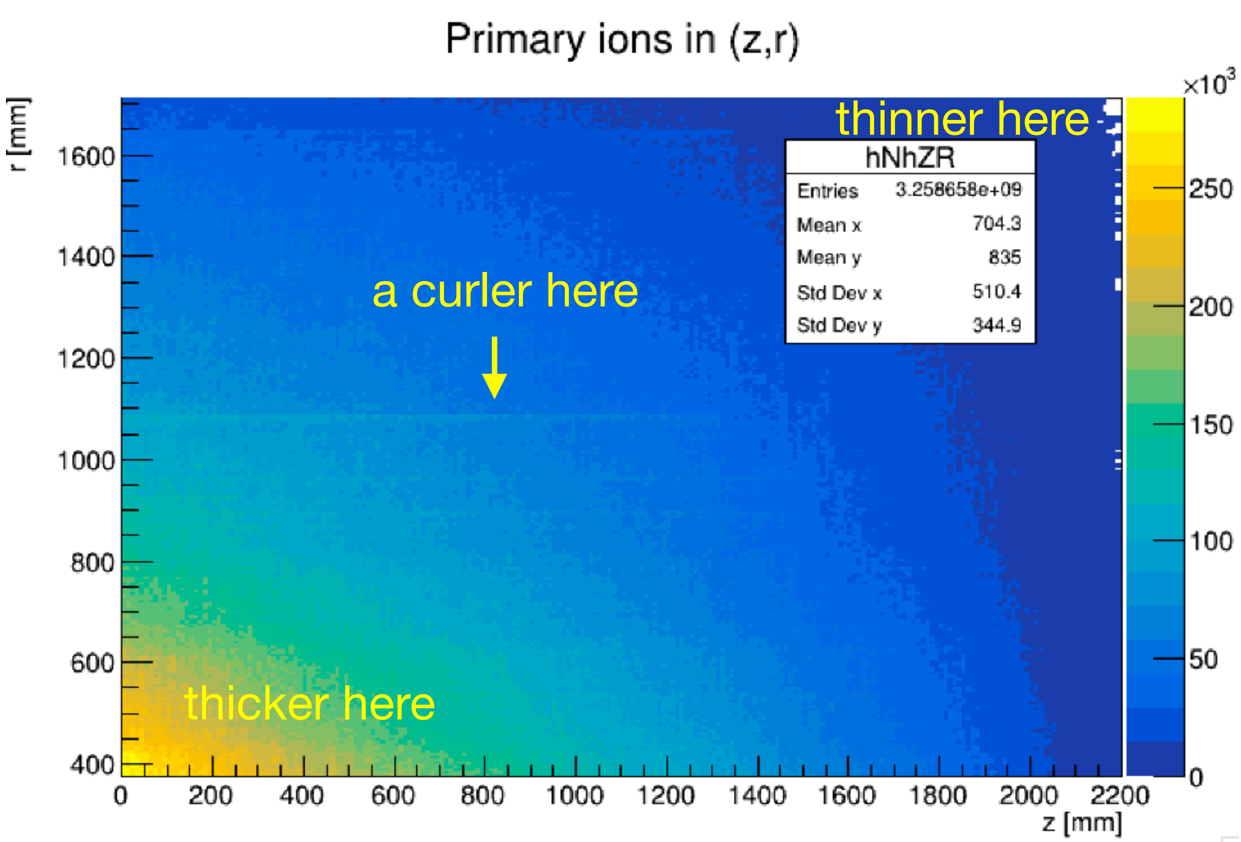
Z pole: **50 [kHz]**

Toy MC using Pythia8

IBF=1

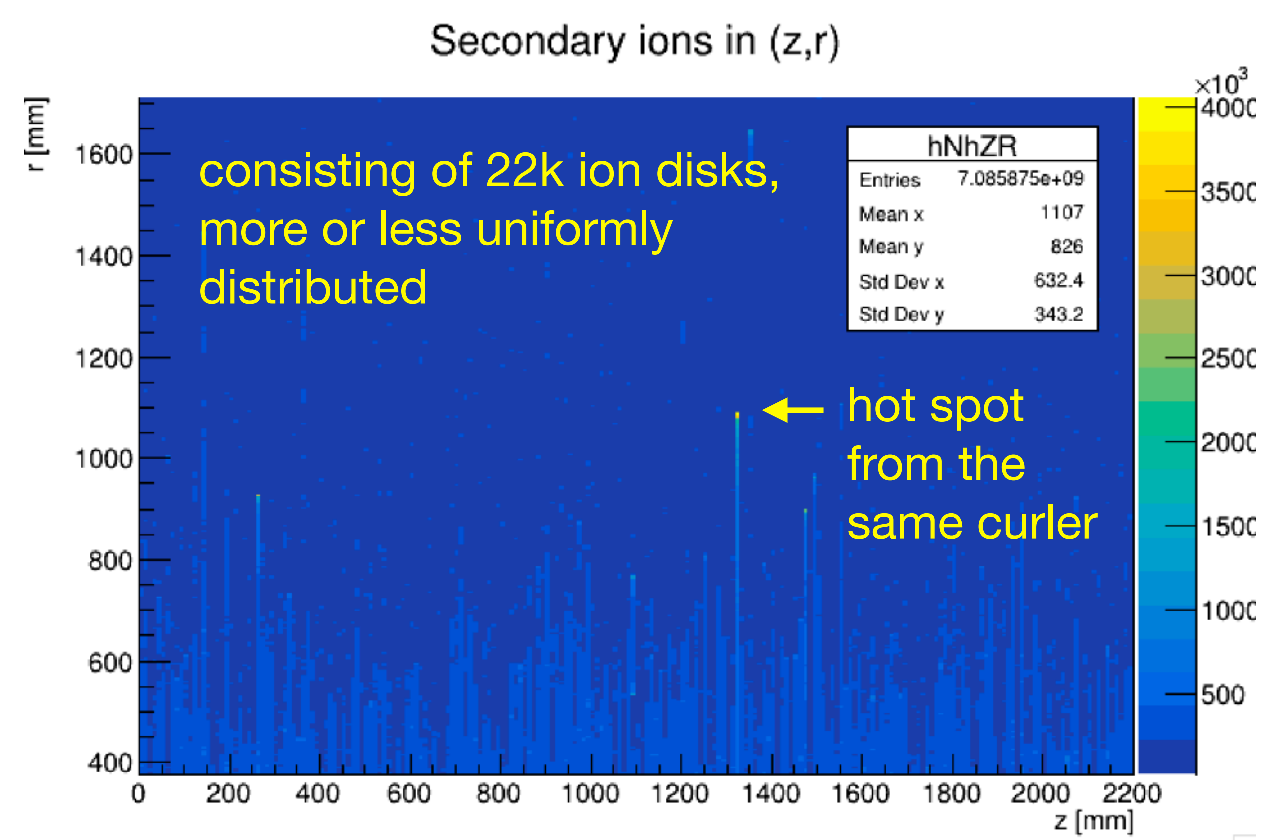
IBF:=# back flow ions / # seed electrons

Primary Ions



cathode at z=0 anode at z=2.2[m]

Ion Back Flow



cathode at z=0 anode at z=2.2[m]

bin size: $(\Delta z, \Delta r)$=(1[cm], 0.5[cm])

rin = 375[mm]
rout = 1720[mm]
len = 2200[mm]
B=2[T], v_{ion} = 5[m/s]

Conversion from ZR hist. to ρ_{ion}(r,z)

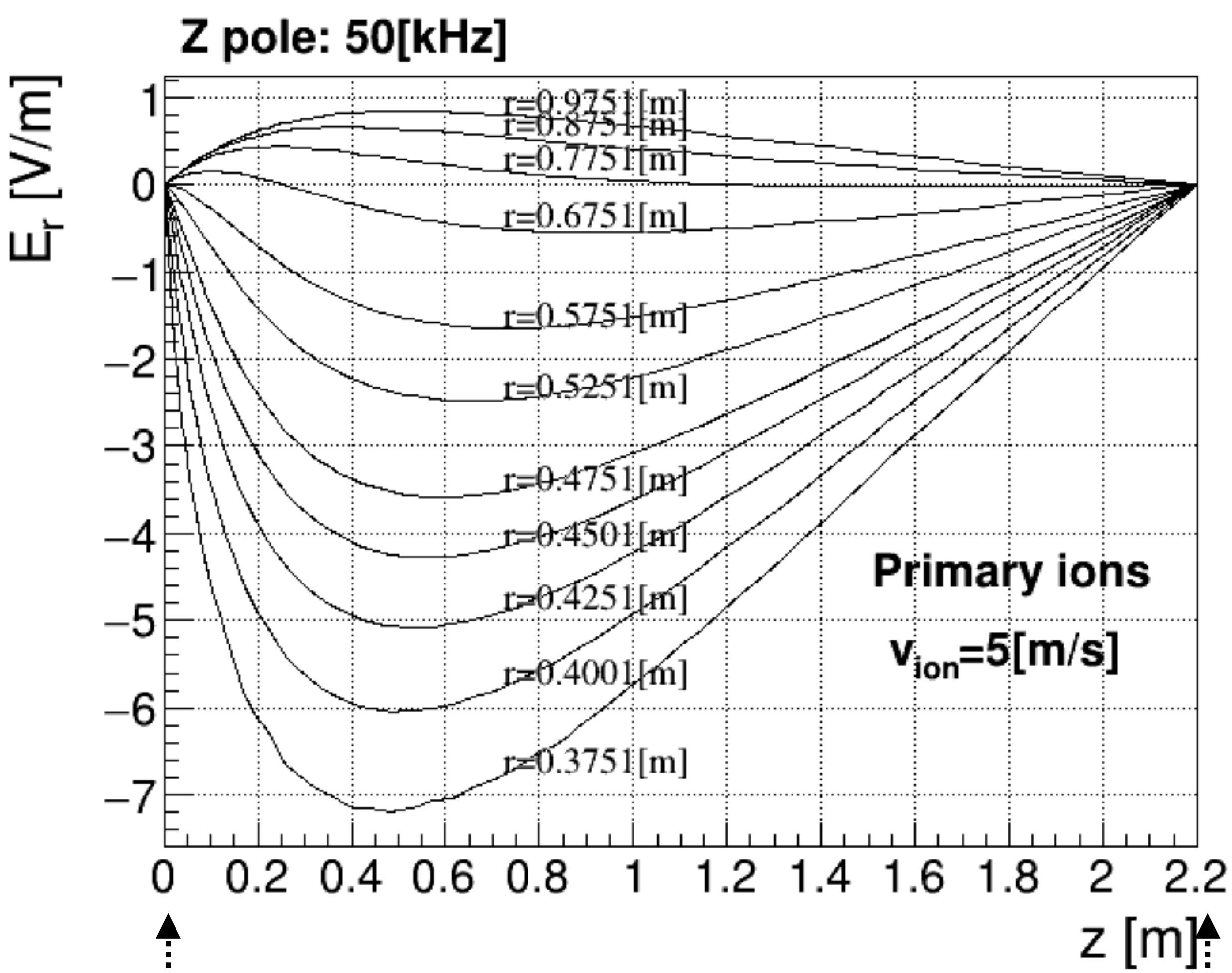
$$\rho_{ion}(r,z) \simeq e \cdot \mathrm{hNhZR}(z,r)/(2\pi r \Delta r \Delta z)$$

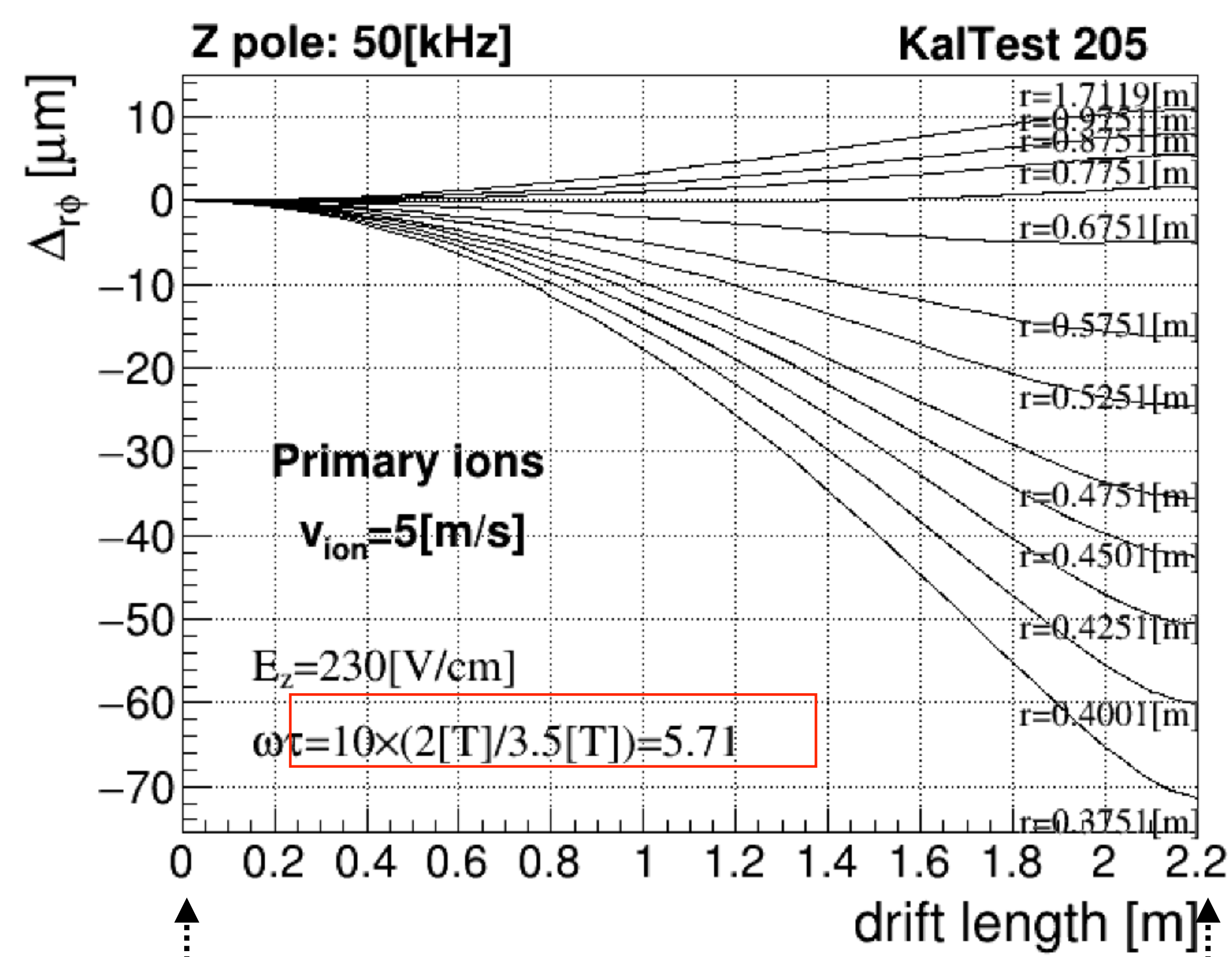
Note: ϕ-symmetry must be broken by curlers

Primary Ions (22k Z pole events)

Z pole run: hadronic Z event rate: **50 [kHz]** (toy MC using pythia8)

v_{ion} = 5 [m/s]





cathode at z=0   anode at z=2.2[m]   anode   cathode

bin size: (Δz, Δr)=(1[cm], 0.5[cm])

Note: $E_\perp$ is constrained to be zero at anode and cathode (conductors).

Ez=230[V/cm] **ωτ=5.71** B=2[T]

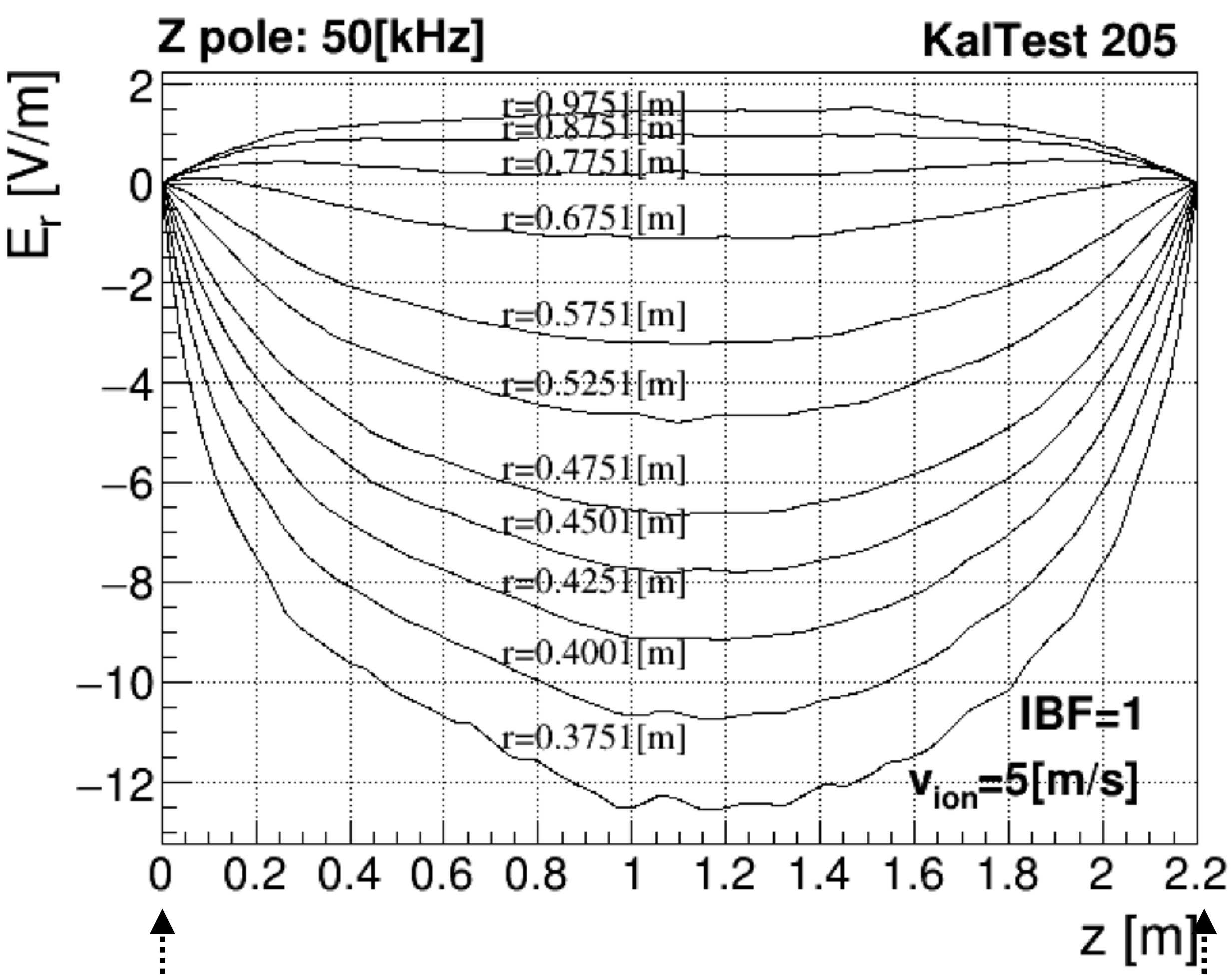
Maximum distortion ~70 [μm] at the innermost region for hadronic Z rate of 50 [kHz]

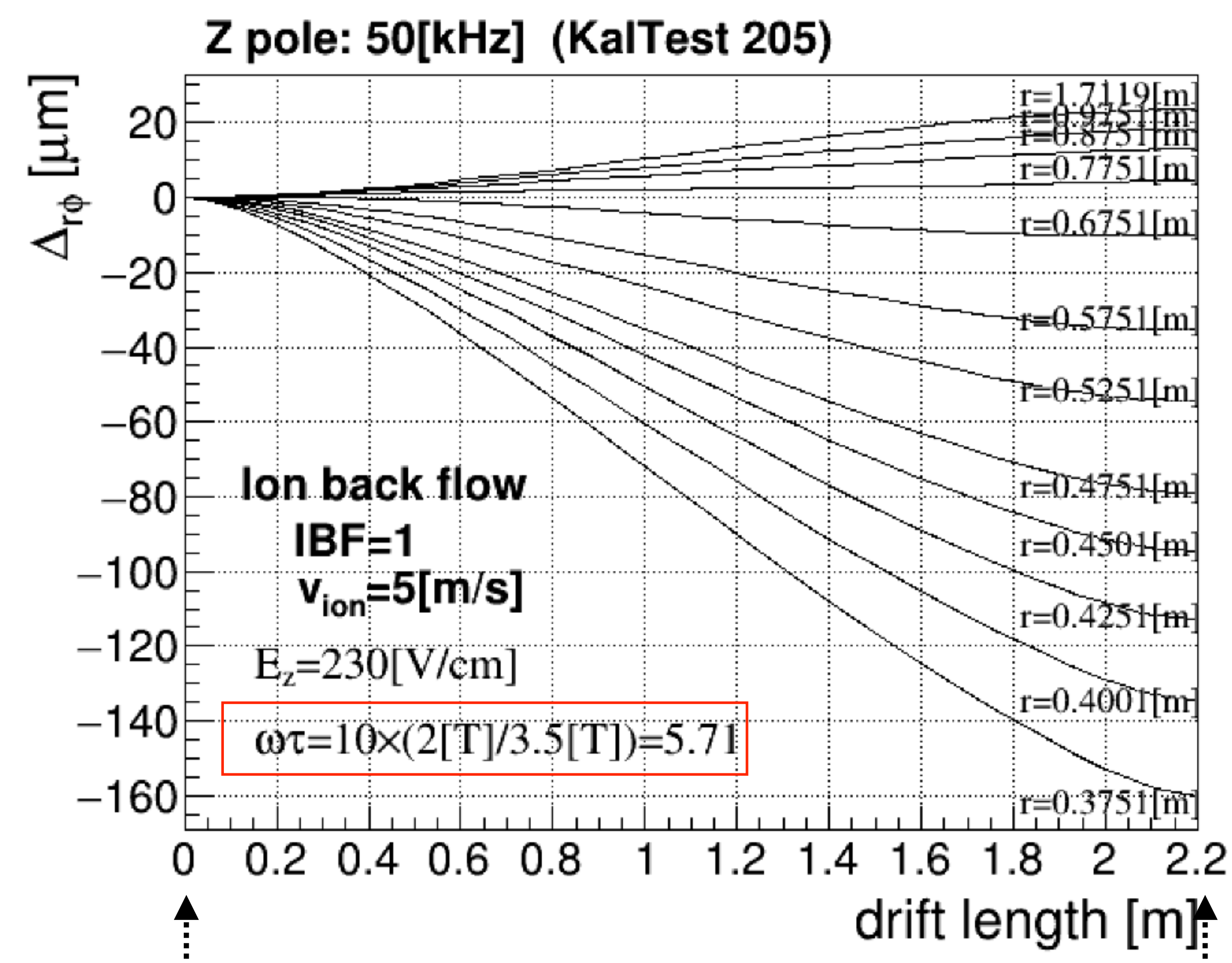
Positive Ion Back Flow (22k Z pole events)

Z pole run: hadronic Z event rate: **50 [kHz]** (toy MC using pythia8)

v_{ion} = 5 [m/s]

IBF = 1





cathode at z=0 anode at z=2.2[m] anode cathode

bin size: (Δz, Δr)=(1[cm], 0.5[cm])

Ez=230[V/cm] **ωτ=5.71** B=2[T]

Glitches correspond to hot spots in ρ_{ion}, which seem to be averaged out in $\Delta r\phi$

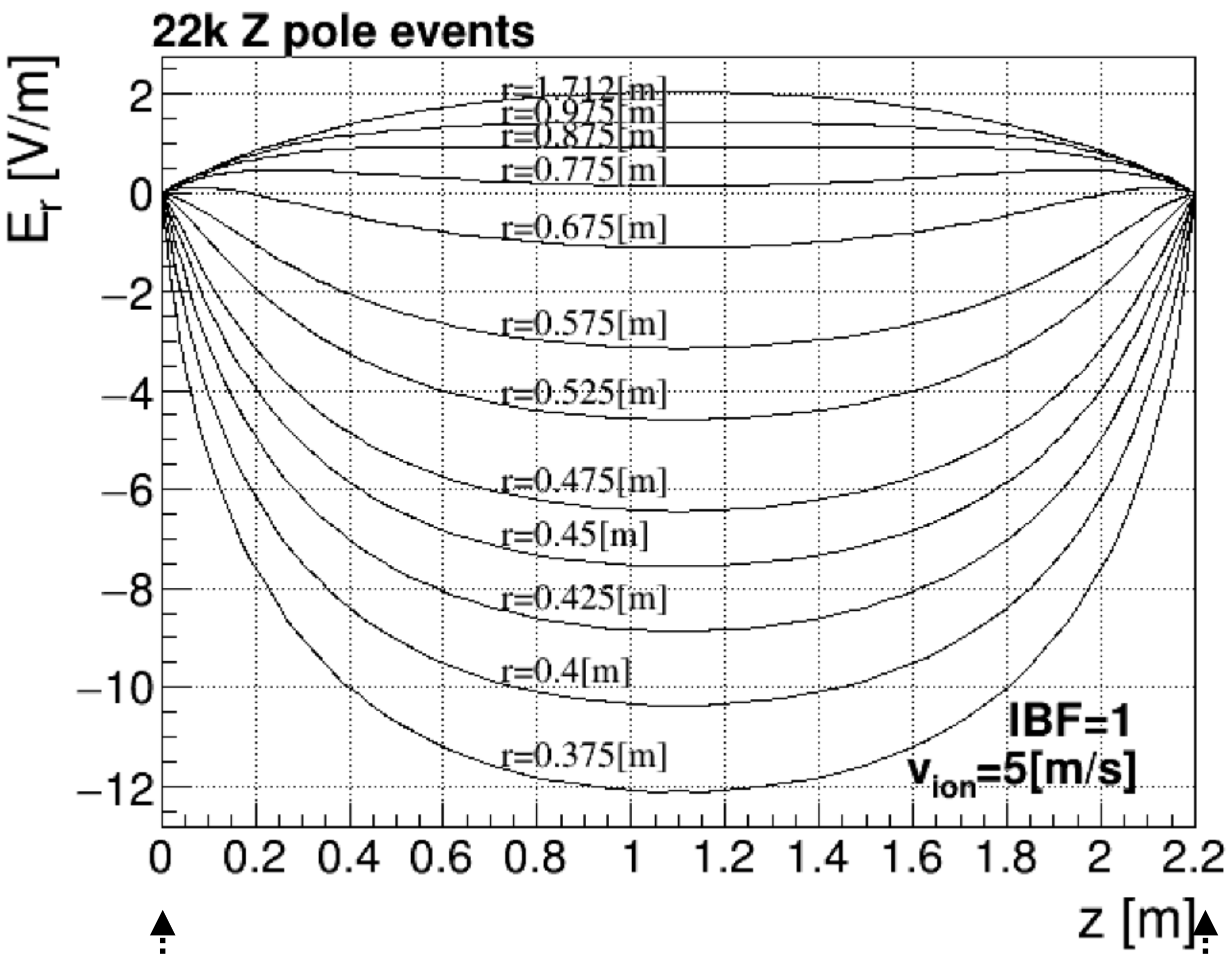
Maximum distortion ~160 [μm] at the innermost region for hadronic Z rate of 50 [kHz]

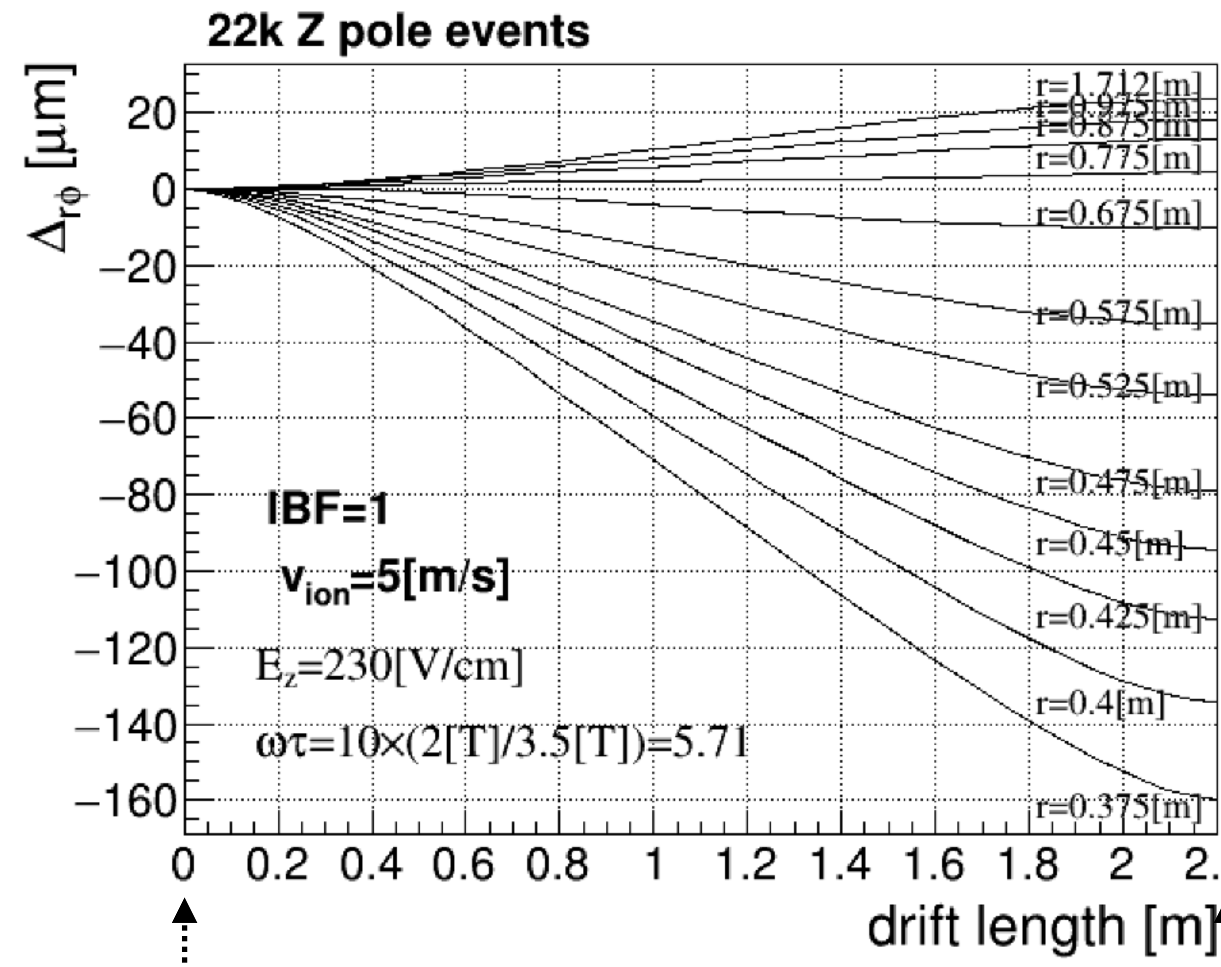
Positive Ion Back Flow (smoothed by proy)

Z pole run: hadronic Z event rate: 50 [kHz] (pythia8)

v_{ion} = 5 [m/s]

IBF = 1





cathode at z=0 anode at z=2.2[m] anode cathode

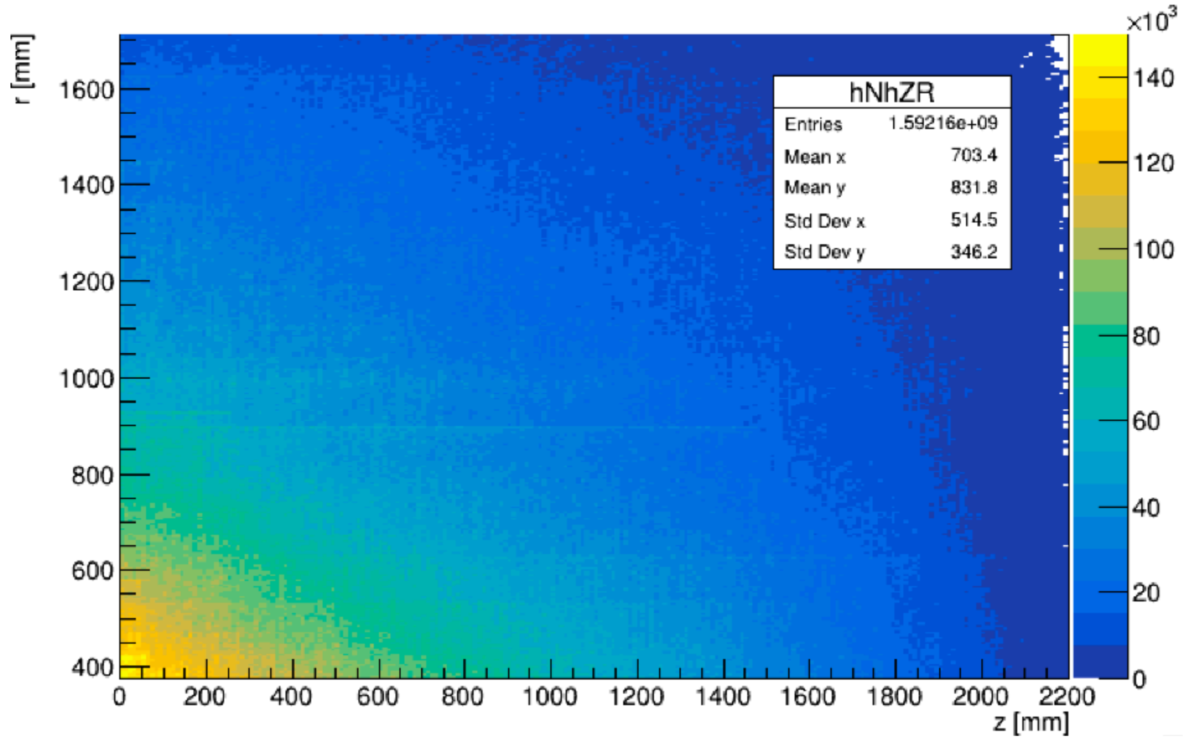
bin size: $(\Delta z, \Delta r)$=(1[cm], 0.5[cm]) Ez=230[V/cm] **ωτ=5.71** B=2[T]

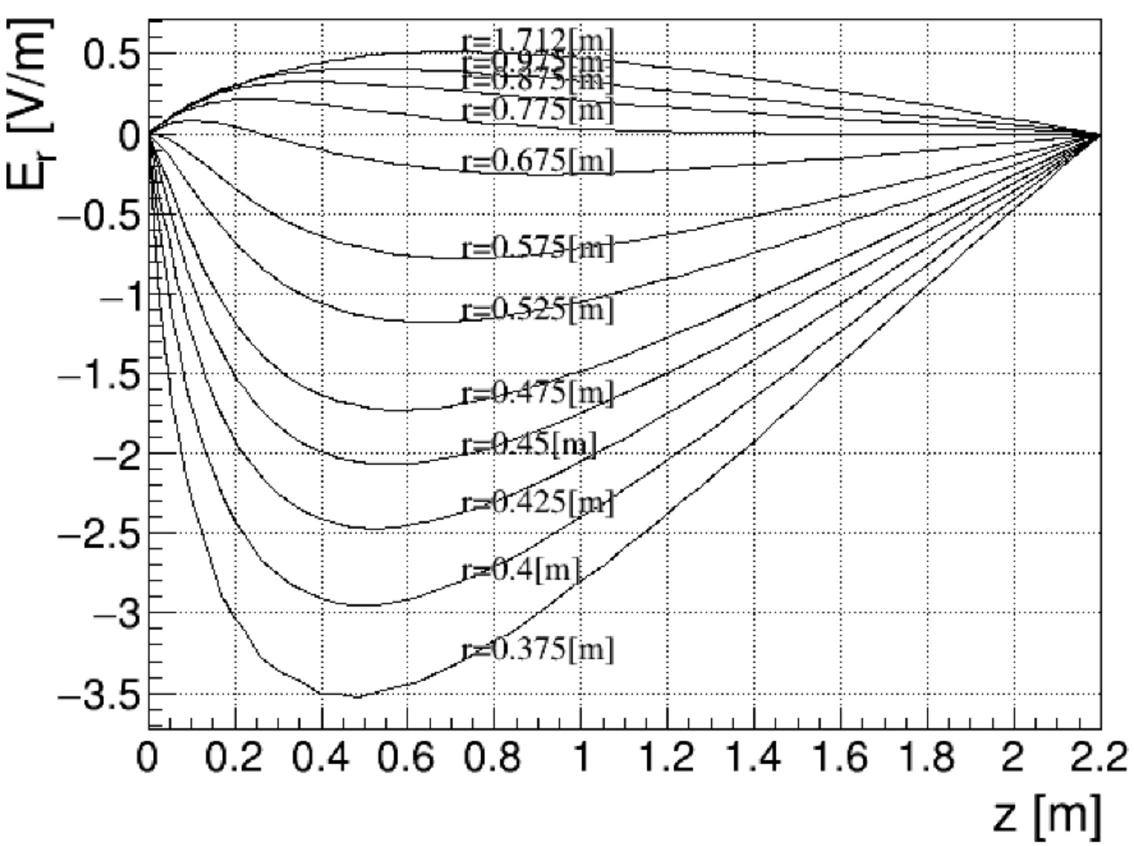
Glitches smoothed as expected

No visible difference in Δrφ!

What happens if the event rate is halved? (11k Z pole events)

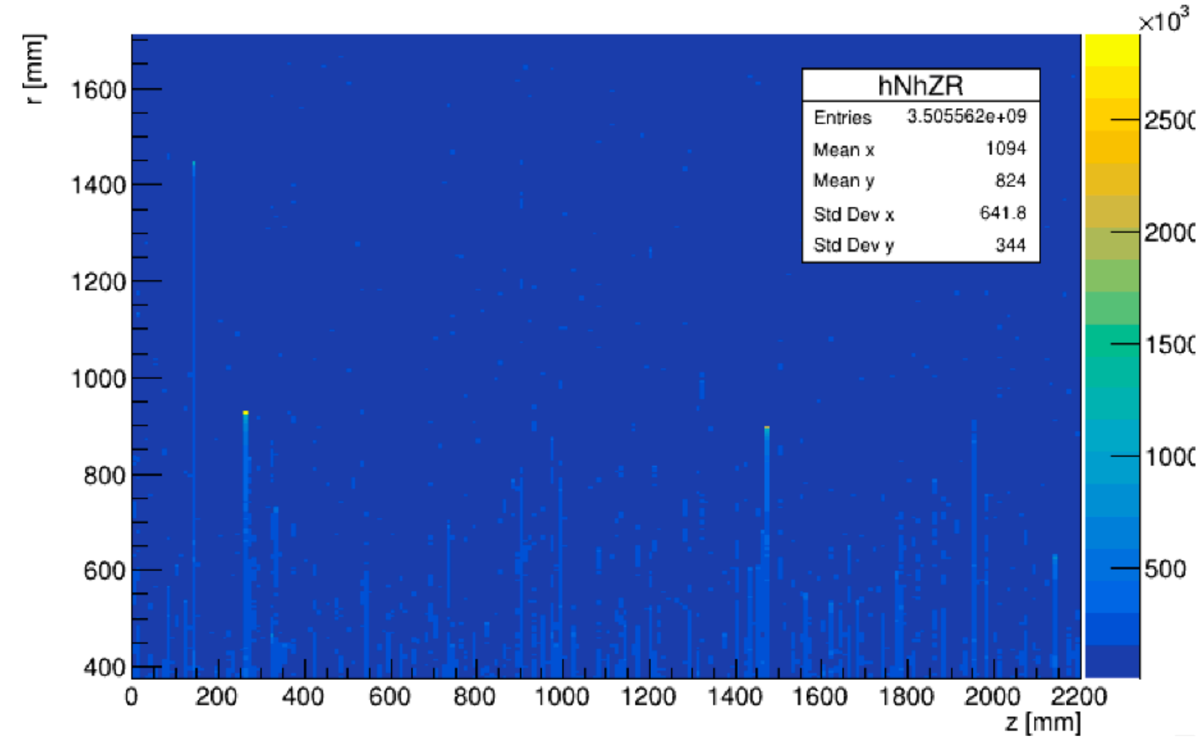
Primary Ions

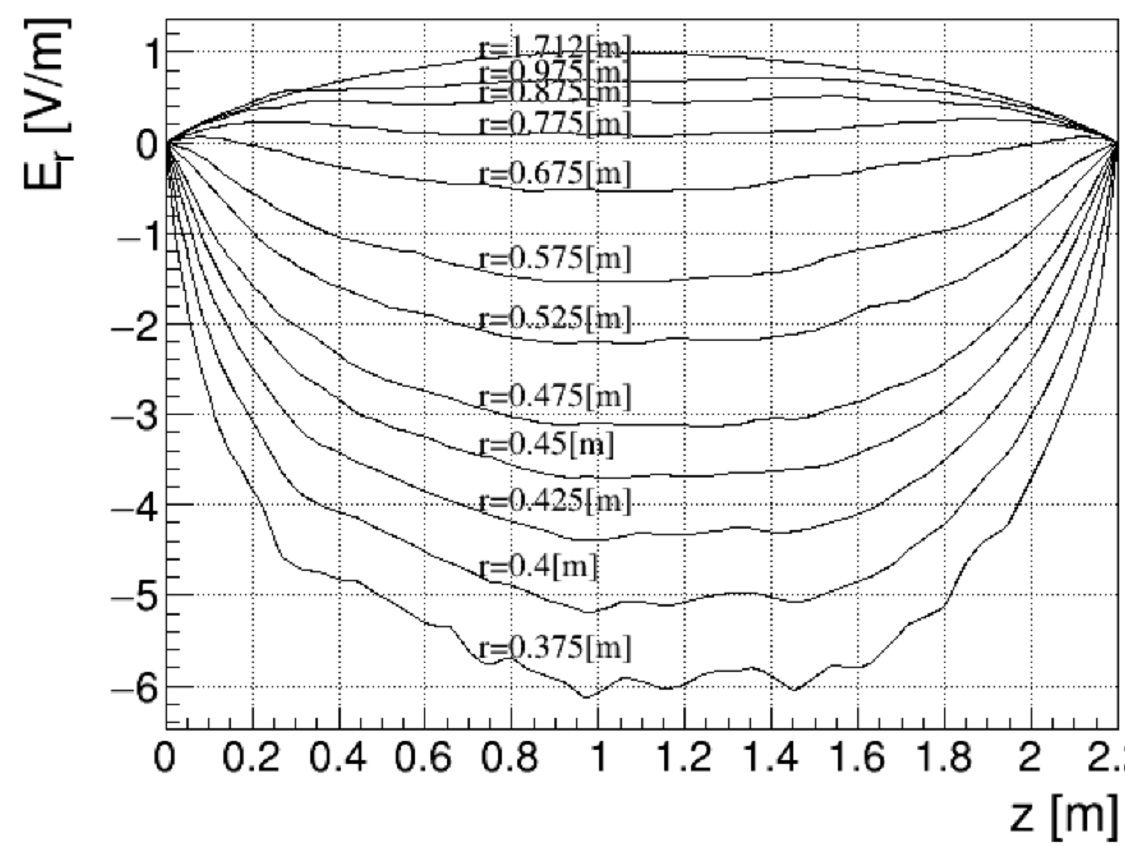




Er halved

Ion Back Flow



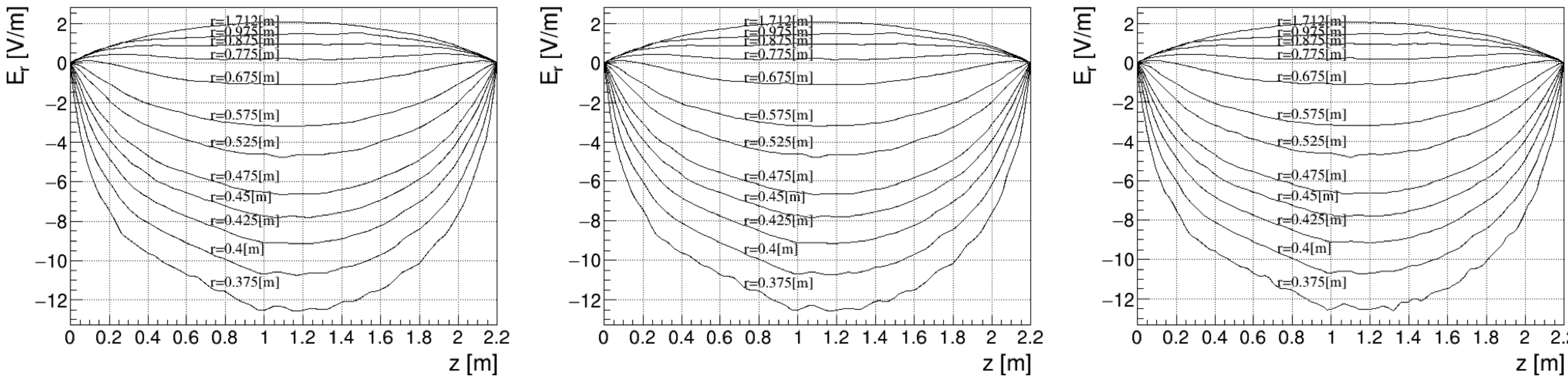


Er halved, more glitches

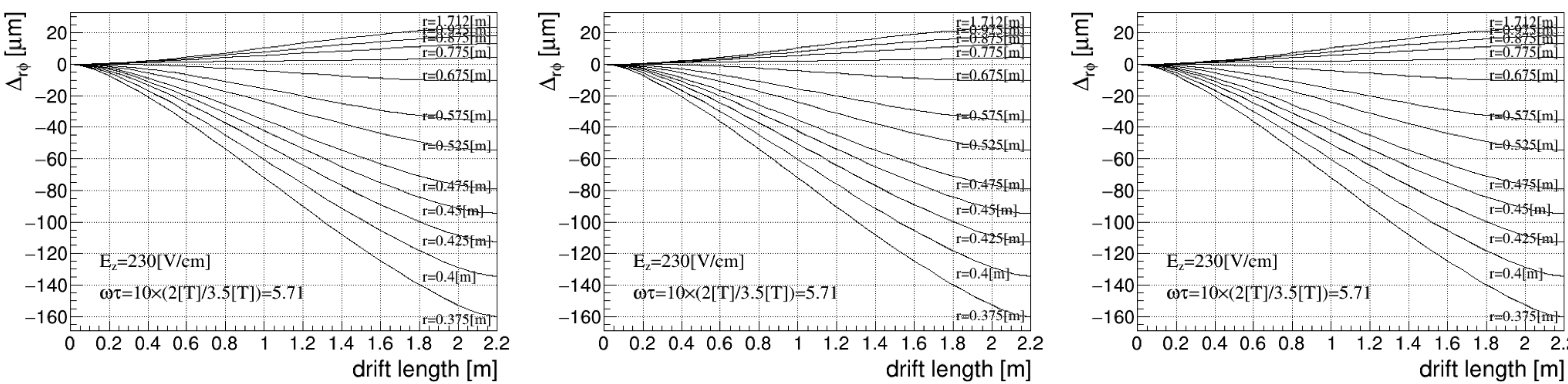
Positive Ion Back Flow (22k Z's): n and n_z high enough?

22k Z pole events

n = 40, nz = 50 → **n = 40, nz = 110** → **n = 220, nz = 110**



Higher n seems necessary to catch fine structure in Er



Nevertheless, glitches in Er seem to be averaged out in Δrϕ

→ For order of mag. Δrϕ estimate, n=40 and nz=50 seem OK.