

SMEFT Interpretation of Higgs Self-Coupling Measurements

see: [arXiv:1910.00012](https://arxiv.org/abs/1910.00012)
Chapter 9

Thanks to the editors:
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Since we have an excellent ability to predict both the signals and the backgrounds to Higgs boson processes within the Standard Model – especially at e^+e^- colliders – it follows that any observed deviation from these predictions implies the presence of physics beyond the Standard Model.

Often we state this the other way around: measurements that agree with the Standard Model predictions restrict the size of new interactions that violate the Standard Model.

At this level, no additional formalism is needed beyond calculations within the Standard Model.

However, often, we want to ask a more specific question:

If we see a deviation from the Standard Model predictions in some process, **to what extent is this evidence of a deviation in the value of the Higgs boson self-coupling ?**

Or, if a measurement of a process sensitive to the Higgs self-coupling agrees with the Standard Model, **to what extent does this measurement restrict the deviation of the Higgs self-coupling from its Standard Model value ?**

To address this question, we need a model.

The answer will be robust to the extent that the underlying model is a robust description of possible new physics.

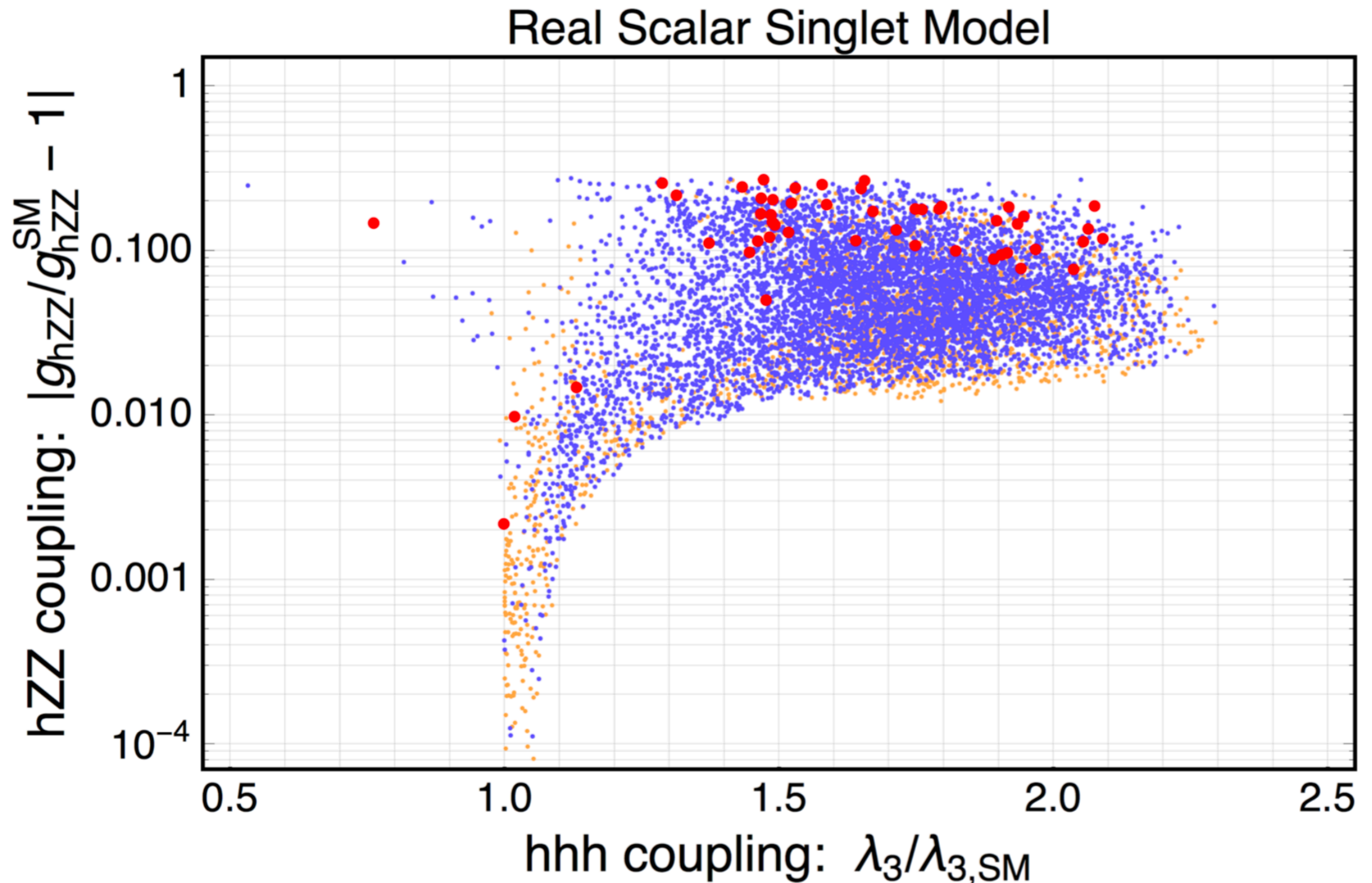
Often, the model is that the Higgs self-coupling takes an arbitrary value

$$\lambda = \kappa_\lambda \cdot \lambda_{SM}$$

while all other Higgs couplings remain at their SM values.

I do not believe that this model has much generality. Once we have modified the Higgs potential by some beyond-SM effect, shouldn't other Higgs couplings also be modified ?

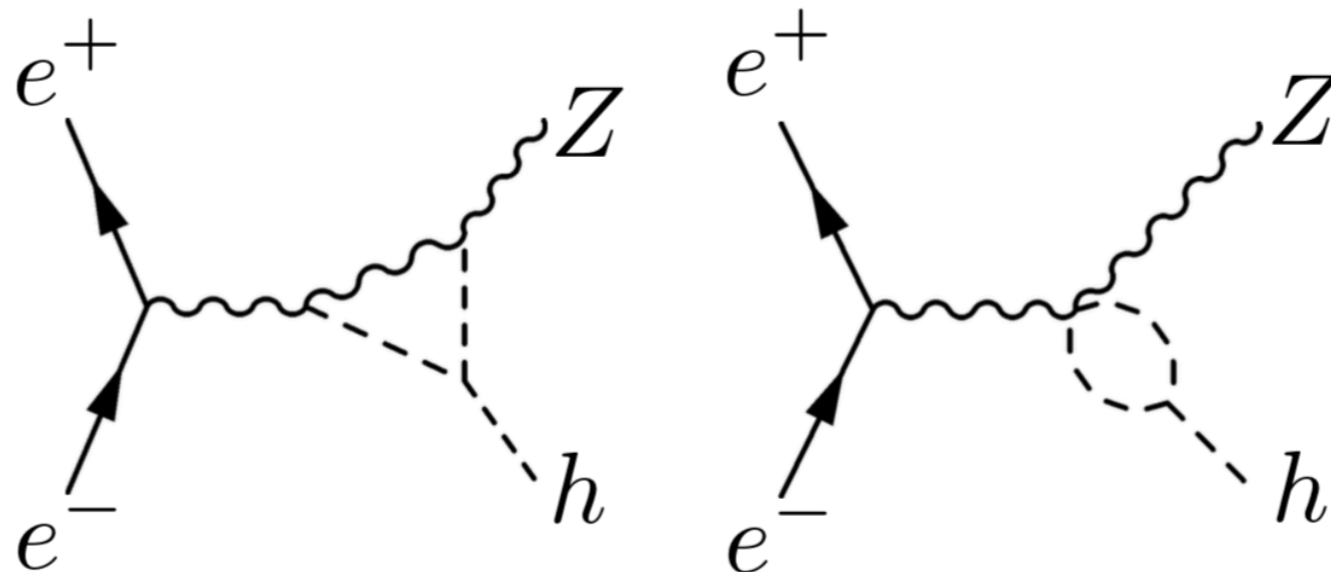
Huang, Long, Wang arXiv:1608.06619 consider a simple class of models in which the Higgs phase transition can become 1st order by mixing a singlet scalar with the SM Higgs doublet:



An example of the question this raises comes in the attempt to restrict κ_λ by precision measurements of single-Higgs production at e^+e^- colliders.

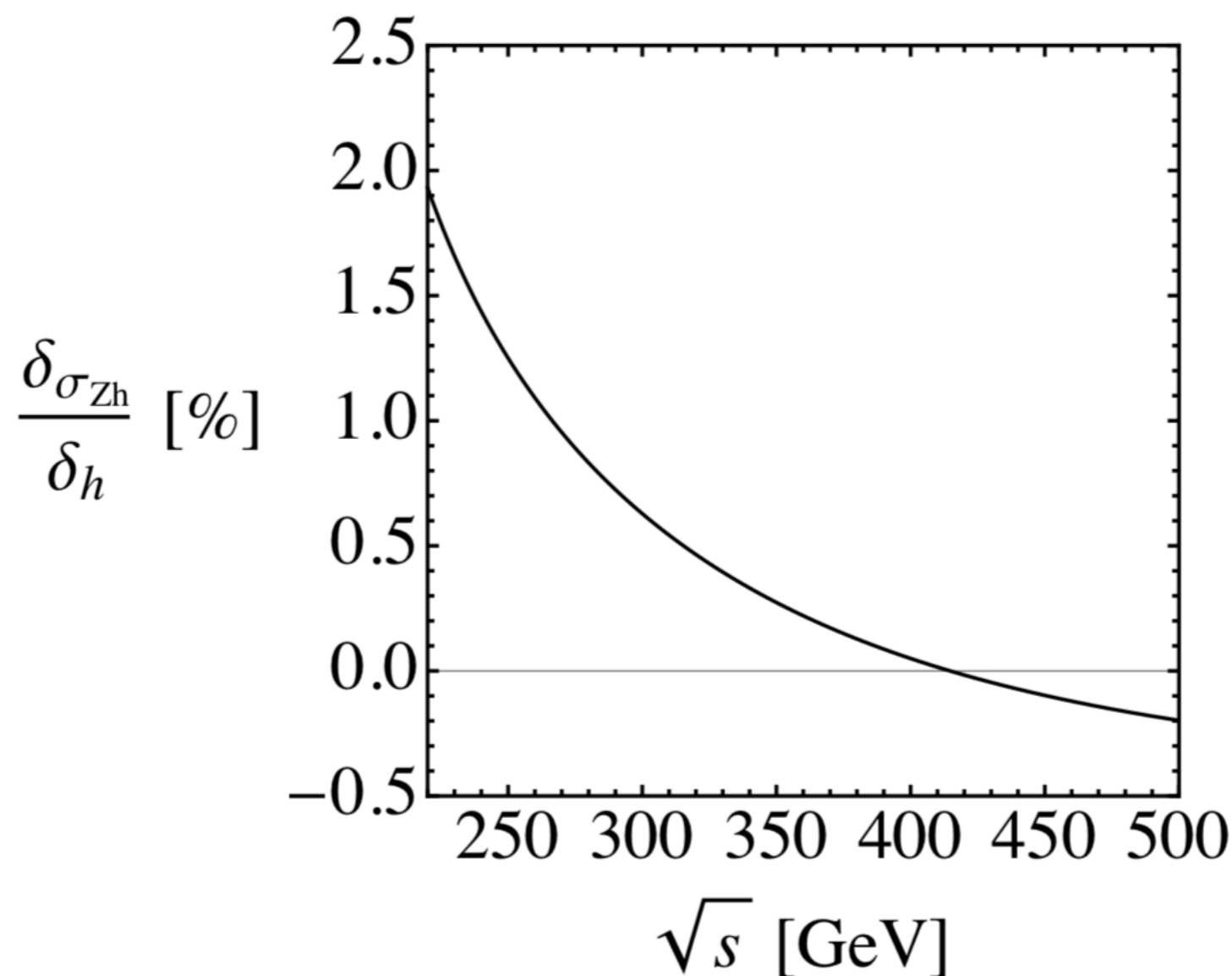
This is possible due to an effect emphasized by McCullough (arXiv:1312.3322):

At e^+e^- colliders, κ_λ does not enter the cross sections for single-h production at tree level. However, at loop level, there are radiative corrections to the Higgstrahlung process



and to the hZZ and hWW decay vertices.

The triangle diagram is singular at the $Z^* \rightarrow hZ$ threshold, leading to a 1.5% enhancement of the cross section near 250 GeV.



This effect is visible with the precision of e^+e^- Higgs factories.

Fitting with 1 parameter κ_λ , and assuming the underlying value to be $\kappa_\lambda = 1$, we find

for ILC (2 ab-1 at 250 GeV)

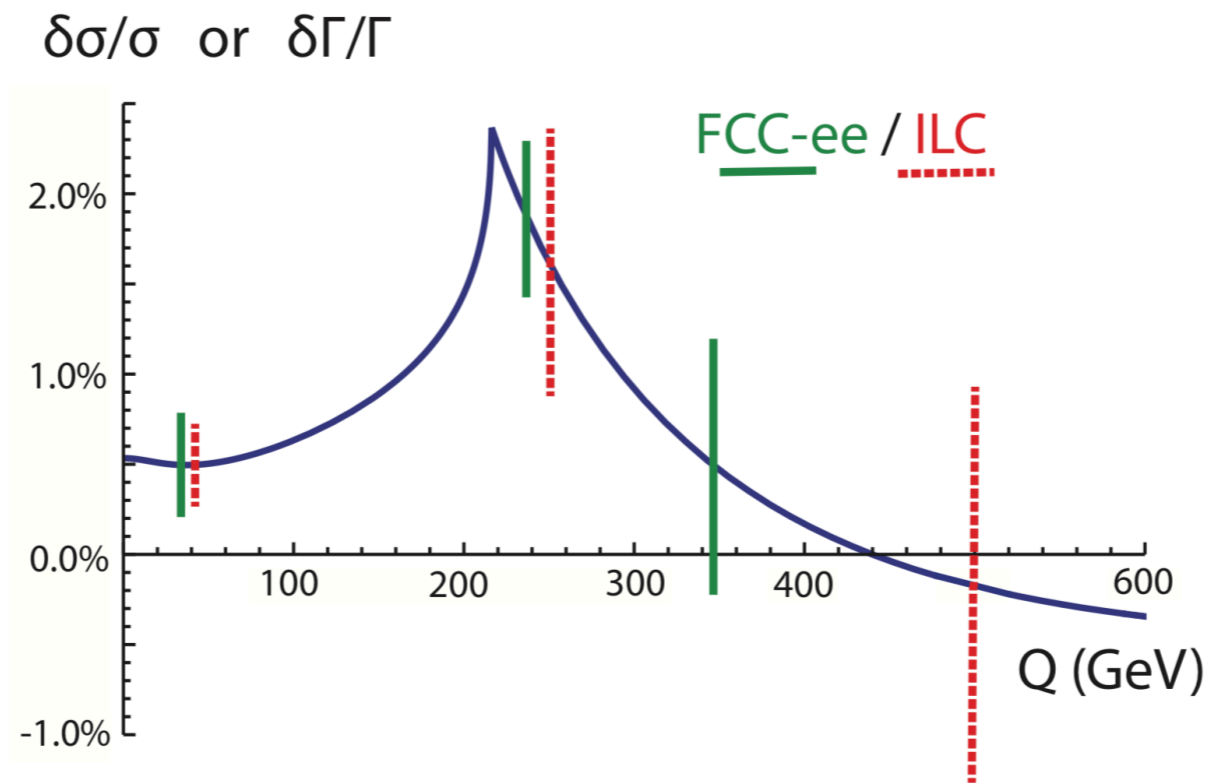
$$\delta\kappa_\lambda = 36\%$$

and for FCC-ee (5 ab-1 at 250 GeV)

$$\delta\kappa_\lambda = 21\%$$

However, an order 2% change in the normalization of the Higgstrahlung cross section can also result from an order 1% change in the hZZ coupling, which appears at the tree level. Adding this second parameter leaves κ_λ almost undetermined.

However, since McCullough's effect depends on CM energy, we still have power to determine κ_λ if we use two different energy points.



To understand the power of the Higgs factory measurements, though, it would be good to use a more general model of possible deviations of the other Higgs couplings.

To provide a very general model of beyond-SM deviations, I would like to use a general extension of the SM to SM Effective Field Theory (SMEFT).

In SMEFT, we add to the SM (already the most general Lagrangian of dimension 4 with the SM gauge symmetries) operators of higher dimension:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{\hat{c}_i}{M^2} \mathcal{O}_i + \sum_j \frac{\hat{d}_j}{M^4} \mathcal{O}_j + \dots ,$$

This is the most general Lagrangian of a model with the SM gauge symmetries and particle content. BSM particles are assumed to be of mass M or larger and are integrated out.

Already at dimension 6, there are 79 independent coefficients for 1 generation and 2499 for 3 generations. However, at e^+e^- colliders, we can restrict this to a manageable number of operators. Then SMEFT can be used as a general model of new physics.

Here, I will impose the following restrictions:

1. Keep only operators of dimension 6, and treat them in linear order only.
2. Keep only operators that enter e^+e^- reactions at the tree level.
3. Ignore CP-violating operators. These affect CP-even observables only in second order.

4. Ignore flavor violating Higgs couplings. Test separately to bound these. More generally, for this talk, it makes very little difference whether we include or not the possibility of Higgs exotic decays.
5. For this talk, I will assume lepton flavor universality and ignore 4-fermion contact operators. It can be shown that including these effects does not change the results.

This leads to a set of **17** operators, plus **4** SM parameters, plus possible parameters for exotic decays, that need to be fit.

Since the SMEFT Lagrangian is **THE** Lagrangian, we can determine these parameters using data from **all** processes measured at e⁺e⁻ colliders:

Higgs, WW, precision electroweak, etc.

For a very general discussion of SMEFT fitting to data from current and future colliders, see

J. de Blas et al., “Global SMEFT Fits at Future Colliders”, arXiv:[2206.08326](https://arxiv.org/abs/2206.08326)

For the study of the Higgs self-coupling, it turns out that the most important of the dimension-6 operators are:

$$\Delta\mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 + \frac{c_{WW}}{2v^2} \Phi^\dagger \Phi W_{\mu\nu}^a W^{a\mu\nu} \\ + i \frac{c_{HL}}{v^2} J_H^\mu (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} J_H^{a\mu} (\bar{L} \gamma_\mu t^a L) + i \frac{c_{HE}}{v^2} J_H^\mu (\bar{e} \gamma_\mu e),$$

where Φ is the SM Higgs doublet and

$$J_H^\mu = \Phi^\dagger \overleftrightarrow{D}_\mu \Phi \quad J_H^{a\mu} = \Phi^\dagger t^a \overleftrightarrow{D}_\mu \Phi$$

The c_{HL}, c'_{HL}, c_{HE} are mainly constrained by precision electroweak measurements.

c_H leads to an overall rescaling of the Higgs decay partial widths.

$$c_6 = (\kappa_\lambda - 1)$$

Using this formalism, we can make quantitative the remarks above on fitting for κ_λ from the rates of single Higgs processes.

At 250 GeV,

$$\begin{aligned} \sigma(Zh)/\sigma(Zh)_{SM} = & 1 + 0.015 c_6 - c_H + 4.7 c_{WW} \\ & + 13.9 (c_{HL} + c'_{HL}) - 12.1 c_{HE} + \dots \end{aligned}$$

This makes clear that, at any one energy point, the effect of κ_λ or c_6 is easily compensated by tree-level effects of other SMEFT operators.

The second line contains the largest numerical coefficients, but the corresponding c terms are highly constrained by precision electroweak measurements.

Including the full proposed program, projected errors on κ_λ are

ILC: 250 + 500 GeV

25% c_6 only

41% $c_6 + c_H$

52% full e+e- SMEFT model

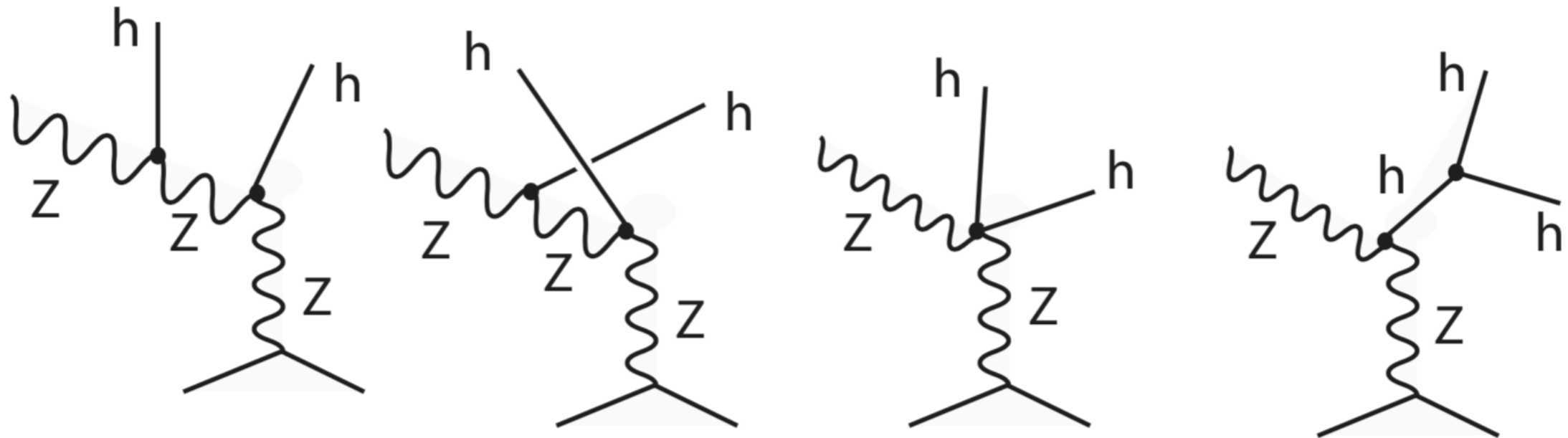
FCC-ee 240 + 365 GeV

16% c_6 only

42% $c_6 + c_H$

47% full e+e- SMEFT model

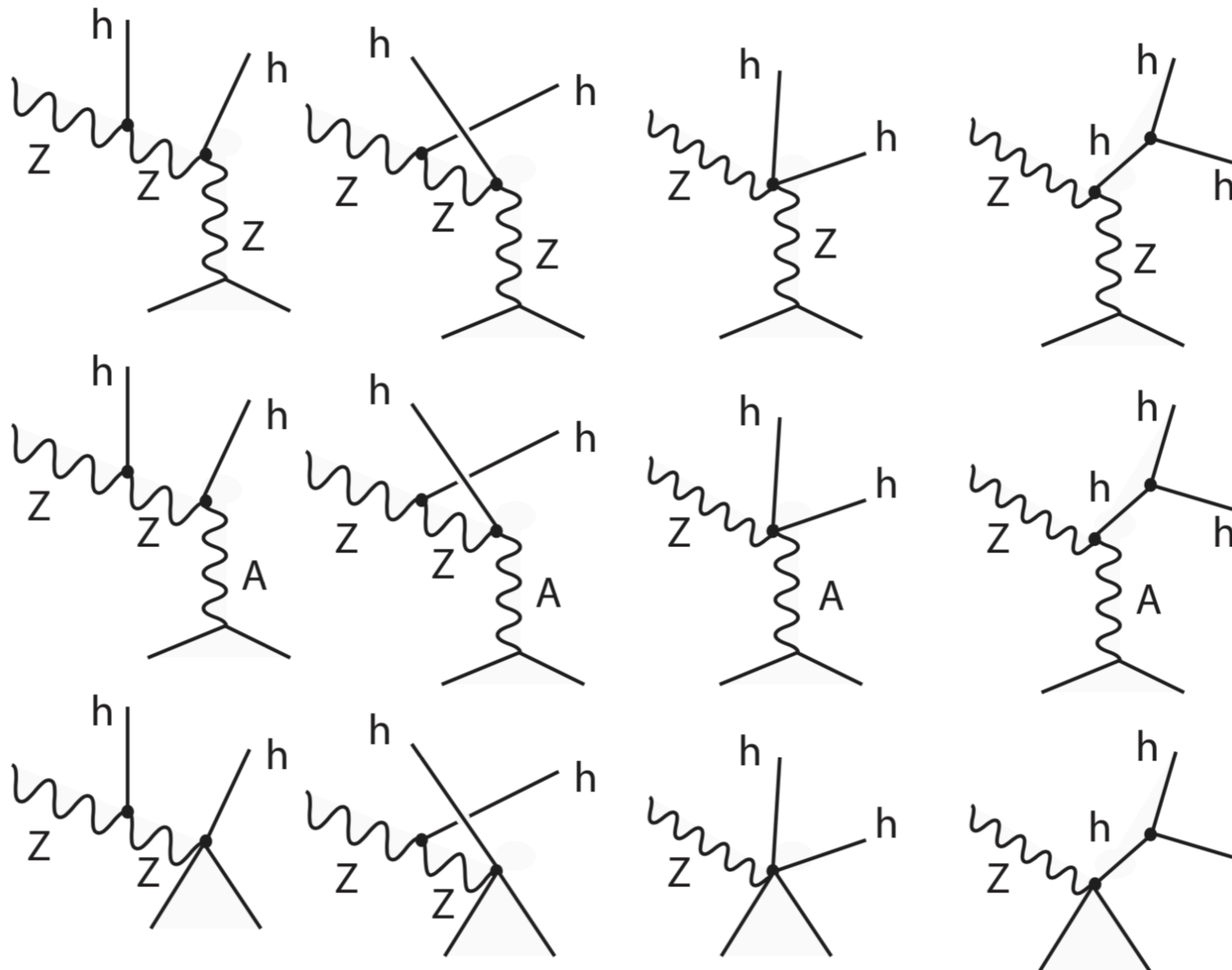
At linear colliders operating at 500 GeV and above, we can also measure double Higgs production through $e^+e^- \rightarrow Zhh$. The SM diagrams are



Julie just showed the latest ILC projections for the determination of κ_λ in a 1-parameter fit.

It is important to ask: **Is this result modified when considered in the full e^+e^- SMEFT model ?**

The tree-level expression for $e^+e^- \rightarrow Zhh$ is more complicated, with more diagrams and additional terms in all vertices:



We find for the major dependences
(Barklow et al. arXiv:1708.09079)

$$\sigma(Zhh)/\sigma(Zhh)_{SM} = 1 + 0.56 c_6 - 4.15 c_H + 15.1 c_{WW} \\ + 62.1 (c_{HL} + c'_{HL}) - 53.5 c_{HE} + \dots$$

Some large numerical coefficients appear. The uncertainties in these terms should be accounted as **additional systematic errors** in the determination of c_6 .

However, the associated c coefficients are constrained by precision electroweak and single-Higgs cross section measurements. We find that, after the projected ILC measurements, the uncertainties in these terms should be

$$\begin{array}{ll} c_H, c_{WW} \text{ terms:} & 2.8\% \\ c_{HL}, c'_{HL}, c_{HE} \text{ terms:} & 0.85\% \end{array}$$

So in fact there is no issue. The quoted error on κ_λ can be taken at face value.

How does this reflect on the LHC extraction of κ_λ ?

At the LHC, the main constraint comes from double Higgs production. Here, the SM term is loop-level in QCD, while the dimension-6 SMEFT terms include the operator

$$\frac{c_{GG}}{v^2} \Phi^\dagger \Phi G_{\mu\nu}^a G^{a\mu\nu}$$

which induces new tree-level diagrams



Fortunately, this operator is constrained to the $\sim 10\%$ level by measurement of the single Higgs ($gg \rightarrow h$) total cross section and should be constrained to the few-percent level at HL-LHC.

Constraints from single-Higgs production, which involve loop-level corrections from κ_λ and tree-level corrections from other SMEFT operators, need to be subjected to the same type of analysis as was used earlier for e+e- processes.

Finally, there is one element missing from both the LHC and the e^+e^- analyses. In both cases, double Higgs production is deeply buried under various SM backgrounds. The systematic errors in the estimation of these backgrounds due to uncertainties in SMEFT coefficients ought also to be included in the analyses.

This is more straightforward in e^+e^- , where the number of SMEFT operators to be considered is smaller, and the coefficients of these operators can be determined from experiment without ambiguity.

In all, I expect that extractions of κ_λ from double Higgs production will be free of model-dependent uncertainties both for HL-LHC (50%-level uncertainty) and for e+e- linear colliders at 500 GeV (20%-level uncertainty).

Experiments at much higher energy seeking few-percent uncertainties on κ_λ must take this source of systematic error into account.

Hopefully, though, those experiments will uncover new physics that contributes to the Higgs potential and dramatically alters the questions that we ask about the physics of electroweak symmetry breaking.