

BSM Triple Higgs Couplings at future e^+e^- colliders

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Based on [arxiv:2005.10576](https://arxiv.org/abs/2005.10576), [arxiv:2106.11105](https://arxiv.org/abs/2106.11105) and [arxiv:2203.12684](https://arxiv.org/abs/2203.12684)



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Motivation

BSM Higgs sectors are still allowed by the measurements of the properties of the 125 GeV Higgs boson

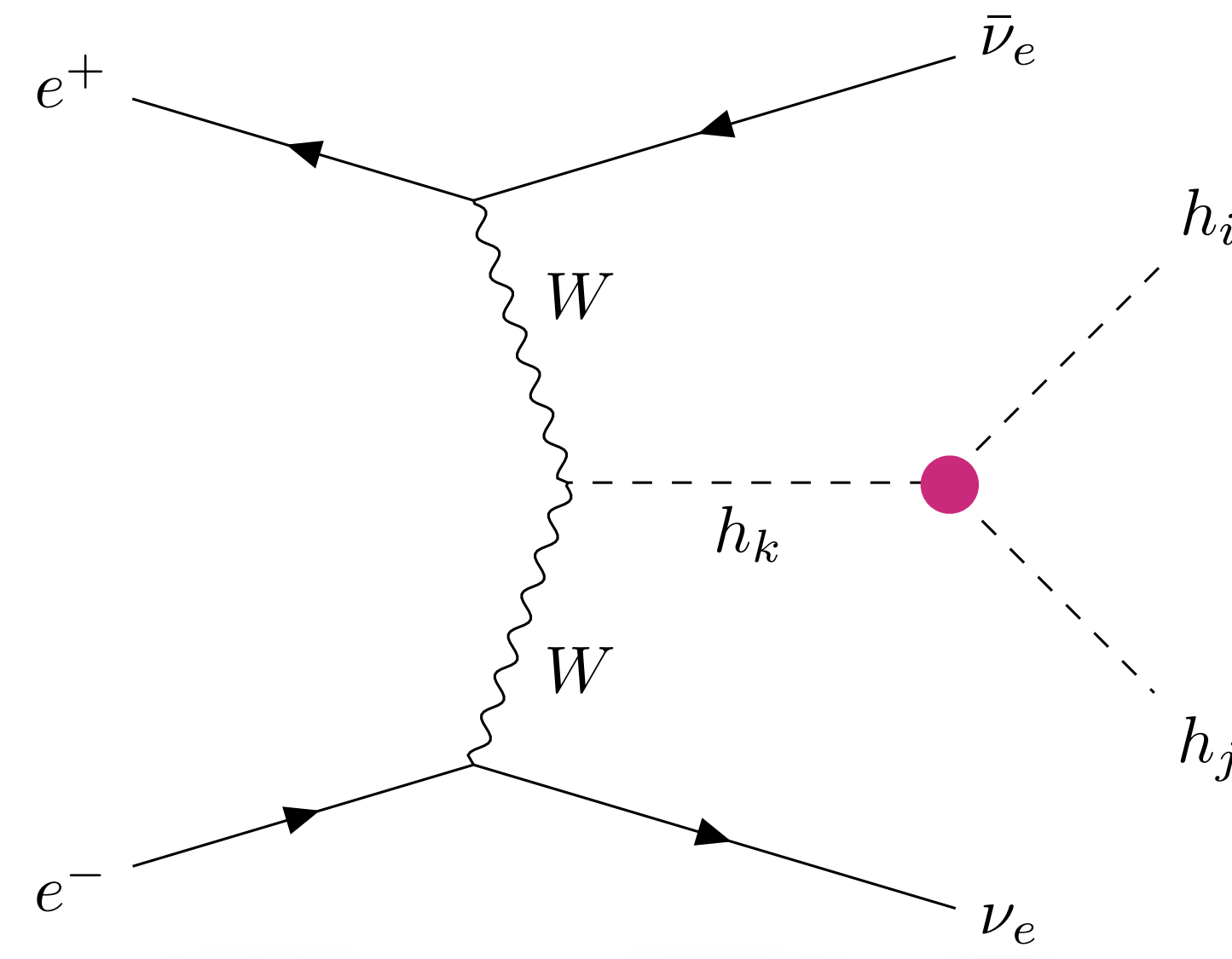
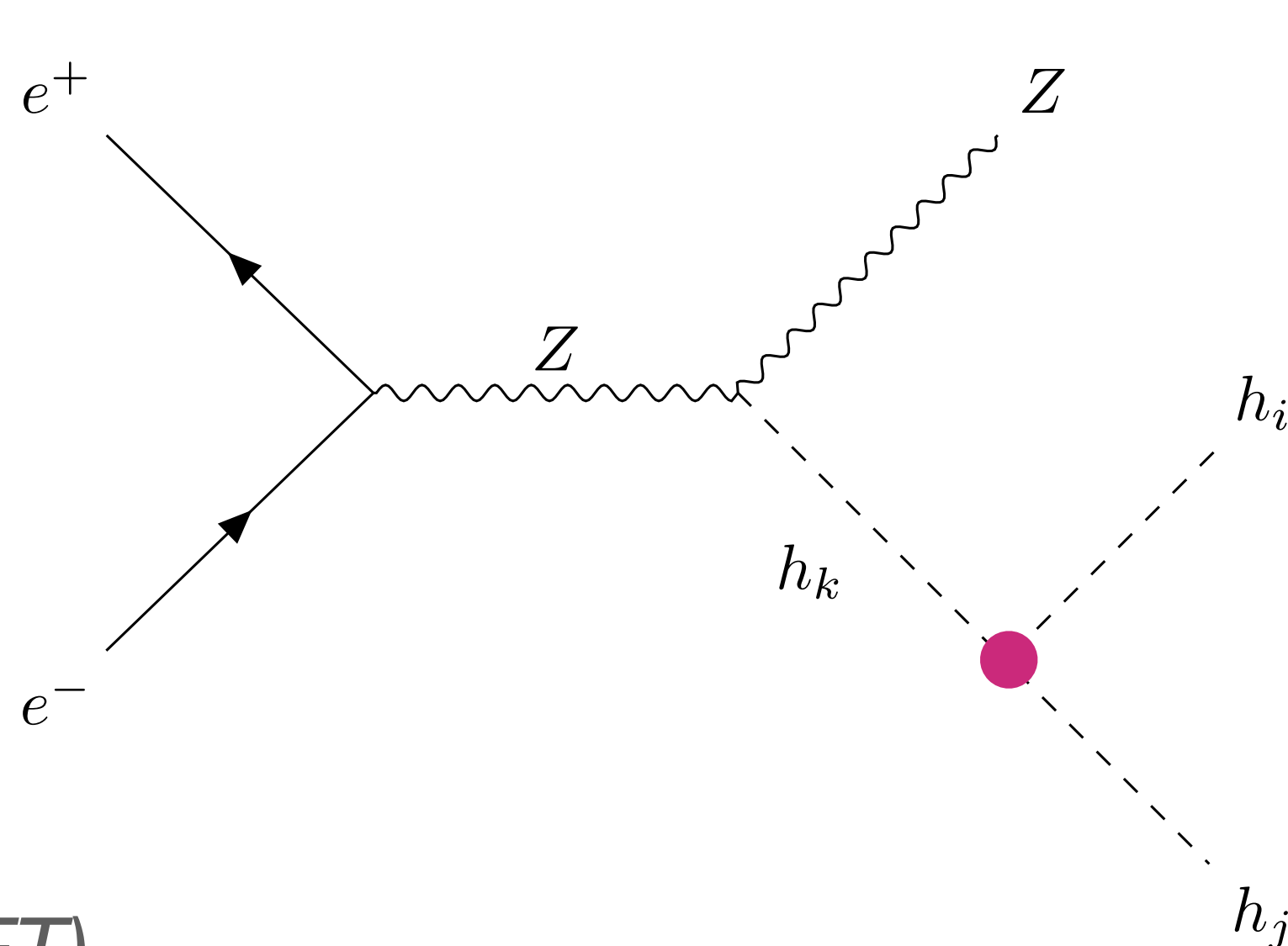
In the 2HDM, triple Higgs couplings $\lambda_{h_i h_j h_k}$ can be large while respecting all the relevant constraints

(Eur.Phys.J.C 80 (2020) 9, 884 [arXiv:2005.10576](#) and Eur.Phys.J.C 82 (2022) 6, 536 [arXiv:2203.12684](#))



Di-Higgs production could give access to $\lambda_{h_i h_j h_k}$ at tree level

Two channels of interest: $e^+ e^- \rightarrow h_i h_j Z$ and $e^+ e^- \rightarrow h_i h_j \nu \bar{\nu}$ with $h_i h_j = hh, hH, HH, AA$



*(Eur.Phys.J.C 81
(2021) 10, 913
[\[arxiv:2106.11105\]](#))*

The Two Higgs Doublet Model (2HDM)

Adding a second Higgs doublet to the SM \implies 5 physical Higgs bosons: h, H, A and H^\pm

POTENTIAL:

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

- CP conservation
- Z_2 symmetry to avoid FCNC: softly broken by m_{12}^2
 - 4 possible Yukawa structures

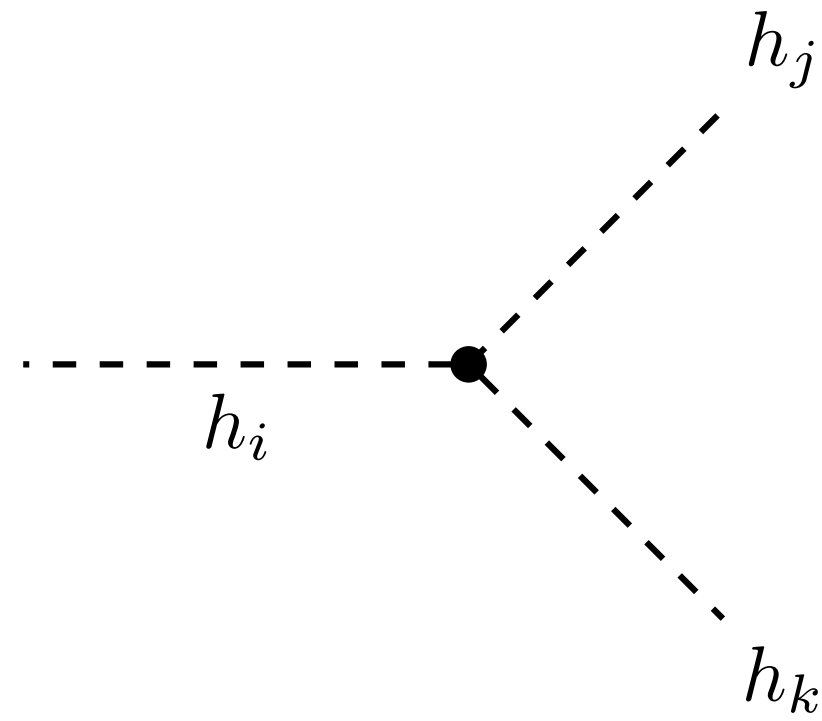
INPUT PARAMETERS:

m_h ($= 125$ GeV), m_H, m_A, m_{H^\pm} , $\tan\beta := v_2/v_1$, $\cos(\beta - \alpha) \equiv c_{\beta-\alpha}$ and m_{12}^2

Alignment limit: $c_{\beta-\alpha} \rightarrow 0$, the SM interactions for h are recovered

Triple Higgs Couplings (THC)

COUPLING DEFINITION



$$= -i v n! \lambda_{h_i h_j h_k}$$

and

$$\kappa_\lambda := \lambda_{hhh} / \lambda_{hhh}^{\text{SM}}$$

at tree level

FINAL ALLOWED RANGES

updated ranges from [arXiv:2203.12684](https://arxiv.org/abs/2203.12684)!

	TYPE I	TYPE II	TYPE III (Y)	TYPE IV (X)
$\kappa_\lambda \in$	$[-0.5, 1.3]$	$[0.6, 1.0]$	$[0.6, 1.0]$	$[0.5, 1.0]$
$\lambda_{hhH} \in$	$[-1.7, 1.6]$	$[-1.8, 1.5]$	$[-1.8, 1.3]$	$[-1.8, 1.4]$
$\lambda_{hHH} \in$	$[-0.7, 15]$	$[-0.5, 16]$	$[-0.3, 16]$	$[-0.6, 9]$
$\lambda_{hH^+H^-} \in$	$[-1.8, 33]$	$[-1.4, 33]$	$[-1.3, 33]$	$[-1.7, 33]$
$\lambda_{hAA} = \lambda_{hH^+H^-} / 2$				

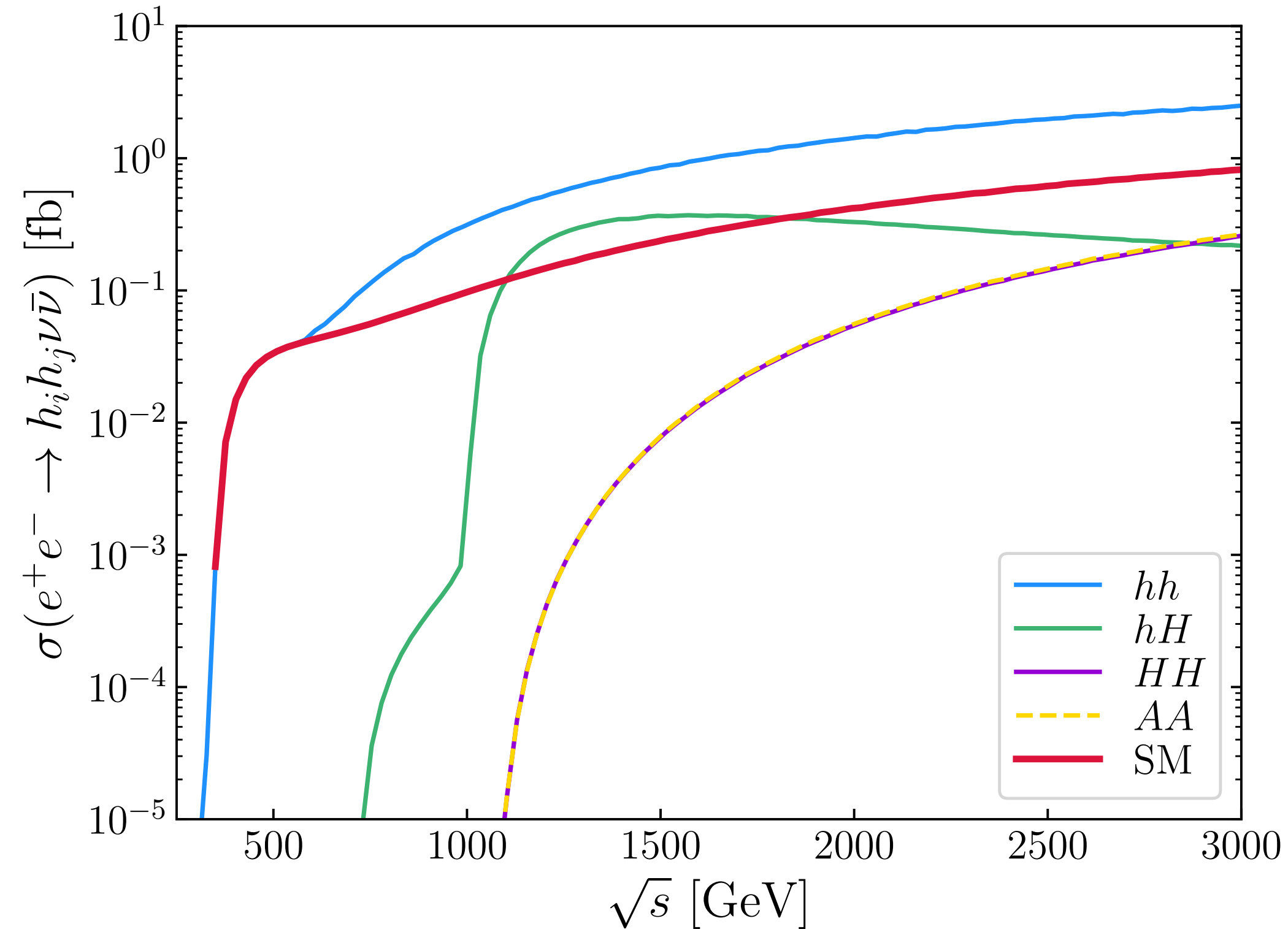
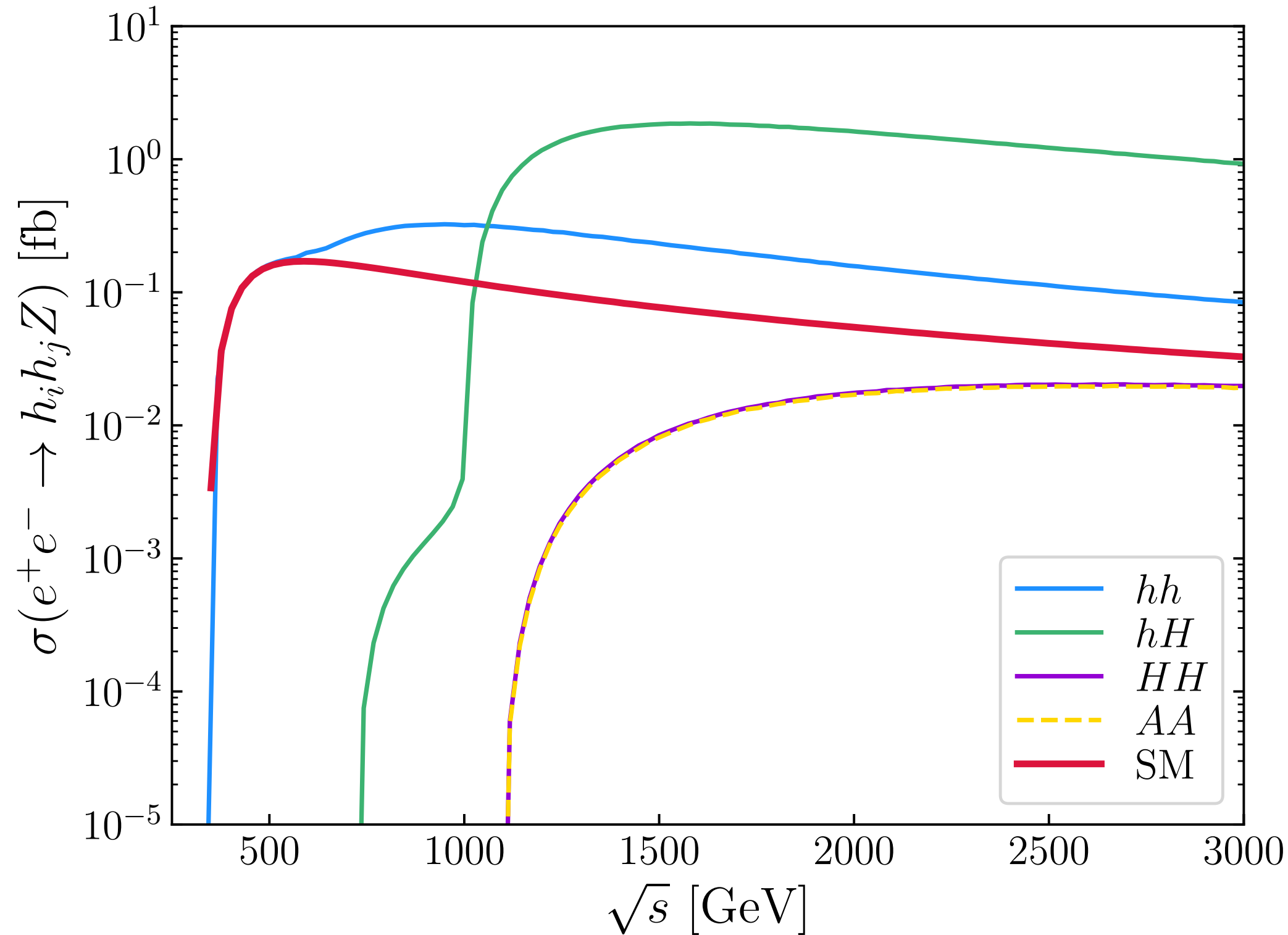
CONSTRAINTS

- Electroweak precision data, T parameter: motivates (before CDF II) scenarios with degenerate masses, $m_H = m_A = m_{H^\pm} \equiv m$
- Tree level **unitarity** and potential **stability**:
 - $m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$ helps to satisfy these constraints
- Collider **measurements of the 125 GeV Higgs**
 - Close to $\cos(\beta - \alpha) = 0$, specially for type II
- **BSM Higgs searches** at LEP, TeVatron and LHC
- **Flavor observables**: $\text{BR}(B \rightarrow X_s \gamma)$ and $\text{BR}(B_s \rightarrow \mu\mu)$

2HDMC, HiggsBounds, HiggsSignals and superISO were used

Dependence with energy

Madgraph was used



Type I
 $m = 500 \text{ GeV}$
 $\tan \beta = 10$
 $c_{\beta-\alpha} = 0.2$
 $m_{12}^2 = 24000 \text{ GeV}^2$

↓

$\kappa_\lambda := \lambda_{hhh} / \lambda_{hhh}^{SM} \simeq 1$
 (num. cancelation)
 $\lambda_{hhH} = -0.5$
 $\lambda_{hHH} = \lambda_{hAA} = 6$
 $\lambda_{hH^+H^-} = 12$

Z channel → decreases with the energy

$\nu\bar{\nu}$ channel → increases with the energy (VBF topologies!)

- hhZ and $hh\nu\bar{\nu}$: ~ 3 times the SM due to resonant diagrams mediated by H (contains λ_{hhH}) and A (without THC)
- hHZ and $hH\nu\bar{\nu}$: dominated by A mediated diagrams but still sensitive to THC at large energies
- $HH\nu\bar{\nu} \sim AA\nu\bar{\nu}$: dominated at large energies by λ_{hHH} (λ_{hAA}) if large enough

Methodology

- XS presented in some **benchmark planes** with large (and allowed) THC
- **ALL** diagrams included (**no NWA!**)
- Access to THC via XS distributions on the **invariant mass of the final $h_i h_j$ pair**
- We studied effects from THC in $m_{h_i h_j}$ for 5 benchmark points (BP) with a wide range of BSM Higgs masses
- In the case of the sensitivity to λ_{hhH} at hh production we propose a theoretical estimator from **4- b jets** events

Collider	\sqrt{s} [GeV]	\mathcal{L}_{int} [ab^{-1}]
ILC	500	4
ILC	1000	8
CLIC	1500	2.5
CLIC	3000	5

Point	Type	m	$\tan \beta$	$c_{\beta-\alpha}$	m_{12}^2
BP1	I	300	10	0.25	Eq. (8)
BP2	I	500	7.5	0.1	32000
BP3	I	600	10	0.2	Eq. (8)
BP4	I	1000	8.5	0.08	Eq. (8)
BP5	II	650	1.5	0.02	10000

(Eq. (8) $\rightarrow m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$)

MadGraph, FeynRules, 2HDMC
and ROOT were used

hh production

Three main effects different from the SM:

1. Deviations from $\kappa_\lambda = 1$

Diagrams with κ_λ has a positive interference in the Z channel and negative in the neutrino channel

2. H mediated resonant diagrams (contains λ_{hhH})

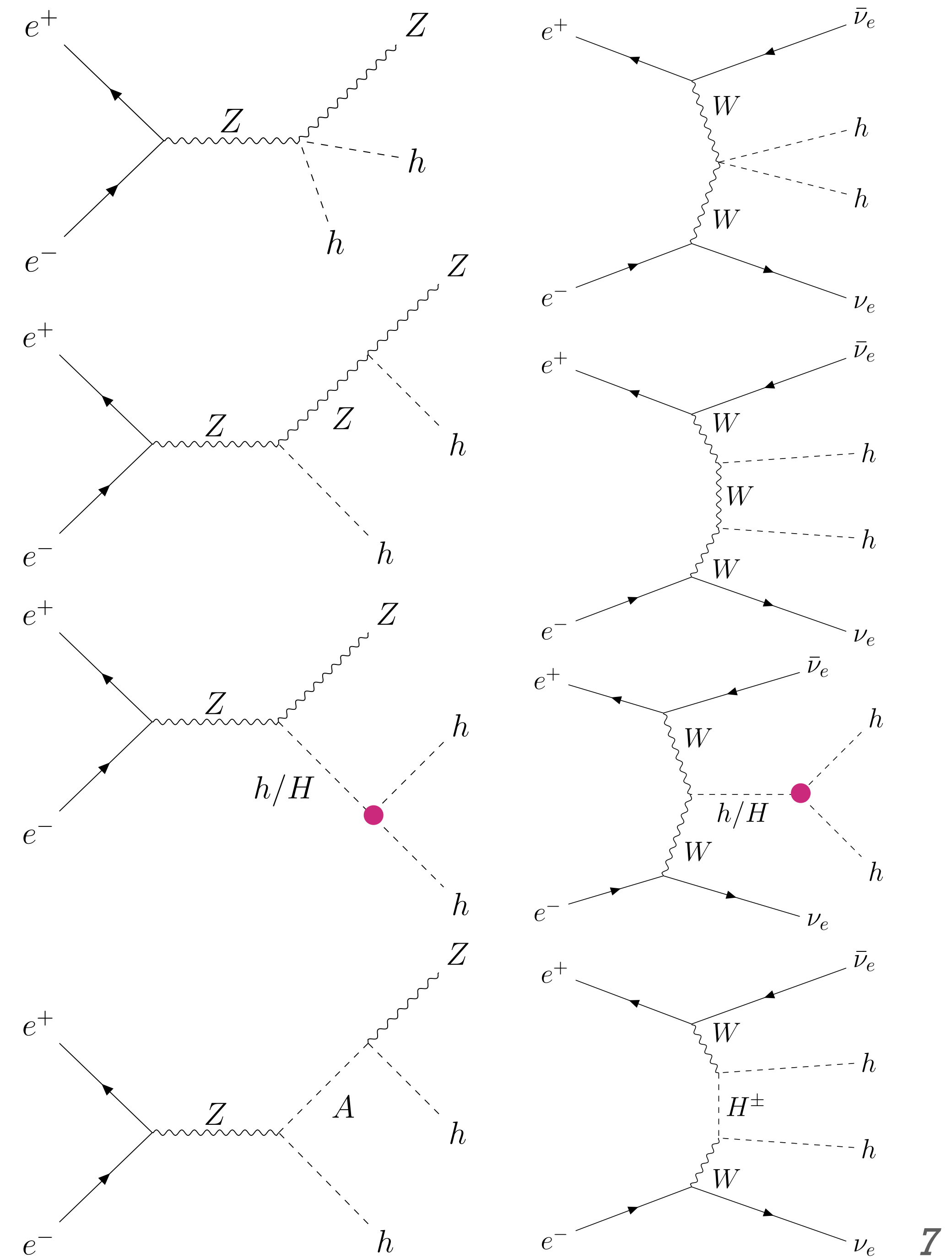
$$e^+e^- \rightarrow H^*Z \rightarrow hhZ \quad (\rightarrow hh\nu\bar{\nu})$$

$$e^+e^- \rightarrow H^*\nu_e\bar{\nu}_e \rightarrow hh\nu_e\bar{\nu}_e$$

3. A mediated resonant diagrams (no sensitivity to THC)

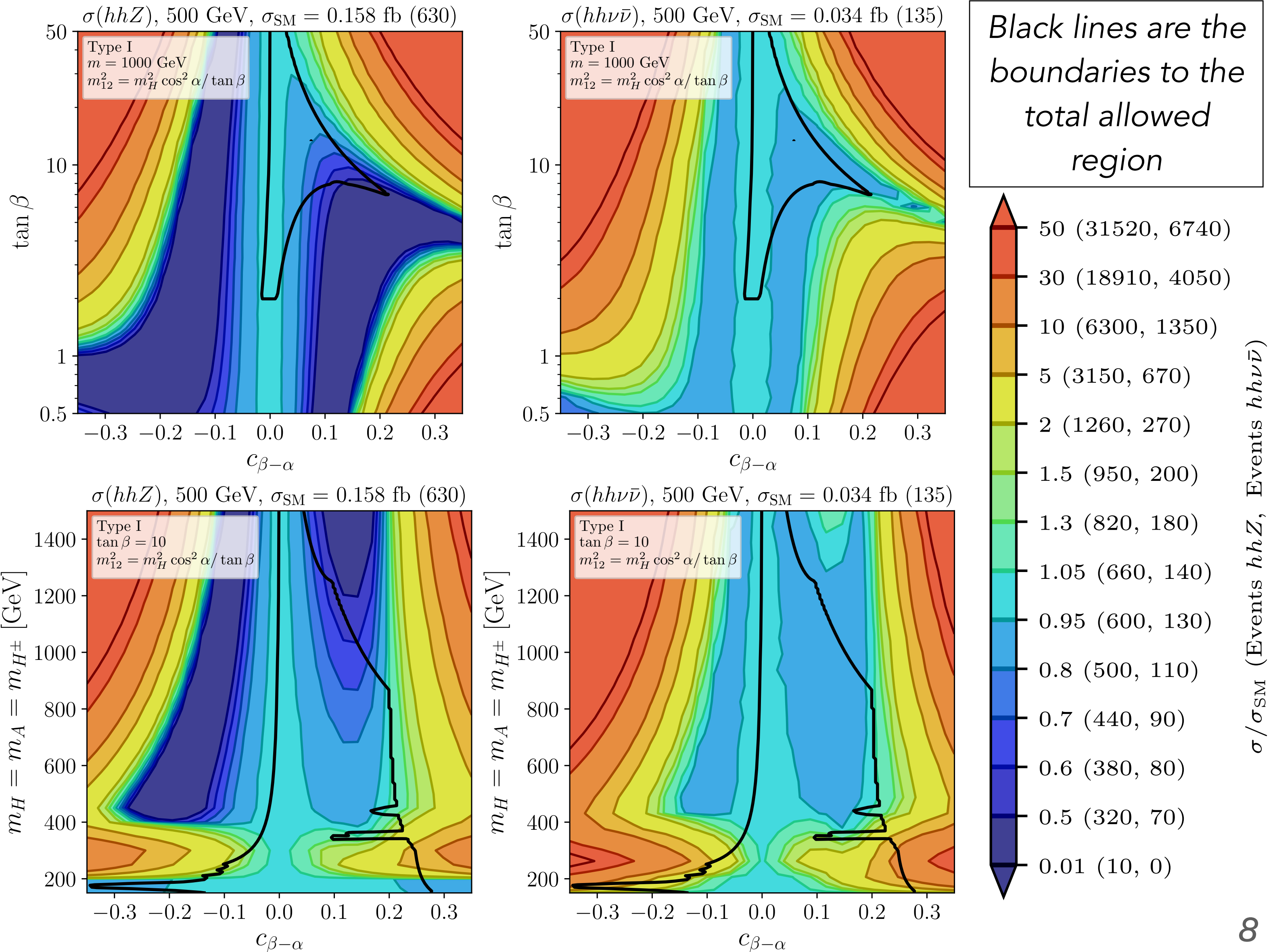
$$e^+e^- \rightarrow hA^* \rightarrow hhZ \quad (\rightarrow hh\nu\bar{\nu}) \quad \text{(More relevant for the } Z \text{ channel)}$$

If $c_{\beta-\alpha} = 0$ we recover the SM prediction



hh production, ILC 500GeV (type I)

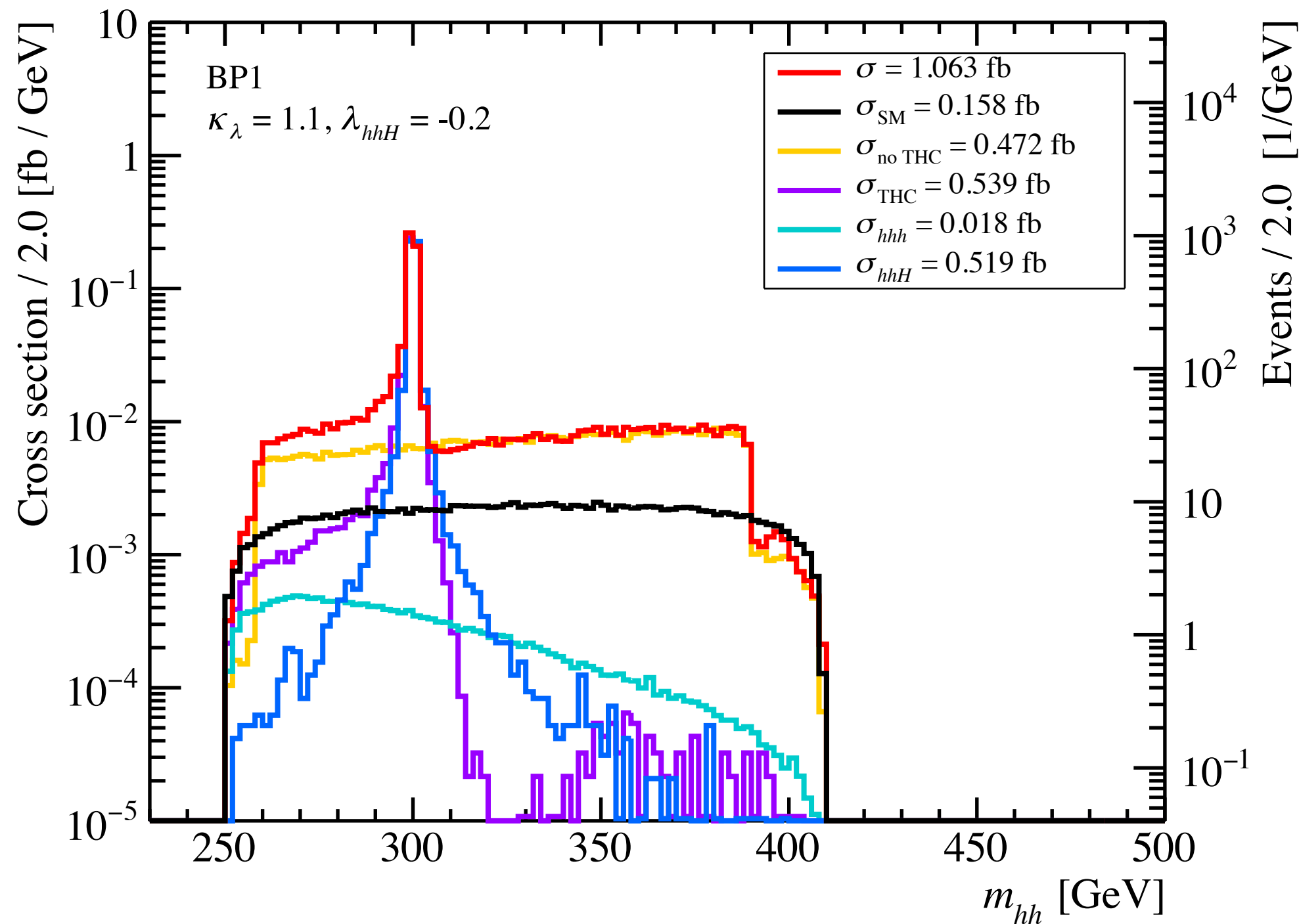
- Production XS for hhZ and $hh\nu\bar{\nu}$ wrt the SM @ILC500GeV
 - hhZ is the dominant channel
 - $c_{\beta-\alpha}$ – $\tan\beta$ plane: H and A are too heavy \rightarrow effect of κ_λ
 - $c_{\beta-\alpha}$ – m plane: the H (λ_{hhH}) and A (no THC) resonances manifest for low masses
 - The enhancement wrt the SM in $hh\nu\bar{\nu}$ is larger, but the absolute XS is smaller
- \Rightarrow Access to THC via the XS distributions on the **invariant mass of hh**



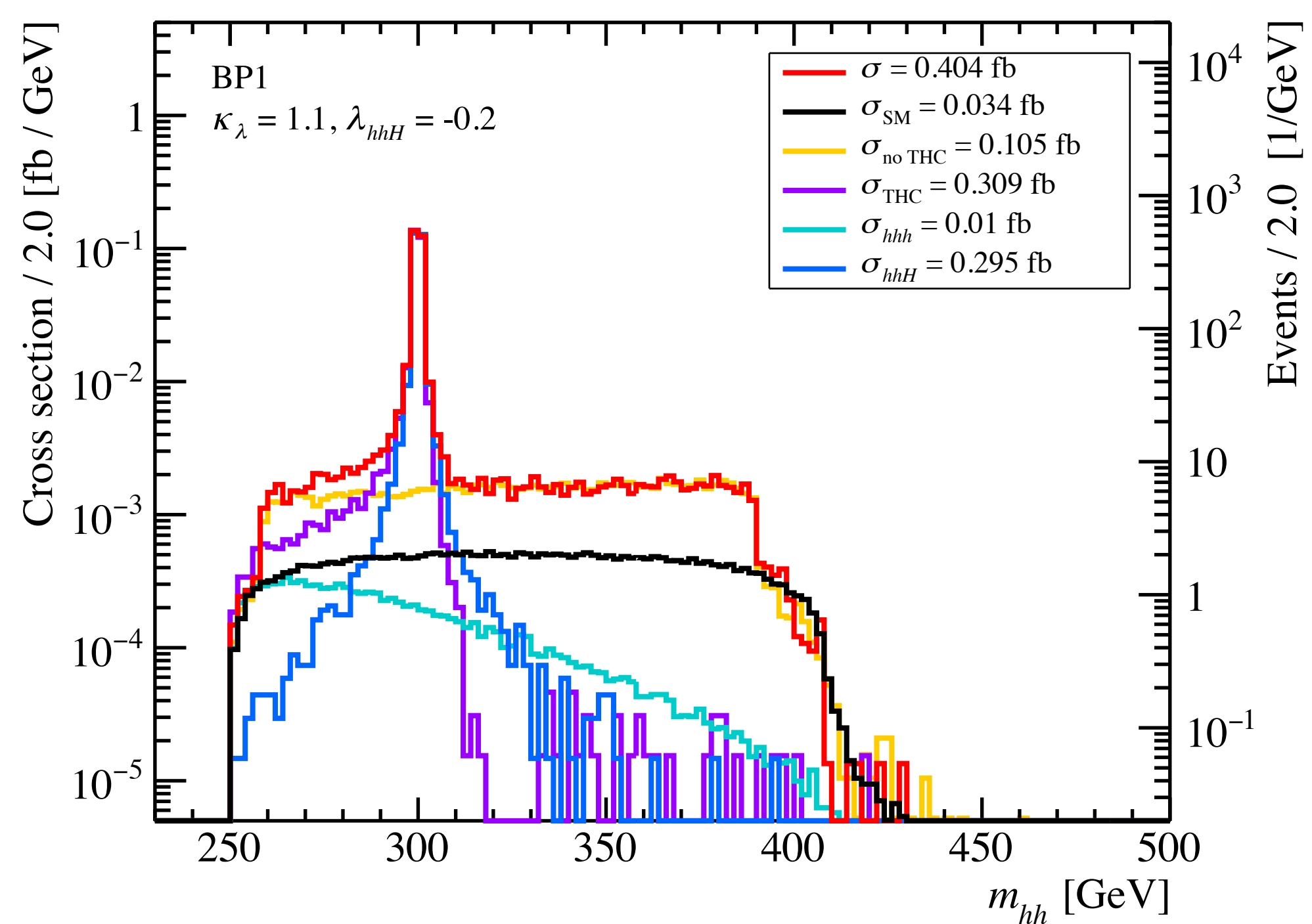
hh production, ILC 500GeV, THC dependence (type I)

Cross section distribution on the invariant mass of hh :

$\sigma(e^+e^- \rightarrow hhZ), \sqrt{s} = 500 \text{ GeV}$



$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 500 \text{ GeV}$



BP1

Type I

$m = 300 \text{ GeV}$

$\tan \beta = 10$

$c_{\beta-\alpha} = 0.25$

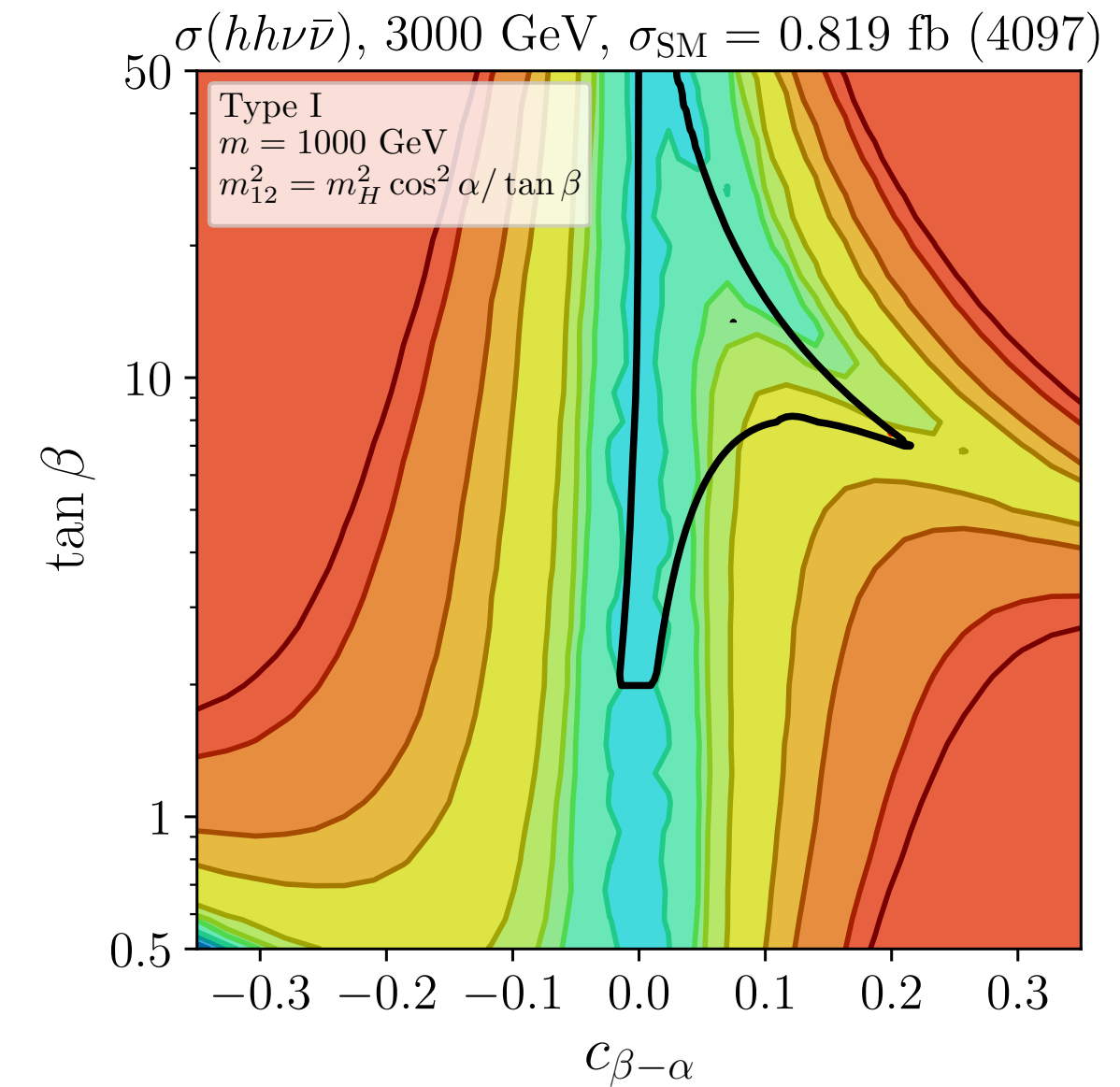
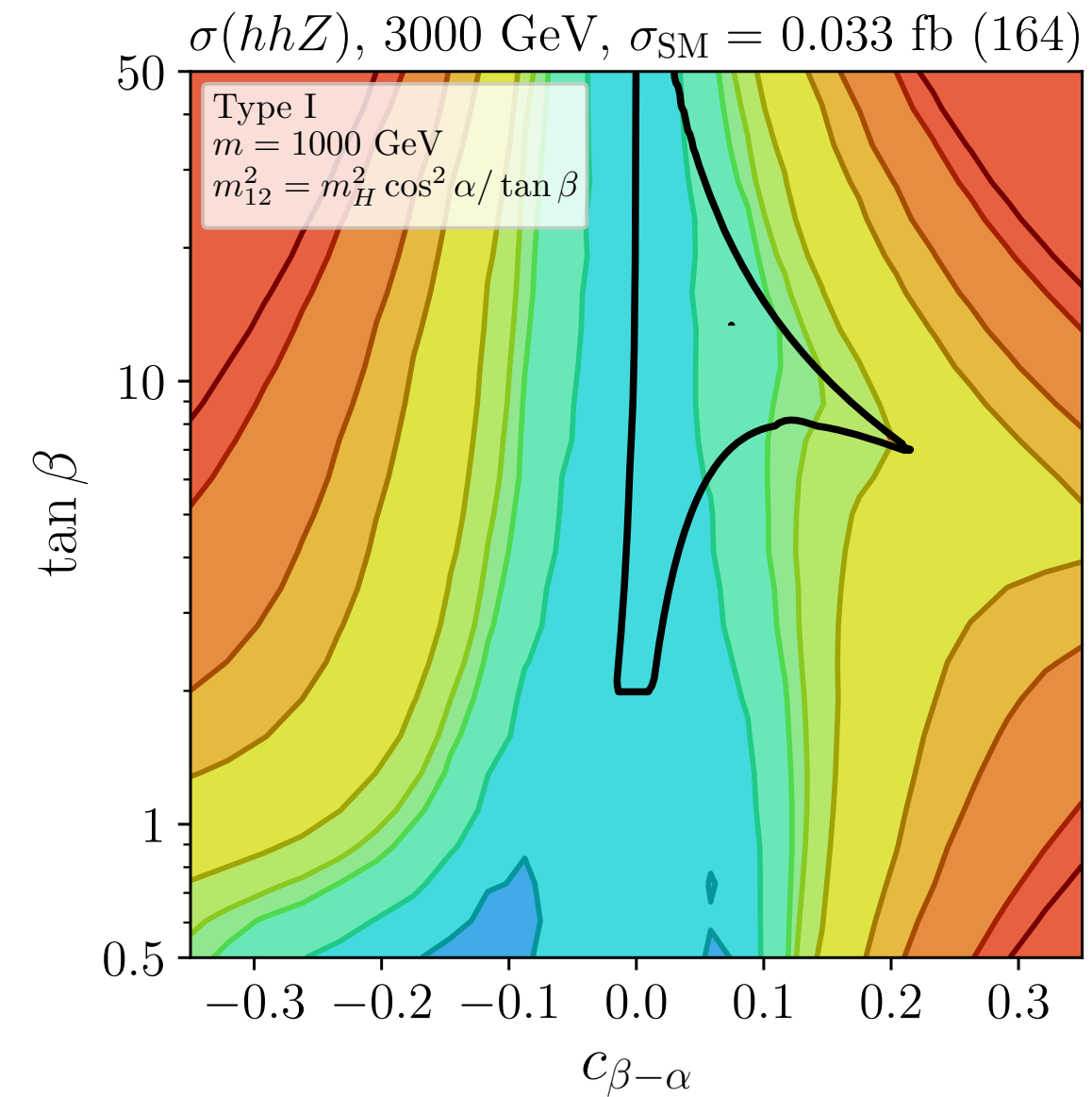
$$m_{12}^2 = \frac{m_H^2 \cos^2 \alpha}{\tan \beta}$$

- Main effect from κ_λ at the threshold of m_{hh} (light blue line)
- H resonance when $m_{hh} \sim m_H = 300 \text{ GeV}$ (dark blue line) \rightarrow access to λ_{hhH}
- Plateau wrt the SM from diagrams without THC (yellow line) \rightarrow A resonant diagrams

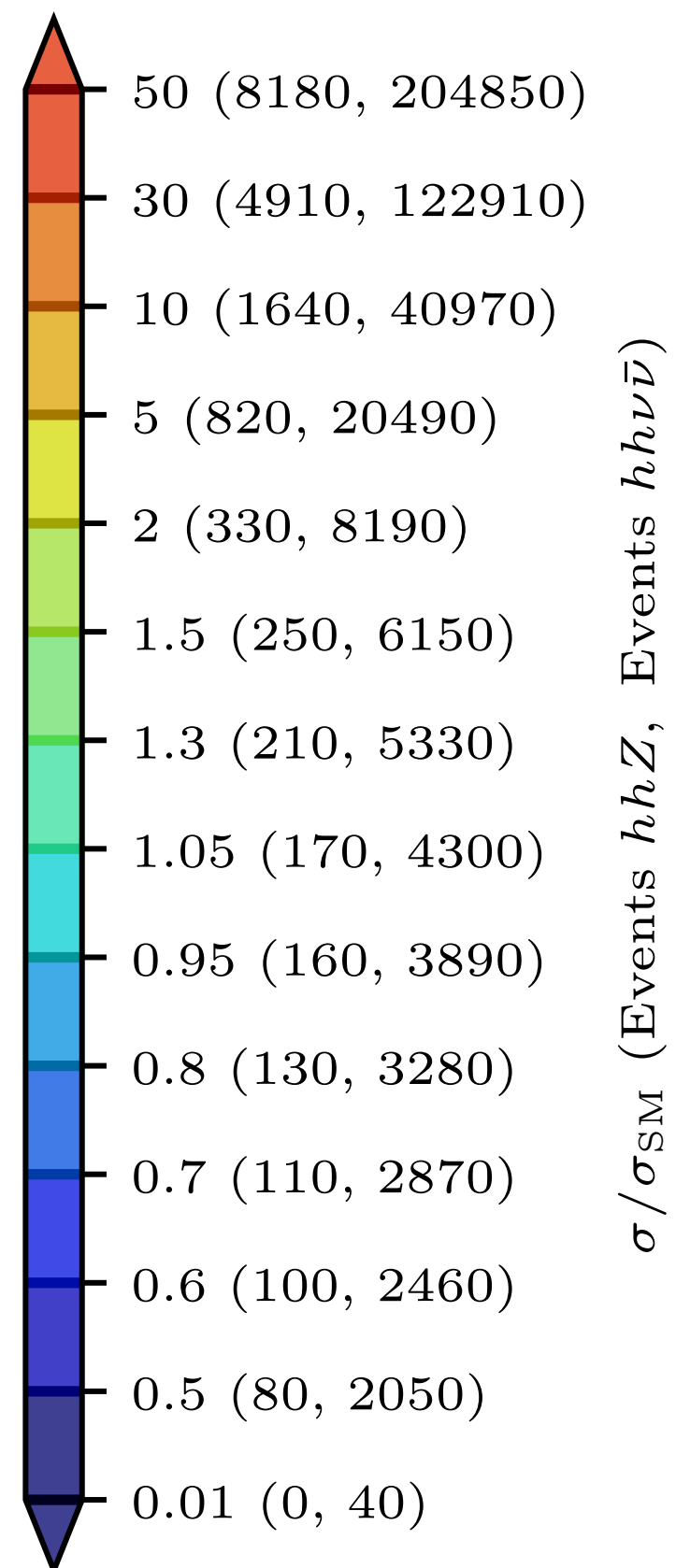
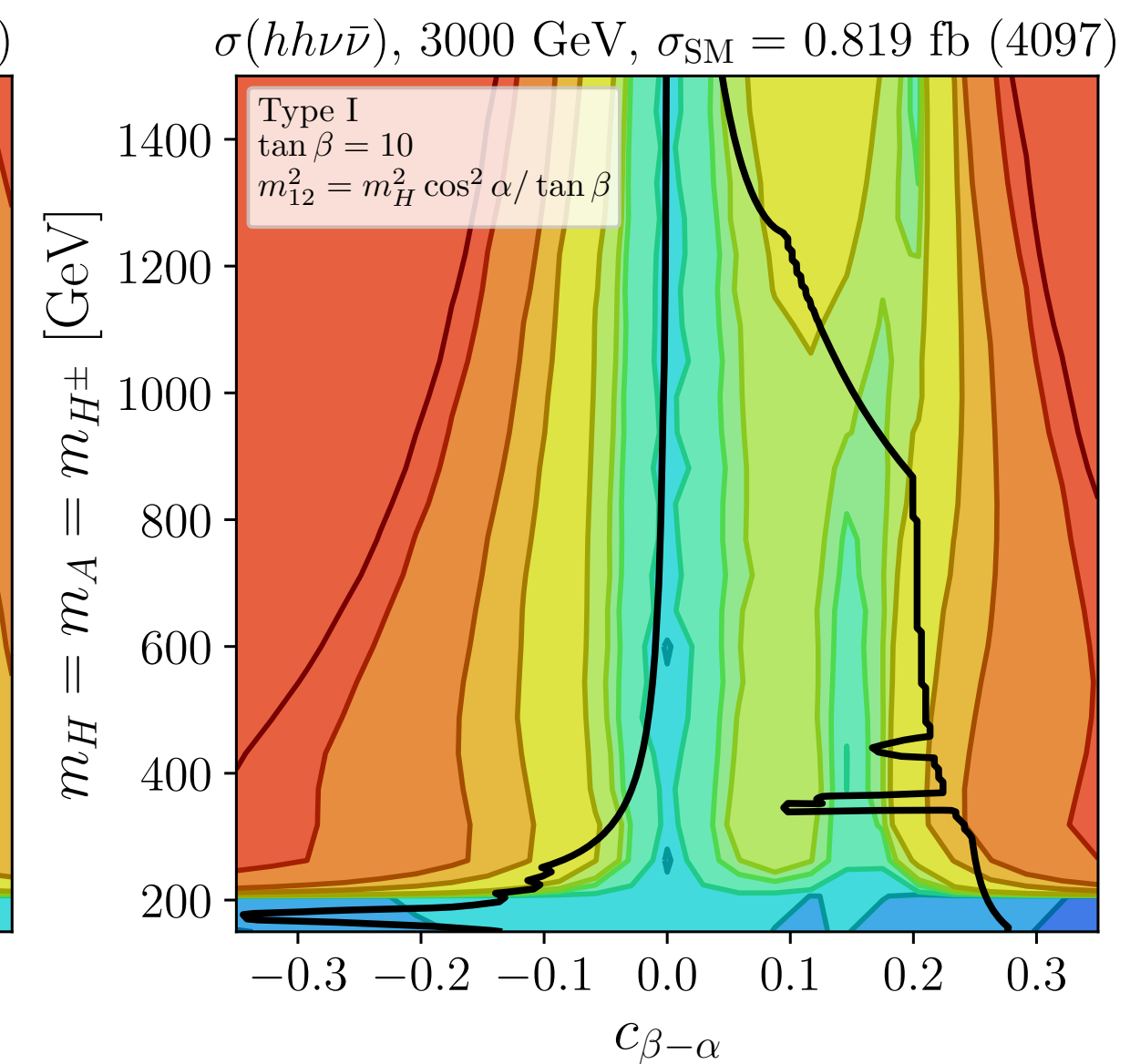
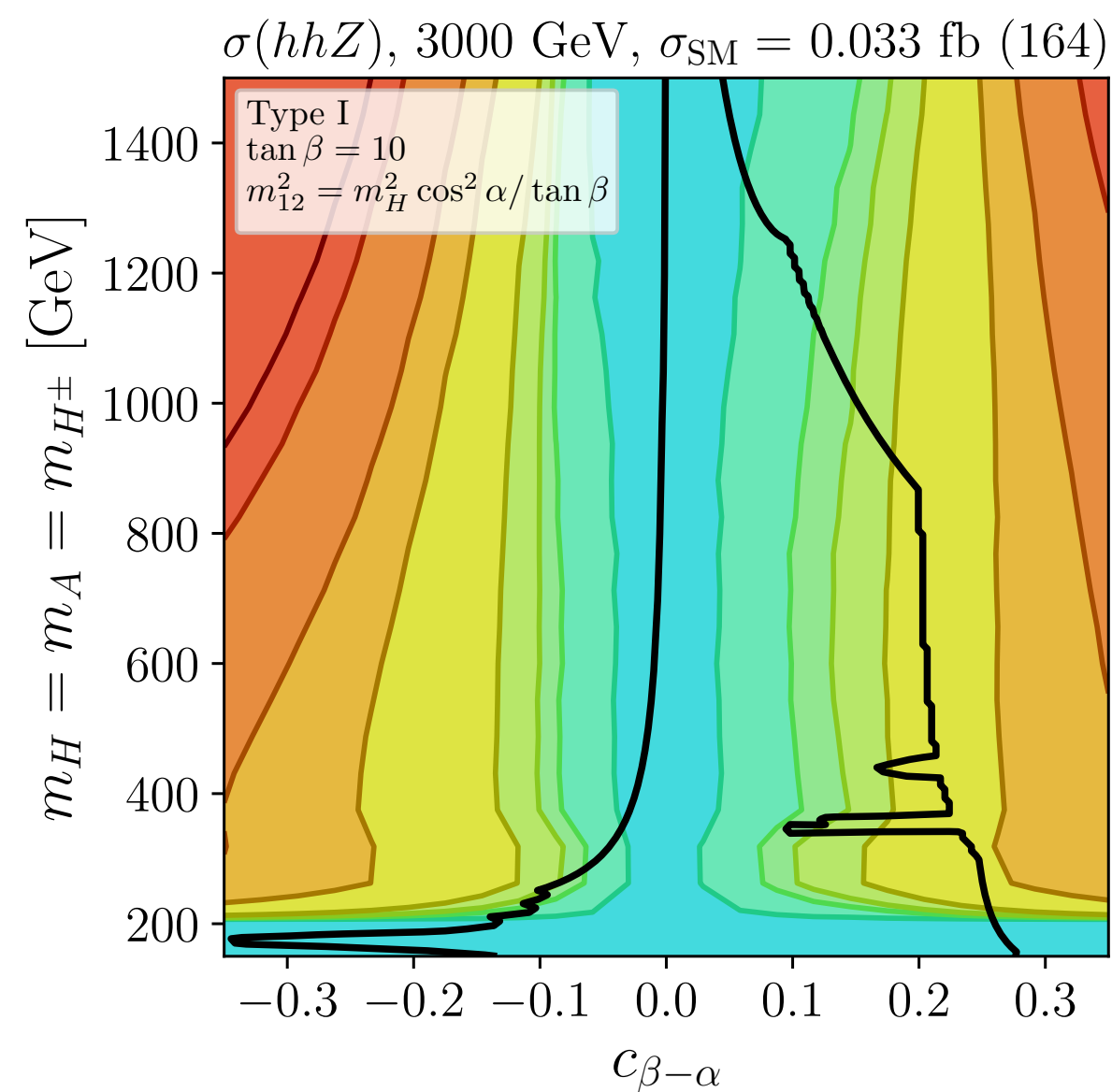
The effect from κ_λ and λ_{hhH} can be “mixed” if m_H is small

hh production, CLIC 3TeV (type I)

- Production XS for hhZ and $hh\nu\bar{\nu}$ wrt the SM @CLIC 3TeV
 - Now $hh\nu\bar{\nu}$ is the dominant channel
- hhZ is dominated by A mediated diagrams \rightarrow no THC info :(
- $hh\nu\bar{\nu}$ the H mediated diagrams are more important \rightarrow access to λ_{hhH}
 - $\sigma(hh\nu\bar{\nu}) \sim 10\sigma_{\text{SM}} = 9 \text{ fb}$ at low masses and $\sim 3\sigma_{\text{SM}}$ for a wide range of masses



Black lines are the boundaries to the total allowed region



hh production, CLIC 3TeV, THC dependence (type I)

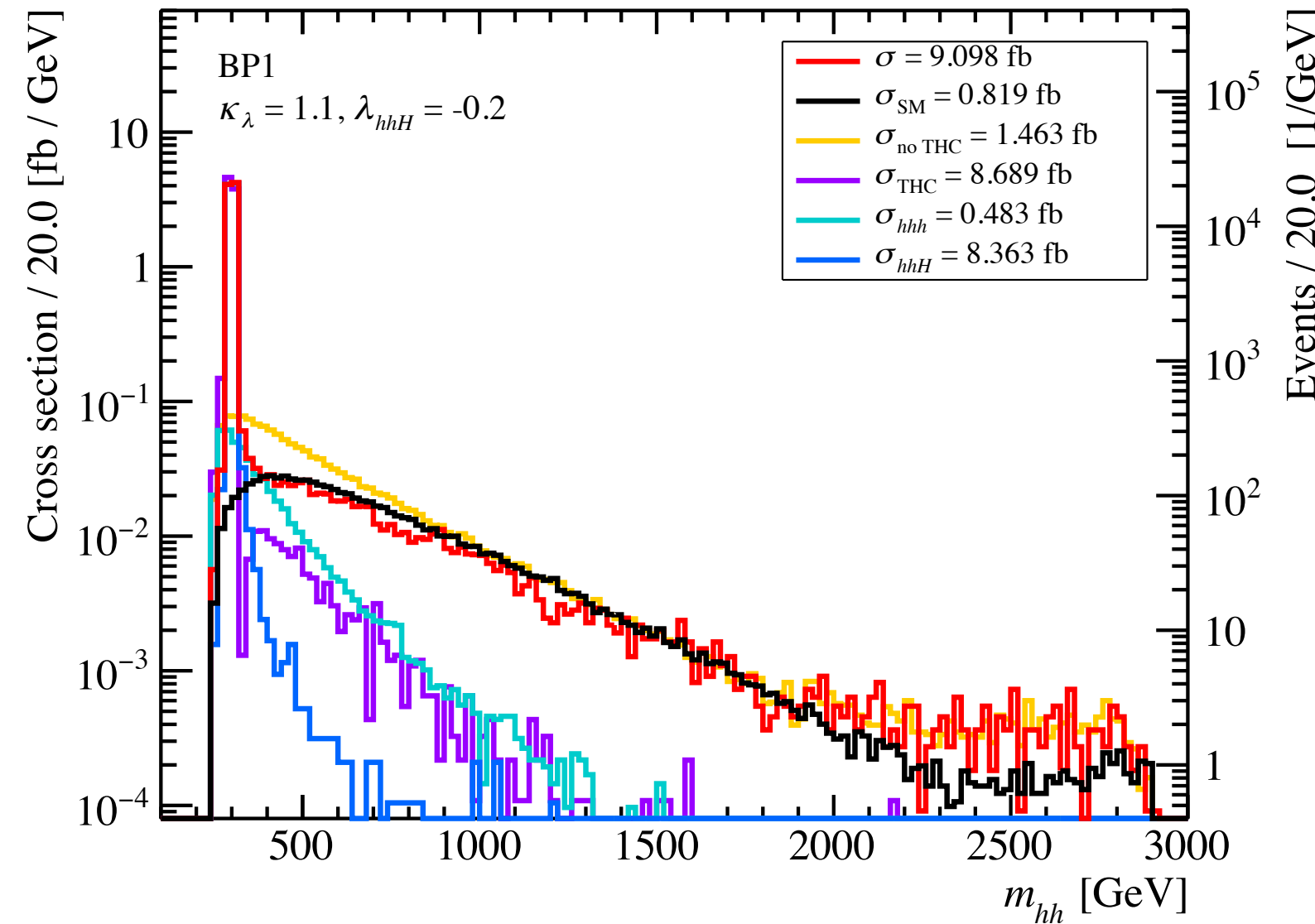
Cross section distributions on m_{hh} for

Point	Type	m	$\tan \beta$	$c_{\beta-\alpha}$	m_{12}^2
BP1	I	300	10	0.25	Eq. (8)
BP2	I	500	7.5	0.1	32000
BP3	I	600	10	0.2	Eq. (8)
BP4	I	1000	8.5	0.08	Eq. (8)

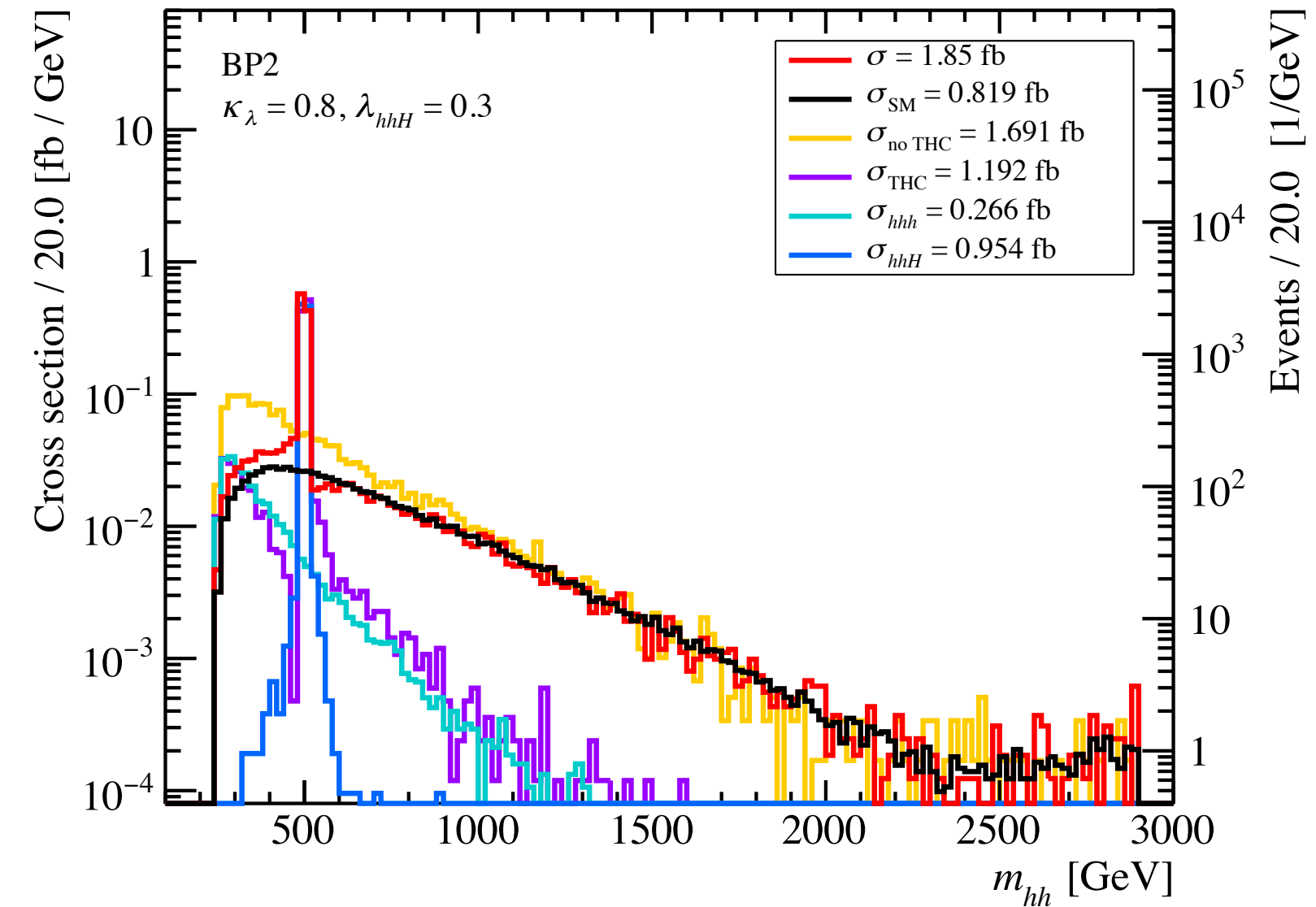
(Eq. (8) $\rightarrow m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$)

- Effect from κ_λ : the region of low invariant mass
- The neutrino channel can provide access to λ_{hhH} via the H resonant peak for a wide range of m_H and $c_{\beta-\alpha}$

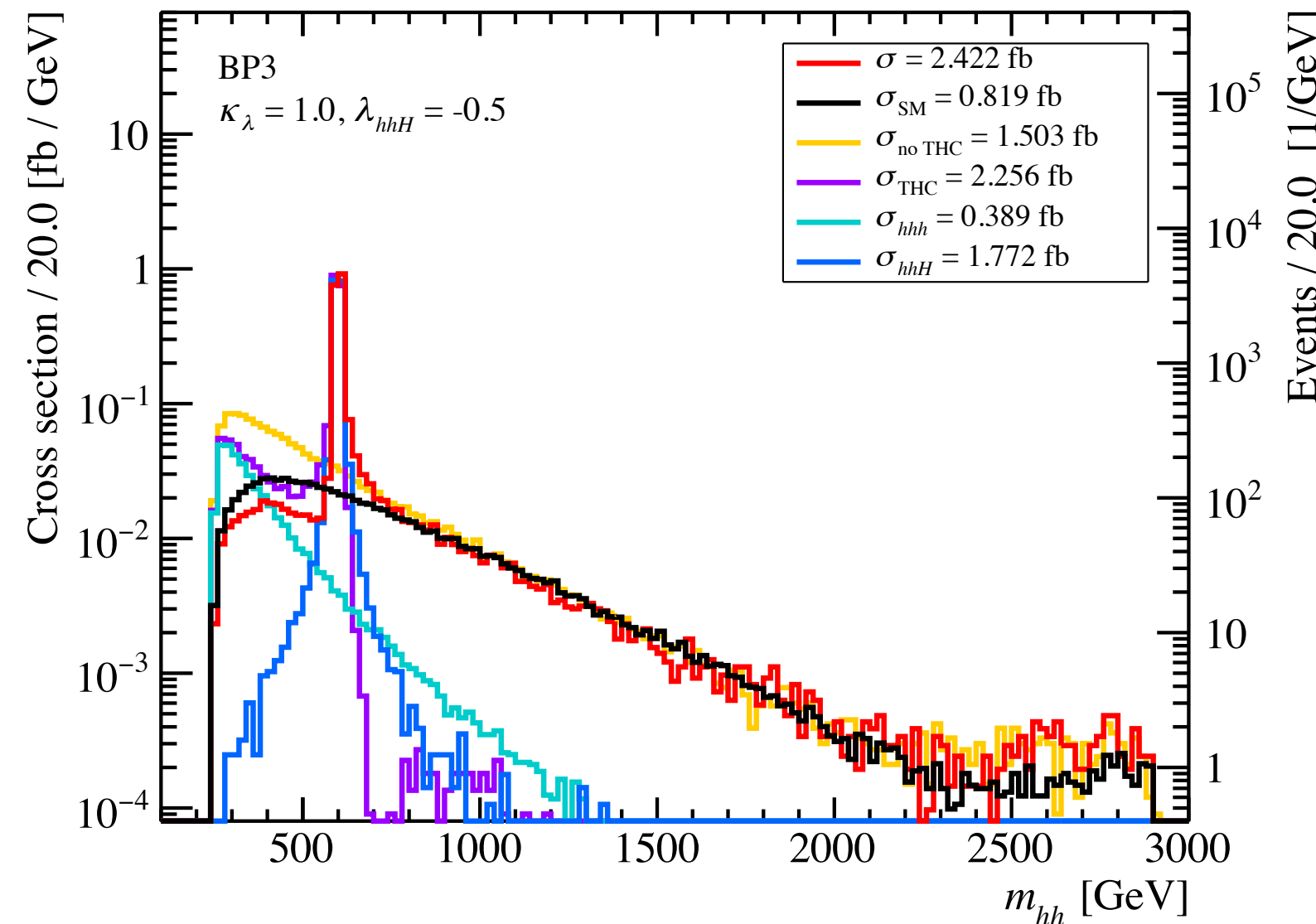
$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



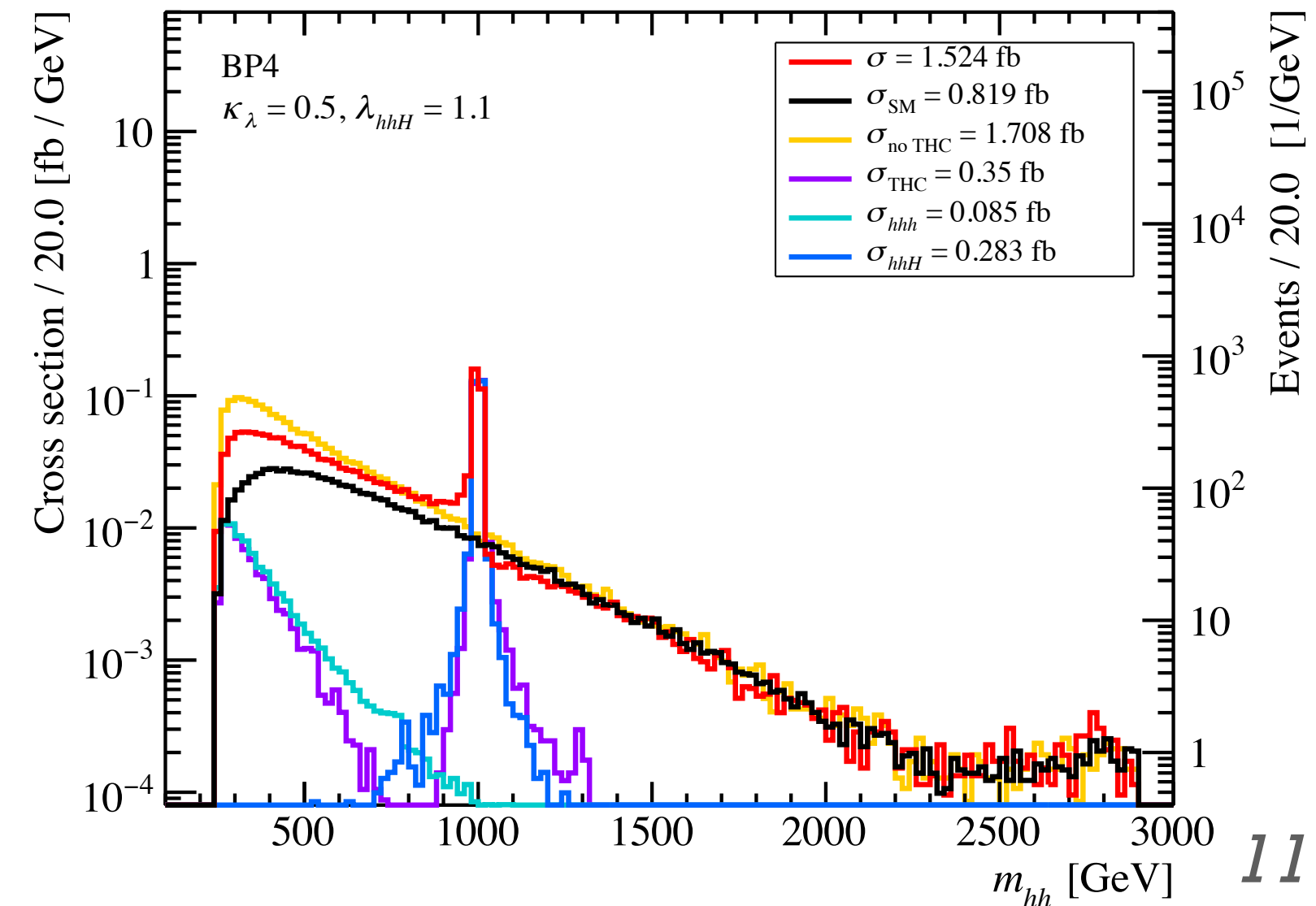
$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



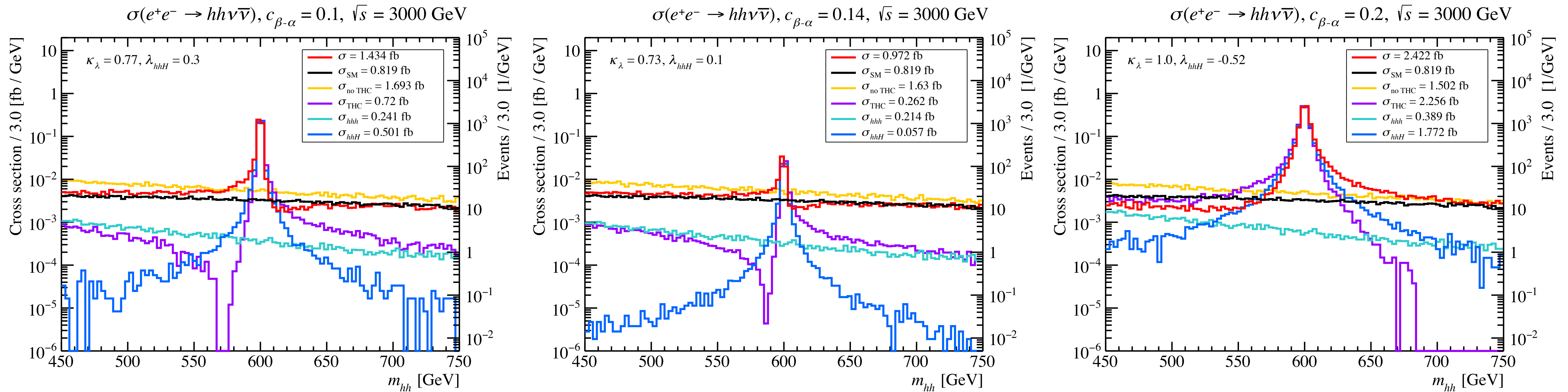
$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



hh production, CLIC 3TeV, THC sign dependence (type I)

Evolution of the H resonance with $c_{\beta-\alpha}$ (and indirectly with λ_{hhH})

Type I, $m = 600$ GeV,
 $\tan \beta = 10$, $m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$



- Height of the resonance depends on λ_{hhH}
- For large $c_{\beta-\alpha}$ the resonance is wider because Γ_H is larger

$\lambda_{hhH} > 0$:

More events at the left of the peak than at the right

$\lambda_{hhH} < 0$:

More events at the right of the peak than at the left

4- b jets in hh production: λ_{hhH} “sensitivity”

- Which collider and channel are best suited to access to λ_{hhH} ?
- We define a theoretical “sensitivity” R ratio: estimation of the final 4- b jets events at a collider close to the H resonance:

$$R = \frac{\bar{N}^R - \bar{N}^C}{\sqrt{\bar{N}^C}}$$

\bar{N}^R are events from the H mediated diagrams
 \bar{N}^C are events from diagrams without THC

where $\bar{N} = N \times \mathcal{A} \times \epsilon_b^4$ with N the number of total 4- b jets events, $\epsilon_b \sim 0.8$ the b -tagging efficiency and $\mathcal{A} = N_{\text{cuts}}/N_{\text{no cuts}}$ is the acceptance with these cuts:

- **Cuts:** $p_T^b > 20 \text{ GeV}$, $|\eta^b| < 2$, $\Delta R_{bb} > 0.4$, $p_T^Z > 20 \text{ GeV}$, $E_T > 20 \text{ GeV}$

λ_{hhH} “sensitivity” at hhZ from 4-bjet events

- Large sensitivity @ILC for BP1 (lighter H)
- 20-30 events @ILC1TeV for BP2 and BP3
- Barely no events @CLIC

hhZ	\sqrt{s} [GeV]	$\sigma_{2\text{HDM}} / \sigma_{\text{SM}}$ [fb]	$\bar{N}_{4bZ}^R / \bar{N}_{4bZ}^C / \bar{N}_{4bZ}^{\text{SM}}$	$\mathcal{A}_{2\text{HDM}} / \mathcal{A}_{\text{SM}}$	R_{4bZ}
BP1	500	1.063 / 0.158	193 / 10 / 3	0.70 / 0.68	58
	1000	0.913 / 0.120	206 / 1 / 4	0.70 / 0.71	205
	1500	0.493 / 0.077	22 / < 1 / 1	0.51 / 0.62	-
	3000	0.147 / 0.033	1 / < 1 / < 1	0.05 / 0.05	-
BP2	1000	0.156 / 0.120	20 / 1 / 1	0.73 / 0.71	19
	1500	0.106 / 0.077	4 / < 1 / < 1	0.65 / 0.62	-
	3000	0.042 / 0.033	< 1 / < 1 / < 1	0.07 / 0.05	-
BP3	1000	0.254 / 0.120	29 / 5 / 2	0.71 / 0.71	11
	1500	0.218 / 0.077	8 / 1 / < 1	0.70 / 0.62	7
	3000	0.086 / 0.033	1 / < 1 / < 1	0.08 / 0.05	-
BP4	1500	0.075 / 0.077	1 / < 1 / < 1	0.64 / 0.62	-
	3000	0.038 / 0.033	< 1 / < 1 / < 1	0.07 / 0.05	-

λ_{hhH} “sensitivity” at $hh\nu\bar{\nu}$ from 4-bjet events

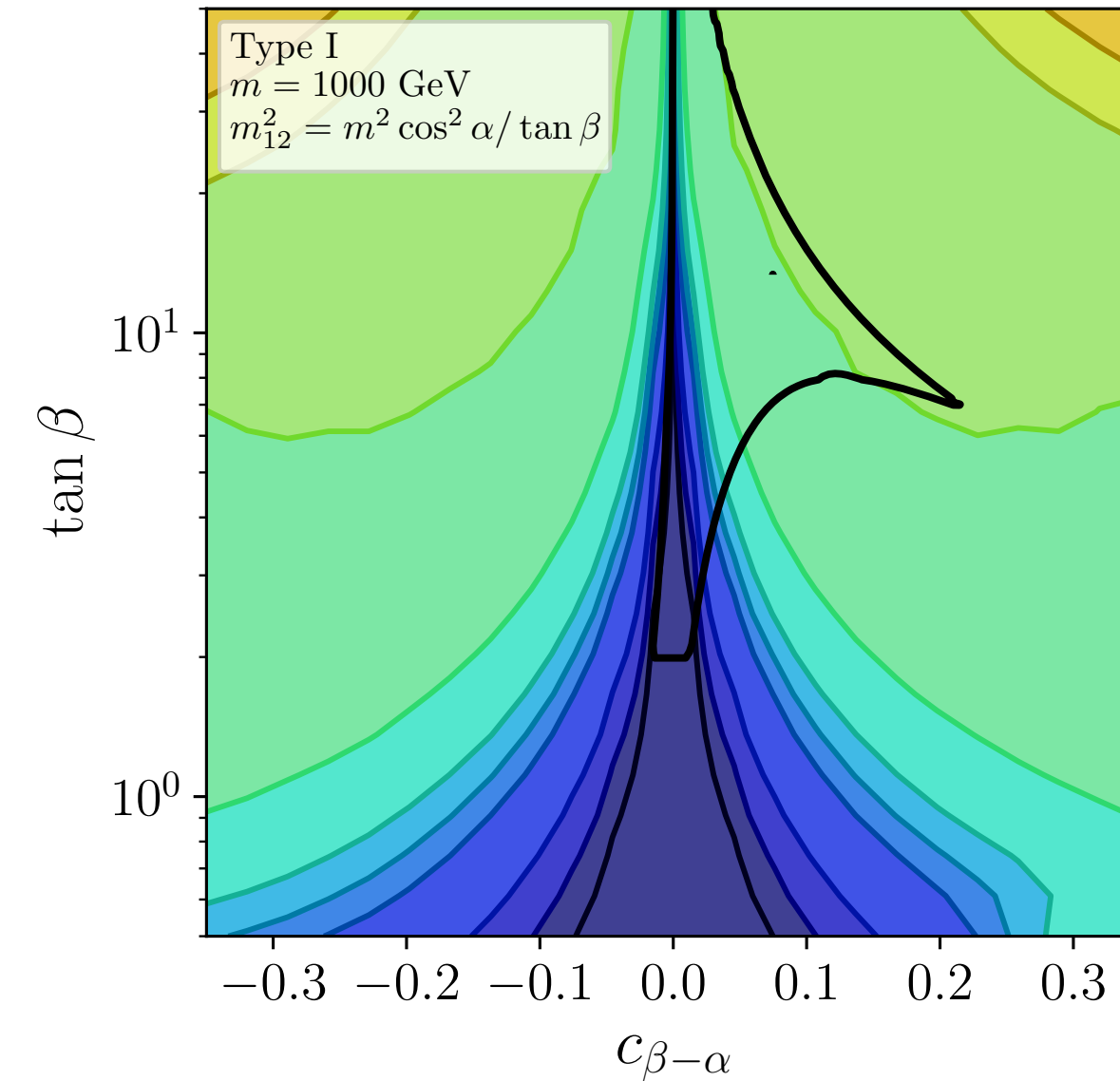
- Better sensitivities for all BPs compared to hhZ
- Better R ratio @CLIC3TeV
- No significant events only for BP4 @CLIC1.5TeV

$hh\nu\bar{\nu}$	\sqrt{s} [GeV]	$\sigma_{2\text{HDM}} / \sigma_{\text{SM}}$ [fb]	$\bar{N}_{4bE_T}^R / \bar{N}_{4bE_T}^C / \bar{N}_{4bE_T}^{\text{SM}}$	$\mathcal{A}_{2\text{HDM}} / \mathcal{A}_{\text{SM}}$	R_{4bE_T}
BP1	500	0.404 / 0.034	119 / 4 / 1	0.70 / 0.68	58
	1000	2.391 / 0.097	1510 / 24 / 0	0.65 / 0.55	303
	1500	4.423 / 0.239	794 / 13 / 2	0.58 / 0.41	217
	3000	9.098 / 0.819	2425 / 46 / 6	0.44 / 0.25	351
BP2	1000	0.234 / 0.097	79 / 3 / 1	0.65 / 0.55	44
	1500	0.625 / 0.239	70 / 3 / 1	0.56 / 0.41	39
	3000	1.850 / 0.819	282 / 28 / 9	0.41 / 0.25	48
BP3	1000	0.208 / 0.097	85 / 5 / 3	0.66 / 0.55	36
	1500	0.709 / 0.239	111 / 5 / 3	0.61 / 0.41	47
	3000	2.422 / 0.819	577 / 30 / 11	0.47 / 0.25	100
BP4	1500	0.428 / 0.239	4 / < 1 / < 1	0.50 / 0.41	-
	3000	1.523 / 0.819	72 / 4 / 3	0.38 / 0.25	34

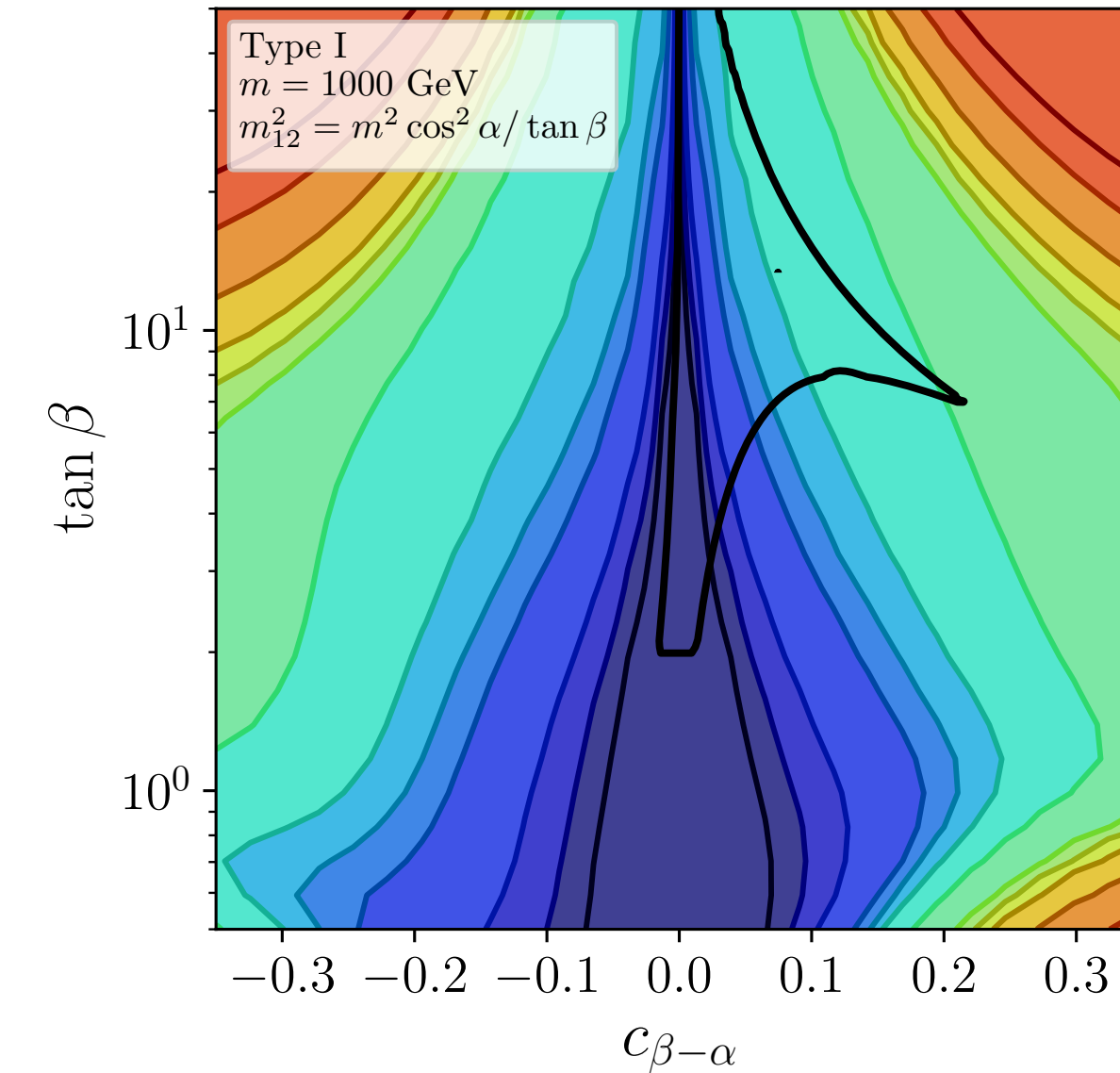
hH production, CLIC 3TeV (type I)

- Production XS for hHZ and $hH\nu\bar{\nu}$ @CLIC 3TeV
- $\sigma = 0$ in the alignment limit
- Very strong contribution from resonant A diagrams in the hHZ channel
- In the $hH\nu\bar{\nu}$ channel, the effects from diagrams with THCs (λ_{hhH} and λ_{hHH}) are mixed with the effects from A mediated diagrams

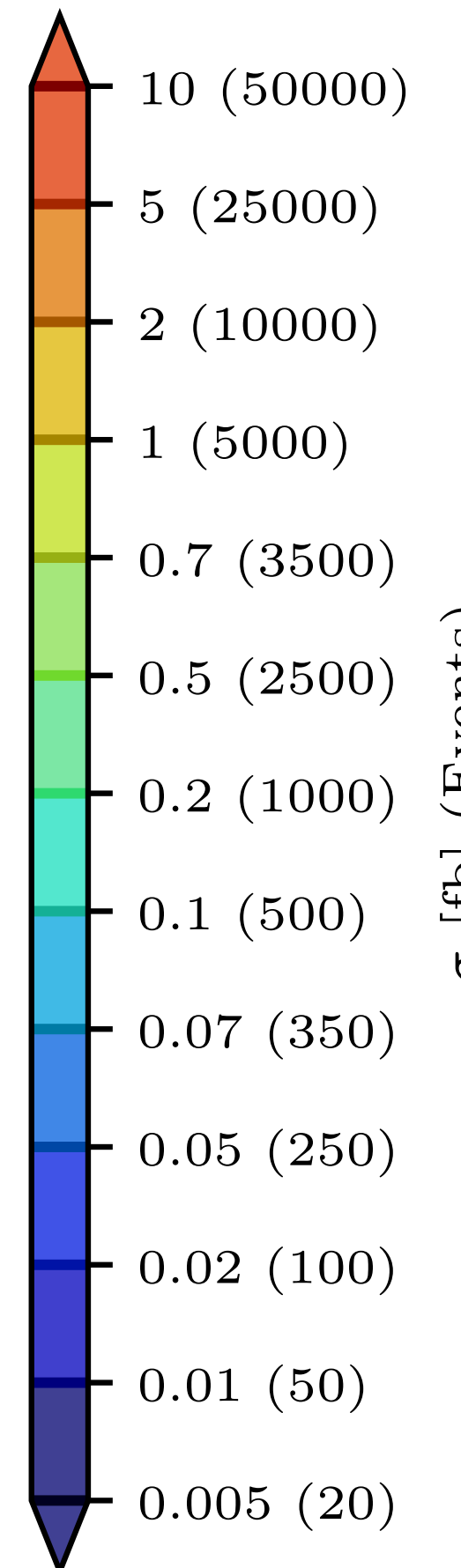
$\sigma(hHZ), \sqrt{s} = 3000 \text{ GeV}$



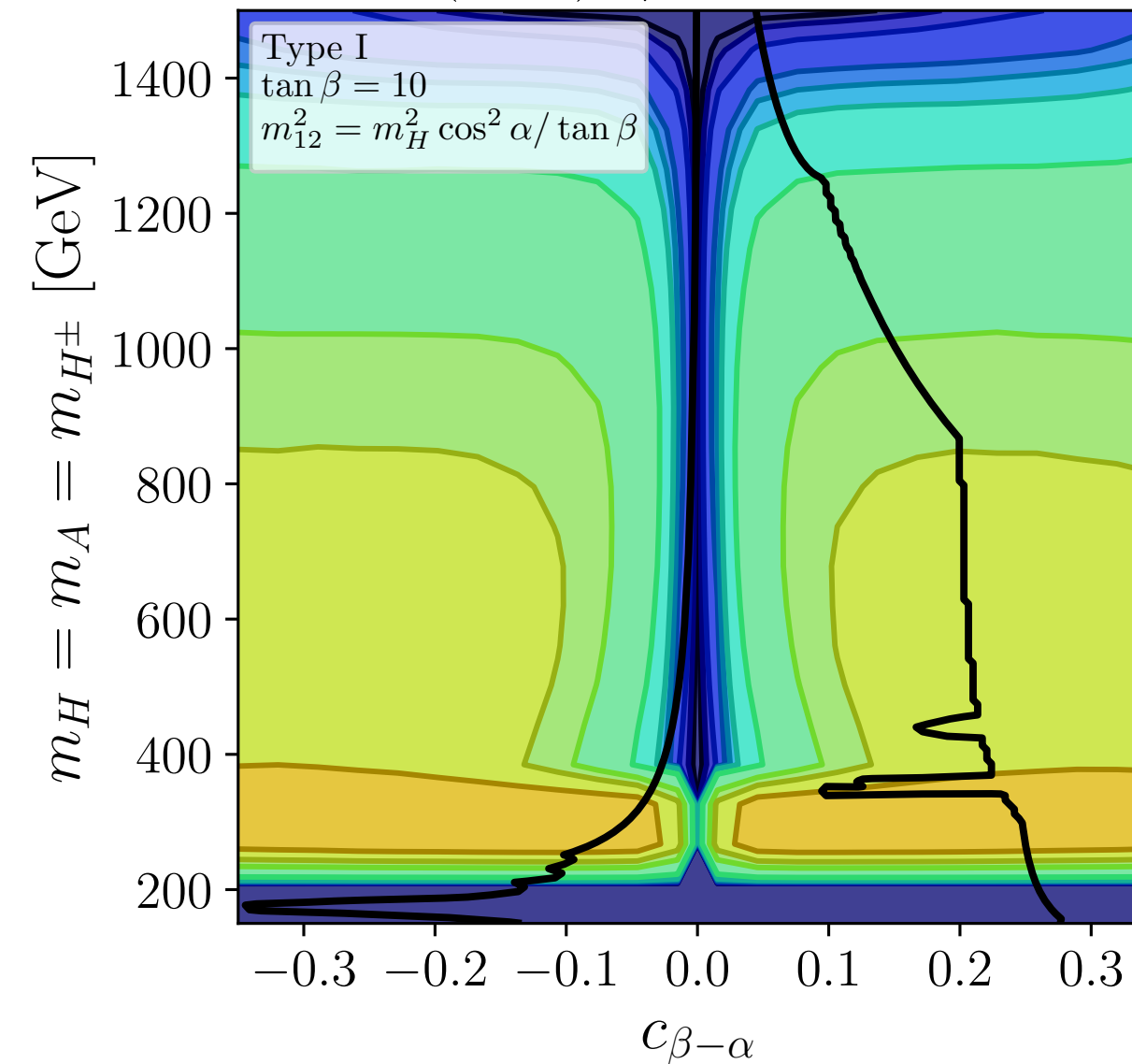
$\sigma(hH\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



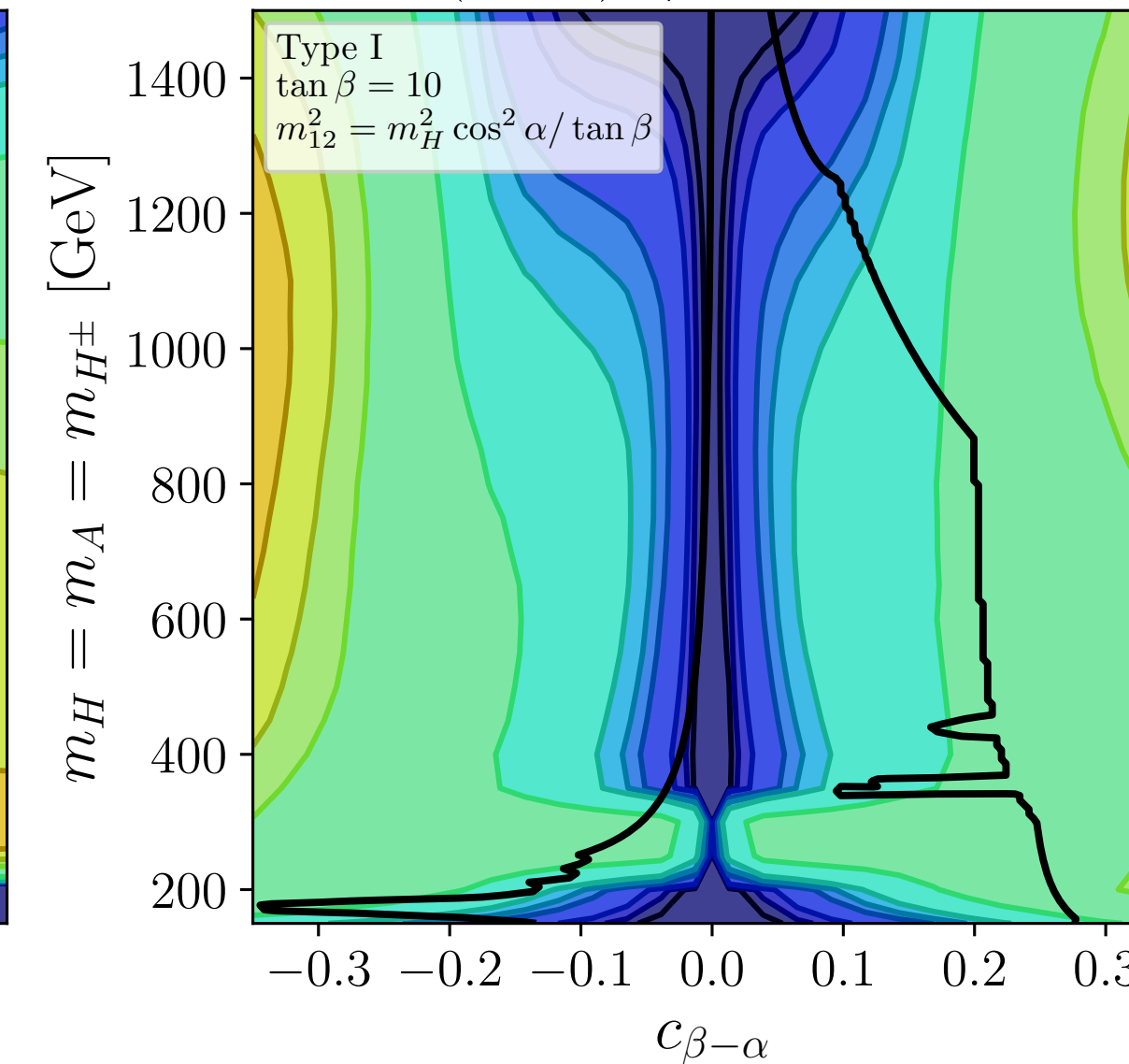
Black lines are the boundaries to the total allowed region



$\sigma(hHZ), \sqrt{s} = 3000 \text{ GeV}$



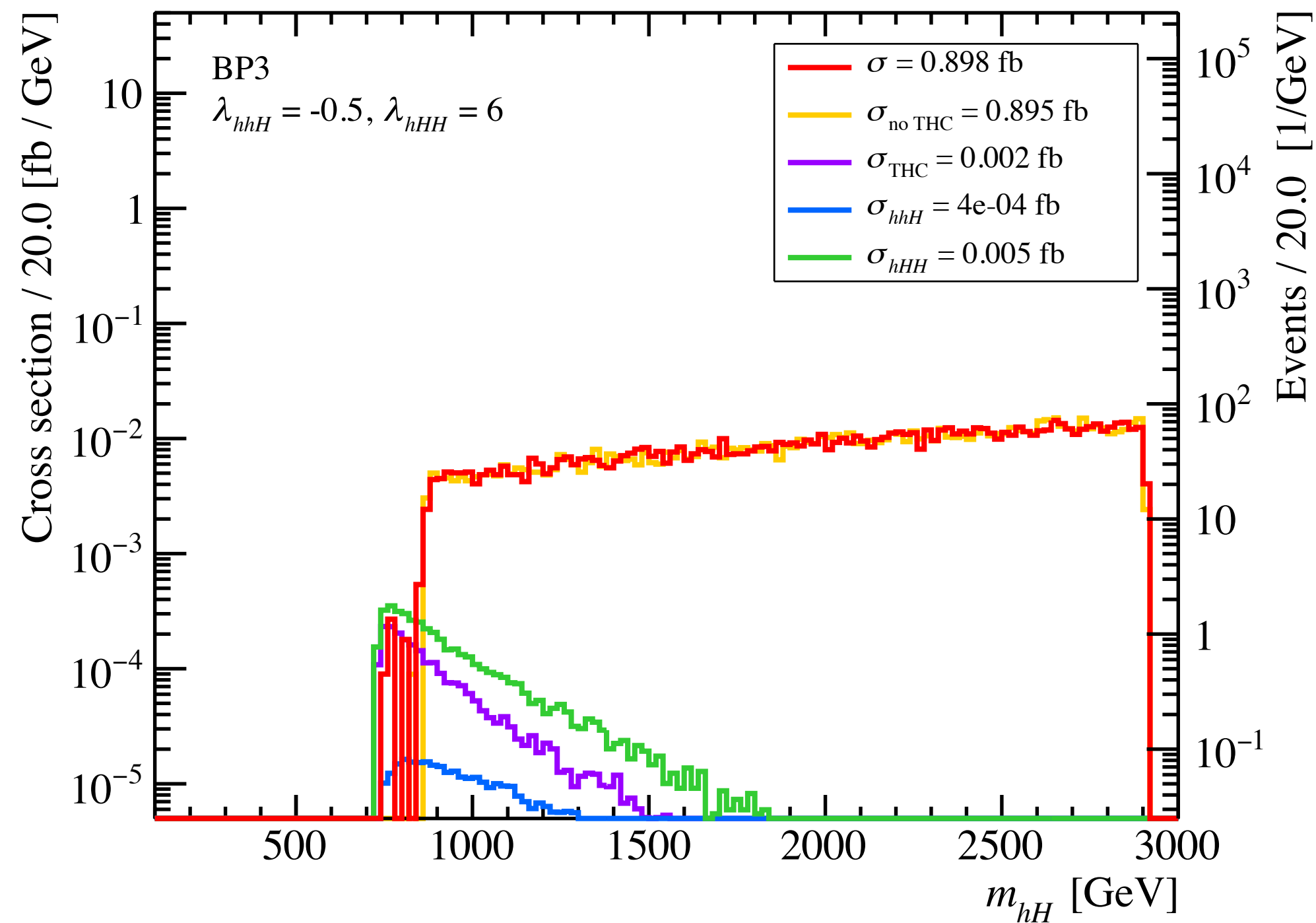
$\sigma(hH\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$



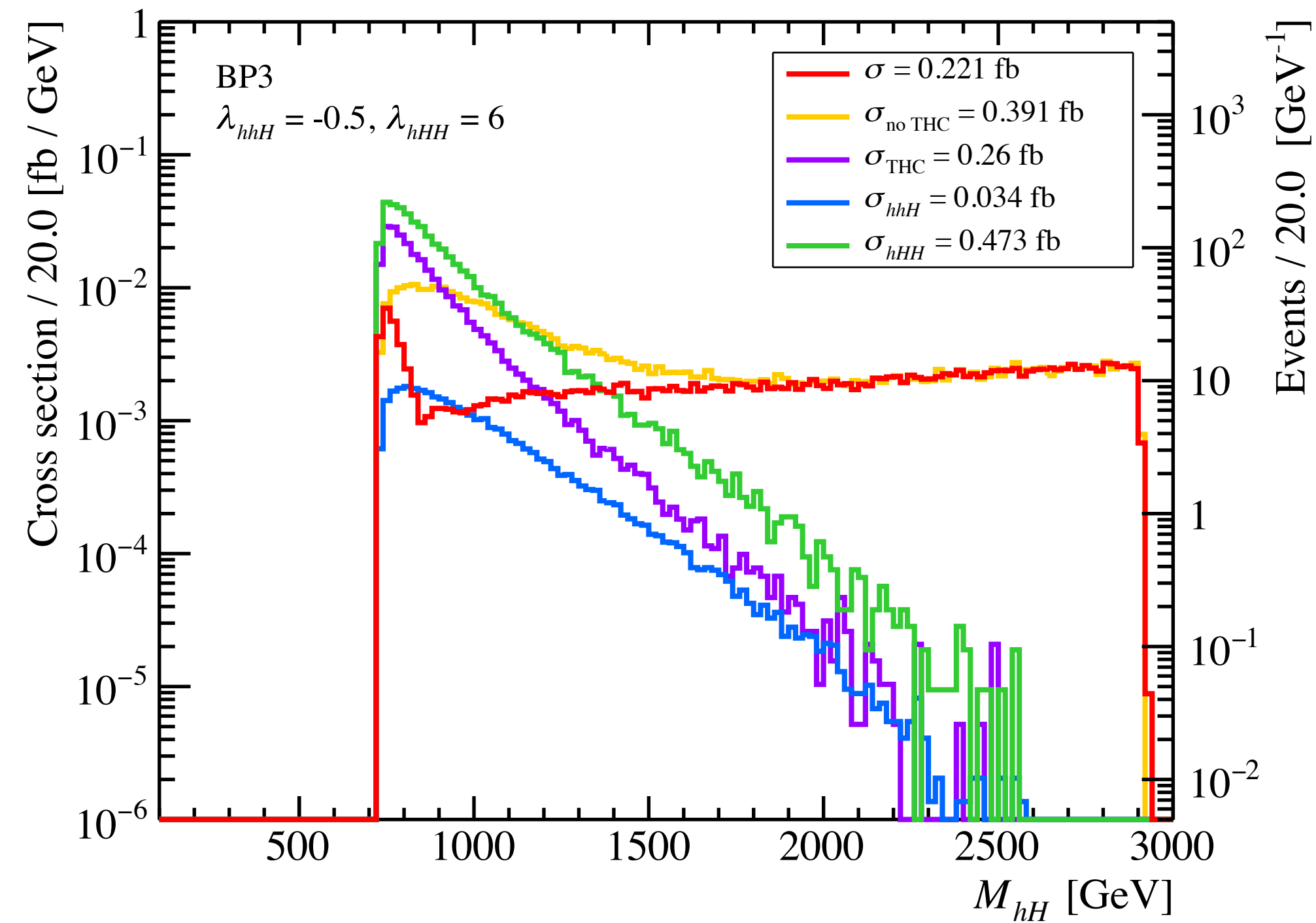
hH production, CLIC 3TeV, THC dependence (type I)

Cross section distribution on the invariant mass of hH :

$$\sigma(e^+e^- \rightarrow hHZ), \sqrt{s} = 3000 \text{ GeV}$$



$$\sigma(e^-e^+ \rightarrow hH\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$$



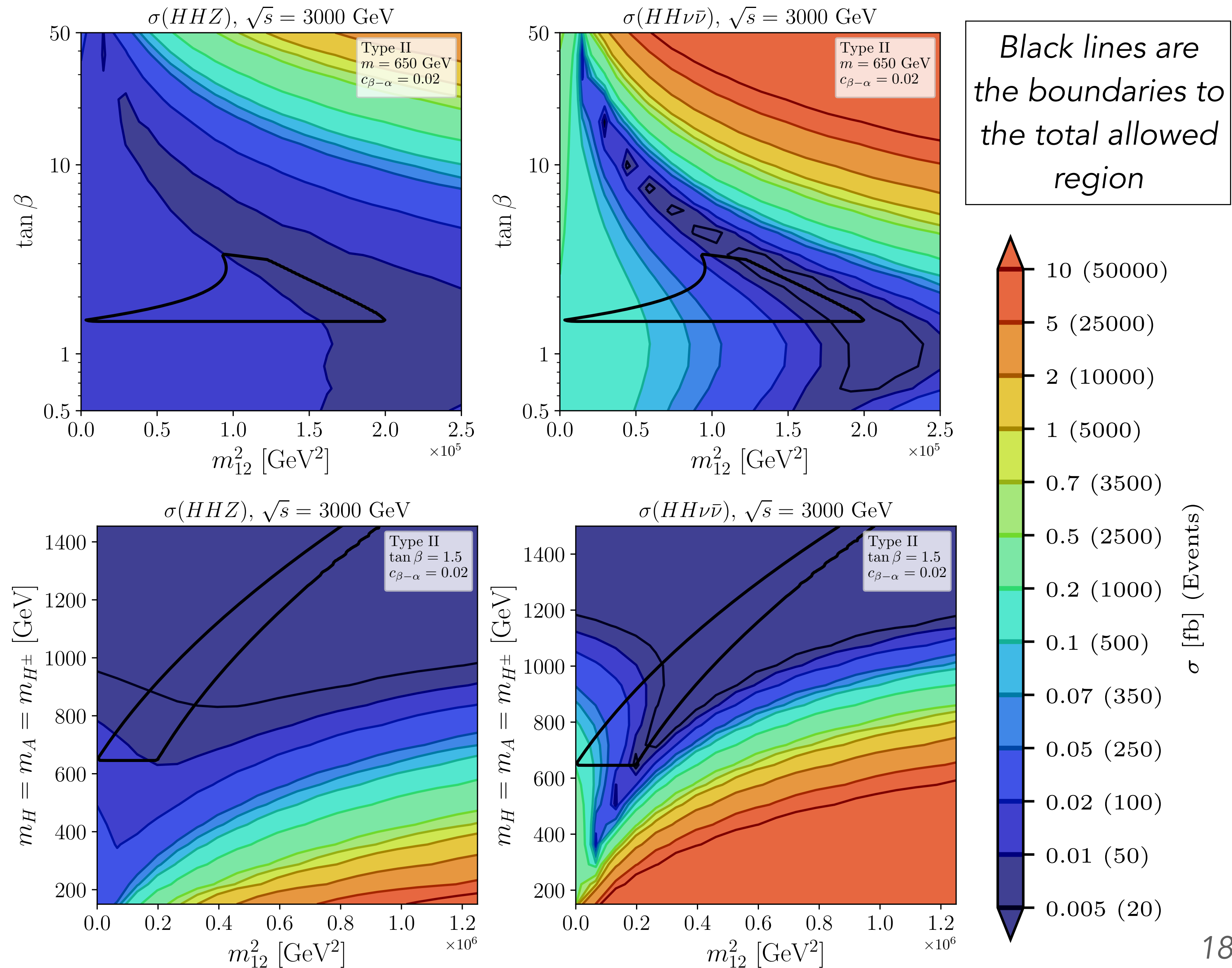
BP3

Type I
 $m = 600 \text{ GeV}$
 $\tan \beta = 10$
 $c_{\beta-\alpha} = 0.2$
 $m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$

- Large plateaus of events coming from A resonant diagrams
- Large effects from diagrams with λ_{hhH} and λ_{hHH} only in the neutrino channel at the m_{hH} threshold
- The combined effect of both THC (purple line) depends on their relative sign

$HH \sim AA$ production, CLIC 3TeV (type II)

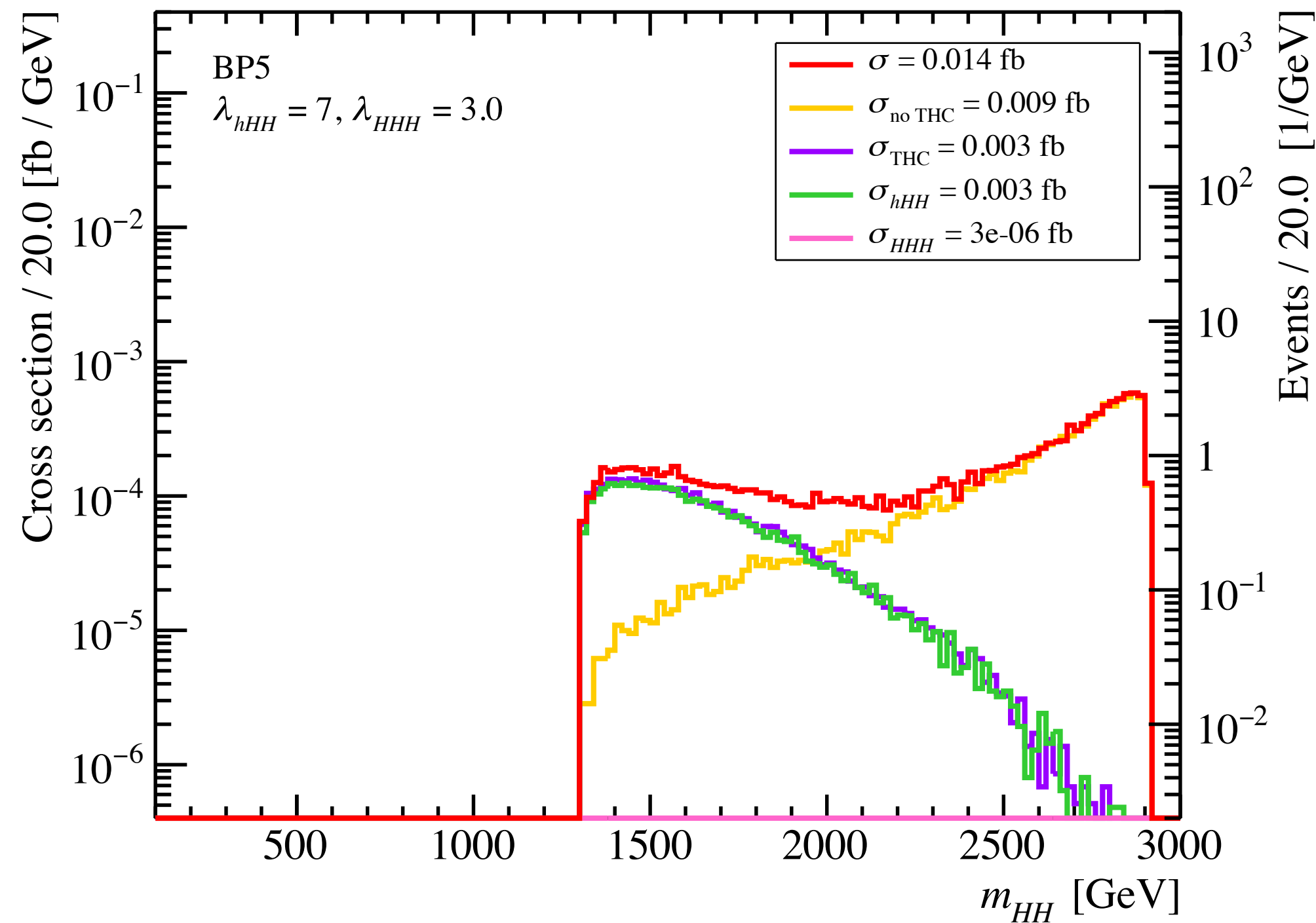
- Production XS for HHZ and $HH\nu\bar{\nu}$ @ILC500GeV
- In type II only $HH \sim AA$ production is relevant (because $c_{\beta-\alpha} \sim 0$)
- $HH\nu\bar{\nu}$ production up to 0.5 fb inside the allowed region
 - Effect coming from λ_{hHH} !
- In type I we can obtain similar XS with similar values for λ_{hHH}



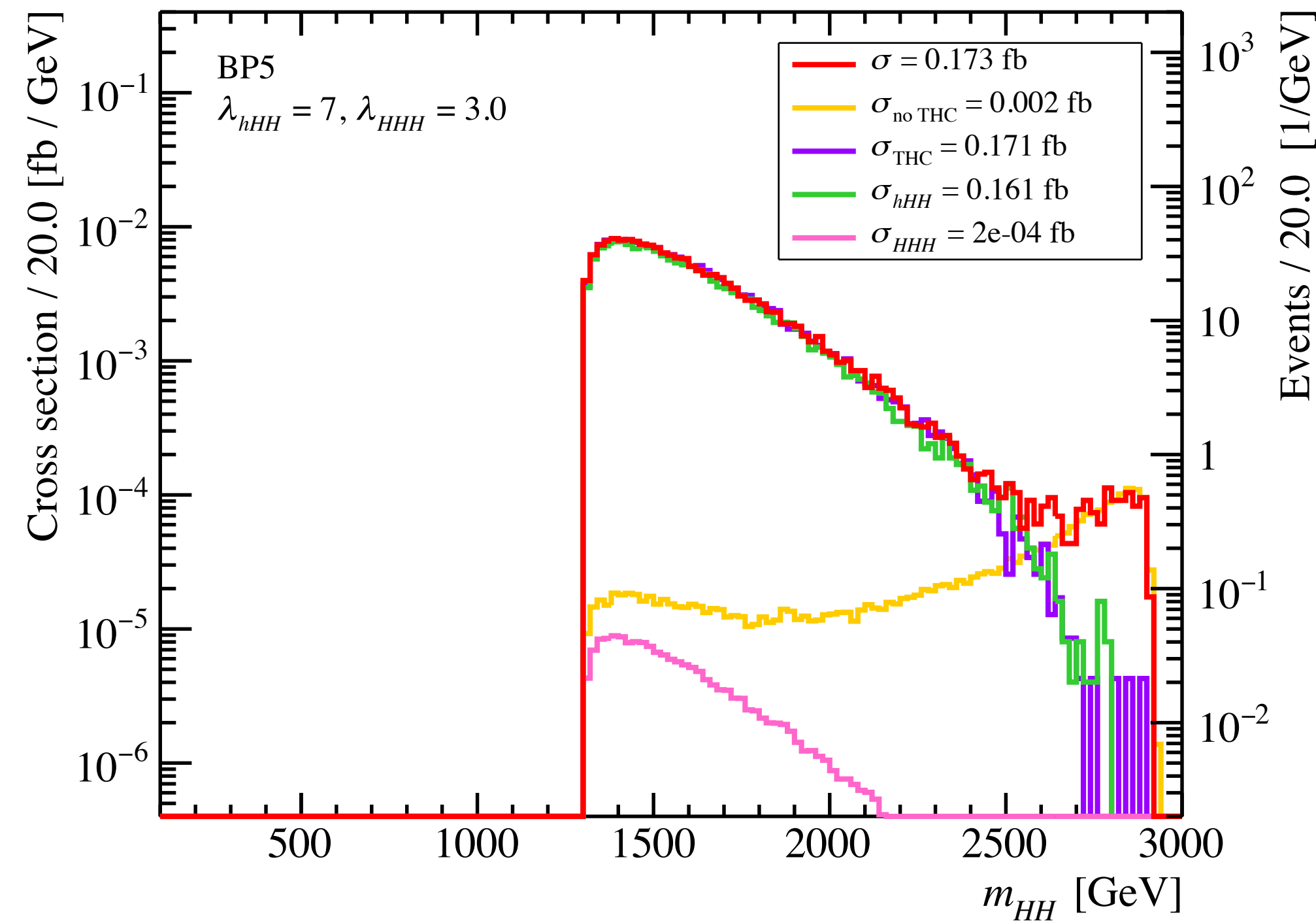
$HH \sim AA$ production, CLIC 3TeV, THC dependence (type II)

Cross section distribution on the invariant mass of HH :

$$\sigma(e^+e^- \rightarrow HHZ), \sqrt{s} = 3000 \text{ GeV}$$



$$\sigma(e^+e^- \rightarrow HH\nu\bar{\nu}), \sqrt{s} = 3000 \text{ GeV}$$



BP5
Type II
 $m = 650 \text{ GeV}$
 $\tan \beta = 1.5$
 $c_{\beta-\alpha} = 0.02$
 $m_{12}^2 = 10000 \text{ GeV}^2$

- Very small XS and number of events in the HHZ channel
- In the neutrino channel the diagram with λ_{hHH} dominates
- Sizable XS in both types if λ_{hHH} is large (and m_H light enough)

Summary & Conclusions

- The di-Higgs production is studied at e^+e^- colliders in the 2HDM, with the aim to find effects coming from BSM triple Higgs couplings (THC)
 - Two production channels were studied: $e^+e^- \rightarrow h_i h_j Z$ and $e^+e^- \rightarrow h_i h_j \nu \bar{\nu}$
- For hh production, only sizable distortions at **type I** (type II is very constrained)
 - From κ_λ , at low invariant mass of the hh pair, similar to what happens in the SM
 - From λ_{hhH} , through a resonant peak due to the H boson:
 - A study of the final 4 b -jets events hints that $hh\nu\bar{\nu}$ channel is better at large energies (specially @CLIC 3TeV)
 - Large #events @ILC energies for a light H at both channels
- Effects from THC on $hH\nu\bar{\nu}$ and $HH\nu\bar{\nu} \sim AA\nu\bar{\nu}$ could be potentially seen @CLIC 3TeV in both types, even at the alignment limit! $\rightarrow \mu$ collider (?)

Thanks for your attention :)

Questions??

Prospects on κ_λ

The shape of the SM Higgs potential is **NOT** measured experimentally and there is not a precise measurement of the Higgs self-coupling

Actual measurements on $\kappa_\lambda = \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$

Prospects on $\kappa_\lambda = \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$

ATLAS	CMS	ILC	CLIC
[-2.3, 10.3] at 95% CL	[-3.3, 8.5] at 95% CL	500GeV: $\pm 27\%$ at 68% CL 1TeV: $\pm 10\%$ at 68% CL	3+1.4 TeV combination: -8% and 11% at 68% CL
[ATLAS-CONF-2019-049]	[arXiv:2011.12373, CMS-HIG-19-018]	[arXiv:1910.11775]	[arXiv:1901.05897]

All the above analysis assume the SM couplings

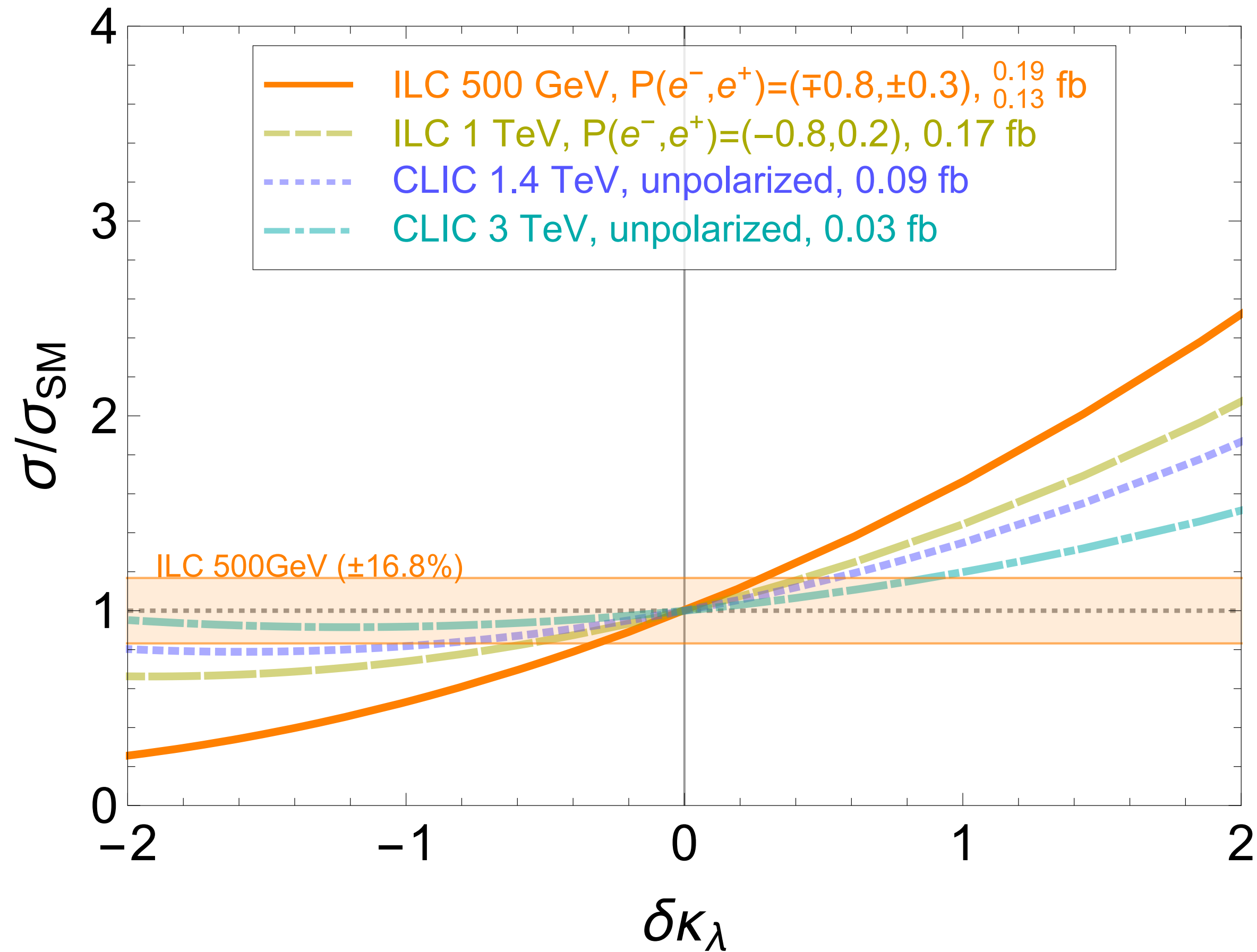
- There are analysis for FCC-hh [arXiv.2004.03505] and ILC [J. List et al., preliminary] with $\kappa_\lambda \neq 1$

Future e^+e^- colliders will play a crucial role to measure λ_{hhh} , but...

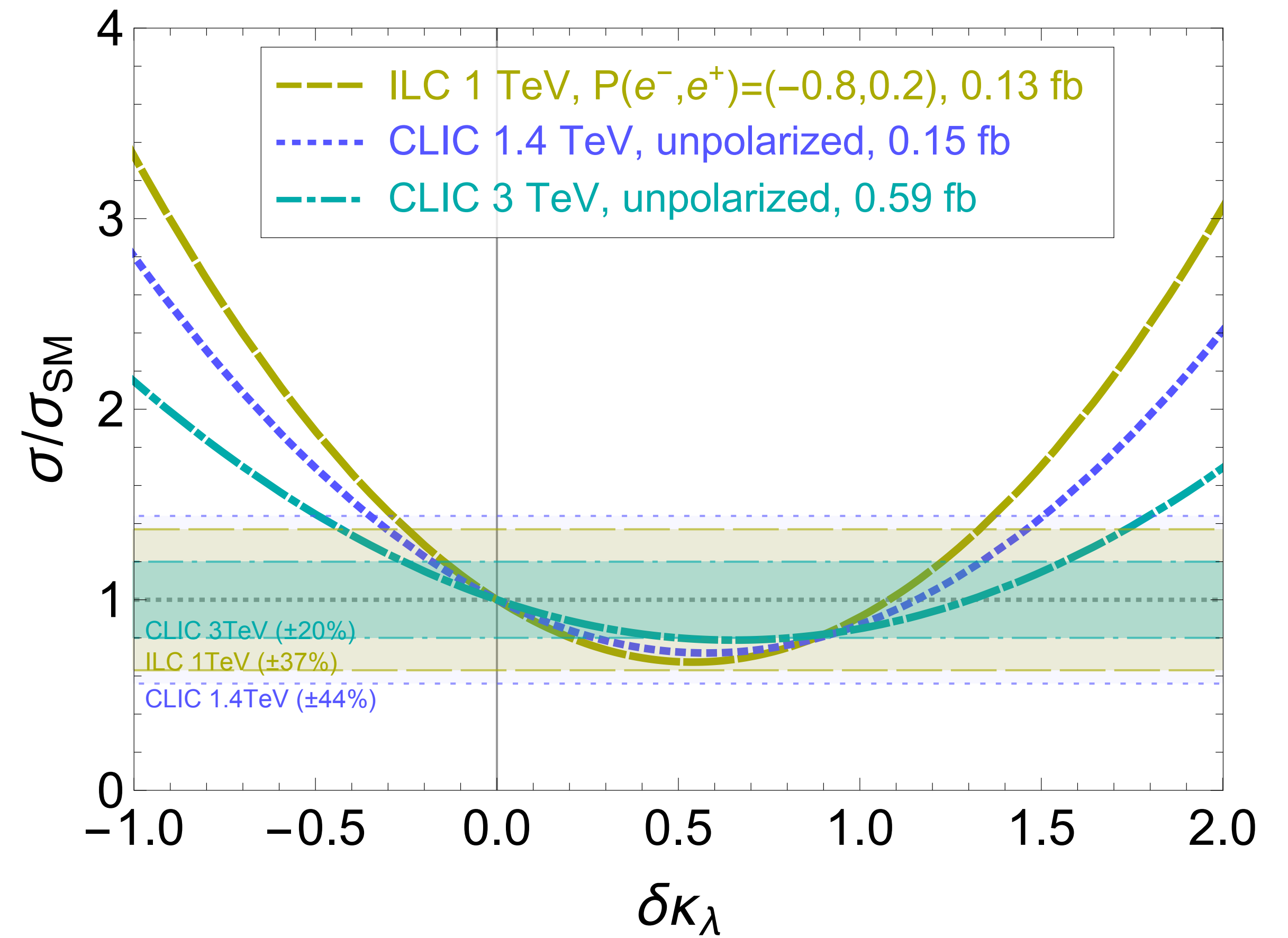
There is room for SM deviations in the scalar sector !!!

Back-up, κ_λ sensitivity on the SM

$e^+e^- \rightarrow Zhh$

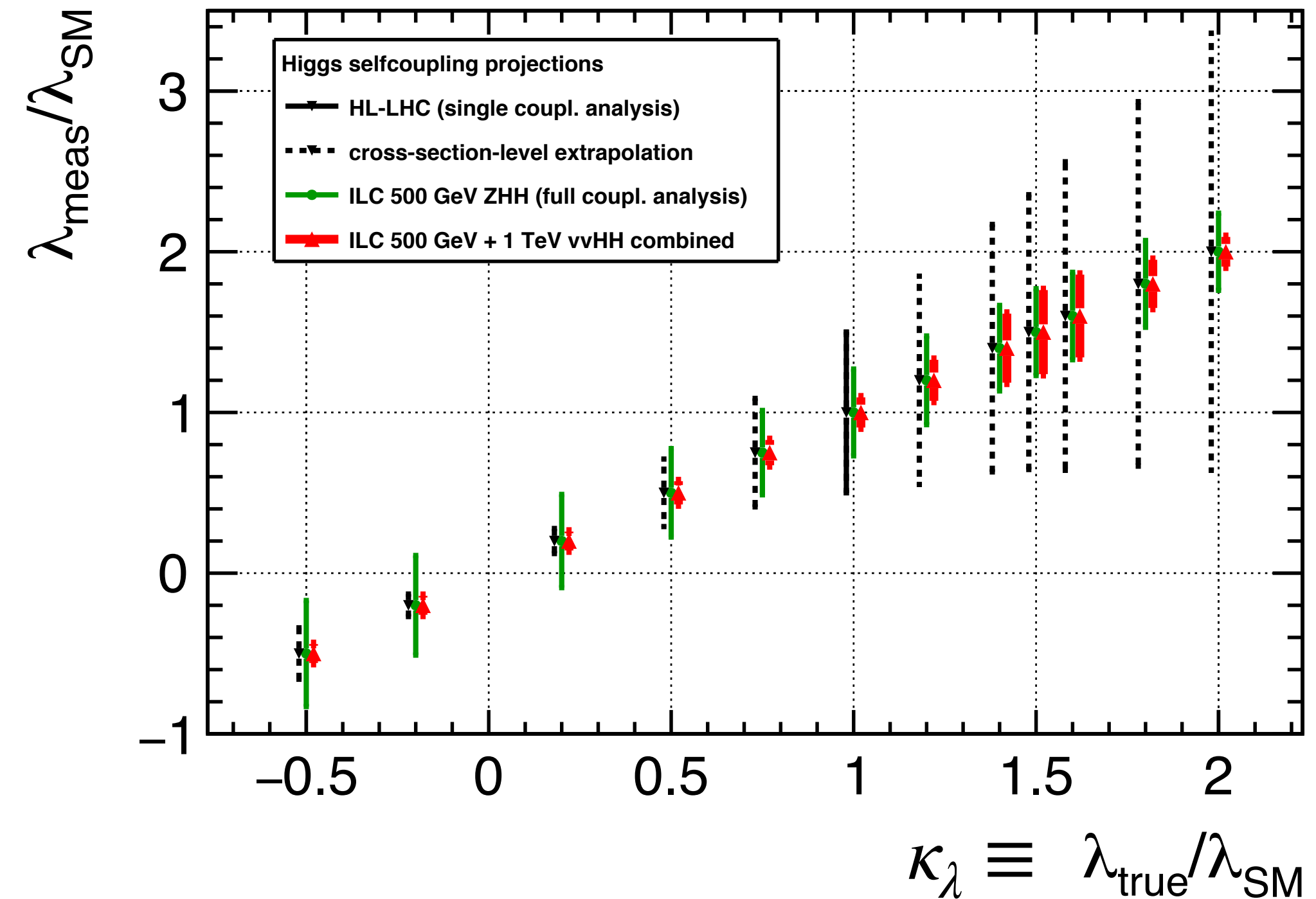
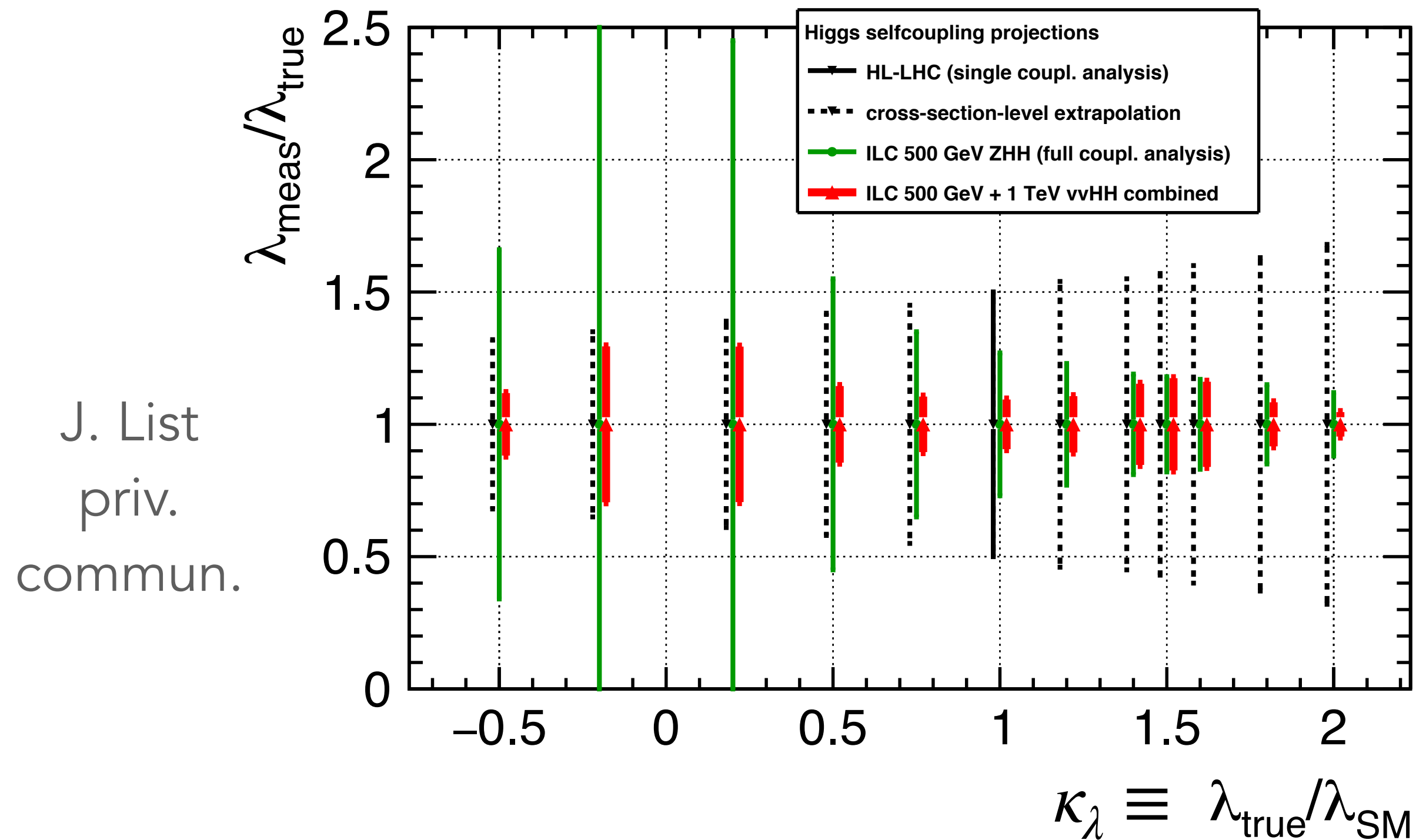


$e^+e^- \rightarrow \nu\bar{\nu}hh$



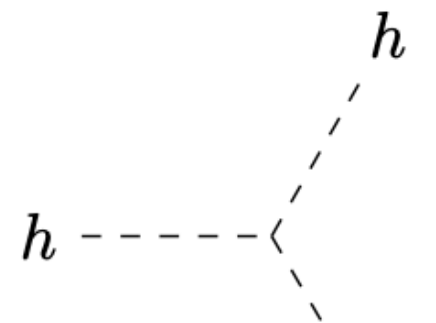
Prospects for κ_λ at ILC

Sensitivity to κ_λ for the di-Higgs production at HL-LHC and ILC, also for $\kappa_\lambda \neq 1$:

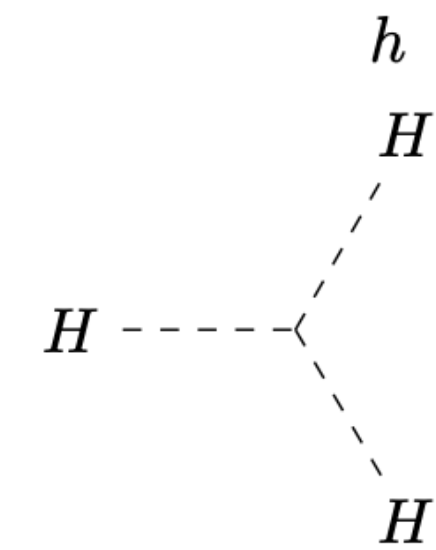


- Allowed ranges by type I and II are included
- **ILC 500 + ILC 1000** is better to measure κ_λ except for $\kappa_\lambda \sim 0$, where HL-LHC competes (no BSM channels are included)

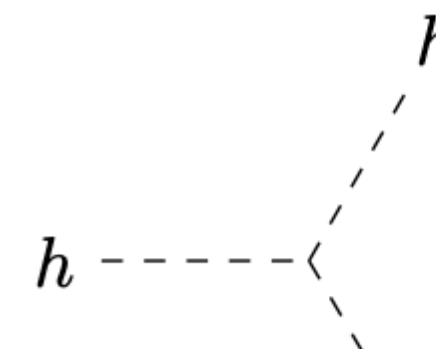
Back-up, Feynman Rules with THC



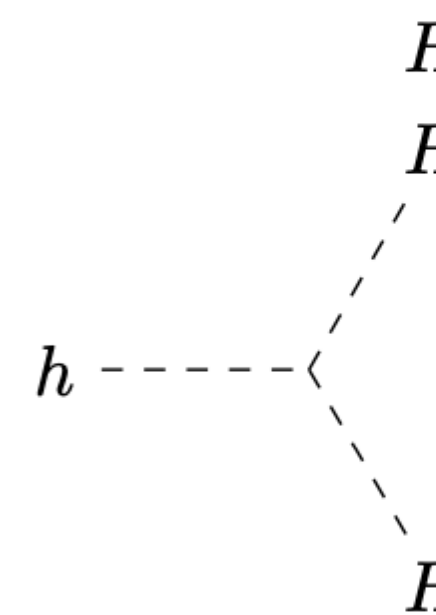
$$= -\frac{3i}{v} \left(2 \cot 2\beta (m_h^2 - \bar{m}^2) c_{\beta-\alpha}^3 + (3m_h^2 - 2\bar{m}^2) c_{\beta-\alpha}^2 s_{\beta-\alpha} + m_h^2 s_{\beta-\alpha}^3 \right)$$



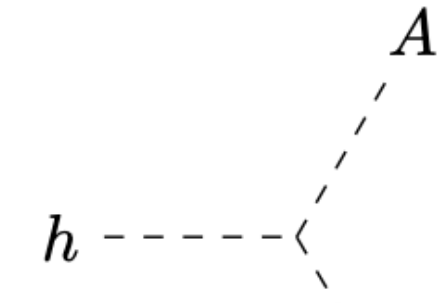
$$= -\frac{3i}{v} \left((3m_H^2 - 2\bar{m}^2) c_{\beta-\alpha} s_{\beta-\alpha}^2 + 2 \cot 2\beta (\bar{m}^2 - m_H^2) s_{\beta-\alpha}^3 + m_H^2 c_{\beta-\alpha}^3 \right)$$



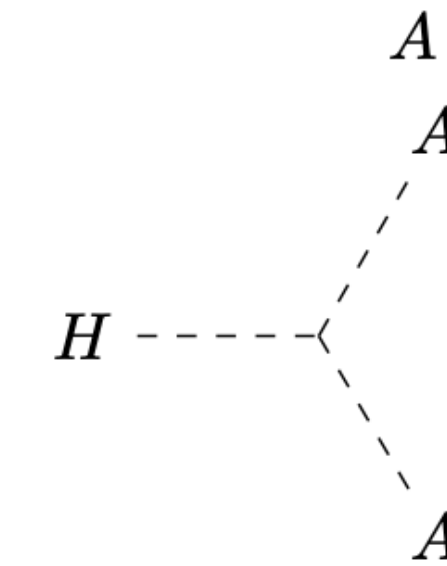
$$= \frac{ic_{\beta-\alpha}}{v} \left(2\bar{m}^2 (c_{\beta-\alpha}^2 - 3 \cot 2\beta c_{\beta-\alpha} s_{\beta-\alpha} - 2s_{\beta-\alpha}^2) + (2m_h^2 + m_H^2) (-c_{\beta-\alpha}^2 + 2 \cot 2\beta c_{\beta-\alpha} s_{\beta-\alpha} + s_{\beta-\alpha}^2) \right)$$



$$= -\frac{is_{\beta-\alpha}}{v} \left((m_h^2 + 2m_H^2) (-c_{\beta-\alpha}^2 + 2 \cot 2\beta c_{\beta-\alpha} s_{\beta-\alpha} + s_{\beta-\alpha}^2) - 2\bar{m}^2 (-2c_{\beta-\alpha}^2 + 3 \cot 2\beta c_{\beta-\alpha} s_{\beta-\alpha} + s_{\beta-\alpha}^2) \right)$$

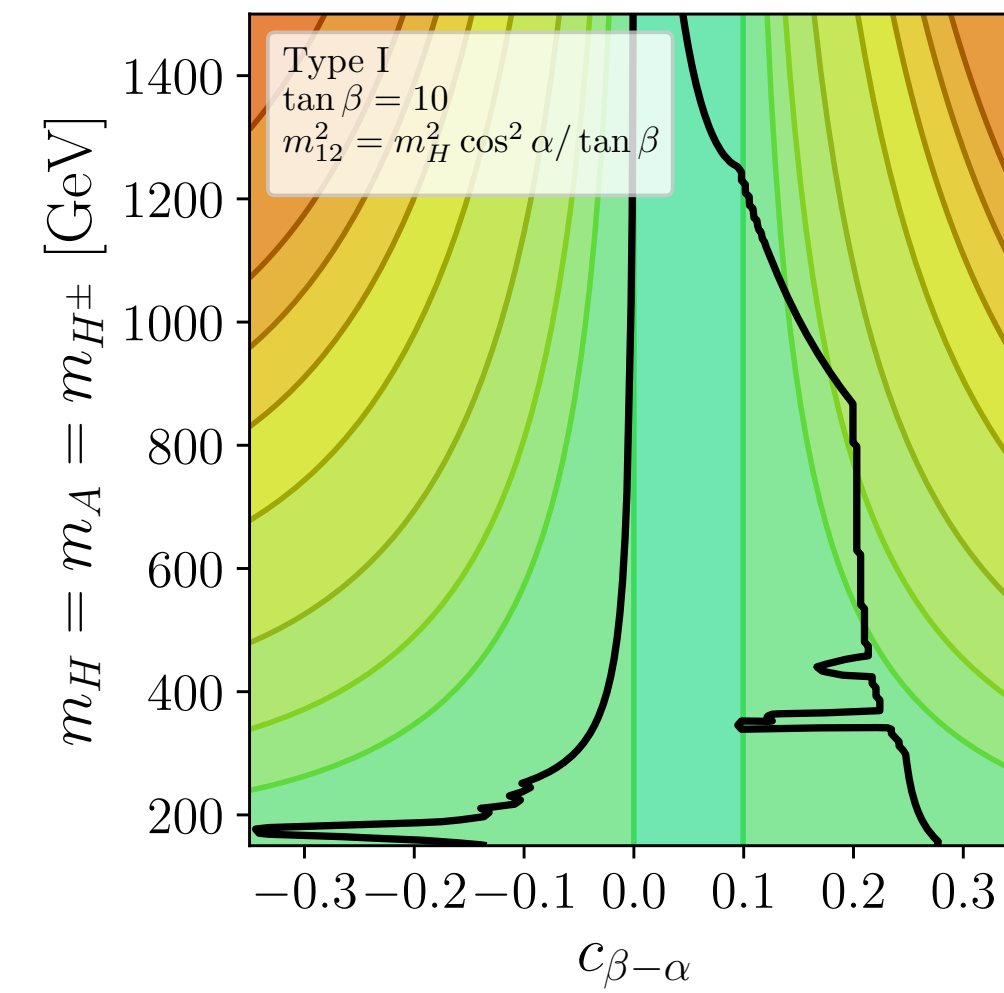
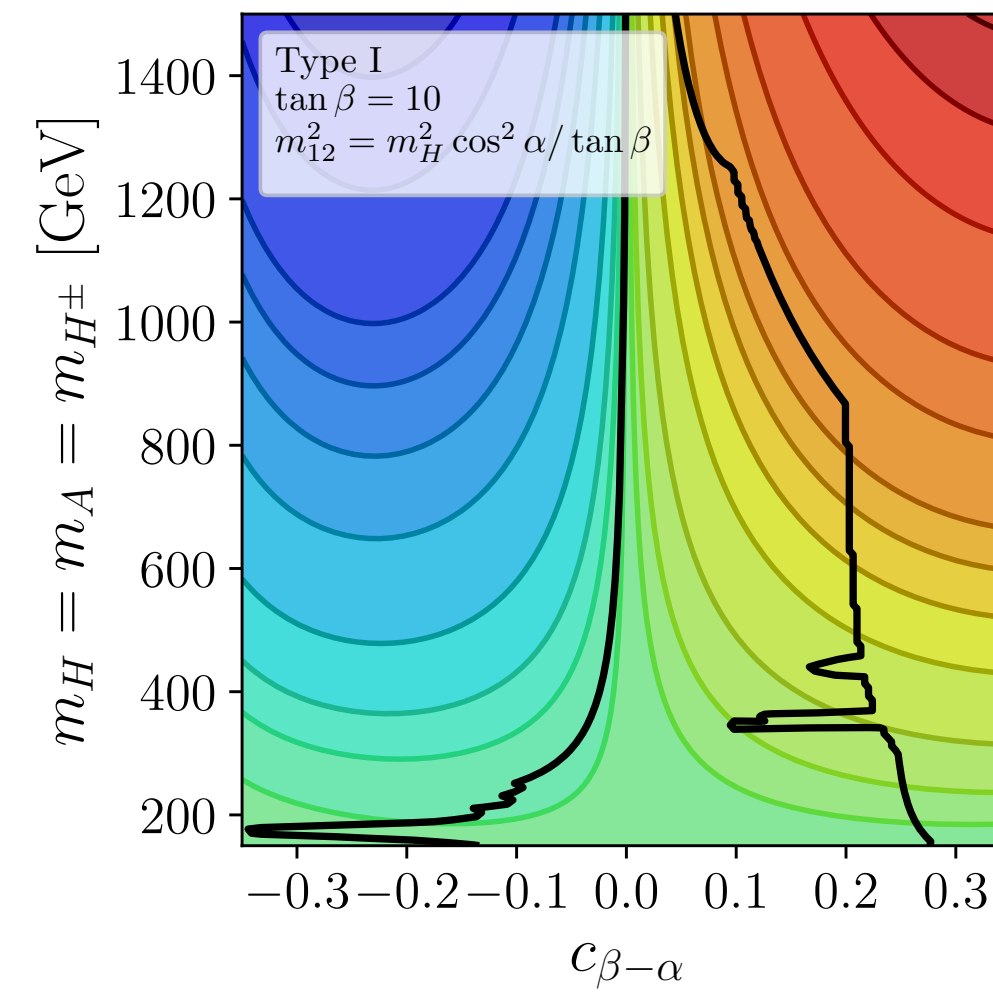
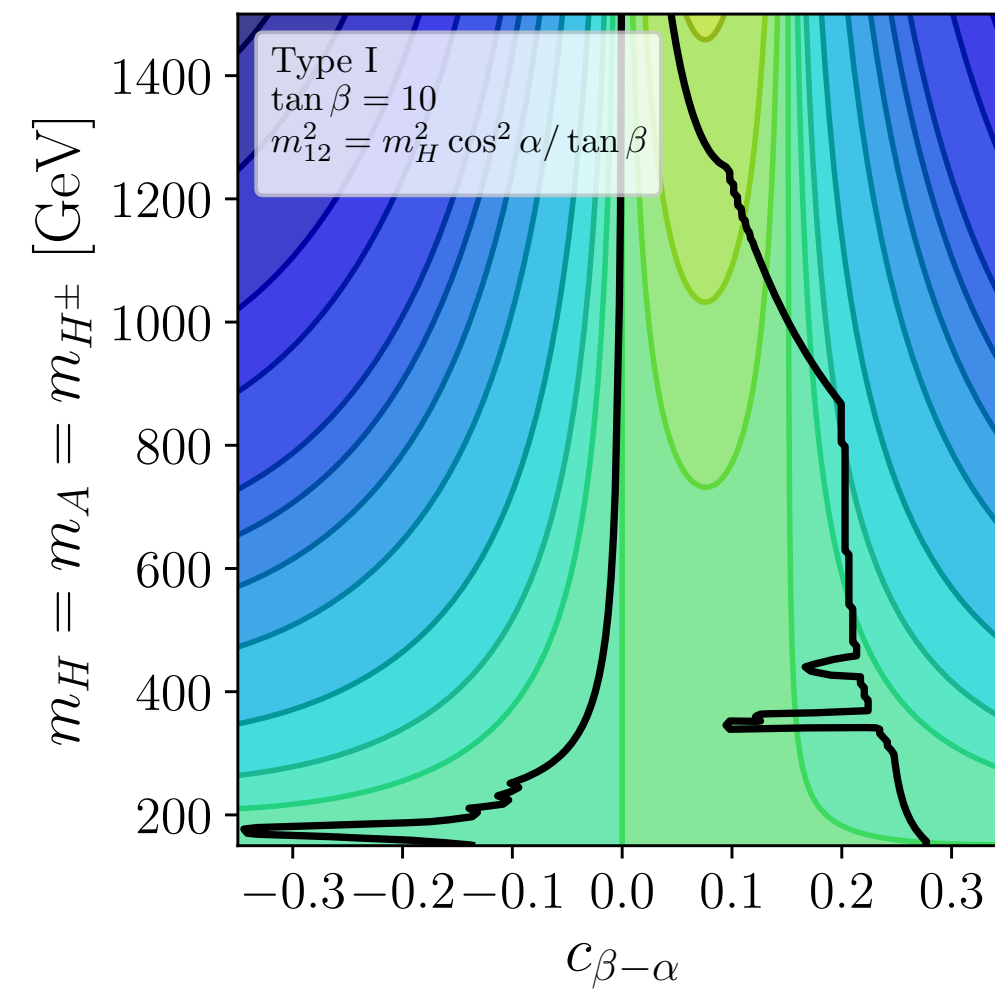
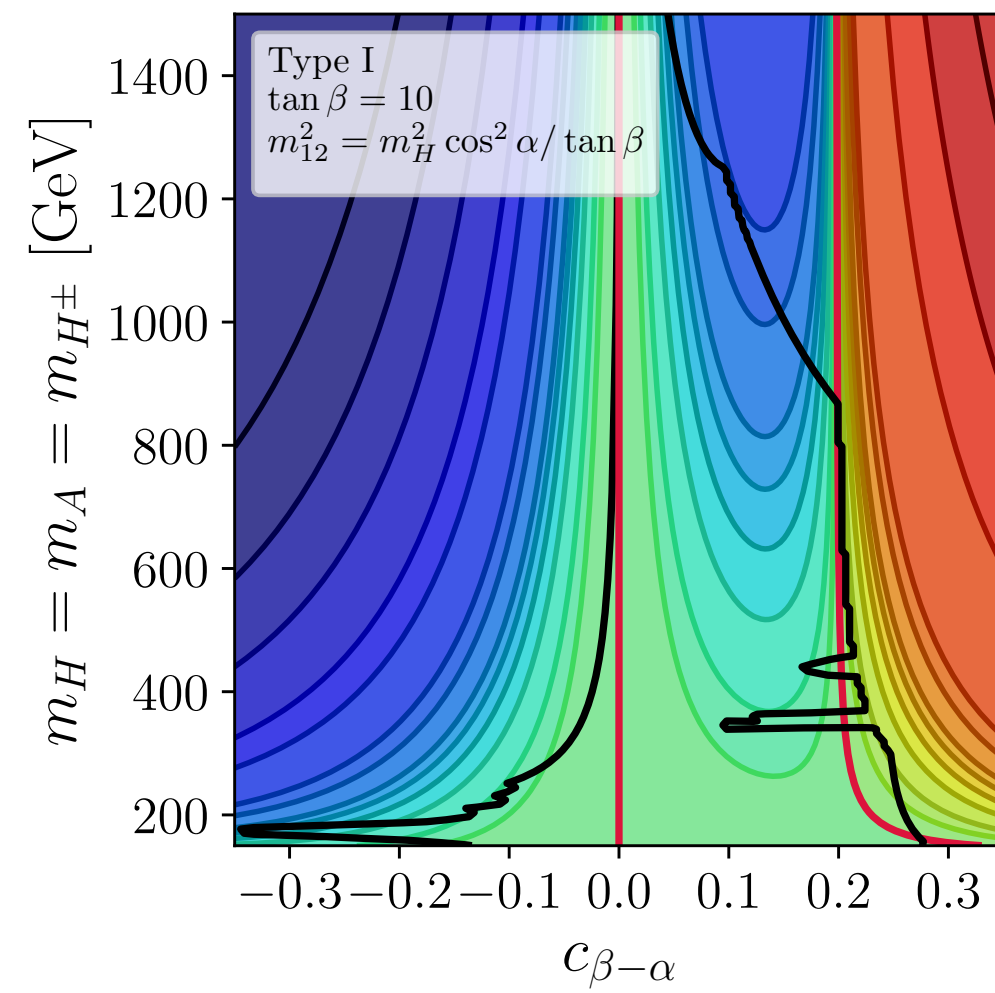
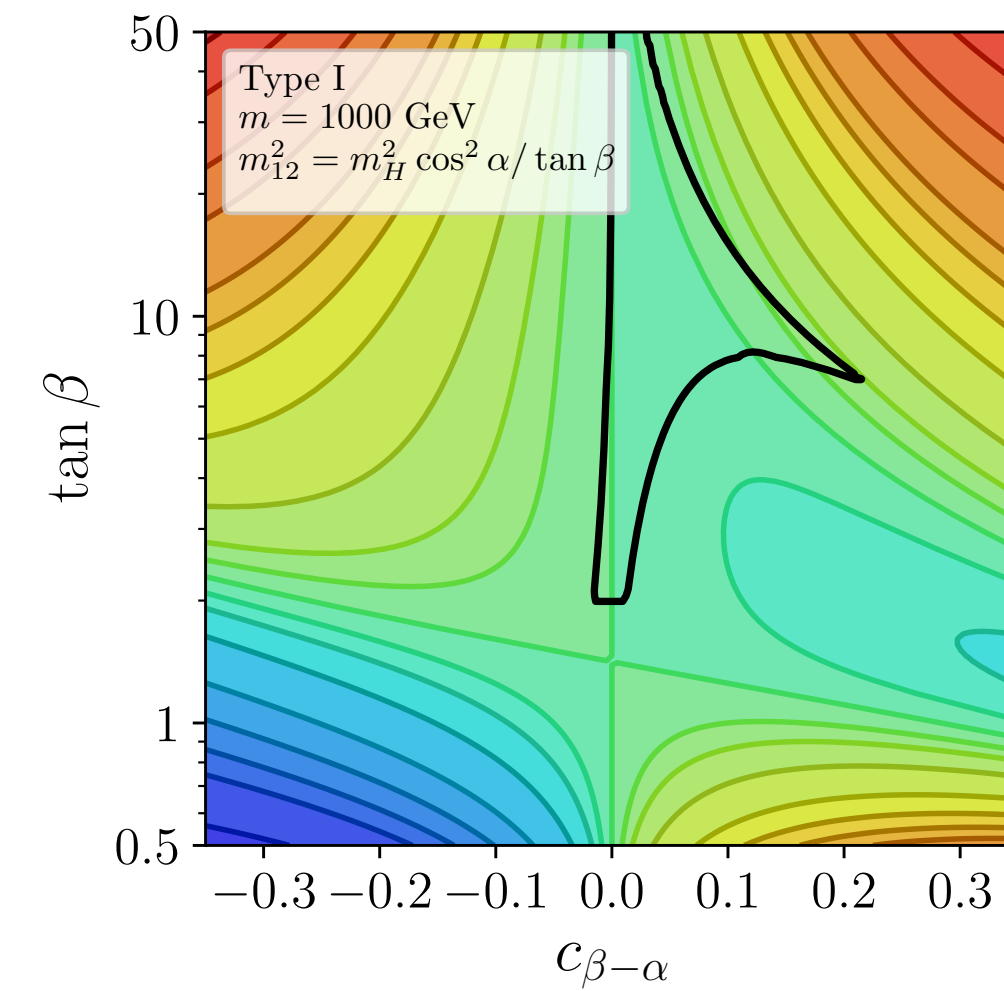
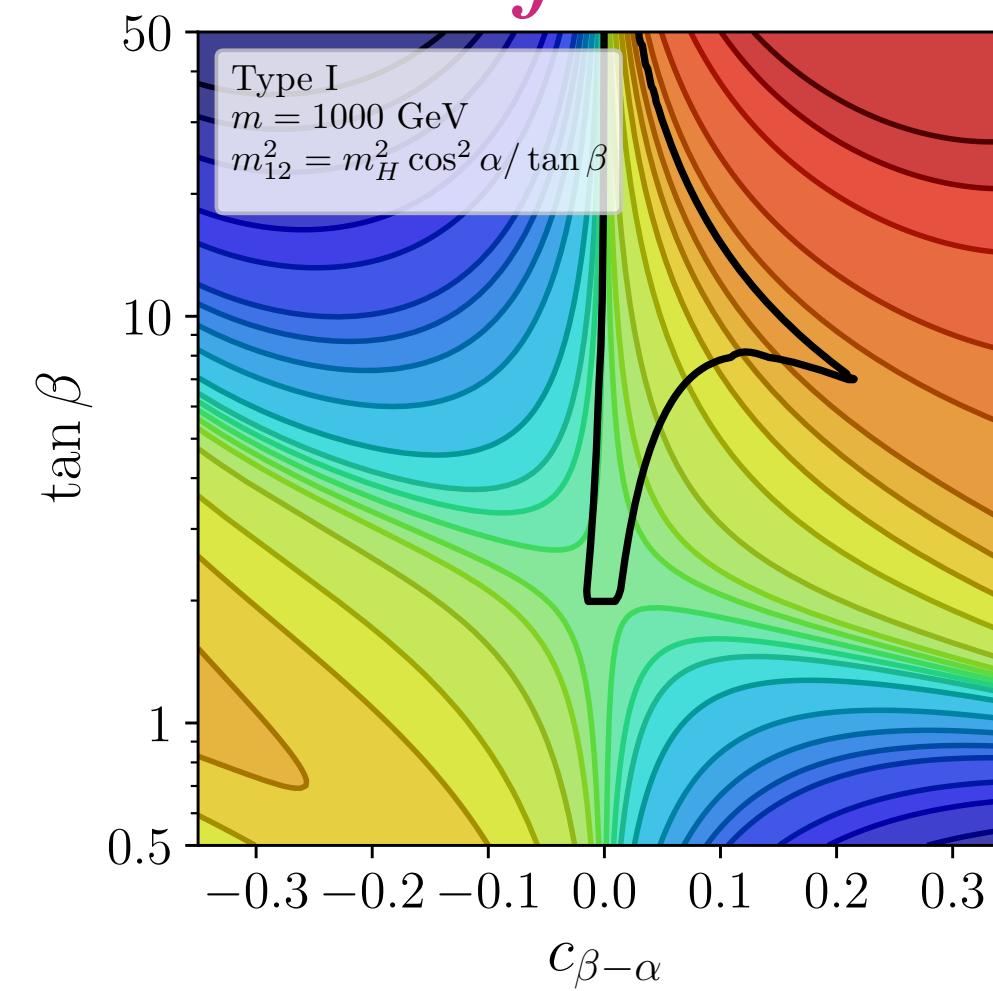
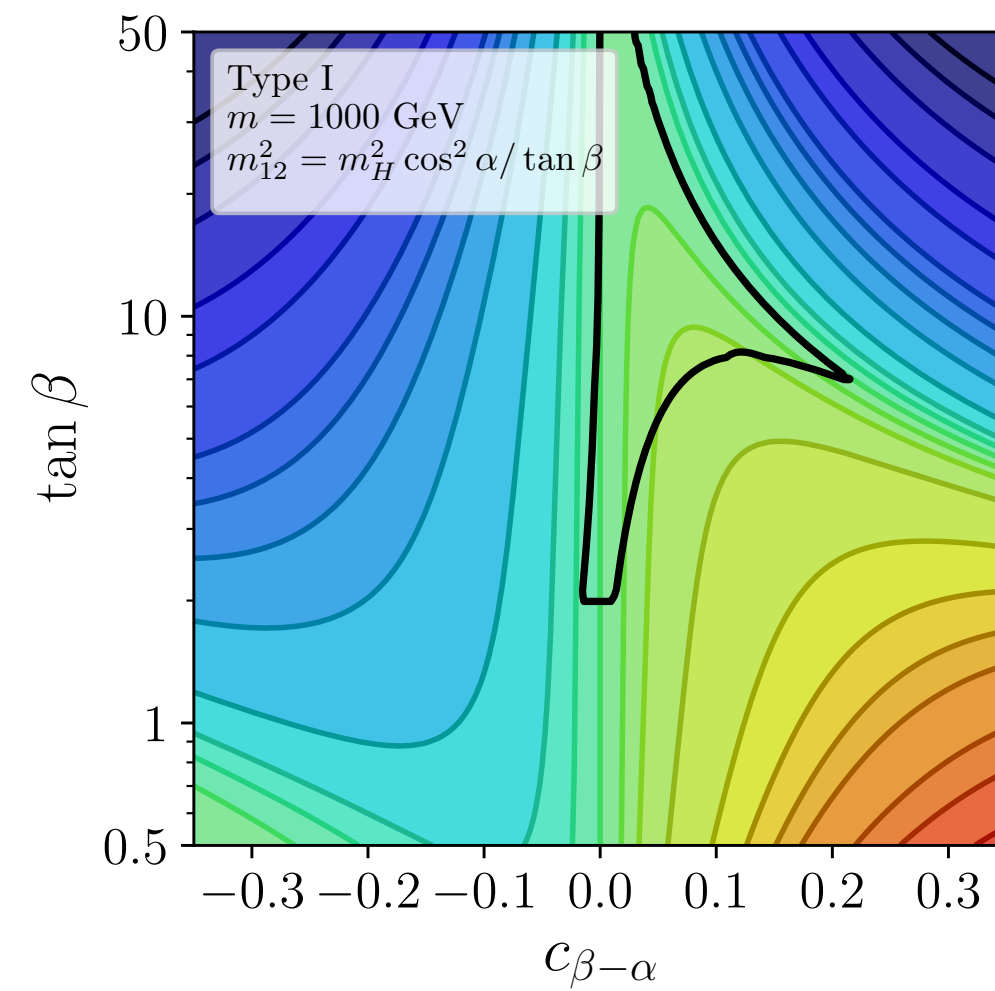
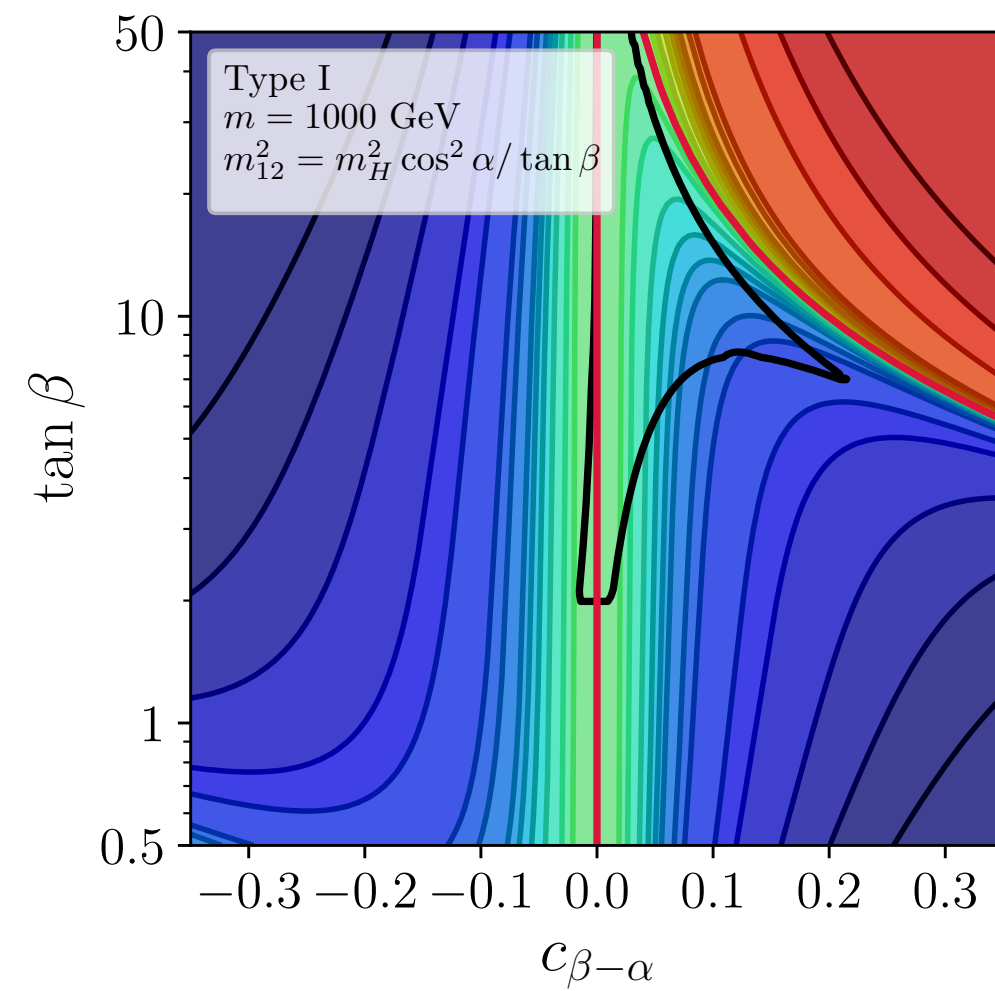


$$= -\frac{i}{v} (s_{\beta-\alpha} (-2\bar{m}^2 + 2m_A^2 + m_h^2) + 2 \cot 2\beta (m_h^2 - \bar{m}^2) c_{\beta-\alpha})$$



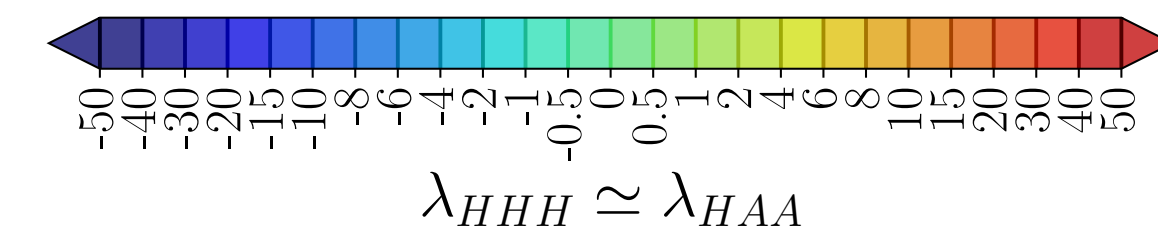
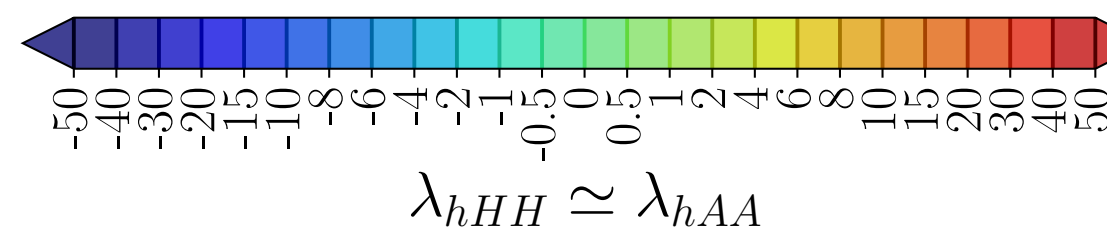
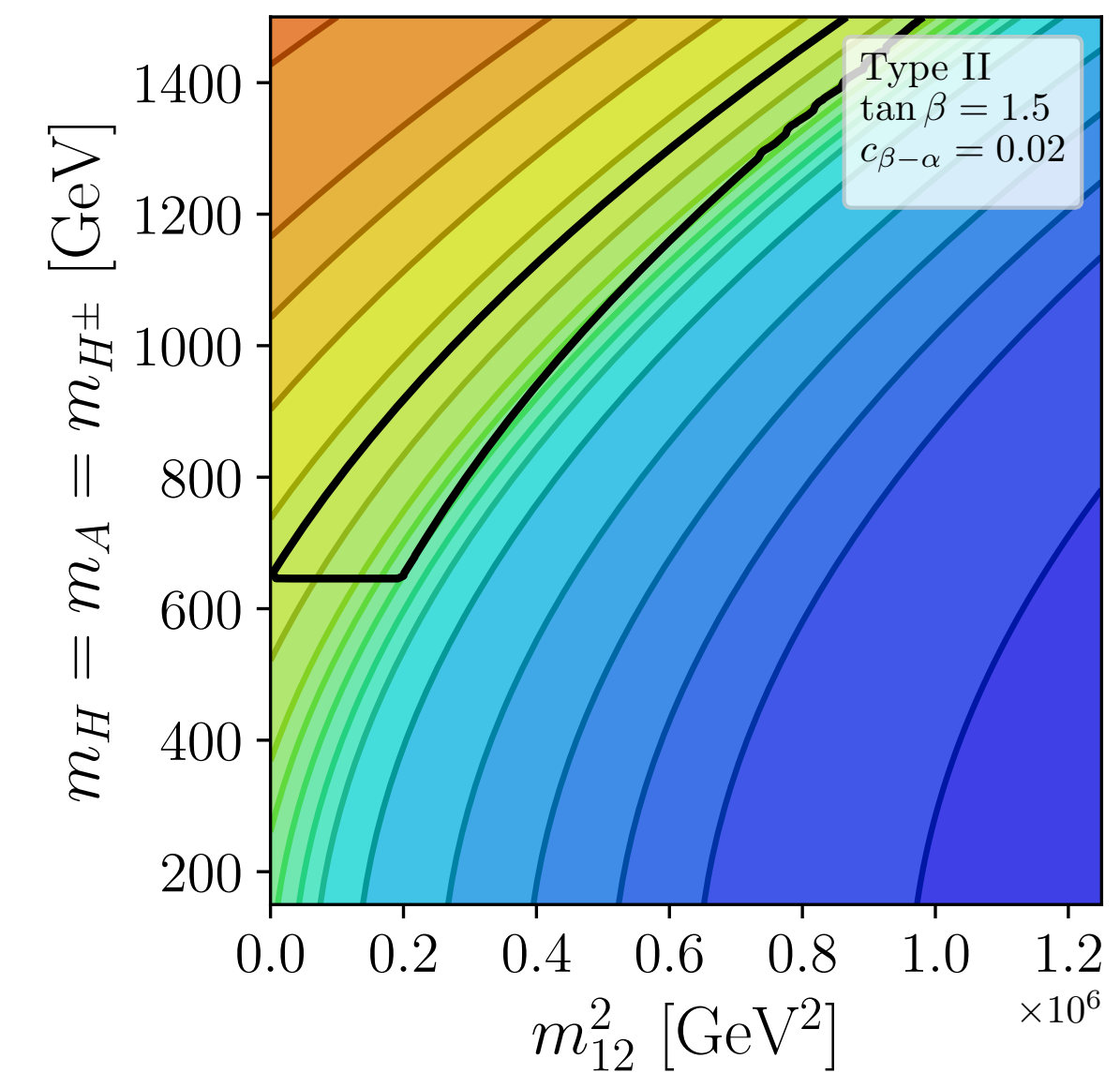
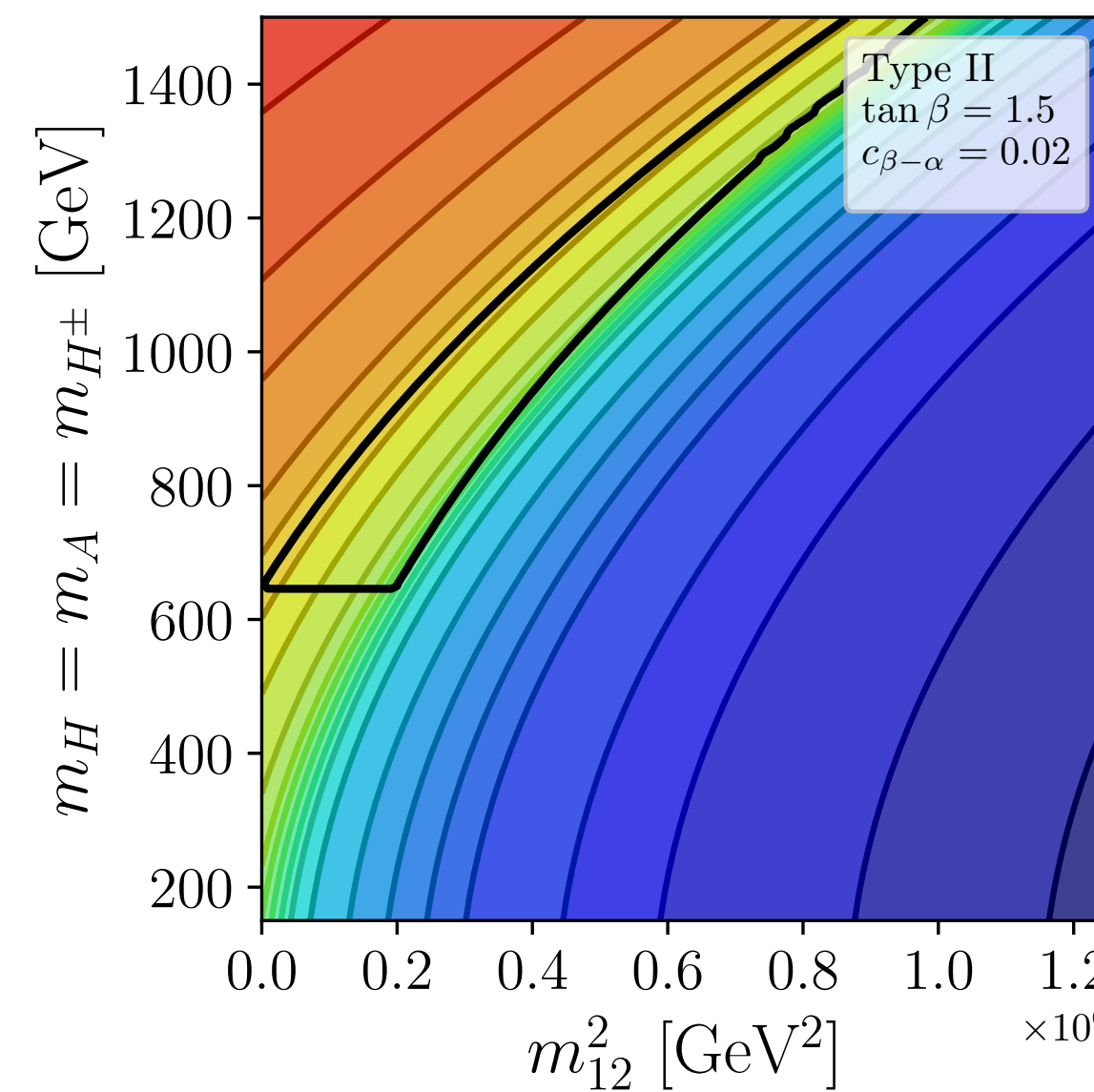
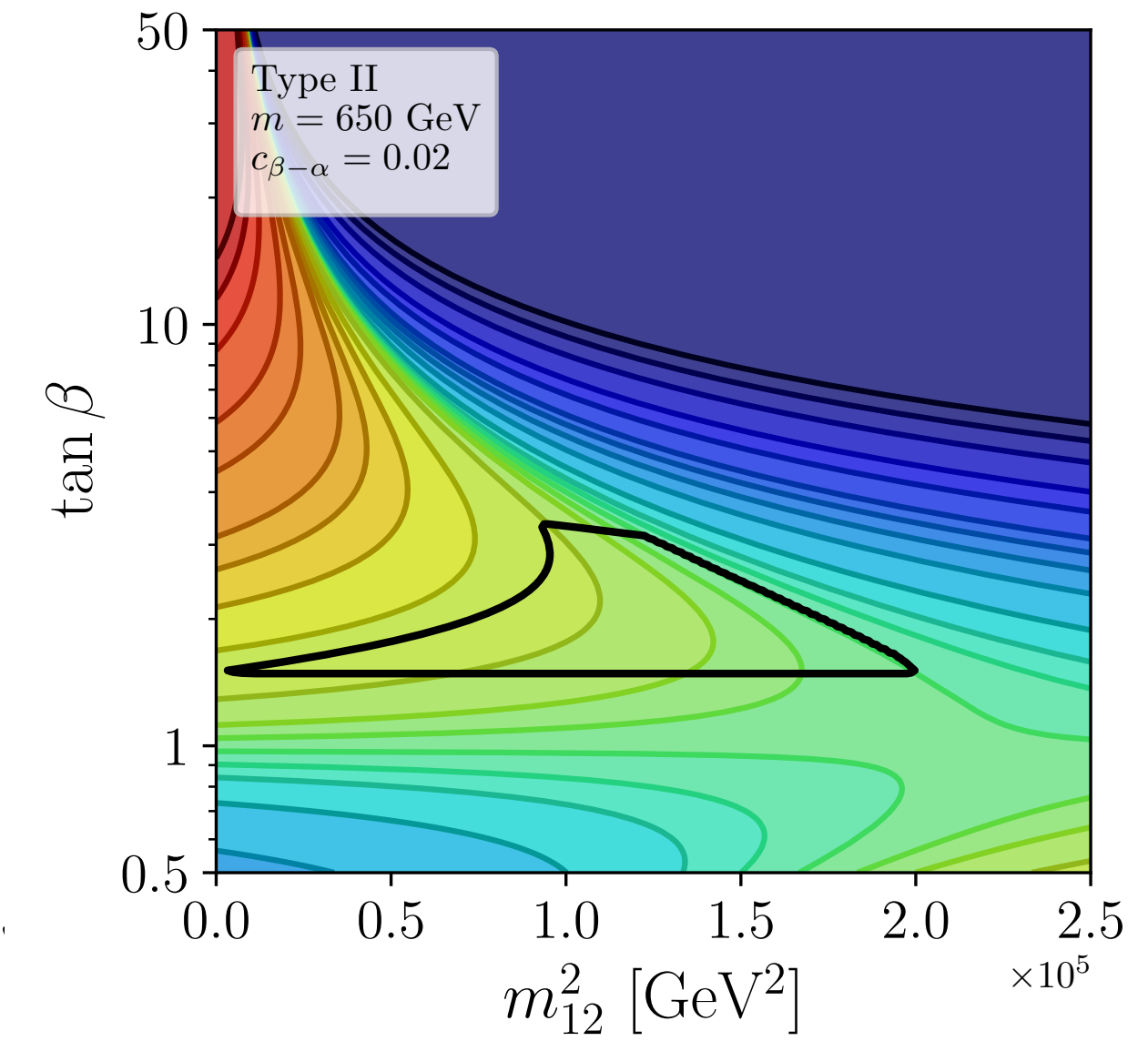
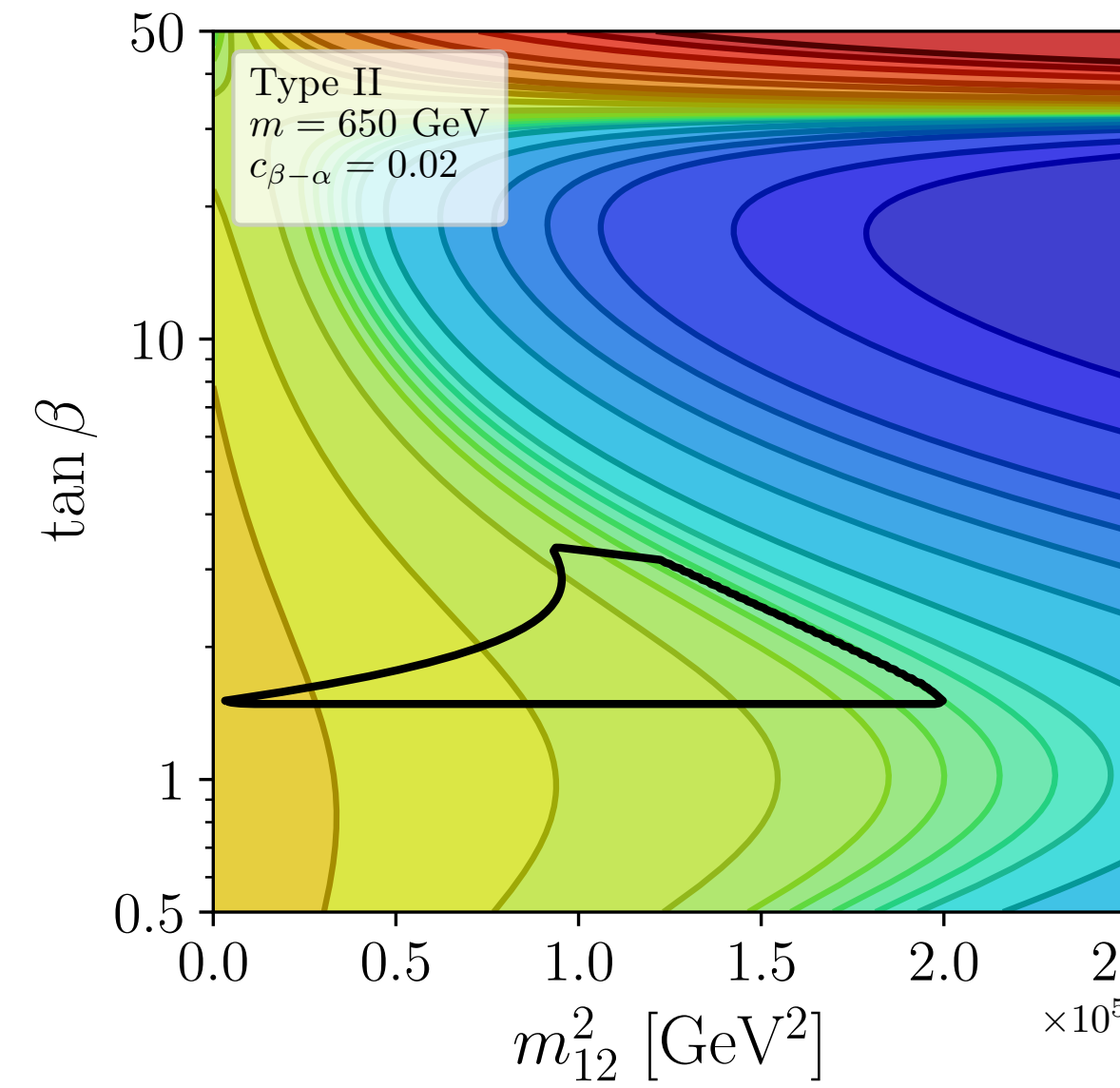
$$= \frac{i}{v} (2 \cot 2\beta (m_H^2 - \bar{m}^2) s_{\beta-\alpha} - c_{\beta-\alpha} (-2\bar{m}^2 + 2m_A^2 + m_H^2))$$

Back-up, $\lambda_{h_i h_j h_k}$



Back-up,

$$\lambda_{h_i h_j h_k}$$



hh production, THC dependence, ILC 1TeV (type I)

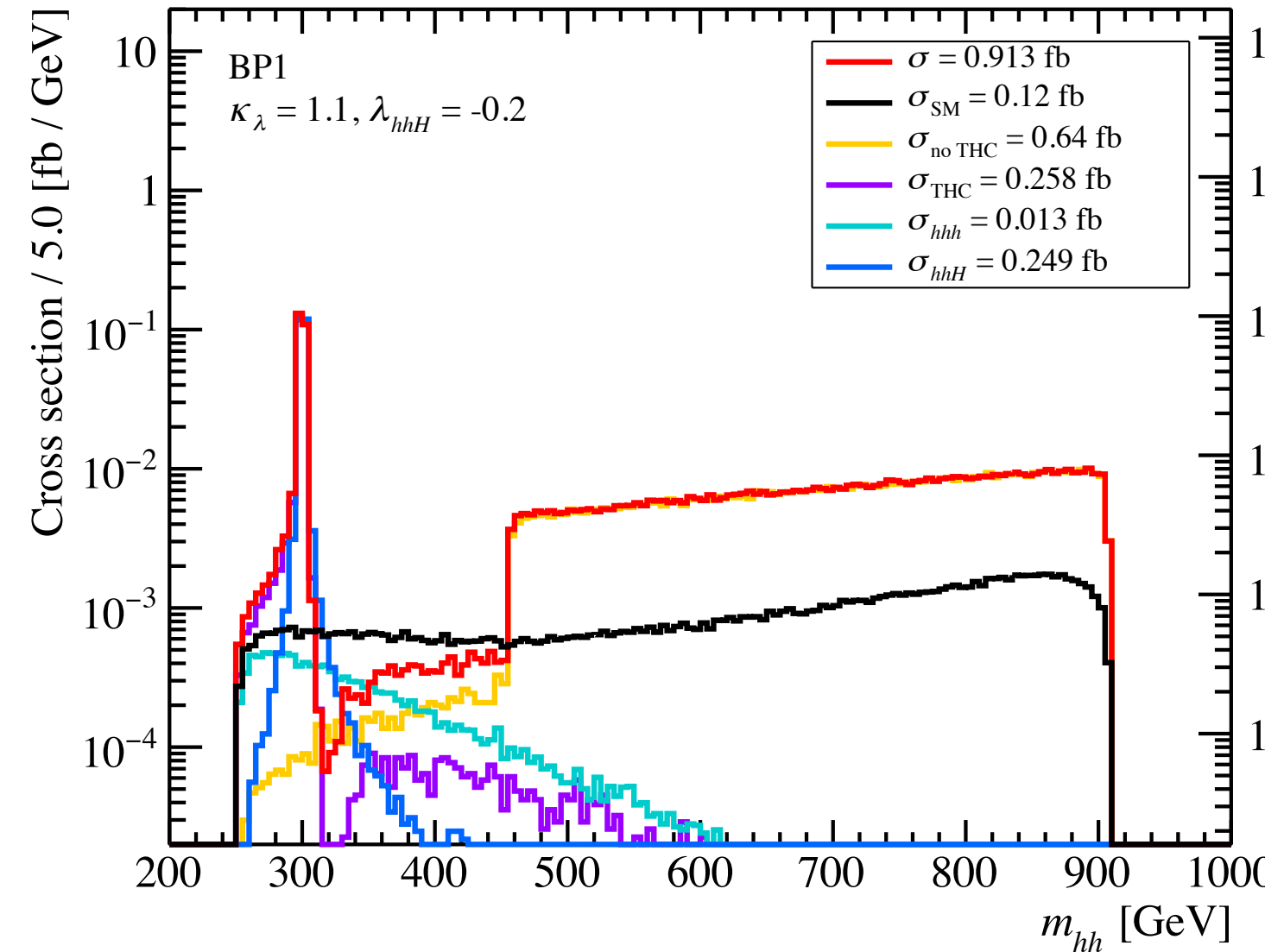
Cross section distributions on m_{hh} for

Point	Type	m	$\tan \beta$	$c_{\beta-\alpha}$	m_{12}^2
BP1	I	300	10	0.25	Eq. (8)
BP2	I	500	7.5	0.1	32000
BP3	I	600	10	0.2	Eq. (8)
BP4	I	1000	8.5	0.08	Eq. (8)

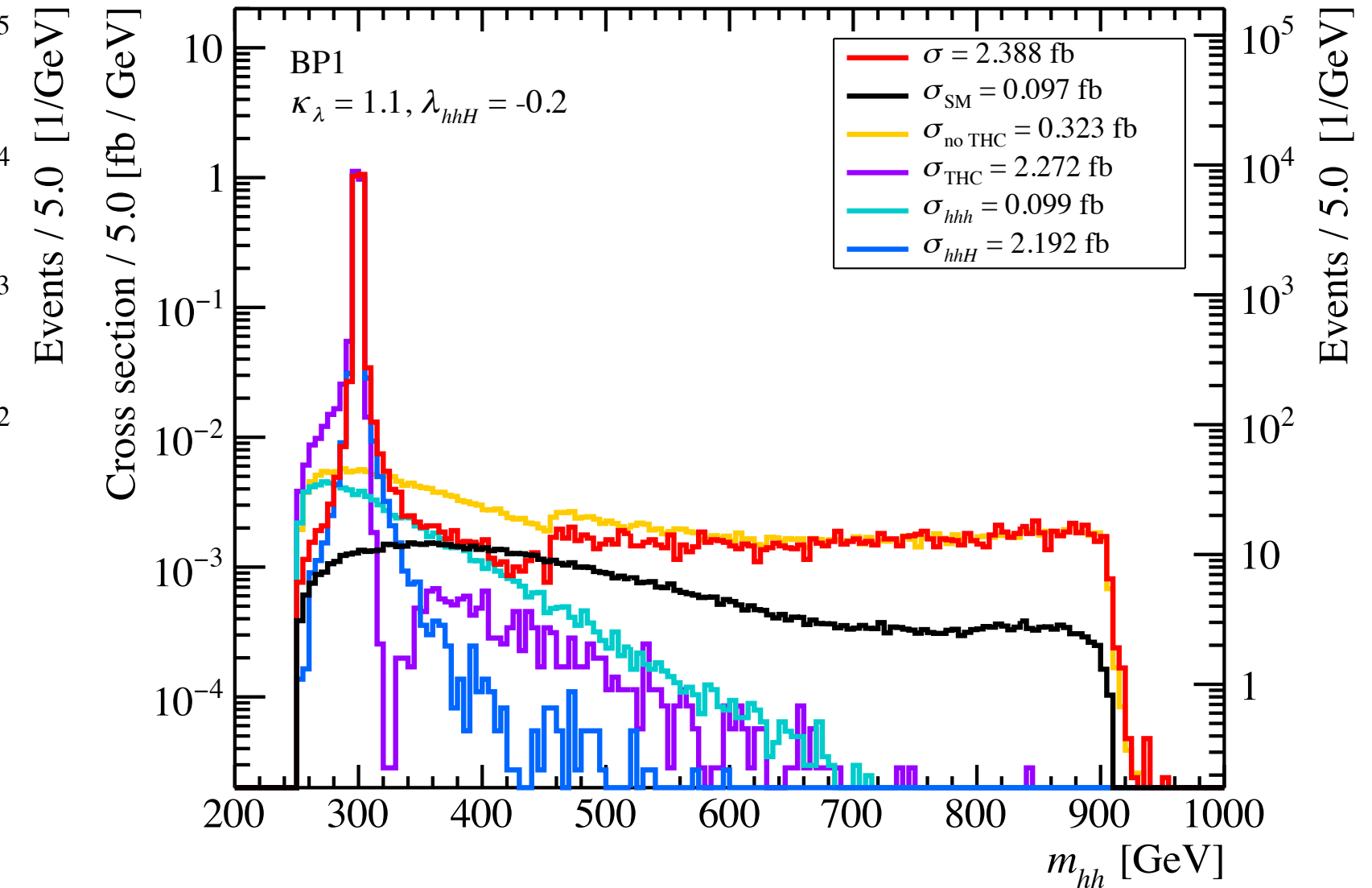
(Eq. (8) $\rightarrow m_{12}^2 = m_H^2 \cos^2 \alpha / \tan \beta$)

- Effect from κ_λ : the region of low invariant mass
- Effect from λ_{hhH} : H resonant peak at $m_{hh} \sim m_H$
- Extra events due to the A resonance

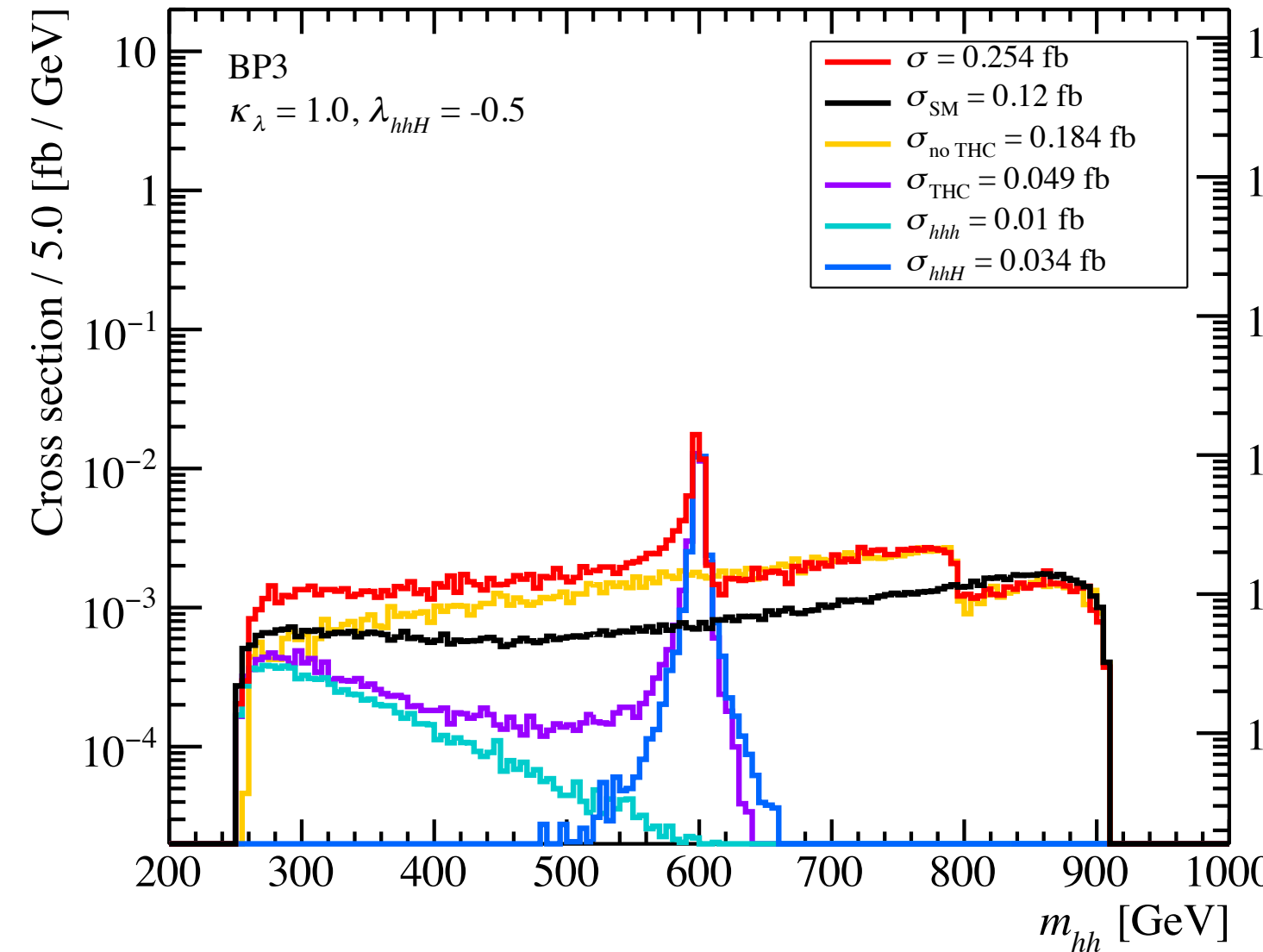
$\sigma(e^+e^- \rightarrow hhZ), \sqrt{s} = 1000 \text{ GeV}$



$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 1000 \text{ GeV}$



$\sigma(e^+e^- \rightarrow hhZ), \sqrt{s} = 1000 \text{ GeV}$



$\sigma(e^+e^- \rightarrow hh\nu\bar{\nu}), \sqrt{s} = 1000 \text{ GeV}$

